

Nash competition for dynamic long-lived information with risk aversion

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Abstract This study investigates adaptive equilibrium strategies for multiple risk-averse informed traders in Almgren–Chriss framework. Dynamic information and transaction costs are taken into account. Using a convex analytic approach, we characterize the open-loop Nash equilibrium in terms of a system of linear forward-backward stochastic differential equations, and further provide an explicit feedback expression of the unique equilibrium. The results show how risk-averse informed traders exploit long-lived information and manage positions in the face of information volatility and inaccuracy.

Keywords Informed trading, Price impact, Stochastic differential game, Nash equilibrium, Risk aversion

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1. Introduction

How information is incorporated into the price has always been a focus of market microstructure. Informed traders rely on long-lived information to know the asset's value and profit from corresponding trades. Other market participants learn the information and adjust the market price based on aggregate order flow, hence promoting price efficiency. In this paper, we extend the results in [14] to explore adaptive equilibrium strategies for multiple risk-averse informed traders in Almgren–Chriss framework [1].

The pioneering work by [9] showed how a risk-neutral informed trader optimally trades and reveals information to the market. Subsequently, a large number of studies started to focus on informed trading and its effect on price dynamics. Important issues such as competition between informed traders, dynamic information, risk aversion and transaction costs have been extensively investigated, producing widely varying results across different cases. For example, while [9] showed that a single risk-neutral informed trader trades uniformly and discloses information

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gradually, [7] and [2] found that competition causes informed traders to trade more rapidly in the beginning, even resulting in instantaneous information disclosure. Meanwhile, [8] found that the risk-averse informed trader concerned about price fluctuations will choose to trade faster in the beginning. However, [11] showed that the risk-averse informed trader faced with dynamic information initially trade slowly, waiting for the information to become accurate. Furthermore, taking transaction costs into account, [4] found incomplete information revelation at the end of the trading horizon, which differs from the result in [9]. Additionally, [14] found that competition leads to the price mean-reverting to the value instead of instantly changing to the value, as described in [7] and [2].

In this paper, we study how multiple risk-averse informed traders compete with dynamic long-lived information in the presence of transaction costs. Existing studies most closely related to ours include [11] and [14]. While [11] adopted the linear pricing rule by [9] and considered CARA utility in the discrete-time model, we use the continuous-time linear price impact model by [1] to additionally investigate the effect of transaction costs on the strategy development and resulting price dynamics. In addition, following [6], we use quadratic running and terminal penalties as an alternative risk measure to explicitly solve the problem. Meanwhile, we extend the case of a single risk-averse trader in [14] to explore adaptive equilibrium strategies for multiple risk-averse informed traders. The common dynamic programming method used in [14] leads to the implicit solution in the case of multiple risk-averse informed traders. Therefore, we use a convex analytic approach and characterize the open-loop equilibrium in terms of a system of linear forward-backward stochastic differential equations (FBSDEs), which can be solved explicitly. This method was first used by [3] to examine how to hedge with transaction costs, and it was further adopted by [13] and [10] to explore complicated order execution games. Using this method allows us to explicitly solve the informed trading game and show how risk-averse informed traders trade based on both information and positions.

Our main result comprises an explicit feedback expression of a unique open-loop Nash equilibrium between multiple risk-averse informed traders. While risk-neutral traders are only concerned about dynamic information as shown in [14], risk-averse traders also need to consider positions. As a result, the predatory behavior that occurs in the liquidation game (see [5, 12]) might also occur in the competition between risk-averse informed traders. More importantly, the aggregate order flow from risk-averse informed traders and the resulting price dynamics are sensitive to the magnitude of risk aversion, number of informed traders, and magnitude of transaction costs. Risk aversion and transaction costs discourage informed traders from fully revealing information to the market within the trading horizon, and informed traders need to balance competition and risk management.

The remainder of this paper is organized as follows. In section 2 we formulate the informed trading game between risk-averse informed traders in Almgren-Chriss framework. Section 3 presents the main result. Section 4 provides some illustrations. Section 5 contains the technical proofs.

2. Model setup

We follow the setup in [14] to formulate a trading game between risk-averse informed traders. Assume that $N(> 1)$ informed traders trade continuously with trading rates v^i ($i = 1, 2, \dots, N$) within the time interval $[0, T]$. Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0 \leq t \leq T}, \mathbb{P})$ be a filtered probability space satisfying the usual conditions. The position for the i th informed trader is driven by

$$dQ_t^i = v_t^i dt, \quad i = 1, 2, \dots, N, \quad (2.1)$$

with an initial position Q_0^i . Note that a positive v is buying and a negative v is selling. The trading rate v^i ($i = 1, 2, \dots, N$) is chosen from the set of admissible strategies

$$\mathcal{A} = \left\{ v : v \text{ progressively measurable s.t. } \mathbb{E} \int_0^T |v_t|^2 dt < +\infty \right\}.$$

Let Y_t denote aggregate trading rates $\sum_{i=1}^N v_t^i$ and X_t denote aggregate positions $\sum_{i=1}^N Q_t^i$. It is evident that

$$dX_t = Y_t dt, \quad X_0 = \sum_{i=1}^N Q_0^i.$$

Informed traders possess dynamic long-lived information and therefore know the “long-run” value of the risky asset. They can trade based on this information, and their trading activities affect price dynamics through permanent price impact. As in [14], we assume the asset’s value V_t follows

$$dV_t = \sigma_1 dB_t^1$$

and the market price S_t follows

$$dS_t = \gamma Y_t dt + \sigma_2 dB_t^2.$$

$\gamma, \sigma_1, \sigma_2$ are three positive constants, and B_t^1, B_t^2 are two independent Brownian motions. Linear permanent price impact $\gamma Y_t dt$ was initially proposed by [1] and empirically supported by [6]. As in [14], we introduce the mispricing $\Lambda_t = V_t - S_t$, which is the spread between the asset’s value and the market price. The mispricing Λ_t is driven by

$$d\Lambda_t = -\gamma Y_t dt + \sigma dW_t, \tag{2.2}$$

where W_t is a Brownian motion satisfying $\sigma W_t = \sigma_1 B_t^1 - \sigma_2 B_t^2$. In addition, we use the intertwined temporary price impact in [5] and [12] to account for transaction costs. The execution price \tilde{S}_t for every informed trader at time t is given by

$$\tilde{S}_t = S_t + \eta Y_t,$$

where η is a positive constant. For the i th informed trader, we use

$$v^{-i} = (v^1, \dots, v^{i-1}, v^{i+1}, \dots, v^N) \in \mathcal{A}^{N-1}$$

to denote other informed traders’ strategies. According to [14], the i th informed trader aims to maximize the performance functional

$$J^i(v^i; v^{-i}) = \mathbb{E} \left[\int_0^T (-\eta Y_t v_t^i + \Lambda_t v_t^i) dt - \lambda \int_0^T (Q_t^i)^2 dt - \frac{\beta}{2} (Q_T^i)^2 \right] \tag{2.3}$$

via the admissible trading strategy $v^i \in \mathcal{A}$. Note that $J^i(v^i; v^{-i}) < \infty$ for any admissible trading strategies $(v^1, \dots, v^N) \in \mathcal{A}^N$. The first term in (2.3) represents the trading revenue after considering transaction costs. The second and third terms in (2.3) implement penalties $\lambda > 0$ and $\beta > 0$ on the running and terminal positions to describe risk aversion to information volatility and inaccuracy, respectively.

Our main goal in this study is to solve a trading game involving multiple risk-averse informed traders and establish Nash equilibrium in the following sense:

Definition 2.1 *A set of trading rates $(\hat{v}^1, \dots, \hat{v}^N) \in \mathcal{A}^N$ is called an open-loop Nash equilibrium if for any $i = 1, 2, \dots, N$ and for any admissible trading rate $v^i \in \mathcal{A}$ it holds that*

$$J^i(v^i; \hat{v}^{-i}) \leq J^i(\hat{v}^i; \hat{v}^{-i}).$$

Remark 2.2 In [14], the single-trader case ($N = 1$) and risk-neutral case ($\lambda = \beta = 0$) were studied by means of dynamic programming principle. Using the same approach, the ODE system derived from HJB equations is difficult to handle in the case of multiple risk-averse informed traders. Thus, [14] only considered deterministic equilibrium strategies. It's well known, however, that informed traders' trading activities adapt to price and value, and solving adaptive equilibrium strategies will provide a better understanding of how informed traders engage in risk management during transactions. Therefore, to address the remaining question in [14], in the next section, we use a convex analytic approach to explicitly solve adaptive equilibrium strategies for risk-averse informed traders.

Remark 2.3 The different risk aversions λ and β of different informed traders can also be considered. However, this will increase computational difficulty and impair the model's tractability. Therefore, as in [8, 10–12], we use identical risk aversion to conveniently study the effect of risk aversion on equilibrium trading strategies and the resulting price dynamics.

3. Main result

Here, we use a convex analytic approach to characterize the open-loop Nash equilibrium in terms of a system of linear FBSDEs and then solve it explicitly. Section 5 contains the technical proofs. First, we establish the strict concavity property of the functional in (2.3) in the following.

Lemma 3.1 For any $i = 1, 2, \dots, N$ and any fixed $v^{-i} \in \mathcal{A}^{N-1}$, the functional $J^i(v^i; v^{-i})$ in (2.3) is strictly concave for $v^i \in \mathcal{A}$. As a result, there exists at most one open-loop Nash equilibrium in the sense of Definition 2.1.

Given $v^{-i} \in \mathcal{A}^{N-1}$, the i th informed trader chooses a trading strategy v^i to maximize the strictly concave functional $J^i(v^i; v^{-i})$ in (2.3) owing to Lemma 3.1. A sufficient condition for the optimality of v^i is given by

$$\langle \nabla J^i(v^i; v^{-i}), w \rangle = \lim_{\varepsilon \rightarrow 0} \frac{J^i(v^i + \varepsilon w; v^{-i}) - J^i(v^i; v^{-i})}{\varepsilon} = 0$$

for all $w \in \mathcal{A}$. Considering the term of ε in $J^i(v^i + \varepsilon w; v^{-i}) - J^i(v^i; v^{-i})$ and applying Fubini's theorem, for every $i = 1, 2, \dots, N$ we have

$$\begin{aligned} & \langle \nabla J^i(v^i; v^{-i}), w \rangle \\ &= \mathbb{E} \left[\int_0^T \left(-\eta w_t (Y_t + v_t^i) + \Lambda_t w_t - \gamma v_t^i \int_0^t w_u du \right) dt - 2\lambda \int_0^T Q_t^i \int_0^t w_u du dt - \beta Q_T^i \int_0^T w_t dt \right] \\ &= \mathbb{E} \left[\int_0^T w_t \left(-\eta (Y_t + v_t^i) + \Lambda_t - \gamma \int_t^T v_u^i du - 2\lambda \int_t^T Q_u^i du - \beta Q_T^i \right) dt \right]. \end{aligned} \tag{3.1}$$

According to Definition 2.1, $(\hat{v}^1, \dots, \hat{v}^N) \in \mathcal{A}^N$ forms an open-loop Nash equilibrium if it holds that $\langle \nabla J^i(\hat{v}^i; \hat{v}^{-i}), w^i \rangle = 0$ for all $w^i \in \mathcal{A}$ and all $i = 1, 2, \dots, N$. Given (3.1), we next characterize the open-loop Nash equilibrium in terms of a system of FBSDEs.

Proposition 3.2 A set of trading strategies $(\hat{v}^1, \dots, \hat{v}^N) \in \mathcal{A}^N$ forms an open-loop Nash equilibrium in the sense of Definition 2.1 if $(\hat{v}^1, \dots, \hat{v}^N)$ satisfies the following coupled FBSDE system

$$\begin{cases} d\Lambda_t = -\gamma Y_t dt + \sigma dW_t, & \Lambda_0 = \Lambda_0, \\ dQ_t^i = v_t^i dt, & Q_0^i = Q_0^i, \quad i = 1, 2, \dots, N, \\ \eta dY_t + \eta dv_t^i = d\Lambda_t + \gamma v_t^i dt + 2\lambda Q_t^i dt + dM_t^i, & i = 1, 2, \dots, N, \\ \eta Y_T + \eta v_T^i = \Lambda_T - \beta Q_T^i, & i = 1, 2, \dots, N \end{cases} \tag{3.2}$$

for suitable square integrable martingales $(M_t^i)_{0 \leq t \leq T}$, $i = 1, 2, \dots, N$.

Note that if we can find a set of trading strategies $(\hat{v}^1, \dots, \hat{v}^N) \in \mathcal{A}^N$ that solves (3.2), $(\hat{v}^1, \dots, \hat{v}^N)$ forms a unique open-loop Nash equilibrium owing to Lemma 3.1. Before giving our result, we first introduce some constants to simplify the expressions:

$$\theta = \frac{\gamma}{2} + \frac{\beta}{N+1}, \quad \alpha = \frac{\gamma}{2} \frac{N-1}{N+1}, \quad \rho = \sqrt{\alpha^2 + \frac{2\eta\lambda}{N+1}}, \quad k = \sqrt{\frac{\gamma^2}{4} + 2\eta\lambda}.$$

Theorem 3.3 *There exists a unique open-loop Nash equilibrium in the sense of Definition 2.1. The aggregate trading rate \hat{Y}_t in feedback form is given by*

$$\hat{Y}_t = \frac{1}{\eta} f(t) \Lambda_t + \frac{1}{\eta} g(t) X_t, \tag{3.3}$$

where

$$\begin{aligned} f(t) &= \frac{N}{N+1} \cdot \frac{\rho e^{\frac{\alpha}{\eta}(T-t)}}{\theta \sinh\left(\frac{\rho}{\eta}(T-t)\right) + \rho \cosh\left(\frac{\rho}{\eta}(T-t)\right)}, & g(t) &= \gamma f(t) - h(t), \\ h(t) &= \alpha + \rho \cdot \frac{\theta \cosh\left(\frac{\rho}{\eta}(T-t)\right) + \rho \sinh\left(\frac{\rho}{\eta}(T-t)\right)}{\theta \sinh\left(\frac{\rho}{\eta}(T-t)\right) + \rho \cosh\left(\frac{\rho}{\eta}(T-t)\right)}. \end{aligned} \tag{3.4}$$

The equilibrium trading strategy is given by

$$\hat{v}_t^i = \frac{1}{N} \hat{Y}_t + \frac{1}{\eta} b(t) \cdot \left(\frac{1}{N} X_t - Q_t^i \right), \quad i = 1, 2, \dots, N, \tag{3.5}$$

where

$$b(t) = \frac{\beta k \cosh\left(\frac{k}{\eta}(T-t)\right) + (2\eta\lambda - \frac{\gamma}{2}\beta) \sinh\left(\frac{k}{\eta}(T-t)\right)}{(\beta + \frac{\gamma}{2}) \sinh\left(\frac{k}{\eta}(T-t)\right) + k \cosh\left(\frac{k}{\eta}(T-t)\right)}. \tag{3.6}$$

Remark 3.4 *In the case of $N = 1$, Theorem 3.3 retrieves the optimal trading strategy for the single risk-averse informed trader in [14].*

Remark 3.5 *In the risk-neutral case ($\lambda = \beta = 0$), our open-loop equilibrium strategy in (3.5) reduces to*

$$\hat{v}_t^i = \frac{1}{\eta} \frac{N-1}{N+1} \cdot \frac{\Lambda_t}{N - e^{-\frac{\gamma}{\eta} \frac{N-1}{N+1}(T-t)}}.$$

A similar expression is also obtained by [15], who studied a Stackelberg-Nash game between risk-neutral informed traders and an institutional investor. Compared with the closed-loop equilibrium strategy in [14], our open-loop equilibrium strategy reacts slightly less strongly to the mispricing Λ_t . This slight difference is similar to the relationship between closed-loop and open-loop equilibrium in [5].

Remark 3.6 As $N \rightarrow \infty$, $\theta \rightarrow \frac{\gamma}{2}$ and $\rho \rightarrow \frac{\gamma}{2}$. Assuming the aggregate initial position $\lim_{N \rightarrow \infty} X_0$ exists, the effect of risk aversion on the aggregate trading rate \hat{Y}_t decreases as N increases. Therefore, in the face of intense competition, the aggregate trading activities of informed traders should tend toward the result in the risk-neutral case. A more comprehensive study of the case when $N \rightarrow \infty$ would need to use a mean-field game setup, which we will focus on in future work.

4. Illustration

In this section, we illustrate our result under different parameter settings and demonstrate the effect of the magnitude of risk aversion, number of informed traders and magnitude of transaction costs on trading behaviors and the resulting price dynamics. The basic parameters are given by

$$\Lambda_0 = 5, \quad T = 1, \quad \gamma = 10.$$

In addition, for simplicity, we remove the fluctuations and only show the expected trajectories.

Figure 1 and 2 show dependence on risk aversion. As noted in [14] and also mentioned in our model setup, λ and β represent the magnitude of risk aversion to information volatility and inaccuracy, respectively. In the risk-neutral case (blue line), competition leads to a race in the beginning; thus, the mispricing quickly moves toward zero, and the information is rapidly incorporated into the price. However, risk aversion to information volatility makes traders less willing to trade at an early stage when information is still relatively inaccurate. As a result, an interesting phenomenon can arise in which informed traders initially trade rapidly because of competition and then slow down; subsequently, they accelerate gradually as information becomes accurate. In addition, risk aversion to information inaccuracy causes informed traders to reduce their overall trading rate, lighten their terminal positions, and reveal less information to the market.

Figure 3 shows dependence on the number of informed traders. The result we obtain using the running penalty is similar to the numerical result [11] obtained using CARA utility. Overall, more intense competition leads to more and faster information disclosure. Owing to transaction costs, however, information is not fully disclosed at the end of the trading horizon, even in the presence of intense competition.

Figure 4 shows dependence on transaction costs. Overall, with high transaction costs, information is disclosed more uniformly, whereas low transaction costs cause informed traders to trade more intensively within a short period of time. With low transaction costs, informed

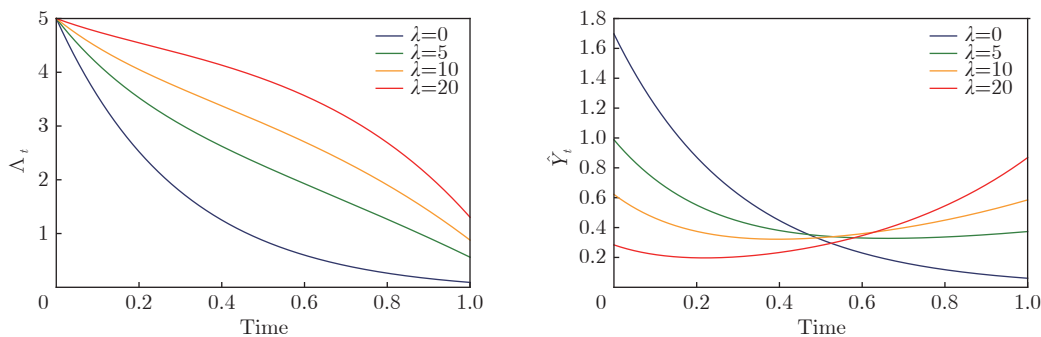


Figure 1 Expected trajectories of mispricing and the aggregate trading rate. $X_0 = 0$, $\eta = 1$, $N = 2$, $\beta = 0$. Blue, green, orange, and red correspond to $\lambda = 0, 5, 10, 20$

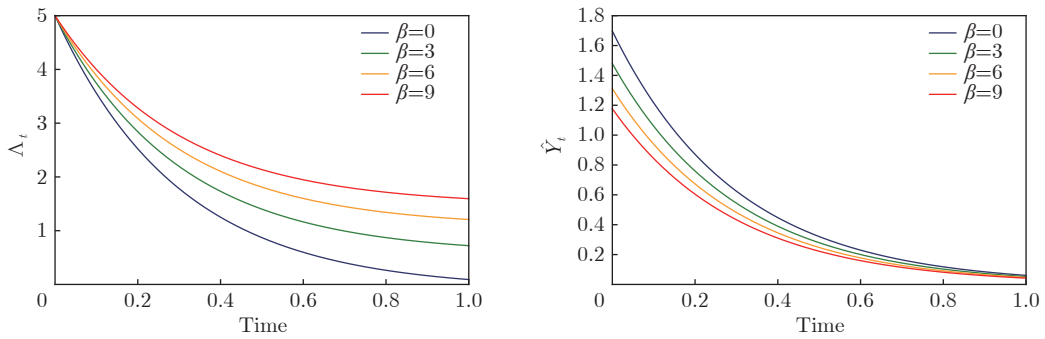


Figure 2 Expected trajectories of mispricing and the aggregate trading rate. $X_0 = 0$, $\eta = 1$, $N = 2$, $\lambda = 0$. Blue, green, orange, and red correspond to $\beta = 0, 3, 6, 9$

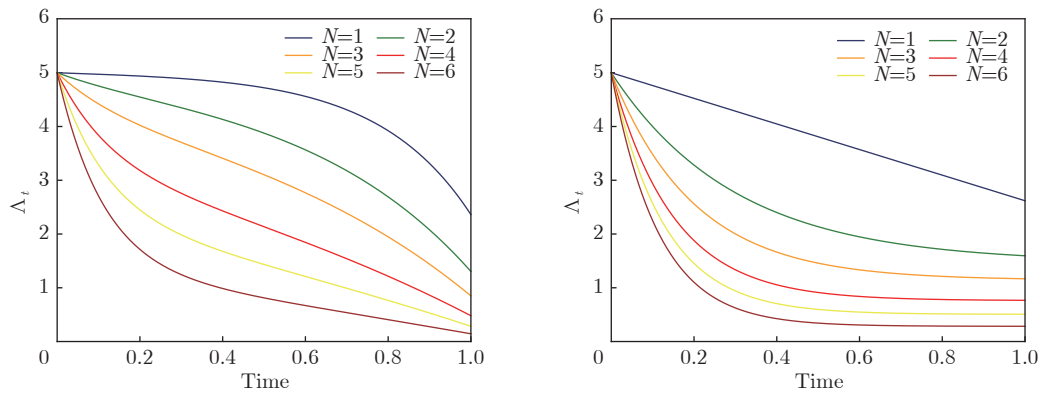


Figure 3 Expected trajectories of mispricing. $X_0 = 0$, $\eta = 1$. $\lambda = 20$, $\beta = 0$ on the left and $\lambda = 0$, $\beta = 9$ on the right. Blue, green, orange, red, yellow, and brown correspond to $N = 1, 2, 3, 5, 8, 15$

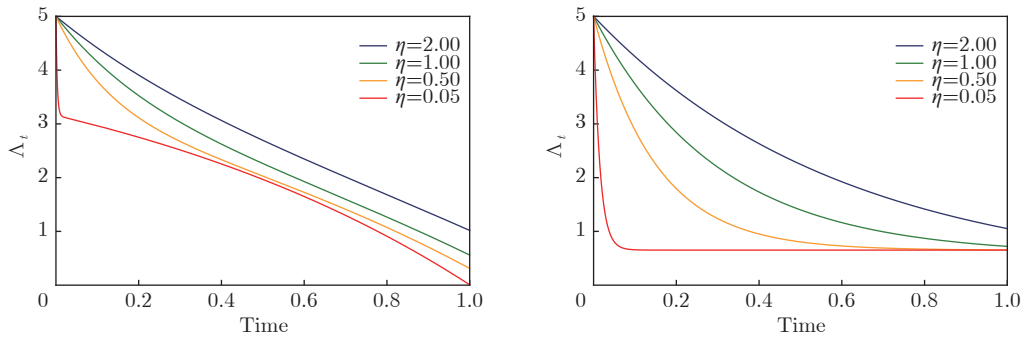


Figure 4 Expected trajectories of mispricing. $X_0 = 0$, $N = 2$. $\lambda = 5$, $\beta = 0$ on the left and $\lambda = 0$, $\beta = 3$ on the right. Blue, green, orange, and red correspond to $\eta = 2, 1, 0.5, 0.05$

traders who are averse to information volatility will immediately build a position at the beginning and disclose a portion of the information. Thereafter, as information becomes more accurate, they slowly disclose more information and fully disclose it at the end of the trading horizon. With low transaction costs, informed traders who are averse to information inaccuracy will disclose a portion of the information rapidly at the beginning and leave the rest undisclosed for the entire trading horizon.

Figure 5 shows the individual behavior of informed traders in the case of a two-player game. Informed traders may change the direction of trading during the trading horizon. On the one hand, this could be attributable to a large initial position (orange line); thus, some positions need to be

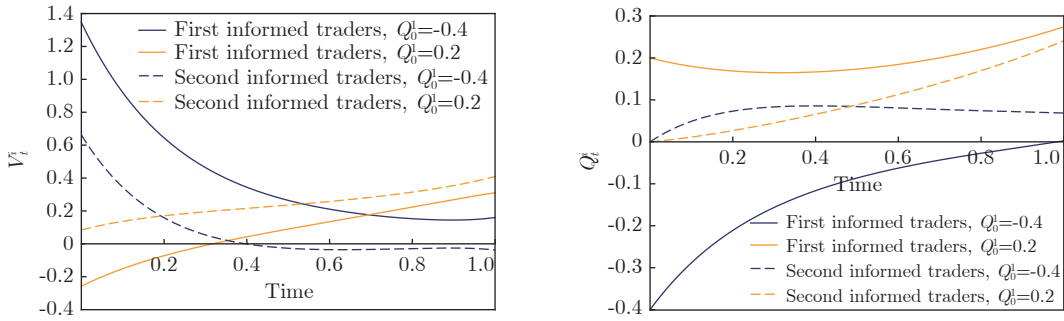


Figure 5 Expected trajectories of trading rates and positions. $Q_0^2 = 0$, $\eta = 1$, $N = 2$, $\lambda = 10$, $\beta = 3$. Solid and dashed lines correspond to the first and second informed traders. Blue and orange correspond to $Q_0^1 = -0.4, 0.2$ closed first to reduce exposure to information volatility. On the other, this could stem from predatory behavior (blue line). Traders who are aware of the trading intentions of other traders can obtain additional revenues through predatory trading.

5. Proofs

5.1 Proof of Lemma 3.1

Let $v^{-i} \in \mathcal{A}^{N-1}$ be fixed. Consider two different $v^i, \tilde{v}^i \in \mathcal{A}$ with the corresponding $\Lambda, \tilde{\Lambda}$ and Q^i, \tilde{Q}^i . For any $\varepsilon \in (0, 1)$ it holds that $\varepsilon v^i + (1 - \varepsilon)\tilde{v}^i \in \mathcal{A}$ with the corresponding $\varepsilon\Lambda + (1 - \varepsilon)\tilde{\Lambda}$ and $\varepsilon Q^i + (1 - \varepsilon)\tilde{Q}^i$. With (2.1) and (2.2) we have

$$\Lambda_t - \tilde{\Lambda}_t = -\gamma(Q_t^i - \tilde{Q}_t^i)$$

and further obtain

$$\begin{aligned} & \varepsilon J^i(v^i; v^{-i}) + (1 - \varepsilon)J^i(\tilde{v}^i; v^{-i}) - J^i(\varepsilon v^i + (1 - \varepsilon)\tilde{v}^i; v^{-i}) \\ &= -\eta\varepsilon(1 - \varepsilon)\mathbb{E} \int_0^T (v_t^i - \tilde{v}_t^i)^2 dt + \varepsilon(1 - \varepsilon)\mathbb{E} \int_0^T (\Lambda_t - \tilde{\Lambda}_t)(v_t^i - \tilde{v}_t^i) dt \\ & \quad - \lambda\varepsilon(1 - \varepsilon)\mathbb{E} \int_0^T (Q_t^i - \tilde{Q}_t^i)^2 dt - \frac{\beta}{2}\varepsilon(1 - \varepsilon)\mathbb{E}(Q_T^i - \tilde{Q}_T^i)^2 \\ &= -\varepsilon(1 - \varepsilon) \left[\eta\mathbb{E} \int_0^T (v_t^i - \tilde{v}_t^i)^2 dt + \lambda\mathbb{E} \int_0^T (Q_t^i - \tilde{Q}_t^i)^2 dt + \frac{\beta + \gamma}{2}\mathbb{E}(Q_T^i - \tilde{Q}_T^i)^2 \right] < 0. \end{aligned}$$

As a result, for any $i = 1, 2, \dots, N$, $J^i(v^i; v^{-i})$ is strictly concave for $v^i \in \mathcal{A}$. In addition, we know there exists at most one open-loop Nash equilibrium by the same argumentation via contradiction in [12, 13].

5.2 Proof of Proposition 3.2

If a set of trading strategies $(\hat{v}^1, \dots, \hat{v}^N) \in \mathcal{A}^N$ together with Λ_t and Q_t^i s satisfy (3.2), for all $i = 1, 2, \dots, N$ we have

$$\begin{aligned} -\eta(Y_t + \hat{v}_t^i) &= \left(\Lambda_T - \Lambda_t + \gamma \int_t^T \hat{v}_u^i du + 2\lambda \int_t^T Q_u^i du + M_T^i - M_t^i \right) - (\eta Y_T + \eta \hat{v}_T^i) \\ &= \beta Q_T^i - \Lambda_t + \gamma \int_t^T \hat{v}_u^i du + 2\lambda \int_t^T Q_u^i du + M_T^i - M_t^i. \end{aligned}$$

Further, with (3.1) we obtain

$$\langle \nabla J^i(\hat{v}^i; \hat{v}^{-i}), w \rangle = \mathbb{E} \left[\int_0^T w_t (M_T^i - M_t^i) dt \right] = \mathbb{E} \left[\int_0^T w_t (\mathbb{E} [M_T^i | \mathcal{F}_t] - M_t^i) dt \right] = 0$$

for all $w^i \in \mathcal{A}$ and all $i = 1, 2, \dots, N$. As a result, $(\hat{v}^1, \dots, \hat{v}^N)$ forms an open-loop Nash equilibrium according to Definition 2.1.

5.3 Proof of Theorem 3.3

With Proposition 3.2 we only need to find a set of trading strategies $(\hat{v}^1, \dots, \hat{v}^N) \in \mathcal{A}^N$ that satisfies the FBSDE system (3.2). We now solve (3.2) through heuristics. Since it is difficult to solve the equilibrium strategy directly, we start with the aggregate trading rate Y_t . By adding N BSDEs in (3.2) together, we obtain the system

$$\begin{cases} d\Lambda_t = -\gamma Y_t dt + \sigma dW_t, & \Lambda_0 = \Lambda_0, \\ dX_t = Y_t dt, & X_0 = \sum_{i=1}^N Q_0^i, \\ \eta(N+1)dY_t = N d\Lambda_t + \gamma Y_t dt + 2\lambda X_t dt + dM_t, \\ \eta(N+1)Y_T = N\Lambda_T - \beta X_T, \end{cases}$$

where $M_t = \sum_{i=1}^N M_t^i$ is a suitable square integrable martingale. Substituting the ansatz (3.3) into this system we have

$$\begin{cases} (N+1)[f'(t)\Lambda_t dt + g'(t)X_t dt] + \sigma[(N+1)f(t) - N]dW_t \\ = \frac{1}{\eta}[(N+1)(\gamma f(t) - g(t)) - \gamma(N-1)](f(t)\Lambda_t + g(t)X_t)dt + 2\lambda X_t dt + dM_t, \\ (N+1)(f(T)\Lambda_T + g(T)X_T) = N\Lambda_T - \beta X_T, \end{cases}$$

which yields the ODE system

$$\begin{cases} (N+1)f'(t) = \frac{1}{\eta}f(t)[(N+1)(\gamma f(t) - g(t)) - \gamma(N-1)], \\ (N+1)g'(t) = \frac{1}{\eta}g(t)[(N+1)(\gamma f(t) - g(t)) - \gamma(N-1)] + 2\lambda, \end{cases}$$

with the terminal conditions

$$f(T) = \frac{N}{N+1}, \quad g(T) = -\frac{\beta}{N+1}.$$

In addition, $M_t = \sigma[(N+1)f(t) - N]W_t$. Define $h(t) = \gamma f(t) - g(t)$, and we have

$$h'(t) = \frac{1}{\eta}h^2(t) - \frac{\gamma}{\eta}\frac{N-1}{N+1}h(t) - \frac{2\lambda}{N+1}, \quad h(T) = \frac{\gamma N + \beta}{N+1} = \theta + \alpha,$$

which can be uniquely solved by $h(t)$ in (3.4). Subsequently, with the ODE

$$f'(t) = \frac{1}{\eta}f(t)[h(t) - 2\alpha],$$

we obtain $f(t)$ in (3.4). With $g(t) = \gamma f(t) - h(t)$ we find the expression of the aggregate trading rate \hat{Y}_t in (3.3) together with (3.4). Next we focus on the specific trading strategy for each informed trader. According to [14], predatory behavior occurs among risk-averse informed traders. Based on the result in [5], we make the ansatz (3.5), which can ensure $\sum_{i=1}^N v_t^i = \hat{Y}_t$. With \hat{Y}_t in (3.3), the system (3.2) and the ansatz (3.5), for each $i = 1, 2, \dots, N$, we have

$$\begin{cases} b'(t) \cdot \left(\frac{1}{N} X_t - Q_t^i \right) dt + \frac{1}{N} dM_t = \left(\frac{1}{\eta} b^2(t) + \frac{\gamma}{\eta} b(t) - 2\lambda \right) \cdot \left(\frac{1}{N} X_t - Q_t^i \right) dt + dM_t^i, \\ b(T) \cdot \left(\frac{1}{N} X_T - Q_T^i \right) = \beta \cdot \left(\frac{1}{N} X_T - Q_T^i \right), \end{cases}$$

which yields the ODE

$$b'(t) = \frac{1}{\eta} b^2(t) + \frac{\gamma}{\eta} b(t) - 2\lambda$$

with the terminal condition $b(T) = \beta$. In addition, $M_t^i = \frac{1}{N} M_t$. The ODE can be uniquely solved by $b(t)$ in (3.6). As a result, we find a set of trading strategies $(\hat{v}^1, \dots, \hat{v}^N)$ that solves (3.2). In addition, since $f(t)$, $g(t)$ and $b(t)$ in (3.4) and (3.6) are all continuous and bounded on $[0, T]$, substituting $(\hat{v}^1, \dots, \hat{v}^N)$ in (3.5) into ODEs (2.1) and (2.2), we know Q_t^i and Λ_t are both mean-square integrable processes on $[0, T]$. Subsequently, it is evident that $(\hat{v}^1, \dots, \hat{v}^N) \in \mathcal{A}^N$ as \hat{v}^i in (3.5) is linear to Λ_t and Q_t^i s. As a result, we can conclude that $(\hat{v}^1, \dots, \hat{v}^N)$ in (3.5) forms the unique open-loop Nash equilibrium according to Lemma 3.1 and Proposition 3.2.

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