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Introduction for recent advances in forward performance processes

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We are pleased to welcome the readers to the Part II of "*Recent Developments in Forward Performance Processes*" and extend our gratitude to the editors of "Probability, Uncertainty and Quantitative Risk" for providing us with this platform to publish the latest advances in this fast-developing area. The special issue contains ten academic papers in distinct topics in the field and a preface article.

The preface article (appeared in Part I) is written by Marek Musiela who co-founded with Thaleia Zariphopoulou and developed the framework of the forward performance processes close to 20 years ago. It provides a comprehensive review on what motivated the introduction of forward utilities and how classical ideas from arbitrage-free pricing influenced the construction of forward preferences. It also formulates some of the main questions in the area, related to the interplay among risk premia, evolution of individual risk preferences and optimality of investment strategies.

The following five papers appeared in Part I.

The article "*Bi-revealed utilities in a defaultable universe: A new point of view on consumption*" written by Nicole El Karoui, Caroline Hillairet, and Mohamed Mrad, analyzes the convex duality of forward performance processes within a defaultable framework with a totally inaccessible default time. Their results provide an equivalence between forward processes of wealth and consumption in a non-default framework and forward processes of wealth in a defaultable framework. The consumption process is, then, interpreted as an accumulation of reserves set aside to accommodate the default event occurring at the default time.

The second work, "*Time-consistent pension policy with minimum guarantee and sustainability constraint*" by Caroline Hillairet, Sarah Kaakai, and Mohamed Mrad, proposes an optimal investment/pension policy for a collective pension scheme with guaranteed payment under sustainability and adequacy constraints. The social planner's preferences are modelled in a forward performance process framework that provides flexibility, time-horizon independence, and heterogeneity. This work contributes to the literature not only on forward performance processes

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but, also, on optimal investment and consumption with labor income.

The rest of the three articles in Part I contribute to the study of forward performance processes under model ambiguity.

The article "*Optimal investment and consumption with forward preferences and uncertain parameters*" by Wing Fung Chong and Gechun Liang analyzes a class of time-monotone robust forward performance criteria in incomplete markets with portfolio constraints. They provide a PDE characterization and a semi-explicit saddle-point construction of the robust forward performance criteria and their optimal policies.

In "*G-forward performance process and representation of the homothetic case via ergodic quadratic G-BSDE*", Yifan Sun and Falei Wang introduce a class of homothetic robust forward performance criteria with non-zero volatility. This is established by utilizing the recently developed ergodic and infinite horizon BSDE driven by G-Brownian motion.

The work "*Forward robust portfolio selection: The binomial case*" by Harrison Waldon introduces, in a binomial tree model, the robust predictable forward performance processes. The agent solves a worst-case portfolio optimization problem over possible market models, which are represented via a Wasserstein neighbourhood centered on a binomial distribution.

Part II contains works on five distinct topics: mean field games, deep learning, rank-dependent forward utilities, project valuation and optimal liquidation.

The article "*Mean field and n-player games in Ito-diffusion markets under forward performance criteria*" by Thaleia Zariphopoulou introduces a new class of relative forward performance criteria for arbitrary time-monotone forward preferences, considerably generalizing the well-studied homothetic families. Such criteria lead to a new class of forward Nash equilibrium games for finitely many players and a new class of mean-field games of common noise with unbounded controls in the volatility. Under minimal assumptions on the forward preferences, explicit solutions for optimal policies, optimal wealth, and optimal game values for both games are provided by leveraging the analytical properties of space-time harmonic functions and the closed-form representation of the optimal processes.

In the work "*Deep-learning scheme for forward utilities using ergodic BSDEs*", Guillaume Broux-Quemerais, Sarah Kaakai, Anis Matoussi, and Wissal Sabbagh propose a deep-learning numerical scheme to study homothetic forward performance processes arising in Markovian models. Such processes may be represented in closed form and are directly related to ergodic BSDE. One of the main difficulties building numerical schemes for ergodic BSDE is the lack of terminal condition, which the authors handle by introducing a new random time approach.

The article "*Rank-dependent predictable forward performance processes*" by Bahman Angoshtari and Shida Duan considers the existence and construction of predictable forward performance processes in the rank-dependent utility framework by incorporating a random and exogenous sequence of probability weighting functions. The discrete-time updating of preferences resolves the conflict between the time-inconsistency of the rank-dependent utility framework and the time-consistency of forward performance criteria.

In "*Forward indifference valuation for dynamically incoming projects*", Haoran Wang investigates the application of forward utilities to the indifference valuation and hedging of dynamically incoming projects, whose flow and payoff structures are not known (not even modelled) at initial time. He investigates two different problems. The first is concerned with how prices and hedging strategies of an existing project are altered as soon as a new, not encountered before, project is initiated. The second problem is on the indifference valuation of a new project in

relation to existing ones. He considers two classes of forward performance processes, predictable and adapted, and studies the related values and strategies.

The last article, "*Optimal liquidation with dynamic parameter update: A forward approach*", written by Haoran Wang and Thaleia Zariphopoulou, proposes a study, using forward performance criteria, of liquidation strategies in the presence of dynamic model updating. The forward liquidation program consists of a series of single-parameter continuous-time forward liquidation problems which are activated at every parameter update and are pasted together in a time-consistent manner. While the forward construction eliminates any model error, it may yield explosions of solutions in finite time and, furthermore, may fail to fully liquidate especially under unfavorable market conditions. They juxtapose these features with the classical approach of adaptive control which delivers full liquidation but, at the heavy cost of time-inconstancy, accumulated model error and loss of value.

We hope that this special issue would provide the readers with a diverse collection of high quality works as well as ideas for new, future research directions in forward performance processes.