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Leader-Following Consensus for a Class of Nonlinear Cascaded Multi-Agent Systems

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Abstract: This paper focuses on the problem of leader-following consensus for nonlinear cascaded multi-agent systems. The control strategies for these systems are transformed into successive control problem schemes for lower-order error subsystems. A distributed consensus analysis for the corresponding error systems is conducted by employing recursive methods and virtual controllers, accompanied by a series of Lyapunov functions devised throughout the iterative process, which solves the leader-following consensus problem of a class of nonlinear cascaded multi-agent systems. Specific simulation examples illustrate the effectiveness of the proposed control algorithm.

Keywords: cascaded multi-agent system; distributed control; consensus; recursive method
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0 Introduction

The research on multi-agent systems, as a comprehensive interdisciplinary field, has important theoretical value and broad application prospects in industrial and military fields. It can be applied to formation control such as multi-robot, drone, military vehicle and satellite formation control, as well as air traffic control and military searching[1-4]. The consensus is the foundation of collaborative control. The goal of achieving consensus among multi-agent systems is to design appropriate control protocols that enable all agents to reach certain quantities of interest.

Generally speaking, cascaded systems can be roughly classified into two types: linear cascaded systems and nonlinear cascaded systems. We are mainly interested in nonlinear cascaded systems. The global asymptotic stability of a class of nonlinear cascaded systems was achieved by using the method of state feedback[5]. The uniform global finite-time stability was discussed for the cascaded time-varying system consisting of two uniformly finite-time stable subsystems[6]. The output feedback control was investigated for nonlinear cascaded systems with external disturbance and asymmetric constraints[7].

Based on the count of leaders in multi-agent systems, the consensus issue can be divided into three types: leaderless consensus, leader-following consensus and containment consensus with multiple leaders. The ultimate states of the followers can reach the trajectories of the leader labeled as the 0th agent. The leader agent can be physical or virtual, and all other agents are referred to as followers.

The majority of research on multi-agent consensus focuses on individual dynamics as first-order integrators and second-order integrators[8-13]. However, when applying the aforementioned theoretical results to real-world applications, such as the coordination of multi-wheeled mobile robots with non-holonomic constraints and consensus of multiple satellite rigid body attitudes, the multi-agent systems could not achieve the desired control effects. The fundamental reason is that the low-order linearized models adopted in the theoretical research are too simplistic and overlook the high-order, nonlinear and non-holonomic characteristics of the systems.

In recent years, there has been increasing attention on the research of collaborative control problems for high-order multi-agent systems. To solve the bipartite consensus problem in high-order multi-agent systems, a new distributed controller was proposed based on the output information from neighboring agents[14]. It was considered that the fixed-time consensus of the high-order chained-form multi-agent systems was subject to non-holonomic constraints by employing the backstepping structure[15]. The finite-time leader-following consensus problem was investigated for a class of high-order multi-agent systems characterized by uncertain nonlinear dynamics[16]. However, there is currently limited research on distributed collaborative control of nonlinear cascaded multi-agent systems. Therefore, it is worthwhile to investigate the consensus of cascaded multi-agent systems for their broad prospects and application value.

The main purpose of this paper is to discuss the consensus problem for nonlinear cascaded multi-agent

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systems. The main contributions are as follows. Firstly, a distributed control strategy based on recursive methods is proposed. The neighbor state information and virtual controllers are utilized. Secondly, it is demonstrated that the nonlinear cascaded multi-agent systems can achieve consensus by providing a proof of the inductive step based on the Lyapunov stability theorem.

The remainder of this paper is arranged as follows. Some preliminaries on graph theory, communication topology, and the necessary definitions and lemmas are given in Section 1. The consensus protocols and the stability analysis of the nonlinear cascaded multi-agent systems are presented in Section 2. Simulation examples are provided in Section 3 to demonstrate the effectiveness of the proposed results. A summary is presented in Section 4.

1 Preliminaries and Problem Formulations

1.1 Preliminaries

The network associated with a multi-agent system can be represented by an undirected graph G. It is represented by a triple, i. e. $G=\{V, E, \boldsymbol{A}\}$, where V is the set of nodes and $V=\{v_1, v_2, \cdots, v_N\}$; E is the set of edges and $E\subseteq V\times V$; $\boldsymbol{A}$ describes the adjacency matrix of G with $a_{ij}=0$ ($i=1, 2, \cdots, N$; $j=1, 2, \cdots, n$) and $\boldsymbol{A}=[a_{ij}]_{n\times n}$. If the agent i can directly receive information from the agent j, $a_{ij}\neq 0$; otherwise $a_{ij}=0$. The edge weights between the leader and the followers are denoted by a_{i0}. $\boldsymbol{D}=\mathrm{diag}(d_1, d_2, \cdots, d_n)$ is defined as the in-degree matrix of the graph with $d_i=\sum_{j=1}^{n}|a_{ij}|$, and the Laplacian matrix $\boldsymbol{L}_s := [l_{ij}] \in \mathbf{R}^{n\times n}$ of the graph is presented as $\boldsymbol{L}_{\mathrm{s}}=\boldsymbol{D}-\boldsymbol{A}$, where $l_{ij}=-a_{ij}, i\neq j$; $\mathbf{R}^{n\times n}$ is Euclidean space with $n\times n$ dimension. If there is a path between any two vertices in the graph, the undirected graph G is connected.

Assumption 1 The communication topology between followers is an undirected connected graph.

Assumption 2 The communication graph, i. e. the undirected graph G, contains a directed spanning tree in which the leader acts as the root node.

Assumption 3 The nonlinear function $f(t, x(t))$ satisfies the following inequality

$$|f(t, x)-f(t, y)|\leqslant\rho|x-y|,\ \forall x, y\in\mathbf{R}, t\geqslant 0, \quad (1)$$

where ρ is a positive constant.

Assumption 4 The input of the leader u_0 is zero, i. e., $u_0(t)=0$.

Definition 1 Define a matrix $\boldsymbol{B}=\mathrm{diag}(a_{10}, a_{20}, \cdots, a_{N0})$, where $a_{i0}>0$ if the leader is a neighbor of an agent i; otherwise, $a_{i0}=0$. The matrix $\boldsymbol{L}$ is defined as $\boldsymbol{L}=\boldsymbol{L}_{\mathrm{s}}+\boldsymbol{B}$.

Lemma 1[17] (Barbalat's Lemma) Let f: $\mathbf{R}\rightarrow\mathbf{R}$ be a uniformly continuous function on $[0,\infty)$. Supposing that $\lim_{t\rightarrow\infty}\int_0^t f(\tau)\mathrm{d}\tau$ exists and is finite, then $\lim_{t\rightarrow\infty}f(t)=0$, where $f(\cdot)$ is a nonlinear function.

Lemma 2[18] Under Assumption 1 and Assumption 2, the matrix $\boldsymbol{L}$ is symmetric positive definite.

1.2 Problem formulation

Considering a nonlinear cascaded multi-agent system consisting of a leader labeled as 0, and N followers labeled as $i=1, 2, \cdots, N$, the dynamic of the ith ($i=0, 1, 2, \cdots, N$) agent is described as follows:

$$\begin{cases}\dot{x}_{i1}=f(t,x_{i1})+x_{i2},\\ \dot{x}_{i2}=x_{i3},\\ \quad\vdots\\ \dot{x}_{in}=u_i,\end{cases} \quad (2)$$

where $\boldsymbol{x}_i=[x_{i1}, x_{i2}, \cdots, x_{in}]^{\mathrm{T}}\in\mathbf{R}^n$ and $\boldsymbol{u}_i\in\mathbf{R}^m$, which are the states of the ith agent and control input, respectively.

The objective of this paper is to make all agents achieve consensus for the nonlinear cascaded multi-agent systems (Eq. (2)). In other words, if the above assumption conditions are satisfied, a distributed controller that only relies on local information can be designed, ensuring that the state of each agent satisfies:

$$\lim_{t\rightarrow\infty}|x_{ij}(t)-x_{0j}|=0,$$
$$i=1, 2, \cdots, N;\quad j=1, 2, \cdots, n. \quad (3)$$

2 Main Results

Based on the local information of the agents, a distributed controller is designed by using a recursive method, and a series of Lyapunov functions are proposed during the iterative process.

Theorem 1 Consider the nonlinear cascaded multi-agent system (Eq. (2)). If **Assumptions** 1 − 4 are satisfied, there exist distributed protocols that solve the consensus tracking problem.

Proof Define the tracking error as $\boldsymbol{e}_i=\boldsymbol{x}_i-\boldsymbol{x}_0$, then the error system can be written as

$$\begin{cases}\dot{e}_{i1}=\Delta f_{i1}+e_{i2},\\ \dot{e}_{i2}=e_{i3},\\ \quad\vdots\\ \dot{e}_{in}=u_i,\ i=1, 2, \cdots, N,\end{cases} \quad (4)$$

where $\Delta f_{i1}=f(t, x_{i1})-f(t, x_{01})$; $\boldsymbol{e}=[e_{1m}, e_{2m}, \cdots, e_{Nm}]^{\mathrm{T}}\in\mathbf{R}^n, m=1, 2, \cdots, n$.

Define a new set of variables

$$\begin{cases}z_{i1}=e_{i1},\\ z_{ij}=e_{ij}-\alpha_{ij},\ i=1, 2, \cdots, N; j=2, 3, \cdots, n,\end{cases} \quad (5)$$

where α_{ij} represents the virtual controller to be designed during the iterative process.

Step 1 Consider the first-order subsystem in the error system (Eq. (4)), i. e.,

$$\dot{e}_{i1}=\Delta f_{i1}+e_{i2},$$

where the error variable is defined as $z_{i1} = e_{i1}$. We choose the following Lyapunov function V_1:

$$V_1 = \frac{1}{2} z_1^{\mathrm{T}} \boldsymbol{L} z_1, \tag{6}$$

where $z_1 = [z_{11}, z_{21}, \cdots, z_{N1}]^{\mathrm{T}}$.

The first distributed virtual controller is defined as

$$\alpha_{i2} = -\beta \sum_{j=1}^{N} [a_{ij}(x_{i1} - x_{j1}) + a_{i0}(x_{i1} - x_{01})], \tag{7}$$

which can be also written as

$$\alpha_{i2} = -\beta \sum_{j=1}^{N} [a_{ij}(e_{i1} - e_{j1}) + a_{i0}(e_{i1} - e_{01})], \tag{8}$$

where $\beta > 0$; $e_{i2} = \alpha_{i2}$, $i = 1, 2, \cdots, N$. Taking the derivative of the Lyapunov function V_1 with respect to Eq. (2) yields

$$\begin{aligned}
\dot{V}_1 &= \boldsymbol{e}_1^{\mathrm{T}} \mathbf{L} \dot{\boldsymbol{e}}_1 = \boldsymbol{e}_1^{\mathrm{T}} \boldsymbol{L}(-\beta \boldsymbol{L} \boldsymbol{e}_1 + \Delta \boldsymbol{f}_1) \\
&\leqslant -\beta \|\boldsymbol{L}\boldsymbol{e}_1\|_2^2 + \frac{1}{2} \boldsymbol{e}_1^{\mathrm{T}} \boldsymbol{L}^2 \boldsymbol{e}_1 + \frac{1}{2} \Delta \boldsymbol{f}_1^{\mathrm{T}} \Delta \boldsymbol{f}_1 \\
&\leqslant -\beta \|\boldsymbol{L}\boldsymbol{e}_1\|_2^2 + \frac{1}{2} \|\boldsymbol{L}_1\|_2^2 + \frac{1}{2} \rho^2 \boldsymbol{e}_1^{\mathrm{T}} \boldsymbol{e}_1 \\
&\leqslant -\beta \|\boldsymbol{L}\boldsymbol{e}_1\|_2^2 + \frac{1}{2} \|\boldsymbol{L}\boldsymbol{e}_1\|_2^2 + \frac{1}{2} \rho^2 \boldsymbol{e}_1^{\mathrm{T}} \frac{\boldsymbol{L}^2}{\lambda_{\min}^2(\boldsymbol{L})} \boldsymbol{e}_1 \\
&\leqslant \|\boldsymbol{L}\boldsymbol{e}_1\|_2^2 \left(-\beta + \frac{1}{2} + \frac{\rho^2}{2\lambda_{\min}^2(\boldsymbol{L})}\right),
\end{aligned} \tag{9}$$

where $\Delta \boldsymbol{f}_1 = [\Delta f_{11}, \Delta f_{21}, \cdots, \Delta f_{N1}]^{\mathrm{T}}$; $\lambda_{\min}(\boldsymbol{L})$ is the minimum eigenvalue of $\boldsymbol{L}$.

Remark 1 According to Lemma 2, $\boldsymbol{L}$ is a positive definite matrix, then $\lambda_{\min}(\boldsymbol{L}) > 0$.

Choose $\beta = \frac{3}{2} + \frac{\rho^2}{2\ \lambda_{\min}^2(\boldsymbol{L})}$, and then we can obtain $\dot{V}_1 \leqslant -\|\boldsymbol{L}\boldsymbol{e}_1\|_2^2$. Integrating both sides of the above inequality, we have that

$$V_1(\boldsymbol{e}_1(t)) - V_1(\boldsymbol{e}_1(0)) \leqslant -\int_0^t \|\boldsymbol{L}\boldsymbol{e}_1(\tau)\|_2^2 \mathrm{d}\tau. \tag{10}$$

It follows that

$$\int_0^t \|\boldsymbol{L}\boldsymbol{e}_1(\tau)\|_2^2 \mathrm{d}\tau \leqslant V_1(\boldsymbol{e}_1(0)) - V_1(\boldsymbol{e}_1(t)) \leqslant V_1(\boldsymbol{e}_1(0)). \tag{11}$$

By **Lemma 1**, we have $\lim_{t\to\infty} \|\boldsymbol{L}\boldsymbol{e}_1(t)\|_2^2 = 0$, i. e., $\lim_{t\to\infty} \boldsymbol{e}_1(t) = 0$, which implies that the tracking error asymptotically converges to 0.

Step 2 Consider the second-order subsystem in the error system (Eq. (4)), i. e.,

$$\dot{e}_{i1} = \Delta f_{i1} + e_{i2},$$

$$\dot{e}_{i2} = e_{i3},$$

and choose the Lyapunov function $V_2 = V_1 + \frac{1}{2} z_2^{\mathrm{T}} \boldsymbol{L} z_2$, where $z_2 = [z_{12}, z_{22}, \cdots, z_{N2}]^{\mathrm{T}}$. The derivative of the Lyapunov function V_2 yields

$$\begin{aligned}
\dot{V}_2 &\leqslant -\|\boldsymbol{L}\boldsymbol{e}_1\|_2^2 + z_2^{\mathrm{T}} \boldsymbol{L} \dot{z}_2 \\
&= -\|\boldsymbol{L}z_1\|_2^2 + z_2^{\mathrm{T}} \boldsymbol{L}(\dot{\boldsymbol{e}}_2 - \dot{\boldsymbol{\alpha}}_2) \\
&= -\|\boldsymbol{L}z_1\|_2^2 + z_2^{\mathrm{T}} \boldsymbol{L}[\boldsymbol{e}_3 + \beta \boldsymbol{L}(\boldsymbol{e}_2 + \Delta \boldsymbol{f}_1)],
\end{aligned} \tag{12}$$

where $\boldsymbol{e}_2 = [e_{12}, e_{22}, \cdots, e_{N2}]^{\mathrm{T}}$.

Define $\boldsymbol{\alpha}_3 = \boldsymbol{e}_3 = -\boldsymbol{L}\boldsymbol{e}_2 - \beta \boldsymbol{L} \rho \|z_1\|_2 - \boldsymbol{L} z_2$, where $\boldsymbol{\alpha}_3 = [\alpha_{13}, \alpha_{23}, \cdots, \alpha_{N3}]^{\mathrm{T}}$, $\|z_1\|_2 = \sqrt{\sum_{i=1}^{N} z_{i1}^2}$, $\alpha_{i3} = -l_{i2} z_{i2} - \beta \sum_{j=1}^{N} l_{ij} e_{i2} - \beta\rho \sum_{j=1}^{N} l_{ij} \|z_1\|_2$. So we have

$$\dot{V}_2 \leqslant -\|\boldsymbol{L}z_1\|_2^2 - \|\boldsymbol{L}z_2\|_2^2. \tag{13}$$

By the inductive method, Formulas (9) and (12) can be recursively extended as follows.

Step *k* Consider the kth-order subsystem in the error system, where $2 \leqslant k \leqslant n-1$, i. e.,

$$\begin{aligned}
\dot{e}_{i1} &= \Delta f_{i1} + e_{i2}, \\
\dot{e}_{i2} &= e_{i3}, \\
&\vdots \\
\dot{e}_{ik} &= e_{i,k+1}, \ i = 1, 2, \cdots, N.
\end{aligned}$$

Assume the Lyapunov function V_{k-1} exists,

$$V_{k-1} = V_1 + \frac{1}{2} \sum_{\mu=2}^{k-1} z_\mu^{\mathrm{T}} \boldsymbol{L} z_\mu \geqslant 0, \tag{14}$$

which satisfies $\dot{V}_{k-1} \leqslant -\sum_{\mu=1}^{k-1} \|\boldsymbol{L}z_\mu\|_2^2$. To prove that when $n = k$, the following results hold, we choose the Lyapunov function $V_k = V_{k-1} + \frac{1}{2} z_k^{\mathrm{T}} \boldsymbol{L} z_k$, where $z_k = [z_{1k}, z_{2k}, \cdots, z_{Nk}]^{\mathrm{T}}$.

Taking the derivative of the Lyapunov function V_k, we have

$$\begin{aligned}
\dot{V}_k &\leqslant \dot{V}_{k-1} + z_k^{\mathrm{T}} \boldsymbol{L} z_{\dot{k}} \\
&\leqslant -\sum_{\mu=1}^{k-1} \|\boldsymbol{L}z_\mu\|_2^2 + z_k^{\mathrm{T}} \boldsymbol{L}(\dot{\boldsymbol{e}}_k - \boldsymbol{\alpha}_{\dot{k}}) \\
&\leqslant -\sum_{\mu=1}^{k-1} \|\boldsymbol{L}z_\mu\|_2^2 + z_k^{\mathrm{T}} \boldsymbol{L}\boldsymbol{e}_{k+1} - z_k^{\mathrm{T}} \boldsymbol{L} \dot{\boldsymbol{\alpha}}_k,
\end{aligned} \tag{15}$$

where $\boldsymbol{\alpha}_k = [\alpha_{1k}, \alpha_{2k}, \cdots, \alpha_{Nk}]^{\mathrm{T}}$;

$\dot{\alpha}_{ik} = -\beta l_{ii} \sum_{\mu=2}^{k-1} \frac{\partial \alpha_{ik}}{\partial e_{i,\mu-1}} e_{i\mu} - \beta \sum_{\mu=2}^{k-1} \sum_{j=1}^{n} l_{ij} \frac{\partial \alpha_{ik}}{\partial e_{j,\mu-1}} e_{j\mu} - \beta \sum_{j=1}^{n} l_{ij} \Delta f_{j1}$; $z_k = \boldsymbol{e}_k - \boldsymbol{\alpha}_k$, $k = 2, 3, \cdots, n$.

If $\alpha_{i,k+1} = e_{i,k+1} = -l_{ik} z_{ik} + \dot{\alpha}_{ik}$, then $\dot{V}_k \leqslant -\sum_{\mu=1}^{k} \|\boldsymbol{L}z_\mu\|_2^2$.

Furthermore, the inductive argument guarantees that for the Lyapunov function $V_n = V_{n-1} + \frac{1}{2} z_n^{\mathrm{T}} \boldsymbol{L} z_n$, and thus

$$\begin{aligned}
\dot{V}_n &\leqslant -\sum_{\mu=1}^{n-1} \|\boldsymbol{L}z_\mu\|_2^2 + z_n^{\mathrm{T}} \boldsymbol{L} \dot{z}_n \\
&\leqslant -\sum_{\mu=1}^{n-1} \|\boldsymbol{L}z_\mu\|_2^2 + z_n^{\mathrm{T}} \boldsymbol{L}(\dot{\boldsymbol{e}}_n - \dot{\boldsymbol{\alpha}}_n)
\end{aligned}$$

$$\leqslant -\sum_{\mu=1}^{n-1}\|Lz_{\mu}\|_2^2 + z_n^{\mathrm{T}}Lu - z_n^{\mathrm{T}}L\dot{\alpha}_n, \tag{16}$$

where $u = [u_1, u_2, \cdots, u_N]^{\mathrm{T}}$; $u_i = -l_{in}z_{in} - \beta l_{ii} \times \sum_{\mu=2}^{n-1}\frac{\partial\alpha_{in}}{\partial e_{j,\mu-1}}e_{i\mu} - \beta\sum_{\mu=2}^{n-1}\sum_{j=1}^{n}l_{ij}\frac{\partial\alpha_{ik}}{\partial e_{j,\mu-1}}\frac{\partial\alpha_{ik}}{\partial e_{j,\mu-1}}e_{j\mu} - \beta\sum_{j=1}^{n}l_{ij}\Delta f_{j1}$; $\alpha_n = [\alpha_{1n}, \alpha_{2n}, \cdots, \alpha_{Nn}]^{\mathrm{T}}$.

The distributed controller is designed as

$$u = -Lz_n + \dot{\alpha}_n.$$

Based on Formula (16), further deduction yields:

$$\dot{V}_n \leqslant -\sum_{\mu=1}^{n}\|Lz_{\mu}\|_2^2.$$

By **Lemma 1**, we have $\lim_{t\to\infty}\|Lz_i\| = 0$, $i = 1, 2, \cdots, n$. Due to $\lim_{t\to\infty} z_1 = 0$, then $\lim_{t\to\infty} e_1 = 0$ and $\lim_{t\to\infty}\alpha_2 = 0$. According to Eq. (5), $e_2 = z_2 + \alpha_2$, so $\lim_{t\to\infty} e_2 = 0$. Based on the inductive discussion, it can be further obtained that for $i = 2, 3, \cdots, n$, $\lim_{t\to\infty} e_i = 0$, and then, $x_i - x_0 = 0$, as $t\to\infty$, which means the leader-following consensus problem is solved.

3 Simulation Example

In this section, a specific numerical example is utilized to validate the effectiveness of the proposed theoretical results.

Considering the multi-agent system consisting of a second-order integrator with one leader and five followers, the dynamic of the ith agent is described as follows:

$$\begin{cases}\dot{x}_{i1} = x_{i1}\sin(0.6t) + x_{i2}, \\ \dot{x}_{i2} = u_i, \quad i = 0, 1, \cdots, 5.\end{cases} \tag{17}$$

According to **Assumption 4**, $u_0(t) = 0$, the error system for the above Eq. (17) can be written as

$$\begin{cases}\dot{e}_{i1} = e_{i1}\sin(0.6t) + e_{i2}, \\ \dot{e}_{i2} = u_i, \quad i = 0, 1, \cdots, 5.\end{cases} \tag{18}$$

According to **Theorem 1**, we can obtain $\beta = 7.4659$ and design $u = [u_1, u_2, u_3, u_4, u_5]^{\mathrm{T}}$ as $-\beta Le_2 - \beta L\rho\|e_1\|_2 - L(e_2 - \alpha_2)$.

The corresponding communication topology among agents is described in Fig. 1

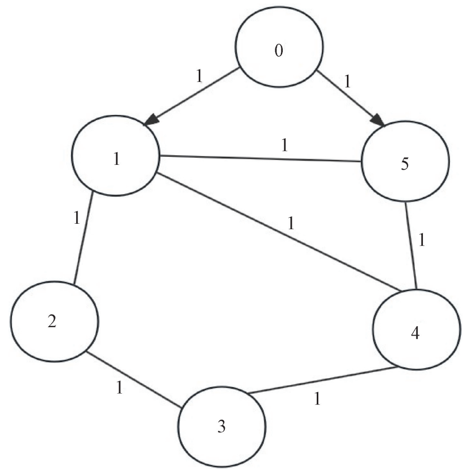

Fig. 1 Communication topology

The Laplacian matrix is expressed as

$$L = \begin{bmatrix} 4 & -1 & 0 & -1 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ -1 & 0 & -1 & 3 & -1 \\ -1 & 0 & 0 & -1 & 3 \end{bmatrix}.$$

The initial states of followers are given as $[x_{11}(0), x_{21}(0), x_{31}(0), x_{41}(0), x_{51}(0)]^{\mathrm{T}} = [-2, 3, 2, -2, -1]^{\mathrm{T}}$ and $[x_{12}(0), x_{22}(0), x_{32}(0), x_{42}(0), x_{52}(0)]^{\mathrm{T}} = [-1, 0, 1, -2, -3]^{\mathrm{T}}$.

The initial states of the leader are given as $x_{01}(0) = 1$ and $x_{02}(0) = -1$.

The tracking errors of the multi-agent system (Eq. (18)) are shown in Fig. 2. It indicates the achievement of leader-following consensus.

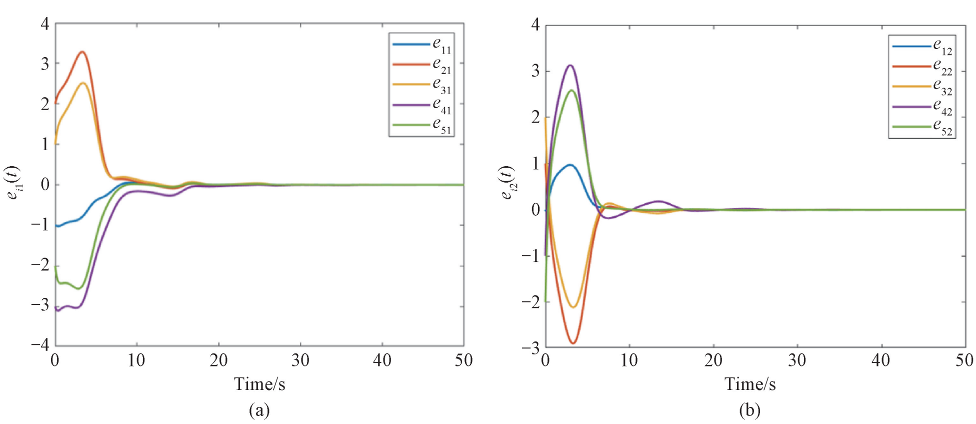

Fig. 2 Trajectories of tracking error: (a) first component; (b) second component

4 Conclusions

In this paper, we considered the leader-following consensus problem for nonlinear cascaded multi-agent systems. The distributed consensus protocols are designed with the aid of virtual controllers in a recursive manner, which can guarantee leader-following consensus. A specific numerical example illustrates the effectiveness of the proposed control strategy. Future work will focus on the bipartite consensus problem for nonlinear cascaded multi-agent systems, with further attention to the consensus for nonlinear cascaded multi-agent systems with time-varying delays.

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一类非线性级联多智能体系统的领导跟随一致性

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摘　要： 研究了非线性级联多智能体系统的领导跟随一致性问题。将非线性级联多智能体系统的控制策略转化为对低阶误差子系统的系列控制。采用递归方法和虚拟控制器，在迭代过程中设计一系列的李雅普诺夫函数，并对相应误差系统的分布式一致性进行分析，解决了一类非线性级联多智能体系统的领导跟随一致性问题。仿真实例验证了所设计控制算法的有效性。

关键词： 级联多智能体系统；分布式控制；一致性；递归方法