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Fixed Points in Generalized Parallel Dynamical System with NAND or NOR Local Functions over Directed Rooted Trees

SUN Yanwen, ZHENG Jie*

College of Science, Donghua University, Shanghai 201620, China

Abstract: In order to study the exact number and states of fixed points in the generalized dynamical system with NAND or NOR local functions over directed rooted trees, structural analysis and classification discussion methods are applied. The exact results of the fixed points in such dynamical systems are obtained. It is proved that the fixed points in such dynamical systems are completely determined by the loops in the rooted trees. This work provides a relevant advance in the knowledge of discrete dynamical systems which constitute mathematical tools to model simulation processes.

Key words: generalized parallel dynamical system; directed rooted tree; Boolean function; fixed point

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0 Introduction

With the development of information technology and computer technology, especially the emergence of ChatGPT, computer simulations have played a more important role in the study of complex natural and human-made systems. They are widely applied to many fields, from biomedicine^[1-2], mathematics^[3-5] to sociology^[6].

Graph dynamical systems can be used to describe generalized processes that occur on graphs or networks. The wide application of these systems is the natural framework for capturing distributed systems such as biological networks, infectious diseases on social networks, and the hottest field, artificial neural networks^[7]. In addition to the fields noted above, machine vision technology whose infrastructure is a complex graph dynamical system is also widely used in textile intellectualization^[8-9] and fabric defect detection^[10-12]. In modern textile manufacturing, textile intellectualization meets consumers' needs in pursuit of smartness, fashion and distinctiveness, while automatic fabric defect detection plays a vital role in ensuring the

quality of textiles^[13]. Both of them are closely related to graph dynamical systems.

Given a finite (non-empty) set of elements X and a function $F: X \rightarrow X$, the pair (X, F) , or simply F , is called a finite dynamical system. In this definition, X is the state space and F is the evolution operator of the system. A Boolean finite dynamical system (BFDS)^[14] is a kind of the simplest finite dynamical system where X and F are Boolean. Two methods can be used in updating the state variables; synchronous and asynchronous. The systems with simple dependency graphs that update through these two methods are called parallel dynamical systems (PDSs)^[15-17] and sequential dynamical systems (SDSs)^[18-24] respectively, while the systems over directed dependency graphs are called parallel dynamical systems over directed graph (PDDSSs)^[25-28] and sequential dynamical systems over directed graph (SDDSSs)^[29-30] respectively. In the above-mentioned systems, it is usually assumed that each state variable influences itself called self-loop during updating. However, this does not entirely conform to real phenomena. So, generalized dynamical systems take both possibilities of having or not having such loops into consideration. These systems are more general^[31].

In this work, we deal with generalized parallel dynamical systems over directed rooted trees induced by the Boolean function NAND or NOR. For this system, we obtain the states of its fixed points and completely determine the number of its fixed points.

1 Definitions and Notations

1.1 Definitions of directed rooted trees

Definition 1 A directed rooted tree $T = (V, D)$ is a rooted tree that is a directed graph in which all tree edges point away from the root r .

In this paper, a vertex $v \in V$ in a directed rooted tree $T = (V, D)$ may have a loop. L is defined as the set of vertexes with a loop.

$$L = \{v \in V \mid v \text{ has a loop in } T = (V, D)\}. \quad (1)$$

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* Correspondence should be addressed to ZHENG Jie, email: jzheng@dhu.edu.cn

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Definition 2 Let $T = (V, D)$ be a directed rooted tree with the root r . For a vertex $v \in V$ and $v \neq r$, if $r \rightarrow v_1 \rightarrow v_2 \cdots \rightarrow v_{k-1} \rightarrow v_k = v$ is the unique directed path in T from r to v , then we call the integer k the distance from the root r to v and denoted by $d(r, v)$.

Definition 3 Partition V into V_r, V_o and V_e as follows.

$$V_r = \{r\},$$

$$V_o = \{v \in V \mid \text{the distance } d(r, v) \text{ is an odd number}\},$$

$$V_e = \{v \in V \mid \text{the distance } d(r, v) \text{ is an even number and } d(r, v) > 0\}.$$

Example 1 Let T be the directed rooted tree given in Fig. 1.

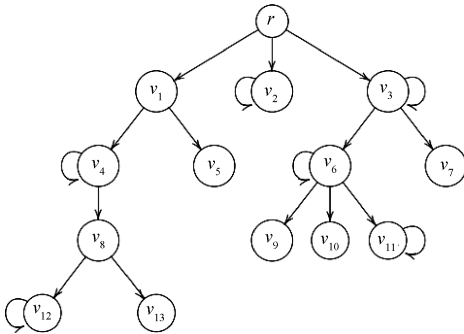


Fig. 1 $T = (V, D)$ of Example 1

Then, we can easily get

$$V = \{r, v_1, v_2, \dots, v_{13}\},$$

$$V_o = \{v_1, v_2, v_3, v_8, v_9, v_{10}, v_{11}\},$$

$$V_e = \{v_4, v_5, v_6, v_7, v_{12}, v_{13}\},$$

$$L = \{v_2, v_3, v_4, v_6, v_{11}, v_{12}\}.$$

1.2 Definitions of generalized parallel dynamical system over directed graph (GPDDS)

Let F_2^n be the set $\{(a_1, a_2, \dots, a_n) \mid a_i = 0 \text{ or } 1, i = 1, 2, \dots, n\}$.

Definition 4 Let $G = (V, D)$ be a directed graph with vertex set $V = \{v_1, v_2, \dots, v_n\}$ whose vertex v_i may have a loop and give a Boolean function $f_i: \{0, 1\}^n \rightarrow \{0, 1\}$. Then a map is expressed as

$$F: F_2^n \rightarrow F_2^n, \tag{2}$$

$$F(a_1, a_2, \dots, a_i, \dots, a_n) = (a_1^1, a_2^1, \dots, a_i^1, \dots, a_n^1), \tag{3}$$

where a_i^1 given by the local functions f_i over the states of the vertexes in $I_D(i) = \{j \in V: (v_j, v_i) \in D\}$, $i = 1, 2, \dots, n$ is the updated state of the vertex v_i . This map constitutes a discrete dynamical system $[F, G]$ called GPDDS when all the vertexes are updated in a synchronous manner.

Definition 5 Let $[F, G]$ be a GPDDS over the graph $G = (V, D)$ associated with F . Let $(a_1, a_2, \dots, a_n) \in F_2^n$ be a binary state satisfying that

$$[F, G](a_1, a_2, \dots, a_n) = (a_1, a_2, \dots, a_n), \tag{4}$$

then (a_1, a_2, \dots, a_n) is called a fixed point of $[F, G]$.

Example 2 Let $[F, G]$ be the GPDDS over the graph $G = (V, D)$ shown in Fig. 2 associated with a map:

$$F: F_2^3 \rightarrow F_2^3, \tag{5}$$

$$F(a_1, a_2, a_3) = (a_1 \vee \overline{a_3}, a_1, a_2). \tag{6}$$

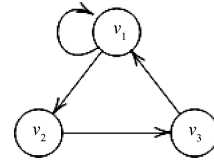


Fig. 2 $G = (V, D)$ of Example 2

The phase portrait of this system is illustrated in Fig. 3, where it can be easily checked that every state converges to the state $(1, 1, 1)$ which is a fixed point of the system.

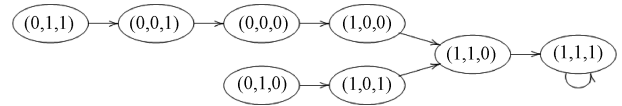


Fig. 3 Phase portrait of Example 2

The above is the definition and example of GPDDS over a directed graph and its fixed point. Now let us define the GPDDS over a directed rooted tree $T = (V, D)$ associated with a Boolean function NAND or NOR.

Definition 6 Let $T = (V, D)$ be a directed rooted tree with $V = \{v_1, v_2, \dots, v_n\}$. Then a map is expressed as

$$[\text{NAND}, T]: F_2^n \rightarrow F_2^n, \tag{7}$$

$$[\text{NAND}, T](a_1, a_2, \dots, a_n) = (a_1^1, a_2^1, \dots, a_n^1), \tag{8}$$

where $a_i^1 = \bigwedge_{j \in I_D(i)} a_j$ and $I_D(i) = \{j \in V: (v_j, v_i) \in D\}$, $i = 1, 2, \dots, n$, constitute a discrete dynamical system called a NAND-GPDDS over T .

Dually, for a GPDDS associated with NOR, we have Definition 7.

Definition 7 Let $T = (V, D)$ be a directed rooted tree with $V = \{v_1, v_2, \dots, v_n\}$. Then a map is expressed as

$$[\text{NOR}, T]: F_2^n \rightarrow F_2^n, \tag{9}$$

$$[\text{NOR}, T](a_1, a_2, \dots, a_n) = (a_1^1, a_2^1, \dots, a_n^1), \tag{10}$$

where $a_i^1 = \bigvee_{j \in I_D(i)} a_j$ and $I_D(i) = \{j \in V: (v_j, v_i) \in D\}$ constitutes a discrete dynamical system called a NOR-GPDDS over T .

Definition 8 Let $[\text{NAND}, T]$ be a generalized parallel dynamical system over a directed rooted tree $T = (V, D)$ associated with NAND. Let $(a_1, a_2, \dots, a_n) \in F_2^n$ be a binary state satisfying that

$$[\text{NAND}, T](a_1, a_2, \dots, a_n) = (a_1, a_2, \dots, a_n). \tag{11}$$

Then (a_1, a_2, \dots, a_n) is called a fixed point of

[NAND, T].

Denote the set of fixed points of [NAND, T] by $Fix[NAND, T]$. That is

$$Fix[NAND, T] = \{(a_1, a_2, \dots, a_n) \mid [NAND, T](a_1, a_2, \dots, a_n) = (a_1, a_2, \dots, a_n)\}. \quad (12)$$

And dually we have Definition 9.

Definition 9 Let [NOR, T] be a generalized parallel dynamical system over a directed rooted tree $T = (V, D)$ associated with NOR. Let $(a_1, a_2, \dots, a_n) \in \mathbf{F}_2^n$ be a binary state satisfying that

$$[NOR, T](a_1, a_2, \dots, a_n) = (a_1, a_2, \dots, a_n). \quad (13)$$

Then (a_1, a_2, \dots, a_n) is called a fixed point of [NOR, T].

Denote the set of fixed points of [NOR, T] by $Fix[NOR, T]$. That is

$$Fix[NOR, T] = \{(a_1, a_2, \dots, a_n) \mid [NOR, T](a_1, a_2, \dots, a_n) = (a_1, a_2, \dots, a_n)\}. \quad (14)$$

2 Fixed Points in NAND-GPDDS and NOR-GPDDS

2.1 Necessary and sufficient conditions for existence of fixed points

Lemma 1 (necessary condition). For a NAND-GPDDS over a directed rooted tree $T = (V, D)$, if $(a_1, a_2, \dots, a_n) \in \mathbf{F}_2^n$ is a fixed point of [NAND, T], then $a_j = 1$ for all $v_j \in L$.

Proof Let [NAND, T] be a GPDDS over a directed rooted tree $T = (V, D)$, and $(a_1, a_2, \dots, a_n) \in \mathbf{F}_2^n$ is a fixed point of [NAND, T]. Suppose that there exists a vertex $v_j \in L$, $a_j = 0$, $(v_i, v_j) \in D$ and $[NAND, T](a_1, a_2, \dots, a_j, \dots, a_n) = (a_1^1, a_2^1, \dots, a_j^1, \dots, a_n^1)$. Then $a_j^1 = \text{NAND}(a_i, a_j) = 1$ whatever $a_i = 0$ or $a_i = 1$. Since $a_j \neq a_j^1$, (a_1, a_2, \dots, a_n) cannot be a fixed point of [NAND, T], it is a contradiction. Therefore, if (a_1, a_2, \dots, a_n) is a fixed point, we have $a_j = 1$ for all $v_j \in L$.

And dually we have Lemma 2.

Lemma 2 (necessary condition). It is for a NOR-GPDDS over a directed rooted tree $T = (V, D)$. If $(a_1, a_2, \dots, a_n) \in \mathbf{F}_2^n$ is a fixed point of [NOR, T], then $a_j = 0$ for all $v_j \in L$.

By the proof, we can further know that Lemma 1 and Lemma 2 are true for NAND-GPDDS and NOR-GPDDS over all directed graphs (not necessary for directed trees).

Theorem 1 Let [NAND, T] be a GPDDS over a directed rooted tree $T = (V, D)$. Then [NAND, T] has a fixed point if and only if $L \subseteq V_o$ or $L \subseteq V_e$.

Proof If $L = \emptyset$, it is easy to get the states $(a_1, a_2, \dots, a_n) \in \mathbf{F}_2^n$ satisfying

$$a_i = \begin{cases} 0, & \text{if } v_i \in V_e \cup V_r \\ 1, & \text{if } v_i \in V_o \end{cases} \text{ or } a_i = \begin{cases} 1, & \text{if } v_i \in V_e \cup V_r, \\ 0, & \text{if } v_i \in V_o, \end{cases} \quad (15)$$

which are two fixed points of [NAND, T].

We consider the case $L \neq \emptyset$.

Firstly, if [NAND, T] has a fixed point $(a_1, a_2, \dots, a_n) \in \mathbf{F}_2^n$, then $r \notin L$. Suppose $V = \{v_1, v_2, \dots, v_n\}$ and v_1 is the root of the tree T . If $v_1 \in L$ and [NAND, T]
 $(a_1, a_2, \dots, a_j, \dots, a_n) = (a_1^1, a_2^1, \dots, a_j^1, \dots, a_n^1)$, then $a_1^1 = \text{NAND}(a_1) = \overline{a_1}$, which implies $a_1 \neq a_1^1$. This is a contradiction to that (a_1, a_2, \dots, a_n) is a fixed point of [NAND, T].

Secondly, if [NAND, T] has a fixed point $(a_1, a_2, \dots, a_n) \in \mathbf{F}_2^n$, then T cannot have the structure as shown in Fig. 4.

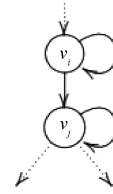


Fig. 4 Impossible structure in T

By Lemma 1, $v_i, v_j \in L$, $a_i = a_j = 1$. Suppose [NAND, T]
 $(a_1, a_2, \dots, a_j, \dots, a_n) = (a_1^1, a_2^1, \dots, a_j^1, \dots, a_n^1)$. Then $a_j^1 = \text{NAND}(a_i, a_j) = \overline{1 \wedge 1} = 0$, which implies $a_j = 1$ and $a_j^1 = 0$. It is a contradiction to that $(a_1, a_2, \dots, a_j, \dots, a_n)$ is a fixed point of [NAND, T].

Thirdly, if $v_i, v_j \in V$, $(v_i, v_j) \in D$ and $(a_1, a_2, \dots, a_n) \in \mathbf{F}_2^n$ is a fixed point of [NAND, T], then T has one of the following cases.

- 1) If $a_j = a_j^1 = 1$, then $a_i = 0$ since $a_j^1 = \text{NAND}(a_i)$.
- 2) If $a_j = a_j^1 = 0$, then $a_i = 1$ since $a_j^1 = \text{NAND}(a_i)$.
- 3) By Lemma 1, $a_j = a_j^1 = 1$, so $a_i = 0$ since $a_j^1 = \text{NAND}(a_i, a_j)$.
- 4) By Lemma 1, $a_i = a_i^1 = 1$. If $a_j = 1$, then $a_j^1 = \text{NAND}(a_i, a_j) = \overline{1 \wedge 1} = 0$. It is a contradiction to that $(a_1, a_2, \dots, a_i, \dots, a_j, \dots, a_n)$ is a fixed point. So $a_j = 0$.

For other cases except for the cases in Fig. 5, it is easy to check they are impossible.

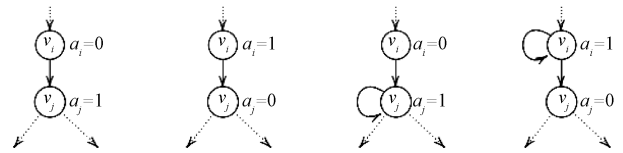


Fig. 5 Four possible cases in T

From the above analysis, we know if [NAND, T] has a fixed point $(a_1, a_2, \dots, a_n) \in \mathbf{F}_2^n$, then every directed path from the root r to a leaf v of T must be a $0 - 1 - 0 - 1 - \dots$ state chain or a $1 - 0 - 1 - 0 - \dots$ state

chain as shown in Fig. 6.

So if $[\text{NAND}, T]$ has a fixed point, then $L \subseteq V_o$ or $L \subseteq V_e$. Conversely, if $L \subseteq V_o$ or $L \subseteq V_e$, we can find a fixed point $(a_1, a_2, \dots, a_n) \in \mathbf{F}_2^n$ shown as Fig. 6(a) or Fig. 6(b).

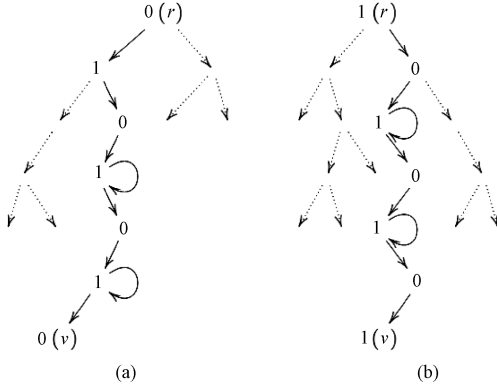


Fig. 6 State chain: (a) $0 - 1 - 0 - 1 - \dots$; (b) $1 - 0 - 1 - 0 - \dots$

Thus we finish the proof for Theorem 1. And dually we have Theorem 2.

Theorem 2 Let $[\text{NOR}, T]$ be a GPDDS over a directed rooted tree $T = (V, D)$. Then $[\text{NOR}, T]$ has a fixed point if and only if $L \subseteq V_o$ or $L \subseteq V_e$.

2.2 Characterization of fixed points

Theorem 3 For a NAND-GPDDS over a directed rooted tree $T = (V, D)$, $(a_1, a_2, \dots, a_n) \in \mathbf{F}_2^n$ is a fixed point if and only if either of the following two conditions is satisfied.

- 1) $L \subseteq V_o$ and $a_i = \begin{cases} 1, & \text{if } v_i \in V_o, \\ 0, & \text{if } v_i \in V_e \cup V_r, \end{cases} i = 1, 2, \dots, n.$
- 2) $L \subseteq V_e$ and $a_i = \begin{cases} 0, & \text{if } v_i \in V_o, \\ 1, & \text{if } v_i \in V_e \cup V_r, \end{cases} i = 1, 2, \dots, n.$

Proof By Theorem 1 and Lemma 1, if $[\text{NAND}, T]$ has a fixed point, then T should satisfy the following conditions.

- 1) $r \notin L$, where r is the root of $T = (V, D)$.
- 2) $L \subseteq V_o$ or $L \subseteq V_e$.

If $(a_1, a_2, \dots, a_n) \in \mathbf{F}_2^n$ is a fixed point of $[\text{NAND}, T]$ where T satisfies the above two conditions, then (a_1, a_2, \dots, a_n) should satisfy the following conditions.

- 3) $a_i = 1$ if $v_i \in L$.
- 4) If v_o is a leaf in T and $r \rightarrow \dots \rightarrow v_i \rightarrow \dots \rightarrow v_j \rightarrow \dots \rightarrow v_o$ is the unique directed path from the root r to v_o , then $a_r - \dots - a_i - \dots - a_j - \dots - a_o$ is a $0 - 1 - 0 - 1 - \dots$ or $1 - 0 - 1 - 0 - \dots$ sequence.

From the above four conditions, we can know that Theorem 3 is true. In fact, if $L = \emptyset$, then we can easily get the state of fixed points as shown in the proof of Theorem 1. If $L \subseteq V_o$ and $L \neq \emptyset$, then $a_i = 1$ for all $v_i \in L$ whose distance $d(r, v_i)$ is always an odd number.

Because of the $0 - 1 - 0 - 1 - \dots$ state chain in condition 4), we can infer that $a_i = 0$ for all vertex $v_i \in V_e$ and the root r , and $a_i = 1$ for all vertex $v_i \in V_o$. Namely, $a_i = \begin{cases} 1, & \text{if } v_i \in V_o, \\ 0, & \text{if } v_i \in V_e \cup V_r, \end{cases} i = 1, 2, \dots, n.$ Similarly, if $L \subseteq V_e$

and $L \neq \emptyset$, we can infer that $a_i = \begin{cases} 0, & \text{if } v_i \in V_o, \\ 1, & \text{if } v_i \in V_e \cup V_r, \end{cases} i = 1, 2, \dots, n.$

Thus we finish the proof for Theorem 3.

And dually we have Theorem 4.

Theorem 4 For a NOR-GPDDS over a directed rooted tree $T = (V, D)$, $(a_1, a_2, \dots, a_n) \in \mathbf{F}_2^n$ is a fixed point if and only if either of the following two conditions is satisfied.

- 1) $L \subseteq V_o$ and $a_i = \begin{cases} 0, & \text{if } v_i \in V_o, \\ 1, & \text{if } v_i \in V_e \cup V_r, \end{cases} i = 1, 2, \dots, n.$
- 2) $L \subseteq V_e$ and $a_i = \begin{cases} 1, & \text{if } v_i \in V_o, \\ 0, & \text{if } v_i \in V_e \cup V_r, \end{cases} i = 1, 2, \dots, n.$

2.3 Number of fixed points

By Theorem 1 and Theorem 3, we can get the number of fixed points in a NAND-GPDDS.

Theorem 5 For a NAND-GPDDS over a directed rooted tree $T = (V, D)$, the number of the fixed points of $[\text{NAND}, T]$ is equal to

$$|Fix[\text{NAND}, T]| = \begin{cases} 2, & \text{if } L = \emptyset, \\ 1, & \text{if } L \neq \emptyset, L \subseteq V_o \text{ or } L \subseteq V_e, \\ 0, & \text{if } L \cap V_o \neq \emptyset \text{ and } L \cap V_e \neq \emptyset, \\ 0, & \text{if } L \cap V_r \neq \emptyset, \end{cases} \quad (16)$$

where $Fix[\text{NAND}, T] = \{(a_1, a_2, \dots, a_n) \in \mathbf{F}_2^n \mid [\text{NAND}, T](a_1, a_2, \dots, a_n) = (a_1, a_2, \dots, a_n)\}$ is the set of fixed points of $[\text{NAND}, T]$.

Proof

Step 1 If $L = \emptyset$, then by Theorem 1, $[\text{NAND}, T]$ has fixed points. By condition 4) in the proof of Theorem 3, we know (a_1, a_2, \dots, a_n) satisfying

$$a_i = \begin{cases} 0, & \text{if } v_i \in V_e \cup V_r, \\ 1, & \text{if } v_i \in V_o, \end{cases} \text{ or } a_i = \begin{cases} 1, & \text{if } v_i \in V_e \cup V_r, \\ 0, & \text{if } v_i \in V_o, \end{cases} i = 1, 2, \dots, n, \quad (17)$$

which are two fixed points of $[\text{NAND}, T]$. So $|Fix[\text{NAND}, T]| = 2$.

Step 2 If $L \neq \emptyset$ and $L \subseteq V_o$, then by Theorem 1, $[\text{NAND}, T]$ has a fixed point. By the conditions 1) - 4) in the proof of Theorem 3, we know (a_1, a_2, \dots, a_n) satisfying

$$a_i = \begin{cases} 0, & \text{if } v_i \in V_e \cup V_r, \\ 1, & \text{if } v_i \in V_o, \end{cases} \quad (18)$$

which is the unique fixed point of $[\text{NAND}, T]$. So $|Fix[\text{NAND}, T]| = 1$.

Step 3 If $L \neq \emptyset$ and $L \subseteq V_e$, we have proof similar to Step 2.

Step 4 If $L \cap V_o \neq \emptyset$ and $L \cap V_e \neq \emptyset$, then by Theorem 1, $[\text{NAND}, T]$ has no fixed point. So $| \text{Fix}[\text{NAND}, T] | = 0$.

Step 5 If $L \cap V_r \neq \emptyset$, then by Theorem 1, $[\text{NAND}, T]$ has no fixed point. So $| \text{Fix}[\text{NAND}, T] | = 0$.

Thus we finish the proof for Theorem 5.

And dually we have Theorem 6.

Theorem 6 For a NOR-GPDDS over a directed rooted tree $T = (V, D)$, the number of the fixed points of $[\text{NOR}, T]$ is equal to

$$| \text{Fix}[\text{NOR}, T] | = \begin{cases} 2, & \text{if } L = \emptyset, \\ 1, & \text{if } L \neq \emptyset, L \subseteq V_o \text{ or } L \subseteq V_e, \\ 0, & \text{if } L \cap V_o \neq \emptyset \text{ and } L \cap V_e \neq \emptyset, \\ 0, & \text{if } L \cap V_r \neq \emptyset, \end{cases} \quad (19)$$

where $\text{Fix}[\text{NOR}, T] = \{ (a_1, a_2, \dots, a_n) \in \mathbf{F}_2^n \mid [\text{NOR}, T](a_1, a_2, \dots, a_n) = (a_1, a_2, \dots, a_n) \}$ is the set of fixed points of $[\text{NOR}, T]$.

3 Example to Explain Theorem 3 and Theorem 5

Example 3 Let T_1, T_2, T_3 and T_4 be the directed rooted trees in Fig. 7. By Theorem 3 and Theorem 5,

$[\text{NAND}, T_1]$ or $[\text{NAND}, T_2]$ has no fixed point;

$[\text{NAND}, T_3]$ has one fixed point $(0, 1, 1, 0, 0, 0, 1, 1, 1) \in \mathbf{F}_2^9$;

$[\text{NAND}, T_4]$ has two fixed points $(0, 1, 1, 0, 0, 0, 1, 1, 1) \in \mathbf{F}_2^9$ and $(1, 0, 0, 1, 1, 1, 0, 0, 0) \in \mathbf{F}_2^9$.

And by Theorem 4 and Theorem 6,

$[\text{NOR}, T_1]$ or $[\text{NOR}, T_2]$ has no fixed point;

$[\text{NOR}, T_3]$ has one fixed point $(1, 0, 0, 1, 1, 1, 0, 0, 0) \in \mathbf{F}_2^9$;

$[\text{NOR}, T_4]$ has two fixed points $(1, 0, 0, 1, 1, 1, 0, 0, 0) \in \mathbf{F}_2^9$ and $(0, 1, 1, 0, 0, 0, 1, 1, 1) \in \mathbf{F}_2^9$.

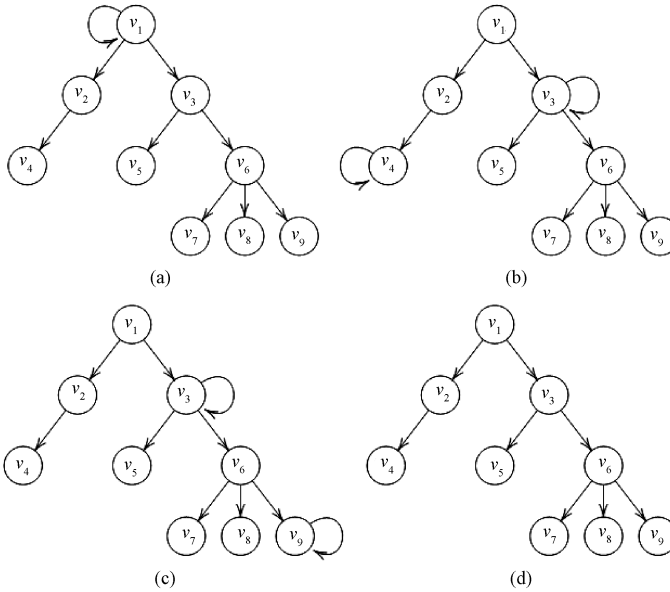


Fig. 7 Directed rooted trees in Example 3: (a) T_1 ; (b) T_2 ; (c) T_3 ; (d) T_4

4 Conclusions and Future Research Directions

In this work, we deal with generalized parallel dynamical systems over directed rooted trees, whose local evolution operator is the Boolean NAND or NOR function. We solve the existence problem of fixed points, get the well-defined state of fixed points, and count the exact number of fixed points in such GPDDS. It is also proved that fixed points in such dynamical systems are completely determined by the loops in the rooted trees.

The future work will focus on studying the periodic orbits problem in such NAND-GPDDS and NOR-GPDDS. Meanwhile, this work lays a foundation for studying the fixed-point problems of GPDDSs (not necessary for directed rooted trees).

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有向根树上局部更新函数为 NAND 或 NOR 的广义并序动力系统的固定点

孙彦文, 郑洁*

东华大学 理学院, 上海 201620

摘要: 研究对象为有向根树上的广义并序动力系统, 其局部更新函数为 NAND 或 NOR。为了研究这类广义并序动力系统固定点的个数及状态, 采用结构分析法和分类讨论法, 得到了此类动力系统中关于固定点的确切结果。证明了这类广义并序动力系统固定点状态完全由有向根树中的自环决定。离散动力系统是数学建模的重要工具, 该文可为离散动力系统的研究提供新的思路。

关键词: 广义并序动力系统; 有向根树; 布尔函数; 固定点

