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Error Analysis of Ellipse Fitting for Incomplete Contour

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Abstract: In computer image processing, owing to the influence of lighting conditions and camera installation locations, incomplete ellipse contour extraction often occurs after the edge extraction of the image. By fitting this residual contour, the result deviates from the original elliptical shape and the fitting error is large, which affects the fitting accuracy. The degree of influence of the characteristics of the incomplete contour on the error was studied, and a numerical simulation method in MATLAB was used to perform incomplete elliptical arc segments at different positions, edge extraction on the arc segment, and ellipse fitting on the arc segment based on the least squares method. The influence of multiple factors, such as the phase angle, the arc length integrity, and the axis ratio of the ellipse on the ellipse fitting error was analyzed, which was significant in understanding the causes of error generation and improving the fitting accuracy. The curves of the fitting error with the three factors yield that all three factors have a significant effect on the fitting error. The effect of contour fitting at different phase angles varies greatly, and the greater the arc length integrity and the smaller the axis ratio of the ellipse, the smaller the fitting error.

Key words: simulation method; least squares method; ellipse fitting; fitting error
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0 Introduction

In the field of computer vision, ellipse fitting serves as a prerequisite for subsequent measurement and object assessment[1]. The commonly employed method for ellipse fitting is the least squares method[2]. Due to lighting conditions and camera shooting angles, circular objects become elliptical, and the extracted ellipses are incomplete with a considerable amount of noise[3]. When fitting incomplete elliptical arcs, the ellipse center and major and minor axes deviate significantly from the original ellipse, resulting in substantial fitting errors[4]. Existing research mainly focuses on improving the fitting algorithms to enhance efficiency or precision. Liang et al.[5] introduced the maximum correntropy criterion into the constrained least squares ellipse fitting method. Kanatani[6] presented a more accurate ellipse fitting method than maximum likelihood (ML) by error analysis of ML followed by subtraction of higher order deviation terms. Lu et al.[7] addressed the detection of incomplete ellipses in strong noise, proposing an iterative random Hough transform that effectively avoided noise interference and was applicable to fetal head detection. Păźtrăźucean et al.[8] proposed a detector combining line segments and elliptical arcs, integrating algebraic distance and gradient direction, thus enhancing the fitting precision of incomplete ellipse contours with noise points. Prasad et al.[9] proposed a non-iterative, least squares-based elliptical fitting method that only allowed data points roughly belonging to the same ellipse to participate in fitting. Compared to other methods, this approach yielded better results for fitting incomplete elliptical arcs and point sets with added noise points. Meanwhile, some studies analyzed the reasons for poor circle or ellipse fitting. Chernov et al.[10] analyzed the instability of traditional least squares fitting for short arc circles and built a new algorithm to improve the fitting accuracy of short arc circles. Kanatani et al.[11] employed HyperLS to eliminate the influence of second-order noise on the incomplete circle and ellipse contour fitting, significantly improving the fitting results. When elliptical arc lengths are less than one-fourth of a complete ellipse, the fitting performance of the HyperLS method is uncertain, depending on the shape and length of the arc. Waibel et al.[12] proposed an algorithm to constrain the center of the fitted ellipse to a straight line, based on this algorithm and two classical ellipse fitting algorithms fitting three arc segments of different lengths, the new algorithm fitted the ellipse with greater overlap and higher accuracy. Meanwhile, all three algorithms fit shorter-length arc segments with higher fitting errors than longer arc segments. Halir et al.[13] proposed a numerically stable non-iterative ellipse fitting algorithm, which fits ellipses in strong noise and arc segments at different locations, with good robustness and fast computation speed compared to other algorithms that may fit hyperbolic curves. Požar et al.[14] proposed an improved

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ellipse fitting method that improves the accuracy of the measurement of displacements of an orthogonal laser interferometer. Other fitting methods are compared to fit arc segments of different lengths, and the method requires shorter arc lengths to achieve the same accuracy. Zakrzewski[15] proposed an Levenberg-Marquardt (LM) algorithm to study the effects of arc length, initial phase angle and noise level on the accuracy of imbalance estimation based on ellipse fitting for imbalance calibration of displacement measurements, and derived the value of the initial phase angle for a higher accuracy of the estimation, and the longer the arc length, the more accurate the estimation. Prasad et al.[16] proposed a least squares method without constrained optimization and with a low false alarm rate, comparing several methods for ellipse fitting of arc segments with the same initial angle, different termination angles and added noise, the proposed method has a higher success rate of ellipse fitting and a better accuracy. These methods significantly improve the performance of ellipse fitting, but do not delve into the sources of least squares ellipse fitting errors, such as the length and curvature of elliptical arcs, and the extent to which these sources affect fitting errors when assuming different values.

When extracting the edge of an ellipse contour based on image processing in the industrial field, it is often not a complete edge, but some fragmentary elliptical edges. Fitting these edges, the accuracy of the ellipses obtained is much less than the accuracy of the ellipses fitted with complete edges, and the difference of ellipses fitted with contours of approximately equal length at different positions is also very large. In this study, the MATLAB simulation method was used to extract the edge of the original elliptical arc to study the influence of three factors on the least squares ellipse fitting error: the phase angle, the arc length integrity, and the axis ratio of the ellipse. This allows us to derive the regular curve of the error variation with the factors, which has a certain guiding significance for improving the accuracy of the ellipse fitting.

1 Simulation Methods

The standard equation of an ideal ellipse was established using MATLAB, from which different arc segments were cut and images were generated. The Canny operator was used for edge detection, and ellipse fitting was conducted on the detected contours using the least squares method[17].

1.1 Major factors affecting fitting error

The closer the phase angle, the major axis and the minor axis in fitting are to those of the original ellipse, the smaller the fitting error is and the higher the accuracy is. The relative errors of the major axis, the minor axis and the deviation distance of the center of the ellipse are the dependent variables in the simulation, reflecting the quality of the fitting results. The relative error of the major axis is denoted as σ_a, and that of the minor axis is denoted as σ_b.

$$\sigma_a = \frac{|a_1 - a|}{a} \times 100\%, \tag{1}$$

$$\sigma_b = \frac{|b_1 - b|}{b} \times 100\%, \tag{2}$$

where a_1 is the length of the major axis of the ellipse fitted from the incomplete contour; a is the length of the major axis of the ideal ellipse; b_1 is the length of the minor axis of the ellipse fitted from the incomplete contour; b is the length of the minor axis of the ideal ellipse.

The deviation distance of the center of the ellipse is the distance between the center of the ellipse fitted from the incomplete contour and the center of the ideal ellipse, denoted as d, in pixels.

The factors affecting the fitting error are the phase angle, the arc length integrity, and the axis ratio of the ellipse.

The phase angle reflects the position of the center point of the arc. It is the angle between the line connecting the midpoint of the arc to the origin and the positive direction of the x-axis, in degrees. As shown in Fig. 1, AC is an elliptic arc, and the phase angle is $\angle BOX$.

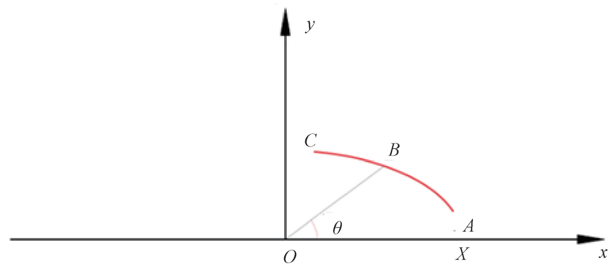


Fig. 1 Phase angle schematic diagram

In addition to these three aforementioned factors, edge extraction can cause fitting errors. By fitting the least squares method to the edges of ideal ellipses with different axis ratios, the deviation in the fitted ellipse's axis ratio was determined to be less than 0.2%. Therefore, the impact of the edge extraction algorithm on fitting errors was not considered.

The midpoint of the contour starts at 0° and moves counterclockwise 1° each time, moving around a circle, creating 360 segments of equal arc length. By using the Canny operator for edge detection and fitting the ellipse, parameters such as the center coordinates and lengths of the major and minor axes of the fitted ellipse can be obtained. According to the obtained ellipse parameter data, this paper produces a line graph and analyzes the change rule to summarize the influence of the phase angle, the arc length integrity, and the axis ratio of the ellipse on the fitting error.

Figure 2 shows some ellipse-fitting effects under different arc lengths, phase angles, and axis ratios. It can be clearly seen that the three factors have a significant impact on the fitting results. In this study, we refer to the proportion of the arc length of the ellipse to the complete ellipse as the arc length integrity of the ellipse.

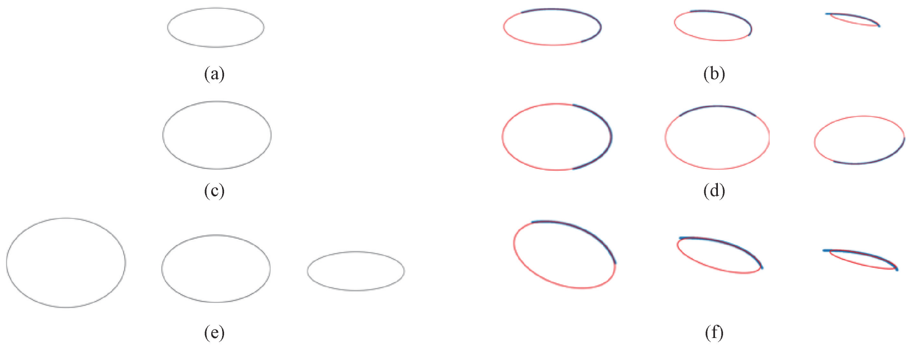


Fig. 2 Ellipse fitting diagram: (a) complete ellipse with an axis ratio of 3∶1; (b) results of ellipse contour fitting with different arc lengths at the same phase angle; (c) complete ellipse with an axis ratio of 2∶1; (d) results of ellipse contour fitting with different phase angles of the same length; (e) complete ellipses with axis ratios of 4∶3, 2∶1 and 3∶1; (f) results of elliptic contour fitting with the same arc length and phase angle and different axis ratios

1.2 Algorithm to maintain equal arc length

The curvature of each point of the ellipse is different, so it is impossible to create equal arc lengths for every 1-degree movement. An algorithm was used to approximate equal elliptical arc lengths.

As shown in Fig. 3, let $\angle DOX = \theta_1$ and $\angle EOX = \theta_2$; then, the length of DE is represented by l, and the calculation formula for the elliptical arc length is

$$l = \int_{\theta_1}^{\theta_2} \sqrt{(-a\sin\theta)^2 + (b\cos\theta)^2}\,\mathrm{d}\theta. \quad (3)$$

The MATLAB numerical calculation gives the circumference L of the ellipse as

$$L = \int_{0}^{2\pi} \sqrt{(-a\sin\theta)^2 + (b\cos\theta)^2}\,\mathrm{d}\theta. \quad (4)$$

The circumference of the ellipse and the elliptical arc length have a relationship

$$l = KL, \quad (5)$$

where K is the proportion of the elliptical arc segment to the complete ellipse.

The length of DE can be obtained by determining the major axis a, minor axis b and starting angle of the arc segment θ_1. However, because Eq. (5) has no elementary function solution, the ending angle of the arc segment cannot be precisely determined. A preset θ_2 value is used in this simulation.

$$\theta_2 = \theta_1 + 360 \times \frac{K}{2}. \quad (6)$$

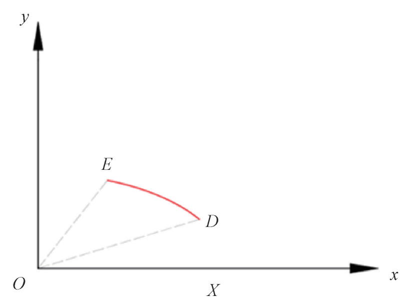
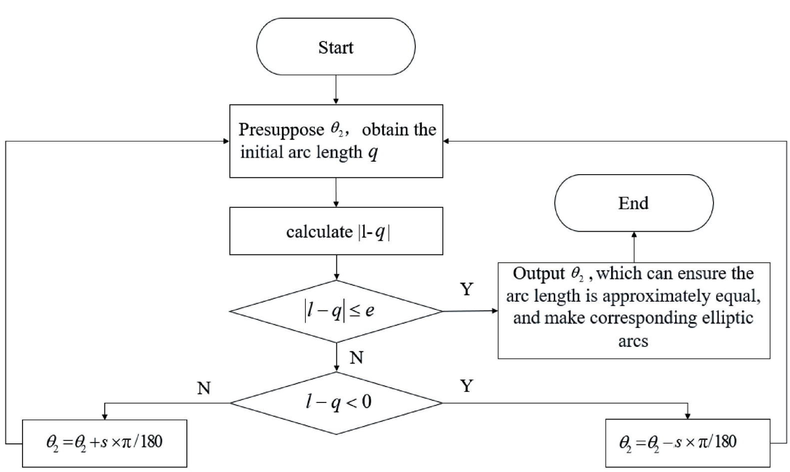


Fig. 3 Schematic diagram of arc equality principle

The initial arc length q is calculated using Eq. (5), and then the difference between l and q is calculated. A threshold e is preset. In this simulation, e is set to 10^{-5}. The subsequent process is shown in Fig. 4, and the arc length is finally calculated.



s ——step.

Fig. 4 Algorithm flow chart

Taking the example of a 1/4 elliptical arc with a major axis length of 3 and a minor axis length of 1, 360 sets of elliptical arc data were obtained. As shown in Table 1, elliptical arc data near phase angles of 0°, 45°, and 90° were selected and organized. These three positions provide a general overview of elliptical arcs.

Table 1 Elliptical arc data

Phase angle/(°)	Starting angle/(°)	Ending angle/(°)	Arc length/ pixel
0	−56. 425 4	56. 425 4	3. 341 235
1	−56. 033 6	56. 815 2	3. 341 215
2	−55. 639 8	57. 204 7	3. 341 225
3	−55. 242 2	57. 594 9	3. 341 232
43	−26. 062 5	79. 320 7	3. 341 224
44	−24. 669 5	80. 060 9	3. 341 231
45	−23. 210 9	80. 810 2	3. 341 222
46	−21. 681 3	81. 568 8	3. 341 229
87	52. 846 1	120. 117 6	3. 341 238
88	54. 053 9	121. 257 0	3. 341 207
89	55. 246 5	122. 409 4	3. 341 219
90	56. 425 4	123. 574 6	3. 341 211

It can be observed from Table 1 that the elliptical arc lengths at different positions are approximately equal, and the accuracy is related to the value of e.

If the length of the arc is not kept unchanged when the phase angle changes, but the angle between the end angle and the start angle of the arc is kept unchanged, and the influence of curvature change on the length of the arc is not considered, this method is denoted as the original method. The arc segment with length to short axis ratio 4 : 3 and arc length integrity 3/4 was fitted, and the deviation distance of the circle center obtained by using the algorithm of arc length equal was compared, as shown in Fig. 5.

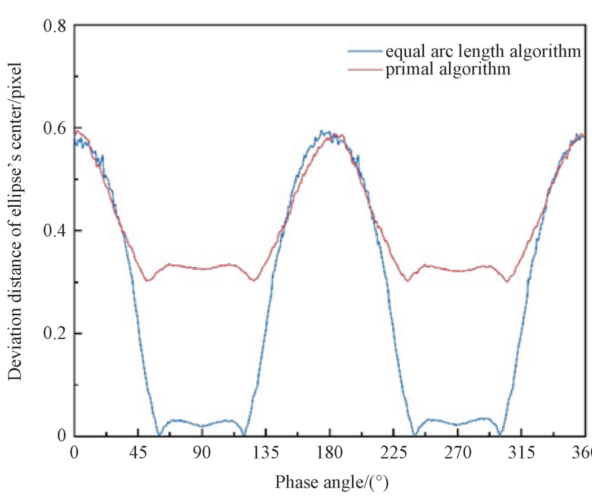


Fig. 5 Comparison of deviation error of ellipse center fitted by algorithm

It can be seen that when the phase angle is between 70° and 110°, the deviation error of the ellipse center fitted by the original algorithm is more than six times that of the algorithm with equal arc length. When the arc length is reduced and the ratio of the long axis to the short axis is increased, the fitting error increases, and the additional error caused by the unequal arc length will be larger, which seriously affects the accuracy of the simulation conclusion. Therefore, the use of an equal arc length algorithm ensures the credibility of the simulation data.

1. 3 Simulation steps

The simulation is all realized by MATLAB, and the specific steps are as follows.

Step 1 Establish the standard equation of the ideal ellipse and make complete ellipses with different ratios of major axis to minor axis, which are 4 : 3, 3 : 2, 2 : 1, 5 : 2 and 3 : 1 respectively.

Step 2 Arcs of different lengths are set, and the integrity of arc length is 3/4, 2/3, 1/2, 1/3 and 1/4, respectively. Taking the integrity of arc length as 3/4 and the ratio of long to short axis as 4 : 3 as an example, a total of 360 arcs with phase angles from 0° to 359° are made based on the algorithm of arc length phase. The ratio of the long axis to the short axis and the arc length integrity are combined in pairs, so 25 groups are made.

Step 3 The Canny operator is used to detect the edge of all the arcs, and the ellipse fitting is performed on the detected contour based on the least square method, and the parameters such as the center coordinates of the fitted ellipse and the length of the major and minor axes are output.

Step 4 Compare the fitted ellipse and the complete ellipse in Step 1, calculate the relative errors of the major and minor axes and the deviation distance of the center of the circle, and make a line chart. The influence of the phase angle, the arc length integrity and the ratio of ellipse length to short axis on fitting error is summarized.

The flow chart is shown in Fig. 6.

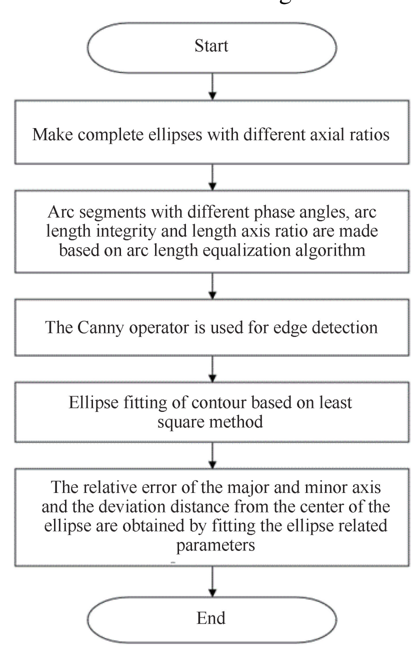


Fig. 6 The general flow chart of the algorithm

2 Simulation Results and Analysis

2.1 Impact of arc length integrity on fitting error

To investigate the impact of arc length integrity on the fitting error of ellipses, ellipses with different degrees of integrity for an axis ratio of 3 : 1 and a phase angle of 60° were fitted, and the results are shown in Fig. 7, with arc length integrity values of 3/4, 2/3, 1/2, 1/3, 7/24, and 1/4.

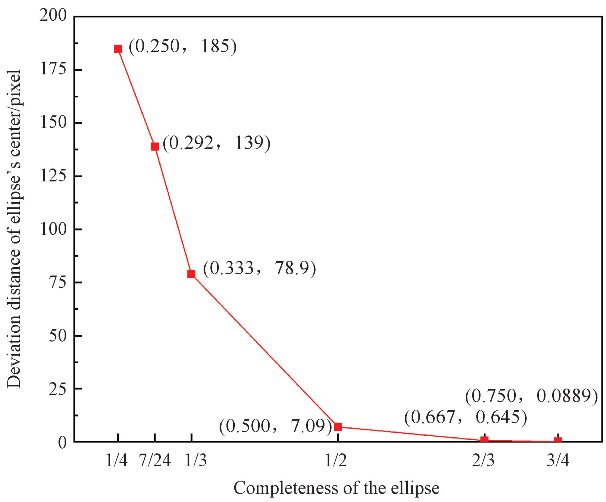


Fig. 7 Deviation distance of ellipse center at six different arc length integrity

The abscissa of the blue coordinate point in the figure represents different arc length integrity, expressed in decimal numbers, corresponding to 1/4, 7/24, 1/3, 1/2, 2/3 and 3/4, respectively. Among them, 7/24 is a supplementary arc length, where the broken line is steep, and the supplementary data point is needed to obtain a more accurate change trend. The ordinate is the distance in pixels from the center of the circle.

As the arc length integrity decreased, the fitting error of the ellipse center increased, indicating a worse fitting result. When the arc length integrity is less than 1/2, the error variation is more evident, and the maximum error can reach about 2 000 times the minimum error.

2.2 Impact of phase angle and arc length integrity on fitting error

The effect of the phase angle on the fitting error of the ellipse is more complex. Ellipses with different degrees of integrity for the axis ratio of 3 : 1 are fitted, as shown in Figs. 8 – 10. Different colors are used to differentiate the arc length integrity: 3/4, 2/3, 1/2, 1/3 and 1/4.

Figure 8 shows that the curves corresponding to contours with lower arc length integrity are always above those with higher arc length integrity, indicating larger fitting errors, which is consistent with the findings in Section 2.1.

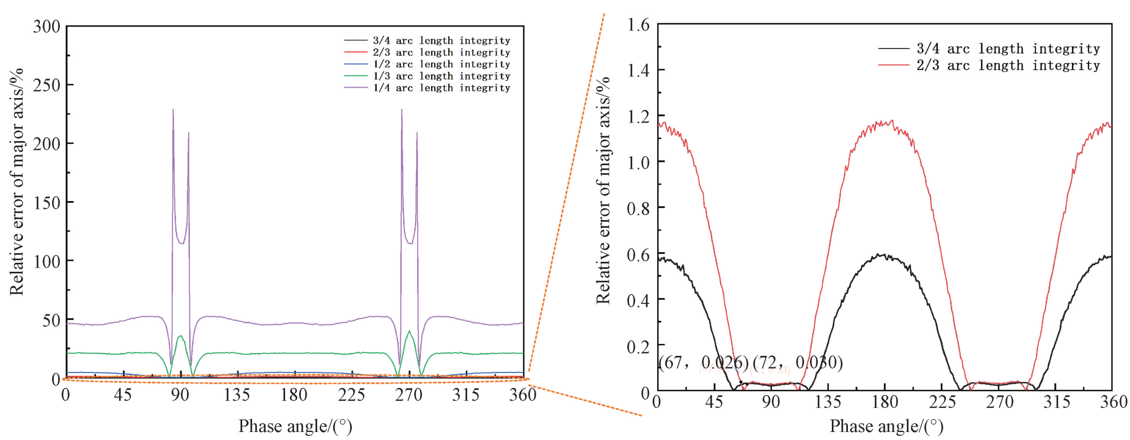


Fig. 8 Distribution of relative error of major axis of ellipses fitted by contours of different arc length integrities at different phase angles

From a single curve, the influence of the phase angle on the relative error of the major axis varies periodically with a period of 180°. It is symmetric about 90° and 270°. For each 180° difference in phase angle, the corresponding two arcs are symmetric about the center of the coordinate origin. After rotation and translation in spatial coordinates, the arcs can coincide, and the fitting error is equal, so they change periodically by 180°. The following analysis is only for fitting errors in a range of 0° to 90°.

For contours with different arc length integrity, the extreme values of the relative error of the major axis and their corresponding phase angles are listed in Table 2.

Table 2 Distribution of maximum and minimum relative errors in major axis fitting for contours with different arc length integrity

Arc length integrity	Maximum value/%	Phase angle corresponding to maximum value/(°)	Minimum value/%	Phase angle corresponding to minimum value/(°)	Ratio of maximum to minimum values
3/4	0.586	0	0.003	60	195.333
2/3	1.168	6	0.004	68	292.000
1/2	4.543	11	0.004	80	1135.750
1/3	35.783	90	0.751	81	47.647
1/4	228.879	84	11.576	82	19.772

The maximum ratio between the maximum and minimum values of the relative error of the major axis is 1 135.750, whereas the minimum ratio is 19.772. It can be observed that the phase angle has a significant impact on the relative error of the major axis. The maximum value of the relative error of the major axis occurs when the phase angle is approximately 0°, and the arc length integrity is greater than 1/2 or when the phase angle is approximately 90° and the arc length integrity is less than 1/2. As the arc length integrity decreases and the relative error of the major axis reaches its minimum, the corresponding phase angle gradually increases.

When the arc length integrity is less than 1/4 and the phase angle is between 80° and 100° or between 260° and 280°, some phase angles are fitted with large fitting errors. In these cases, the fitting does not produce an ellipse, but a hyperbola or a straight line[18-20]. These points do not follow the periodic and symmetric changes observed in the curves. Figure 8 shows that when the phase angle is in a range of 67° to 72°, there are cases in which contours with lower arc length integrity have smaller fitting errors.

Figure 9 shows that the phase angle also has a periodic and symmetric effect on the relative error of the minor axis. The maximum ratio between the maximum and minimum values of the relative error of the minor axis is about 223, and the minimum ratio is about 35. As the arc length integrity decreases and the relative error of the minor axis reaches its minimum, the corresponding phase angle gradually increases. Unlike the relative error of the major axis, the maximum value of the relative error of the minor axis occurs at a phase angle of approximately 90°, regardless of the arc length integrity.

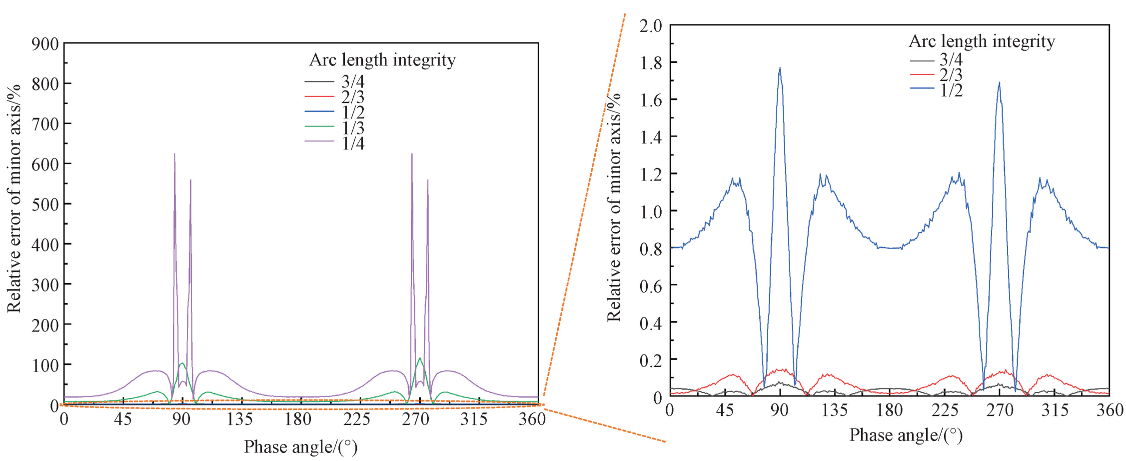


Fig. 9 Distribution of relative error of minor axis of ellipses fitted by contours of different arc length integrities at different phase angles

As shown in Fig. 10, the variation pattern of the deviation distance of the center with respect to the phase angle is generally similar to that of the relative error of the major axis. The maximum ratio between the maximum and minimum values of the deviation distance of the center is 103, whereas the minimum ratio is 7. When the phase angle is approximately 0° and the arc length integrity is greater than 1/2 or when the phase angle is approximately 90° and the arc length integrity is less than 1/2, the deviation distance of the center reaches its maximum value. As the arc length integrity decreases and the deviation distance of the center reaches its minimum, the corresponding phase angle gradually increases.

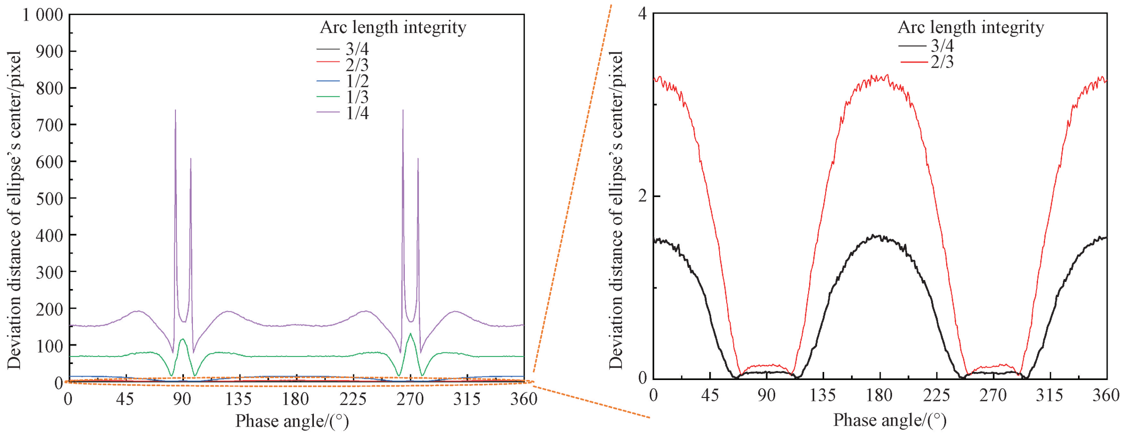


Fig. 10 Distribution of the distance between the center of the ellipse fitted by contours of different arc length integrities at different phase angles and the original ellipse

2.3 Impact of phase angle and ratio of major and minor axis on fitting error

Fitting is performed on ellipse contours with an arc length integrity of 1/3 and different axis ratios. The results are shown in Figs. 11–13, with axis ratios of 4 : 3, 3 : 2, 2 : 1, 5 : 2 and 3 : 1.

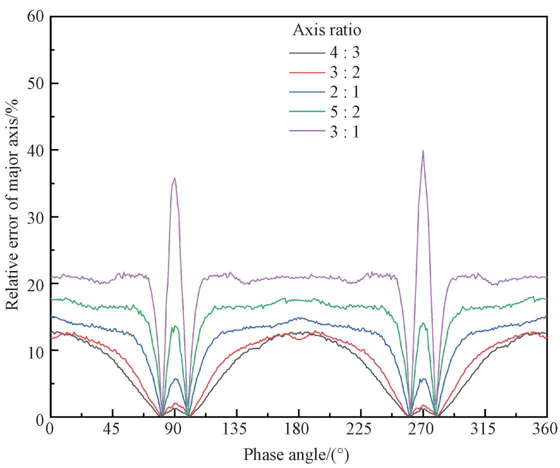


Fig. 11 Distribution of relative error of the major axis of ellipses fitted by contours of different axis ratios at different phase angles

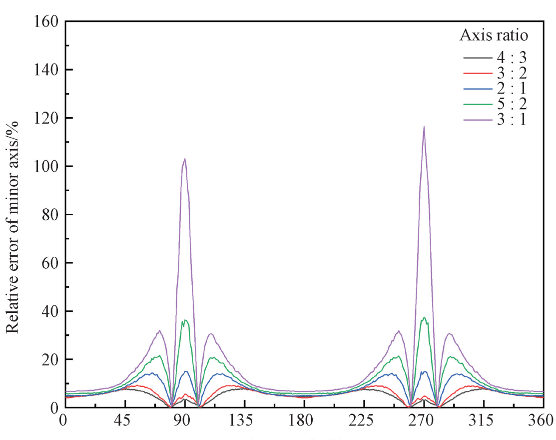


Fig. 12 Distribution of relative error of the minor axis of ellipses fitted by contours of different axis ratios at different phase angles

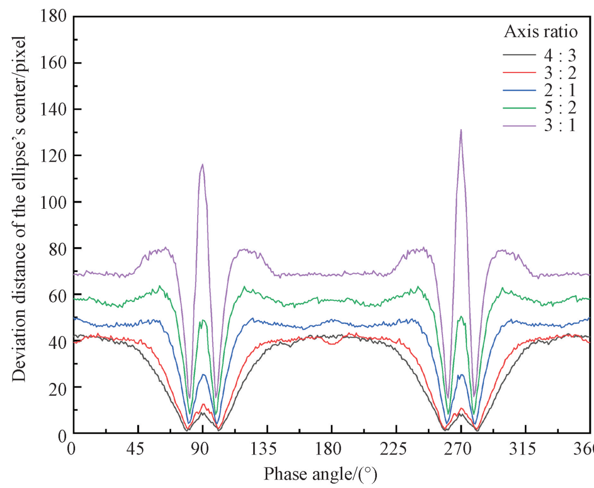


Fig. 13 Distribution of the distance between the center of the ellipse fitted by contours of different axis ratios at different phase angles and the original ellipse

It can be observed that the curves corresponding to contours with larger axis ratios are always above the curves corresponding to contours with smaller axis ratios. This implies that when the phase angle and arc length integrity are constant, a larger axis ratio results in a larger relative error in the major axis.

The extreme values of the relative error of the major axis and their corresponding phase angles for contours with different axis ratios are listed in Table 3.

Table 3 shows that the maximum ratio between the maximum and minimum values of the relative error of the major axis is 125.284, while the minimum ratio is 44.134. When the phase angle is approximately 0° and the axis ratio is less than 2.5 or when the phase angle is approximately 90° and the axis ratio is greater than 2.5, the relative error of the major axis reaches its maximum value. For contours with different axis ratios, the relative error of the major axis is minimized around 81°, and the minimum value is extremely small.

Table 3 Distribution of maximum and minimum relative errors in major axis fittingfor contours with different axis ratios

Axis ratios	Maximum value/%	Phase angle corresponding to maximum value/(°)	Minimum value/%	Phase angle corresponding to minimum value/(°)	Ratio of maximum to minimum values
4/3	12. 779	1	0. 102	81	125. 284
3/2	12. 715	17	0. 114	80	111. 535
2/1	15. 138	0	0. 343	81	44. 134
5/2	17. 871	11	0. 279	81	64. 054
3/1	35. 783	90	0. 751	81	47. 647

It can be observed that when the phase angle and arc length integrity are constant, a larger axis ratio leads to a larger relative error in the minor axis. The maximum ratio between the maximum and minimum values of the relative error of the minor axis is 223, and the minimum ratio is 18. For contours with different axis ratios, the relative error of the minor axis is minimized to approximately 81°. Unlike the relative error of the major axis, the maximum value of the relative error of the minor axis occurs when the phase angle is approximately 50° and the axis ratio is less than 2 or when the phase angle is approximately 90° and the axis ratio is greater than 2.

It can be observed that when the phase angle and arc length integrity are constant, a larger axis ratio results in a larger deviation distance from the center. The maximum ratio between the maximum and minimum values of the deviation distance of the center is 45, whereas the minimum ratio is 7. For contours with different axis ratios, the deviation distance of the center is minimized to approximately 80°. When the phase angle is approximately 0° and the axis ratio is less than 2, the maximum value of the deviation distance of the center is obtained.

2. 4 Analysis of variance

In order to investigate whether the phase angle, the arc length integrity and the axis ratio have significant influences on the fitting error, the analysis of variance (ANOVA) function of MATLAB is used to analyze the variance of part of the data, where the phase angle is named *A* and divided into four levels, which are 0–44, 45–89, 90–134 and 135–179, respectively. The full arc length integrity is called *B* and is divided into two levels, 0. 25 and 0. 75 respectively. The axis ratio is named *C* and divided into two levels: 1. 33 and 3. 00. The ANOVA is shown in Table 4.

Table 4 ANOVA table

	Mean square	Mean square error	df	F	p
A	8 339. 362	1 214. 624	3	6. 866	< 0.001
B	3 871 260. 888	1 214. 624	1	3 187. 210	< 0.001
C	65 067. 435	1 214. 624	1	53. 570	< 0.001
Error	1 214. 624		704		
Total			720		

As can be seen from Table 4, the significance p of A, B and C are all less than 0. 01, so the main effects of the three independent variables are significant. The phase angle, the arc length integrity, and the axis ratio all have significant effects on the fitting error.

When solving practical problems based on image processing, the above conclusions can be used to improve the fitting accuracy. When the fragmented contour is extracted from the edge, the point set of the contour can be stored by the container, and the least squares ellipse fitting can be performed on the point set of multiple contours, which is equivalent to improving the arc length integrity to improve the fitting accuracy. Through the simulation curve, it can be seen that when the arc length integrity reaches 1/2, the fitting accuracy is significantly greater when the phase angle is from 70° to 110°. By moving the camera, the lens is as close to the measured object as possible to reduce the axis ratio. At the same time, the edge extraction threshold is set reasonably, so that the center point of the detected contour is close to the end point of the short axis, and the fitting accuracy is improved.

3 Conclusions

To address the issue of large errors in least squares ellipse fitting, this study takes a different approach from previous research by analyzing the impact of phase angle, arc length integrity and axis ratio on fitting errors. The deviations of the center and the major and minor axes are used to measure the errors, and an arc length equalization algorithm is employed to ensure the credibility of the simulation data. The following key findings are obtained.

1) The smaller the arc length integrity of the ellipse,

the larger the fitting error. This finding is consistent with the conclusions of previous studies.

2) The larger the axis ratio of the ellipse, the larger the fitting error. When a camera cannot directly face a circular object, it is advisable to avoid excessive deviation of the lens direction from the vertical direction of the object to prevent a significant increase in the axis ratio and fitting error.

3) By comparing the ratios of the maximum and minimum values of the different fitting errors, it can be observed that the phase angle has a significant impact on the fitting error of the ellipse. The fitting results for the different phase angles exhibit significant variations, and the curves exhibit periodic changes.

References

[1] DAI C S, RONG W B, FAN Z H, et al. A novel detection algorithm of microspheres based on Hough transform and ellipse fitting[C]//2015 IEEE International Conference on Mechatronics and Automation (ICMA), Beijing, China. New York: IEEE, 2015: 999-1004.

[2] GUO S Y, WU Y D. Improved ellipse fitting algorithm with outlier removal [J]. *Computer Science*, 2022, 49 (4): 188-194. (in Chinese)

[3] QUAN Y M, LING Q. Ellipse fitting algorithm based on geometric symmetry[J]. *Automation & Instrumentation*, 2020, 35 (5): 51-55. (in Chinese)

[4] BUTTERS L, XU Z Z, KLETTE R. Using machine vision to command a 6-axis robot arm to act on a randomly placed zinc die cast product [C]//Proceedings of the 2nd International Conference on Control and Computer Vision, Jeju, Republic of Korea. New York: A CM, 2019: 8-12.

[5] LIANG J L, WANG Y L, ZENG X J. Robust ellipse fitting via half-quadratic and semidefinite relaxation optimization [J]. *IEEE Transactions on Image Processing*, 2015, 24 (11): 4276-4286.

[6] KANATANI K. Ellipse fitting with hyperaccuracy [J]. *IEICE Transactions on Information and Systems*, 2006, E89-D (10): 2653-2660.

[7] LU W, TAN J L. Detection of incomplete ellipse in images with strong noise by iterative randomized Hough transform (IRHT) [J]. *Pattern Recognition*, 2008, 41(4): 1268-1279.

[8] PĂŹTRĂŹUCEAN V, GURDJOS P, GIOI R G. A parameter lessline segment and elliptical arc detector with enhanced ellipse fitting [C]// Proceedings of the 12th European Conference on Computer Vision. New York: ACM, 2012 (7573): 572-585.

[9] PRASAD D K, LEUNG M K H, QUEK C. ElliFit: an unconstrained, non-iterative, least squares based geometric ellipse fitting method [J]. *Pattern Recognition*, 2013, 46(5): 1449-1465.

[10] CHERNOV N, LESORT C. Least squares fitting of circles[J]. *Journal of Mathematical Imaging and Vision*, 2005, 23(3): 239-252.

[11] KANATANI K, RANGARAJAN P. Hyper least squares fitting of circles and ellipses [J]. *Computational Statistics & Data Analysis*, 2011, 55(6): 2197-2208.

[12] WAIBEL P, MATTHES J, GRÖLL L. Constrained ellipse fitting with center on a line [J]. *Journal of Mathematical Imaging and Vision*, 2015, 53(3): 364-382.

[13] HALIR R, FLUSSER J. Numerically stable direct least squares fitting of ellipses [C]// Proceedings of the 6th International Conference in Central Europe on Computer Graphics and Visualization (WSCG 98). Plzen-Bory: CiteSeer, 1998, 98: 125-132.

[14] POŽAR T, MOŽINA J. Enhanced ellipse fitting in a two-detector homodyne quadrature laser interferometer [J]. *Measurement Science and Technology*, 2011, 22(8): 085301.

[15] ZAKRZEWSKI M, SINGH A, YAVARI E, et al. Quadrature imbalance compensation with ellipse-fitting methods for microwave radar physiological sensing[J]. *IEEE Transactions on Microwave Theory and Techniques*, 2014, 62 (6): 1400-1408.

[16] PRASAD D K, QUEK C, LEUNG M K H. A precise ellipse fitting method for noisy data[C]// International Conference Image Analysis and Recognition. Berlin: Springer, 2012: 253-260.

[17] CANNY J. A computational approach to edge detection [J]. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 1986, 8(6): 679-698.

[18] GANDER W, GOLUB G H, STREBEL R. Least-squares fitting of circles and ellipses[J]. *BIT Numerical Mathematics*, 1994, 34(4): 558-578.

[19] ZHU S R, LIU J H. Improved fast algorithm for elliptic fitting based on least square method[J]. *Ship Electronic Engineering*, 2022, 42(1): 33-35. (in Chinese)

[20] FITZGIBBON A, PILU M, FISHER R B. Direct least square fitting of ellipses [J]. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 1999, 21(5): 476-480.

不完整轮廓的椭圆拟合误差分析

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摘　要：在计算机图像处理过程中，受光照条件和摄像机安装位置的影响，对图像进行边缘提取后，经常存在椭圆轮廓提取不完整的情况。拟合此残缺轮廓结果与原椭圆形状出现偏差，拟合误差较大，影响拟合精度。该文研究了残缺轮廓的特征对误差的影响程度，利用 MATLAB 数值仿真的方法依次做不同位置的残缺椭圆弧段，对弧段进行边缘提取，基于最小二乘法对弧段进行椭圆拟合，分析相角、弧长完整度和椭圆长短轴之比三个因素对椭圆拟合误差的影响。拟合误差随三个因素变化的曲线表明，三个因素都对拟合误差有显著影响。不同相角处的轮廓拟合效果差异大；弧长完整度越大，椭圆长短轴之比越小，则拟合误差越小。

关键词：仿真方法；最小二乘法；椭圆拟合；拟合误差