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Robust H_∞ Bipartite Consensus for Uncertain Nonlinear Multi-Agent Systems with Disturbances

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Abstract: The robust H_∞ bipartite consensus problem is studied for a class of nonlinear time-varying multi-agent systems (MASs) with parameter uncertainties and external disturbances over signed networks. Following the thought of dealing with uncertainties in robust control, the considered system is transformed into a time-invariant dynamical model with norm-bounded parameter uncertainties. The robust bipartite consensus is converted to a reduced-order H_∞ control problem. Based on the Lyapunov stability theory, sufficient conditions in linear matrix inequalities (LMIs) are obtained for the robust bipartite consensus of MASs with desired H_∞ performance. Moreover, the design procedure of a distributed static output feedback controller is shown. Furthermore, an application for two-degree-of-freedom (2-DOF) planar mobile robots is presented to illustrate the effectiveness of the proposed controller.

Key words: robust H_∞ control; multi-agent system (MAS); bipartite consensus; parameter uncertainty; output feedback
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0 Introduction

Distributed cooperative control of multi-agent systems (MASs) has become an active research domain in the past few decades, and has potential applications in wireless sensor networks, environmental monitoring and autonomous vehicle formation[1-3]. Consensus is the fundamental research of cooperative control, it means that each agent only interacts with its neighbors to achieve a common value under a proper control protocol. So far, abundant creative results have been obtained on the consensus problem of MASs[4-6].

Most of the corresponding results have been concerned with the ideal system with time-invariant parameters. In practice, however, most controlled systems are time-varying and nonlinear owing to mutative working conditions[7]. Analysis tools for the time-invariant systems, such as the eigenvalue method and the linear matrix inequality (LMI) theory[8-9], are no longer applicable. Among the literature on nonlinear time-varying MASs[10-12], a novel time-varying strategy is proposed to compensate for an unknown nonlinear Lipschitz rate[10], and adaptive dynamic surface control schemes are developed for high-order MASs[11-12].

It is inevitable for an MAS to be affected by parameter uncertainties and external disturbances. Until now, great progress has been made in robust consensus control of MASs with parametric uncertainties[13-14]. In addition to providing the system internal stability, the control system also aims to achieve a certain performance index[15-17]. By converting the robust consensus into the simultaneous H_∞ issues, the consensus of MASs with parameter uncertainties and external disturbances is addressed, while guaranteeing the predetermined H_∞/H_2 performance[16-17]. However, most of the existing literatures assume that agents have linear time-invariant dynamics and are subject to the same norm-bounded parameter uncertainty.

Signed graphs are effective tools for describing interactions among agents, where positive and negative adjacency weights represent collaborative and antagonistic relationships, respectively[18]. The bipartite consensus is put forward for MASs on signed graphs, and it means that all agents converge to a value with the same modulus but opposite signs[19-22]. Among them, the H_∞ bipartite consensus for nonlinear MASs with external disturbances is addressed through an output feedback controller[20]. Based on the distributed observer, Bhowmick et al.[21] investigate the leader-following bipartite consensus for linear MASs subject to parameter uncertainties and external disturbances. Additionally, the practical fixed-time bipartite consensus for uncertain nonlinear MASs is tackled by utilizing a barrier Lyapunov function-based approach[22]. The robust H_∞ bipartite consensus of nonlinear time-varying MASs subject to parameter uncertainties and external disturbances has not yet been investigated, which motivates this study.

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Compared with Ref. [20], this paper takes parameter uncertainties into consideration. At the same time, the strong restriction of nonlinearity is relaxed to Lipschitz-type, which has broader applications. Compared with Ref. [21], this paper further considers the robust H_∞ performance of the consensus. Furthermore, this paper addresses more challenging cases in which the control input matrices of MASs are subject to different parameter uncertainties and the nominal system is a more practical time-varying system, which is different from the existing results in Refs. [13, 16-17, 23].

The remainder of this paper is arranged as follows. Some preliminaries on the graph theory, the necessary lemmas and the system formulation are given in Section 1. Sufficient conditions for robust H_∞ bipartite consensus of MASs are analyzed in Section 2. A numerical example is presented in Section 3. The conclusions are drawn in Section 4.

1 Preliminaries and Problem Formulation

1.1 Preliminaries

In this subsection, some basic concepts of the graph theory are reviewed.

The communication network of an MAS with N agents can be modeled by a graph where the nodes/vertices correspond to the agents and the edges correspond to the communication links between agents. A graph $\mathcal{G}$ is represented by a triple $\mathcal{G}=\{\mathcal{V},\mathcal{E},\mathcal{A}\}$, where $\mathcal{V}=\{v_1, v_2, \cdots, v_N\}$ is the set of nodes, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the set of edges, and $\mathcal{A}=[a_{ij}]_{N\times N}$ is the adjacency matrix of $\mathcal{G}$ with $a_{ii}=0$, $i=1, 2, \cdots, N$. If agent i can directly receive information from agent j, $a_{ij}\neq 0$; otherwise $a_{ij}=0$. If $a_{ij}=a_{ji}$, $i, j=1, 2, \cdots, N$, the graph is undirected; otherwise it is directed. An undirected graph $\mathcal{G}$ is connected if there is a path between any two vertices in $\mathcal{G}$.

Definition 1[19] The signed graph $\mathcal{G}(\mathcal{A})$ is structurally balanced if there is a bipartition of $\mathcal{V}$ with the property that $\mathcal{V}=\mathcal{V}_1\cup\mathcal{V}_2$ and $\mathcal{V}_1\cap\mathcal{V}_2=\emptyset$, such that $a_{ij}\geqslant 0$ when nodes v_i and v_j are in the same subgroup, and $a_{ij}\leqslant 0$ when nodes v_i and v_j are in the different subgroups.

Lemma 1[20] Consider a connected and structurally balanced signed graph $\mathcal{G}=\{\mathcal{V},\boldsymbol{\mathcal{E}},\mathcal{A}\}$ with the bipartition $\mathcal{V}_1$ and $\mathcal{V}_2$ according to Definition 1. Then $\tilde{\boldsymbol{A}}=[\tilde{a}_{ij}]_{N\times N}=\boldsymbol{DAD}$ has all nonnegative entries, i. e., $\tilde{a}_{ij}=|a_{ij}|$, if and only if the gauge transformation $\boldsymbol{D}=\mathrm{diag}\{d_1, d_2. . . , d_N\}\in\mathcal{D}$, where $d_i\in\{1,-1\}$, $i=1, 2, \cdots, N$, and $d_i=d_j$ if and only if $q_i=q_j$. Here, $q_i=s$, $s\in\{1, 2\}$, $i=1, 2, \cdots, N$, and agent i belongs to $\mathcal{V}_s$.

Lemma 2[24] Given matrices $\boldsymbol{Q}$, $\boldsymbol{H}$, $\boldsymbol{F}$ and $\boldsymbol{E}$ of appropriate dimensions and $\boldsymbol{Q}$ is symmetric, then $\boldsymbol{Q}+\boldsymbol{HF}(t)\boldsymbol{E}+\boldsymbol{E}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{H}^{\mathrm{T}}<0$ for all $\boldsymbol{F}(t)$ satisfying $\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t)\leqslant\rho^2\boldsymbol{I}$ if and only if there exists a constant $\varepsilon>0$ such that $\boldsymbol{Q}+\varepsilon\rho^2\boldsymbol{HH}^{\mathrm{T}}+\frac{1}{\varepsilon}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E}<0$.

1.2 Problem formulation

Consider a time-varying MAS consisting of N agents subject to unknown nonlinear dynamics and parameter uncertainties. The model of the ith agent is described as

$$\begin{aligned}\dot{\boldsymbol{x}}_i(t)&=\boldsymbol{A}(t)\boldsymbol{x}_i(t)+(\boldsymbol{B}(t)+\Delta\boldsymbol{B}_i(t))\boldsymbol{u}_i(t)+\boldsymbol{D}_1\boldsymbol{f}(t,\boldsymbol{x}_i(t))+\boldsymbol{E}\boldsymbol{w}_i(t),\\ \boldsymbol{y}_i(t)&=\boldsymbol{C}\boldsymbol{x}_i(t),\ i=1, 2, \cdots, N,\end{aligned}\tag{1}$$

where $\boldsymbol{x}_i(t)\in\mathbf{R}^n$, $\boldsymbol{y}_i(t)\in\mathbf{R}^p$, $\boldsymbol{u}_i(t)\in\mathbf{R}^m$ and $\boldsymbol{w}_i(t)\in\mathcal{L}_2[0,\infty)$ represent state, output, control input and external disturbance of the ith agent, respectively; $\boldsymbol{f}(t,\boldsymbol{x}_i(t)):\mathbf{R}_+\times\mathbf{R}^n\rightarrow\mathbf{R}^n$ is an unknown nonlinear function; $\boldsymbol{A}(t):\mathbf{R}\rightarrow\mathbf{R}^{n\times n}$ is the state matrix; $\boldsymbol{B}(t):\mathbf{R}\rightarrow\mathbf{R}^{n\times m}$ is the input matrix; $\Delta\boldsymbol{B}_i(t)$ is an unknown matrix, representing the uncertainty associated with the input matrix of the agent i; $\boldsymbol{D}_1$, $\boldsymbol{E}$ and $\boldsymbol{C}$ are constant matrices of appropriate dimensions.

Assuming that $\boldsymbol{A}(t)$ and $\boldsymbol{B}(t)$ are piecewise continuous and norm-bounded matrix-valued time-varying functions. $\boldsymbol{A}(t)$ and $\boldsymbol{B}(t)$ have the decomposition of[23]

$$[\boldsymbol{A}(t)\quad\boldsymbol{B}(t)]=[\boldsymbol{A}\quad\boldsymbol{B}]+\boldsymbol{M\Delta}(t)[\boldsymbol{N}_1\quad\boldsymbol{N}_2],$$

where $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{M}_1$, $\boldsymbol{N}_1$ and $\boldsymbol{N}_2$ are constant matrices, $\boldsymbol{\Delta}(t)\in\mathbf{R}^{r_1\times r_2}$ is Lebesgue measurable, and

$$\boldsymbol{\Delta}^{\mathrm{T}}(t)\ \boldsymbol{\Delta}(t)\leqslant\delta_1^2\boldsymbol{I}_{r_2}\tag{2}$$

holds for all $t\geqslant 0$. $\Delta\boldsymbol{B}_i(t)$ has the form of $\Delta\boldsymbol{B}_i(t)=\boldsymbol{HF}_i(t)\boldsymbol{G}$, where $\boldsymbol{H}$ and $\boldsymbol{G}$ are known constant matrices that depict the structure of uncertainties, and $\boldsymbol{F}_i(t)\in\mathbf{R}^{q_1\times q_2}$ is unknown time-varying and Lebesgue measurable. $\boldsymbol{F}_i(t)$ satisfies

$$\boldsymbol{F}_i^{\mathrm{T}}(t)\boldsymbol{F}_i(t)\leqslant\delta_2^2\boldsymbol{I}_{q_2},\ (t\geqslant 0),\tag{3}$$

where δ_1 and δ_2 are positive scalars; $\boldsymbol{I}_{r_2}$ and $\boldsymbol{I}_{q_2}$ are r_2 and q_2 dimensional identity matrices, respectively.

The MAS (1) can be expressed as

$$\begin{aligned}\dot{\boldsymbol{x}}_i(t)=&(\boldsymbol{A}+\boldsymbol{M\Delta}(t)\boldsymbol{N}_1)\boldsymbol{x}_i(t)+\\&(\boldsymbol{B}+\bar{\boldsymbol{H}}\boldsymbol{\Delta}_i(t)\bar{\boldsymbol{G}})\boldsymbol{u}_i(t)+\boldsymbol{D}_1\boldsymbol{f}(t,\boldsymbol{x}_i(t))+\boldsymbol{E}\boldsymbol{w}_i(t),\end{aligned}\tag{4}$$

where $\bar{\boldsymbol{H}}=[\boldsymbol{M}\quad\boldsymbol{H}]$, $\boldsymbol{\Delta}_i(t)=\begin{bmatrix}\boldsymbol{\Delta}(t) & 0\\ 0 & \boldsymbol{F}_i(t)\end{bmatrix}$, and $\bar{\boldsymbol{G}}=\begin{bmatrix}\boldsymbol{N}_2\\ \boldsymbol{G}\end{bmatrix}$. In light of conditions (2) and (3), it can be obtained that $\boldsymbol{\Delta}_i^{\mathrm{T}}(t)\boldsymbol{\Delta}_i(t)\leqslant\delta^2\boldsymbol{I}_{r_2+q_2}$, $\forall t\geqslant 0$, $\delta=max\{\delta_1,\delta_2\}$.

Assumption 1[25] The unknown nonlinear function $\boldsymbol{f}(t,\boldsymbol{x}_i(t))$ satisfies the Lipschitz-like condition:

$$\|\boldsymbol{f}(t,\boldsymbol{x})-d_i\boldsymbol{f}(t,\boldsymbol{y})\|\leqslant\alpha\|\boldsymbol{x}-d_i\boldsymbol{y}\|,$$

$$\forall \boldsymbol{x},\ \boldsymbol{y} \in \mathbf{R}^n,\ d_i \in \{1, -1\},\ t \geqslant 0,$$

where α is a positive constant.

Assumption 2 The interaction topology $\mathcal{G}$ is an undirected, connected and structurally balanced signed graph.

Define the consensus performance variable as

$$z_i(t) = \boldsymbol{C}_1\left(\boldsymbol{x}_i(t) - \frac{1}{N}\sum_{j=1}^{N} d_i d_j \boldsymbol{x}_j(t)\right), \quad (5)$$

where $\boldsymbol{C}_1$ is a given constant matrix.

In order to suppress the effect of the external disturbance $\boldsymbol{w}(t)$ on $\boldsymbol{z}(t)$, the following H_∞ performance index $J(w)$ is considered[15]:

$$J(\boldsymbol{w}) = \int_0^\infty [z^{\mathrm{T}}(t)z(t) - \gamma^2 \boldsymbol{w}^{\mathrm{T}}(t)\boldsymbol{w}(t)]\, dt < 0, \quad (6)$$

i.e., $\|\boldsymbol{T}_{zw}\|_\infty = \sup_{0\neq w(t)\in\mathcal{L}_2[0,\infty)} \frac{\|z(t)\|_2}{\|\boldsymbol{w}(t)\|_2} < \gamma$, where $\boldsymbol{T}_{zw}$ is the transfer function from $\boldsymbol{w}$ to z, and γ is a given positive scalar.

Definition 2 The MAS (1) achieves robust H_∞ bipartite consensus, if the bipartite consensus is achieved when $\boldsymbol{w}(t) = \boldsymbol{0}$, i.e., $\lim_{t\to\infty}\left\|\boldsymbol{x}_i(t) - \frac{1}{N}\sum_{j=1}^{N} d_i d_j \boldsymbol{x}_j(t)\right\| = 0$ for all i, and the desired H_∞ performance (6) is satisfied when $\boldsymbol{w}(t) \neq \boldsymbol{0}$ under parameter uncertainties satisfying conditions (2) and (3).

2 Main Results

In this section, the objective is to design an output feedback controller such that nonlinear time-varying MAS (1) achieves the robust bipartite consensus asymptotically with the desired H_∞ performance.

The following distributed output feedback controller is developed:

$$\boldsymbol{u}_i = \boldsymbol{K}\sum_{j\in\mathcal{N}_i} |a_{ij}| [\mathrm{sgn}(a_{ij})\boldsymbol{y}_i(t) - \boldsymbol{y}_j(t)], \quad (7)$$

where $\boldsymbol{K}$ is the feedback gain matrix to be determined.

The network dynamics of the system resulted from Eqs. (4), (5) and (7) can be rewritten as

$$\begin{aligned}\dot{\boldsymbol{x}}(t) = {} & (\boldsymbol{I}_N \otimes (\boldsymbol{A} + \boldsymbol{M\Delta}(t)\boldsymbol{N}_1) + \boldsymbol{L}\otimes \boldsymbol{BKC} + \\ & (\boldsymbol{I}_N \otimes \overline{\boldsymbol{H}})\tilde{\boldsymbol{\Delta}}(t)(\boldsymbol{L}\otimes\overline{\boldsymbol{G}}\boldsymbol{KC}))\boldsymbol{x}(t) + \\ & (\boldsymbol{I}_N \otimes \boldsymbol{D}_1)\boldsymbol{f}(t,\boldsymbol{x}(t)) + (\boldsymbol{I}_N\otimes \boldsymbol{E})\boldsymbol{w}(t), \\ z(t) = {} & (\boldsymbol{R}\otimes \boldsymbol{C}_1)\boldsymbol{x}(t),\end{aligned} \quad (8)$$

where $\boldsymbol{R} = \boldsymbol{I}_N - \frac{1}{N}\boldsymbol{dd}^{\mathrm{T}} \in \mathbf{R}^{N\times N}$; $\boldsymbol{d} = [d_1, d_2, \cdots, d_N]^{\mathrm{T}}$;
$\boldsymbol{x}(t) = [\boldsymbol{x}_1^{\mathrm{T}}(t), \boldsymbol{x}_2^{\mathrm{T}}(t), \cdots, \boldsymbol{x}_N^{\mathrm{T}}(t)]^{\mathrm{T}}$;
$z(t) = [z_1^{\mathrm{T}}(t), z_2^{\mathrm{T}}(t), \cdots, z_N^{\mathrm{T}}(t)]^{\mathrm{T}}$;
$\boldsymbol{w}(t) = [\boldsymbol{w}_1^{\mathrm{T}}(t), \boldsymbol{w}_2^{\mathrm{T}}(t), \cdots, \boldsymbol{w}_N^{\mathrm{T}}(t)]^{\mathrm{T}}$;
$\tilde{\boldsymbol{\Delta}}(t) = \mathrm{diag}\{\boldsymbol{\Delta}_1(t), \boldsymbol{\Delta}_2(t), \cdots, \boldsymbol{\Delta}_N(t)\}$;
$\boldsymbol{f}(t, \boldsymbol{x}(t)) = [\boldsymbol{f}^{\mathrm{T}}(t, \boldsymbol{x}_1(t)), \boldsymbol{f}^{\mathrm{T}}(t, \boldsymbol{x}_2(t)), \cdots, \boldsymbol{f}^{\mathrm{T}}(t, \boldsymbol{x}_N(t))]^{\mathrm{T}}$.

By Lemma 1 and Assumption 2, we consider the transformation

$$\tilde{\boldsymbol{x}}(t) = (\boldsymbol{D}\otimes\boldsymbol{I})\boldsymbol{x}(t),\ \tilde{z}(t) = (\boldsymbol{D}\otimes\boldsymbol{I})z(t),\ \tilde{\boldsymbol{w}}(t) = (\boldsymbol{D}\otimes\boldsymbol{I})\boldsymbol{w}(t).$$

The system (8) can be rewritten as

$$\begin{aligned}\dot{\tilde{\boldsymbol{x}}}(t) = {} & (\boldsymbol{I}_N \otimes (\boldsymbol{A} + \boldsymbol{M\Delta}(t)\boldsymbol{N}_1) + \tilde{\boldsymbol{L}}\otimes \boldsymbol{BKC} + \\ & (\boldsymbol{I}_N \otimes \overline{\boldsymbol{H}})\tilde{\boldsymbol{\Delta}}(t)(\tilde{\boldsymbol{L}}\otimes\overline{\boldsymbol{G}}\boldsymbol{KC}))\tilde{\boldsymbol{x}}(t) + \\ & (\boldsymbol{D}\otimes \boldsymbol{D}_1)\boldsymbol{f}(t,\boldsymbol{x}(t)) + (\boldsymbol{I}_N\otimes \boldsymbol{E})\tilde{\boldsymbol{w}}(t), \\ \tilde{z}(t) = {} & (\tilde{\boldsymbol{R}}\otimes \boldsymbol{C}_1)\tilde{\boldsymbol{x}}(t),\end{aligned} \quad (9)$$

where $\tilde{\boldsymbol{L}} = \boldsymbol{DLD}$, $\tilde{\boldsymbol{R}} = \boldsymbol{I}_N - \frac{1}{N}\boldsymbol{1}_N\boldsymbol{1}_N^{\mathrm{T}}$, and $\boldsymbol{1}_N$ is an N-dimensional column vector with all ones.

Let $\tilde{\boldsymbol{\xi}}(t) = (\tilde{\boldsymbol{R}}\otimes\boldsymbol{I})\tilde{\boldsymbol{x}}(t)$, by virtue of the properties $\tilde{\boldsymbol{R}}^2 = \tilde{\boldsymbol{R}}$, $\tilde{\boldsymbol{R}}\tilde{\boldsymbol{L}} = \tilde{\boldsymbol{L}} = \tilde{\boldsymbol{L}}\tilde{\boldsymbol{R}}$, and then we have

$$\begin{aligned}\dot{\tilde{\boldsymbol{\xi}}}(t) = {} & (\boldsymbol{I}_N \otimes (\boldsymbol{A} + \boldsymbol{M\Delta}(t)\boldsymbol{N}_1) + \tilde{\boldsymbol{L}}\otimes \boldsymbol{BKC} + \\ & (\tilde{\boldsymbol{R}} \otimes \overline{\boldsymbol{H}})\tilde{\boldsymbol{\Delta}}(t)(\tilde{\boldsymbol{L}}\otimes\overline{\boldsymbol{G}}\boldsymbol{KC}))\tilde{\boldsymbol{\xi}}(t) + \\ & (\tilde{\boldsymbol{R}}\boldsymbol{D}\otimes \boldsymbol{D}_1)\boldsymbol{F}(t,\boldsymbol{x}(t)) + (\tilde{\boldsymbol{R}}\otimes \boldsymbol{E})\tilde{\boldsymbol{w}}(t), \\ \tilde{z}(t) = {} & (\tilde{\boldsymbol{R}}\otimes \boldsymbol{C}_1)\tilde{\boldsymbol{\xi}}(t).\end{aligned} \quad (10)$$

where $\boldsymbol{F}(t, \boldsymbol{x}(t)) = \boldsymbol{f}(t, \boldsymbol{x}(t)) - \boldsymbol{D1}_N \otimes \boldsymbol{f}\left(t, \frac{1}{N}\sum_{j=1}^{N} d_j\boldsymbol{x}_j(t)\right)$.

According to Ref. [26], there exists an orthogonal matrix $\boldsymbol{V}\in\mathbf{R}^{N\times N}$ such that $\boldsymbol{V}^{\mathrm{T}}\tilde{\boldsymbol{R}}\boldsymbol{V} = \hat{\boldsymbol{R}} = \mathrm{diag}\{0, 1, \cdots, 1\}$ and $\boldsymbol{V}^{\mathrm{T}}\tilde{\boldsymbol{L}}\boldsymbol{V} = \hat{\boldsymbol{L}} = \mathrm{diag}\{0, \lambda_2, \cdots, \lambda_N\}$, where $\boldsymbol{V} = [\boldsymbol{V}_1, \boldsymbol{V}_2]$, $\boldsymbol{V}_1 = \frac{\boldsymbol{1}_N}{\sqrt{N}}$. Let $\hat{\boldsymbol{\xi}}(t) = (\boldsymbol{V}^{\mathrm{T}}\otimes\boldsymbol{I})\tilde{\boldsymbol{\xi}}(t)$, $\hat{z}(t) = (\boldsymbol{V}^{\mathrm{T}}\otimes\boldsymbol{I})\tilde{z}(t)$, and $\hat{\boldsymbol{w}}(t) = (\boldsymbol{V}^{\mathrm{T}}\otimes\boldsymbol{I})\tilde{\boldsymbol{w}}(t)$, and then it follows from Eq. (10) that

$$\begin{aligned}\dot{\hat{\boldsymbol{\xi}}}(t) = {} & (\boldsymbol{I}_N \otimes (\boldsymbol{A} + \boldsymbol{M\Delta}(t)\boldsymbol{N}_1) + \hat{\boldsymbol{L}}\otimes \boldsymbol{BKC} + \\ & (\hat{\boldsymbol{R}} \otimes \overline{\boldsymbol{H}})\hat{\boldsymbol{\Delta}}(t)(\hat{\boldsymbol{L}}\otimes\overline{\boldsymbol{G}}\boldsymbol{KC}))\hat{\boldsymbol{\xi}}(t) + \\ & (\boldsymbol{V}^{\mathrm{T}}\tilde{\boldsymbol{R}}\boldsymbol{D}\otimes \boldsymbol{D}_1)\boldsymbol{F}(t,\boldsymbol{x}(t)) + (\hat{\boldsymbol{R}}\otimes \boldsymbol{E})\hat{\boldsymbol{w}}(t), \\ \hat{z}(t) = {} & (\hat{\boldsymbol{R}}\otimes \boldsymbol{C}_1)\hat{\boldsymbol{\xi}}(t),\end{aligned} \quad (11)$$

where $\hat{\boldsymbol{\Delta}}(t) = (\boldsymbol{V}^{\mathrm{T}}\otimes\boldsymbol{I})\tilde{\boldsymbol{\Delta}}(t)(\boldsymbol{V}\otimes\boldsymbol{I})$ and $\hat{\boldsymbol{\Delta}}^{\mathrm{T}}(t)\hat{\boldsymbol{\Delta}}(t) \leqslant \delta^2\boldsymbol{I}$.

Notice that $\hat{\boldsymbol{\xi}}(t) = (\boldsymbol{V}^{\mathrm{T}}\otimes\boldsymbol{I})(\tilde{\boldsymbol{R}}\otimes\boldsymbol{I})\tilde{\boldsymbol{x}}(t) = \left(\begin{bmatrix}\boldsymbol{V}_1^{\mathrm{T}}\tilde{\boldsymbol{R}} \\ \boldsymbol{V}_2^{\mathrm{T}}\tilde{\boldsymbol{R}}\end{bmatrix}\otimes\boldsymbol{I}\right)\tilde{\boldsymbol{x}}(t)$. Therefore, $\hat{\boldsymbol{\xi}}_1(t) = 0$.

Denoting that $\bar{\boldsymbol{\xi}}(t) = [\hat{\boldsymbol{\xi}}_2^{\mathrm{T}}(t), \hat{\boldsymbol{\xi}}_3^{\mathrm{T}}(t), \cdots, \hat{\boldsymbol{\xi}}_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\bar{z}(t) = [\hat{z}_2^{\mathrm{T}}(t), \hat{z}_3^{\mathrm{T}}(t), \cdots, \hat{z}_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, and $\overline{\boldsymbol{w}}(t) = [\hat{\boldsymbol{w}}_2^{\mathrm{T}}(t), \hat{\boldsymbol{w}}_3^{\mathrm{T}}(t), \cdots, \hat{\boldsymbol{w}}_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, the system (11) can be transformed into the following reduced-order system

$$\begin{aligned}\dot{\bar{\xi}}(t) = &(I_{N-1} \otimes (A + M\Delta(t)N_1) + \bar{L} \otimes BKC + \\ &(I_{N-1} \otimes \bar{H})\bar{\Delta}(t)(\bar{L} \otimes \bar{G}KC))\bar{\xi}(t) + \\ &(V_2^{\mathrm{T}}D \otimes D_1)F(t, x(t)) + (I_{N-1} \otimes E)\bar{w}(t), \\ \bar{z}(t) = &(I_{N-1} \otimes C_1)\bar{\xi}(t), \end{aligned} \quad (12)$$

where $\bar{L} = \mathrm{diag}\{\lambda_2, \lambda_3, \cdots, \lambda_N\}$; $\bar{\Delta}(t) = \hat{\Delta}(t)_{(r_1+q_1+1):(N-1)(r_1+q_1);(r_2+q_2+1):(N-1)(r_2+q_2)}$ representing the submatrix of $\hat{\Delta}(t)$, and $\bar{\Delta}^{\mathrm{T}}(t)\bar{\Delta}(t) \leqslant \delta^2 I$.

Then the following result is obtained.

Theorem 1 Under Assumptions 1 and 2, the desired robust H_∞ bipartite consensus for MAS (1) can be achieved asymptotically via the feedback controller (7), if there exists a matrix $P > 0$ with appropriate dimension and a scalar $\varepsilon > 0$ such that the following LMI holds:

$$\begin{bmatrix} A_c + C_1^{\mathrm{T}}C_1 & PM & P\bar{H} & PD_1 & PE \\ * & -\varepsilon I & 0 & 0 & 0 \\ * & * & -\varepsilon I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0,$$

$$i = 2, 3, \cdots, N, \quad (13)$$

where $A_c = A^{\mathrm{T}}P + PA + \lambda_i PBKC + \lambda_i C^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}P + \alpha^2 I_n + \varepsilon\delta^2 N_1^{\mathrm{T}}N_1 + \varepsilon\delta^2\lambda_i^2 C^{\mathrm{T}}K^{\mathrm{T}}\bar{G}^{\mathrm{T}}\bar{G}KC$, and $\lambda_i (i = 2, 3, \cdots, N)$ is nonzero eigenvalues of $\tilde{L}$.

Proof The following analysis is divided into two parts.

Part I. Consider the asymptotic stability of system (12) when $w(t) = 0$.

For system (12), construct the following Lyapunov function

$$V(t) = \bar{\xi}^{\mathrm{T}}(t)\,\mathcal{P}\,\bar{\xi}(t),$$

where $\mathcal{P} > 0$ and $\mathcal{P} = I_{N-1} \otimes P$.

Along the trajectory of system (12) with $w(t) = 0$, the derivative of $V(t)$ is

$$\begin{aligned}\dot{V}(t) = &2\bar{\xi}^{\mathrm{T}}(t)\,\mathcal{P}\dot{\bar{\xi}}(t) = \\ &2\bar{\xi}^{\mathrm{T}}(t)(I_{N-1} \otimes PA + \bar{L} \otimes PBKC)\bar{\xi}(t) + \\ &2\bar{\xi}^{\mathrm{T}}(t)(I_{N-1} \otimes P)\begin{bmatrix} I_{N-1} \otimes M & I_{N-1} \otimes \bar{H} \end{bmatrix} \\ &\begin{bmatrix} I_{N-1} \otimes \Delta(t) & 0 \\ 0 & \bar{\Delta}(t) \end{bmatrix}\begin{bmatrix} I_{N-1} \otimes N_1 \\ \bar{L} \otimes \bar{G}KC \end{bmatrix}\bar{\xi}(t) + \\ &2\bar{\xi}^{\mathrm{T}}(t)(V_2^{\mathrm{T}}D \otimes PD_1)F(t, x(t)). \end{aligned} \quad (14)$$

By applying the Young inequality in Eq. (14), it yields

$$\begin{aligned}&2\bar{\xi}^{\mathrm{T}}(t)(V_2^{\mathrm{T}}D \otimes PD_1)F(t, x(t)) \leqslant \\ &\bar{\xi}^{\mathrm{T}}(t)(I_{N-1} \otimes PD_1D_1^{\mathrm{T}}P)\bar{\xi}(t) + F^{\mathrm{T}}(t, x(t))F(t, x(t)) \\ &\leqslant \bar{\xi}^{\mathrm{T}}(t)(I_{N-1} \otimes PD_1D_1^{\mathrm{T}}P)\bar{\xi}(t) + \\ &\alpha^2\|(R \otimes I_n)x(t)\|^2. \end{aligned} \quad (15)$$

Notice that $\|\bar{\xi}(t)\| = \|\hat{\xi}(t)\| = \|(R \otimes I_n)x(t)\|$. Substituting inequality (15) into Eq. (14) yields

$$\begin{aligned}\dot{V}(t) \leqslant &\bar{\xi}^{\mathrm{T}}(t)\{I_{N-1} \otimes A^{\mathrm{T}}P + I_{N-1} \otimes PA + \bar{L} \otimes PBKC + \bar{L} \otimes C^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}P + \begin{bmatrix} I_{N-1} \otimes N_1^{\mathrm{T}} & \bar{L} \otimes C^{\mathrm{T}}K^{\mathrm{T}}\bar{G}^{\mathrm{T}} \end{bmatrix} \cdot \\ &\begin{bmatrix} I_{N-1} \otimes \Delta^{\mathrm{T}}(t) & 0 \\ 0 & \bar{\Delta}^{\mathrm{T}}(t) \end{bmatrix}\begin{bmatrix} M^{\mathrm{T}}P \\ \bar{H}^{\mathrm{T}}P \end{bmatrix} + \begin{bmatrix} I_{N-1} \otimes PM & I_{N-1} \otimes P\bar{H} \end{bmatrix}\begin{bmatrix} I_{N-1} \otimes \Delta(t) & 0 \\ 0 & \bar{\Delta}(t) \end{bmatrix}\begin{bmatrix} I_{N-1} \otimes N_1 \\ \bar{L} \otimes \bar{G}KC \end{bmatrix} + \\ &I_{N-1} \otimes PD_1D_1^{\mathrm{T}}P + \alpha^2 I_{N-1} \otimes I_n\}\bar{\xi}(t) = \bar{\xi}^{\mathrm{T}}(t)\,\Lambda\bar{\xi}(t). \end{aligned}$$

If $\Lambda < 0$, it is obtained that $\dot{V}(t) < 0$. In light of the Lyapunov stability theorem, the asymptotic stability of system (12) can be achieved.

According to Lemma 2, $\Lambda < 0$ if and only if there exists a scalar $\varepsilon > 0$ such that

$$\begin{aligned}&A^{\mathrm{T}}P + PA + \lambda_i PBKC + \lambda_i C^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}P + PD_1D_1^{\mathrm{T}}P + \alpha^2 I_n + \varepsilon\delta^2 N_1^{\mathrm{T}}N_1 + \\ &\varepsilon\delta^2\lambda_i^2 C^{\mathrm{T}}K^{\mathrm{T}}\bar{G}^{\mathrm{T}}\bar{G}KC + \frac{1}{\varepsilon}PMM^{\mathrm{T}}P + \frac{1}{\varepsilon}P\bar{H}\,\bar{H}^{\mathrm{T}}P < 0,\ i = 2, 3, \cdots, N, \end{aligned} \quad (16)$$

which ensures MAS (1) to achieve the robust bipartite consensus.

Part II. Consider H_∞ performance when $w(t) \neq 0$.

Similar to inequality (15), it is obtained that

$$\begin{aligned}&2\bar{\xi}^{\mathrm{T}}(t)(I_{N-1} \otimes PE)\bar{w}(t) \leqslant \\ &\frac{1}{\gamma^2}\bar{\xi}^{\mathrm{T}}(t)(I_{N-1} \otimes PEE^{\mathrm{T}}P)\bar{\xi}(t) + \gamma^2\bar{w}^{\mathrm{T}}(t)\bar{w}(t), \end{aligned}$$

where scalar $\gamma > 0$ describes the H_∞ performance level.

By the aid of zero initial conditions and the system stability, it can be concluded that

$$\begin{aligned}J_T(\bar{w}) = &\int_0^T [\bar{z}^{\mathrm{T}}(t)\bar{z}(t) - \gamma^2\bar{w}^{\mathrm{T}}(t)\bar{w}(t) + \dot{V}(t)]\,dt - V(T) + V(0) \leqslant \\ &\int_0^T \left[\bar{\xi}^{\mathrm{T}}(t)(I_{N-1} \otimes C_1^{\mathrm{T}}C_1)\bar{\xi}(t) + \bar{\xi}^{\mathrm{T}}(t)\,\Lambda(t)\bar{\xi}(t) + \frac{1}{\gamma^2}\bar{\xi}^{\mathrm{T}}(t)(I_{N-1} \otimes PEE^{\mathrm{T}}P)\bar{\xi}(t)\right]\mathrm{d}t. \end{aligned}$$

By applying Schur Complement Lemma[27], it is derived that $J_T(\bar{w}) < 0$ if inequality (13) holds, which implies that the desired H_∞ performance is achieved. This completes the proof of Theorem 1.

Furthermore, we discuss how to design the controller gain matrix $\boldsymbol{K}$.

Theorem 2 Under Assumptions 1 and 2, the desired robust H_∞ bipartite consensus for MAS (1) is achieved via the output feedback controller (7), if there exist matrices $\boldsymbol{P} > 0$ and $\bar{\boldsymbol{K}}$ with appropriate dimensions satisfying the following LMIs:

$$\begin{bmatrix} \bar{\boldsymbol{A}}_c + \boldsymbol{C}_1^{\mathrm{T}}\boldsymbol{C}_1 & \boldsymbol{PM} & \boldsymbol{P}\bar{\boldsymbol{H}} & \boldsymbol{PD}_1 & \boldsymbol{PE} & \boldsymbol{C}^{\mathrm{T}}\bar{\boldsymbol{K}}^{\mathrm{T}}(\boldsymbol{W}^{-1})^{\mathrm{T}}\bar{\boldsymbol{G}}^{\mathrm{T}} \\ * & -\varepsilon\boldsymbol{I} & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon\boldsymbol{I} & 0 & 0 & 0 \\ * & * & * & -\boldsymbol{I} & 0 & 0 \\ * & * & * & * & -\gamma^2\boldsymbol{I} & 0 \\ * & * & * & * & * & -(\varepsilon\delta^2\lambda_i^2)^{-1}\boldsymbol{I} \end{bmatrix} < 0,\ i = 2,3,\cdots,N, \tag{17}$$

$$\begin{bmatrix} -\eta\boldsymbol{I} & (\boldsymbol{PB}-\boldsymbol{BW})^{\mathrm{T}} \\ * & -\boldsymbol{I} \end{bmatrix} < 0, \tag{18}$$

where $\bar{\boldsymbol{A}}_c = \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{PA} + \lambda_i\boldsymbol{B}\bar{\boldsymbol{K}}\boldsymbol{C} + \lambda_i\boldsymbol{C}^{\mathrm{T}}\bar{\boldsymbol{K}}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}} + \alpha^2\boldsymbol{I}_n + \varepsilon\delta^2\boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{N}_1$, and $\eta > 0$. Then, the designed controller is proposed by $\bar{\boldsymbol{K}} = \boldsymbol{WK}$, and $\boldsymbol{W}$ is an invertible square matrix with compatible dimensions.

Proof Consider $\boldsymbol{PB} = \boldsymbol{BW}$. Then it follows that $(\boldsymbol{PB}-\boldsymbol{BW})^{\mathrm{T}}(\boldsymbol{PB}-\boldsymbol{BW}) - \eta\boldsymbol{I} < 0, \eta > 0$, which can be converted into the LMI (18). Let $\bar{\boldsymbol{K}} = \boldsymbol{WK}$. Then inequality (13) is equivalent to the LMI (17). Therefore, LMIs (17) and (18) can guarantee the desired robust H_∞ bipartite consensus.

Remark 1 $\boldsymbol{W}$ is an intermediate matrix that can be arbitrarily selected. In fact, in order to ensure that the term in $\boldsymbol{A}_c$ satisfies $\lambda_i\boldsymbol{PBKC} = \lambda_i\boldsymbol{B}\bar{\boldsymbol{K}}\boldsymbol{C}$, only $\boldsymbol{PBK} = \boldsymbol{B}\bar{\boldsymbol{K}}$ needs to be established. By considering $\boldsymbol{PB} = \boldsymbol{BW}$, the control gain can be selected as $\boldsymbol{K} = \boldsymbol{W}^{-1}\bar{\boldsymbol{K}}$. Denote $\boldsymbol{W} = \boldsymbol{B}^{+}\boldsymbol{PB}$, where $\boldsymbol{B}^{+}$ represents the pseudo-inverse matrix of $\boldsymbol{B}$.

3 Simulation Examples

The proposed control law can be applied to control the single-link flexible joint manipulator[28], simple pendulum[29], spacecraft attitude alignment[30], etc. For example, consider a group of four two-degree-of-freedom (2-DOF) planar mobile robots. The kinematics of the ith robot is described by the system (1) with $\boldsymbol{x}_i = [x_{i1} \quad x_{i2}]^{\mathrm{T}}$ denoting the position and the velocity, respectively[31]. The communication topology is depicted in Fig. 1. Here, the numbers 1, 2, 3 and 4 in the circle represent the labels of the four robots, respectively. The solid (dashed) line between agents i and j indicates that the adjacency weights are $a_{ij} = a_{ji} = 1$ ($a_{ij} = a_{ji} = -1$) for $(j, i) \in \mathcal{E}$, representing a cooperative (antagonistic) relationship between agents i and j. Let $\mathcal{V}_1 = \{1,2\}$ and $\mathcal{V}_2 = \{3, 4\}$ be two opposing camps, and $\boldsymbol{D} = \mathrm{diag}\{1,1,-1,-1\}$. It can be calculated that $\lambda_2 = 1$, $\lambda_3 = 3$ and $\lambda_4 = 4$.

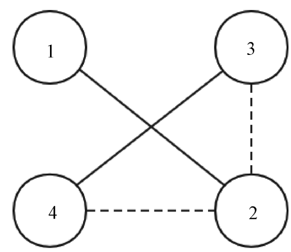

Fig. 1 Communication topology

The nominal dynamic parameters are described below.

$$\boldsymbol{A}(t) = \begin{bmatrix} -\frac{5}{8} + \frac{3}{4}\cos^2 t & 1 - \frac{3}{4}\sin t\cos t \\ -1 - \frac{3}{4}\sin t\cos t & -\frac{5}{8} + \frac{3}{4}\sin^2 t \end{bmatrix},\ \boldsymbol{B}(t) = \begin{bmatrix} 1 - \frac{1}{2}\sin(2t) \\ 1 - \frac{1}{2}\cos(2t) \end{bmatrix},\ \boldsymbol{E} = \begin{bmatrix} 0.1 & -0.1 \\ 0.2 & -0.2 \end{bmatrix},\ \boldsymbol{D}_1 = \boldsymbol{C} = \boldsymbol{I}_2.$$

Therefore, we obtain

$$\boldsymbol{A} = \begin{bmatrix} -\frac{1}{4} & 1 \\ -1 & -\frac{1}{4} \end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ \boldsymbol{M} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix},\ \boldsymbol{N}_1 = \begin{bmatrix} \frac{3}{4} & 0 \\ 0 & \frac{3}{4} \end{bmatrix},\ \boldsymbol{N}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ \boldsymbol{\Delta}(t) = \begin{bmatrix} \cos(2t) & -\sin(2t) \\ -\sin(2t) & -\cos(2t) \end{bmatrix}.$$

Suppose the nonlinear function $\boldsymbol{f}(t, \boldsymbol{x}_i(t)) = [0.2x_{i1}\sin t \quad 0.2x_{i1}\cos t]^{\mathrm{T}}$ with the Lipschitz-like constant $\alpha = 0.2$ and the external disturbance $\boldsymbol{w}_i(t) = [\sin t \quad \cos t]^{\mathrm{T}}$, $i = 1, 2, \cdots, 4$.

The parameter uncertainty associated with agent i is $\Delta\boldsymbol{B}_i(t) = \boldsymbol{HF}_i(t)\boldsymbol{G}$, with

$$\boldsymbol{H}=\begin{bmatrix}0.25 & 0\\ 0 & 0.25\end{bmatrix},\ \boldsymbol{G}=\begin{bmatrix}0\\ 0.5\end{bmatrix},$$

$$\boldsymbol{F}_1(t)=\begin{bmatrix}0.5 & 0\\ 0 & 0.5\cos t\end{bmatrix},$$

$$\boldsymbol{F}_2(t)=\begin{bmatrix}0.5\sin(2t) & 0\\ 0 & 0.5\end{bmatrix},$$

$$\boldsymbol{F}_3(t)=\begin{bmatrix}0.75\cos(3t) & 0\\ 0 & 0.25\end{bmatrix},$$

$$\boldsymbol{F}_4(t)=\begin{bmatrix}0.25\sin t & 0\\ 0 & 0.5\end{bmatrix}.$$

It then follows that $\delta = 1$. The parameters are chosen as $\gamma = 1.5$, $\varepsilon = 0.2$ and $\eta = 0.1$. For simplicity, we select $\boldsymbol{W} = 1$. According to Theorem 2, the gain parameter of controller (7) can be calculated as $\boldsymbol{K} = [-1.3857 \quad -1.3827]$.

Take $\boldsymbol{x}_1(0)=[1 \quad 2]^{\mathrm{T}}$, $\boldsymbol{x}_2(0)=[-3 \quad -4]^{\mathrm{T}}$, $\boldsymbol{x}_3(0)=[-2 \quad 0.3]^{\mathrm{T}}$, and $\boldsymbol{x}_4(0)=[0.1 \quad -0.4]^{\mathrm{T}}$. The state trajectories $x_{i1}(t)$ and $x_{i2}(t)$, $i = 1, 2, \cdots, 4$ under controller (7) are shown in Figs. 2 and 3. Figure 4 shows that the robust bipartite consensus is achieved asymptotically when $\boldsymbol{w}(t) = \boldsymbol{0}$. Figure 5 depicts the evolution of the measure function $\boldsymbol{z}(t)$ in the presence of external disturbances $\boldsymbol{w}(t)$.

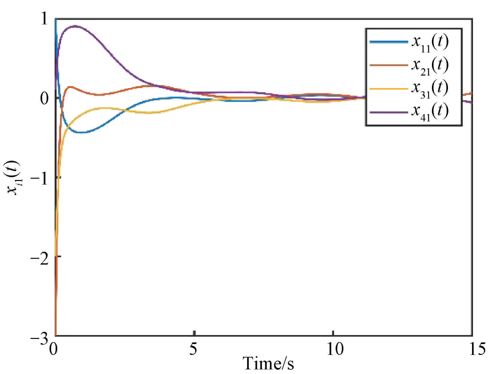

Fig. 2 State trajectories of $x_{i1}(t)$

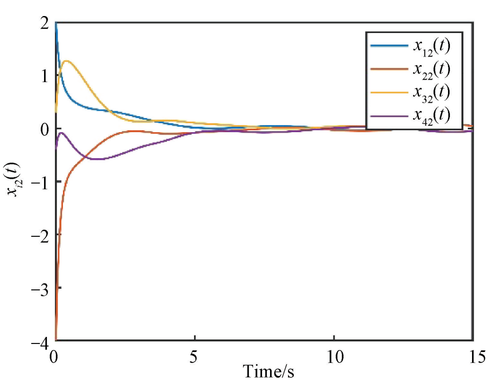

Fig. 3 State trajectories of $x_{i2}(t)$

The energy of $\boldsymbol{z}(t)$ at the steady state is 1.3342, so that the desired H_∞ performance $\int_0^\infty \boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{z}(t)\,\mathrm{d}t < 2.25\int_0^\infty \boldsymbol{w}^{\mathrm{T}}(t)\boldsymbol{w}(t)\,\mathrm{d}t$ is achieved.

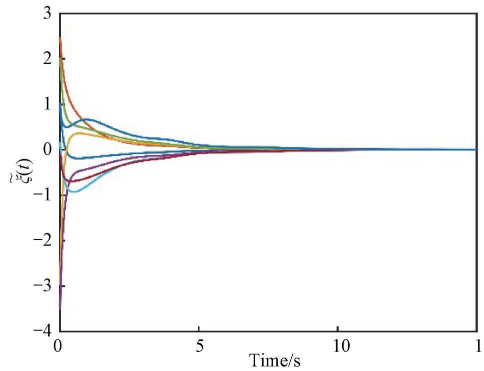

Fig. 4 Evolution of bipartite consensus error $\tilde{\xi}(t)$

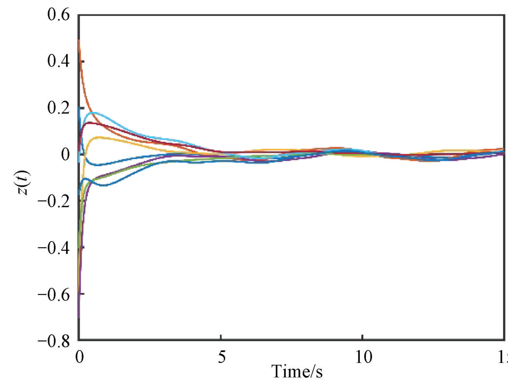

Fig. 5 Evolution of measure function $z(t)$

4 Conclusions

In this paper, we have studied the robust H_∞ bipartite consensus control for nonlinear time-varying MASs with parameter uncertainties and external disturbances over signed topologies. A distributed static output feedback controller is constructed and LMI sufficient conditions for consensus are developed by employing the robust H_∞ control theory and the Lyapunov stability theory. Our future work will focus on extending the theoretical results to the discrete-time case.

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具有扰动的不确定非线性多智能体系统的鲁棒 H_∞ 二分一致性

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摘　要：研究了符号网络上具有参数不确定性和外部扰动的一类非线性时变多智能体系统（multi-agent system，MAS）的鲁棒 H_∞ 二分一致性问题。采用鲁棒控制中处理不确定性的思想，将所考虑的系统转化为具有范数有界参数不确定性的时不变动力学模型。将鲁棒一致性转化为降阶 H_∞ 控制问题。基于李雅普诺夫稳定性理论，得到了具有期望 H_∞ 性能的多智能体系统鲁棒二分一致性的线性矩阵不等式（linear matrix inequality，LMI）充分条件。此外，给出了分布式静态输出反馈控制器的设计过程。进一步以二自由度（two-degree-of-freedom，2-DOF）的平面移动机器人为例，验证了所提出的控制器的有效性。
关键词：鲁棒 H_∞ 控制；多智能体系统（MAS）；二分一致性；参数不确定性；输出反馈