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RESEARCH ARTICLE

Modeling and analysis of the dynamics of an excessive gambling problem with modified fractional operator

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ABSTRACT

This work introduces a fractional-order model of gambling addiction using the modified Atangana-Baleanu-Caputo operator. We establish solution existence/uniqueness, derive the reproduction number $\mathcal{R}_0$, and analyze stability. Numerical results demonstrate how fractional order υ influences addiction dynamics. The model identifies key intervention parameters through sensitivity analysis. The optimal control strategy is proposed to reduce progression to addiction. These approaches provide new tools for understanding and managing problem gambling behaviors.

1. Introduction

Games of chance have a long history dating back to Egyptian civilization.[1,2] Monetary rewards significantly contribute to popularity, as the primary objective of gambling is to generate financial rewards for lucky participants. These rewards enhance the appeal of chance-based games and act as psychological and biological factors in the development of gambling addiction in many individuals. Recently, gambling has grown into a rapidly expanding industry with millions of dollars in cash flows. The world focus on gambling has highlighted its uncontrolled distribution in modern society. There are several ways to gamble, ranging from state lotteries and sports betting (including predicting football matches) to more sophisticated casinos with slot machines, roulette wheels, and poker tables. Technology has diversified gambling options, enabling various electronic video gambling activities. Many people benefit from gambling, but many also incur unwanted losses. This can lead to a difficult way of life, as individuals may try to spend more money to recover from their losses. Every regular gambler encounters what is known as a "near-miss," a failure that appears extremely close to success. This often leads to repeated gambling, ultimately transforming gamblers into addicted ones. Surprisingly, most games have the lowest probability of winning. Anyone from any walk of life can lead to gambling addiction. Several factors, including social, biological, and cognitive-educational aspects, may contribute to the progression from gambling

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to problem gambling, which is a form of addiction. Problem gambling reflects a loss of control over gambling that persists despite often severe negative consequences.[3]

Gambling addiction occurs when gambling is the daily focus of participants. Skitch and Hodgins[3] defined problem gambling as a less judgmental way of describing a repetitive pattern of gambling that leads to serious destructive consequences and is beyond voluntary control. In recent decades, problem gambling has become a major public health issue. Problem gambling is a repetitive behavior, despite harm and negative consequences. It is an addictive behavior with high comorbidity with alcohol problems.[4] In the past two decades, the world community has gradually come to accept that addiction is a type of disease. Recently, gambling problems have been recognized as a major public health issue in modern society. The clinical definition of problem gambling was determined using the Diagnostic and Statistical Manual, 4th Edition, Test Revision (DSM-IV-TR) symptom checklist (American Psychiatric Association [APA]).[5] According to a study in Room et al.,[6] problem gambling leads to several social and psychological problems, including depression and alcohol and drug dependence. Research findings related to problem gambling suggest that this disorder is a real issue that needs to be addressed as a public health concern.

Over the last two decades, excessive gambling has been a primary subject of analysis and investigation in mathematics and statistics. Several mathematical models have been formulated to understand the dynamics of excessive gambling better. For example, Do and Lee[7] used a simple compartmental model to explore gambling dynamics by focusing on young people's gambling. In that study, the basic reproduction number was $\mathcal{R}_0 = 1.466$. The authors of Kong et al.[8] analyzed online gambling on scale-free networks. Shaffer and Korn[9] viewed problems associated with gambling as a socially transmitted disease. In Young and Tae,[10] Lee and Do examined the dynamics of gambling among older adults. The authors of ref.[11] investigated impulsivity trajectories and gambling in adolescence. The authors of ref.[12] focused on the reduction of attentional blink for gambling-related stimuli in problem gamblers. Brown et al.[13] investigated the differences between problem and non-problem gamblers in electronic machine games. Coping strategies in adolescents with gambling problems were the main study objective of the paper in Gupta et al.[14] The authors of Michalczuk et al.[15] studied pathological gamblers' impulsivity and cognitive distortions at the United Kingdom's national problem gambling clinic. Sharpe et al.[16] formulated a cognitive-behavioral model of problem gambling from a biopsychosocial perspective. An author in C˘at˘alin[17] provides a mathematician's advice for safe and rational gambling, one of which is "understanding your game."

Recently, mathematical models involving non-integer order derivatives have gained significant attention because they are more accurate and realistic than classical models.[18–20] An important question that led to the birth of fractional-order calculus in 1695 came from the letter L'Hopital sent to Leibniz. The Riemann and Liouville definition fails to explain the importance of starting conditions. The Caputo derivative addresses this problem, but when defined with a power-law kernel, it has a notable limitation owing to its singular nature.[21] The exponential decay kernel definition by Caputo and Fabrizio addressed the issue of singularity but introduced a locality problem. The first definition of a non-singular arbitrary order derivative by Caputo and Fabrizio significantly contributed to the concept of non-singular kernel fractional calculus. A novel fractional-order Caputo–Fabrizio operator, derived in 2015,[22] addresses several linear and nonlinear issues. The fractional operator is frequently used in various branches of mathematics and engineering. Many authors have applied this fractional operator to investigate the behavior of mathematical models (see[23] and references cited therein). Non-local and non-singular operators are key reasons why fractional calculus is becoming increasingly popular. A few years ago, significant efforts were made to find more interesting and novel non-singular arbitrary order derivatives based on kernels. Atangana and Baleanu studied a well-known non-singular derivative within the Mittag-Leffler kernel in 2016 and used it to solve various science and engineering problems.[24] This operator was recently updated.[25]

In recent years, many global issues have been modeled using arbitrary order calculus.[26] Motivated by the advancement of arbitrary order calculus, many authors have focused on studying the results of nonlinear differential equations with a fractional operator by developing various analytical, semi-analytical, and fully numerical techniques to find approximate solutions.

The primary reason for using non-integer order derivative models is that many systems exhibit memory, history, or non-local effects, which are challenging to model with integer order derivatives. Recent studies have confirmed the effectiveness of the modified Atangana-Baleanu-Caputo (mABC) fractional derivative in solving linear and nonlinear problems, such as the linear time-fractional advection-diffusion equation,[27] the spread of the polio model including the vaccination effect,[28] the hepatitis C model,[29] novel solutions of fractional differential equations,[30] the fractional-order leukemia model[31] and the waterborne disease model.[32]

This study focuses on fractional modeling of the problem of excessive gambling using the mABC non-integer order derivative operator, which is a modified version of the Atangana-Baleanu-Caputo fractional derivative operator.

Systems of fractional differential equations are used to represent many real-life issues. Nonlinear problems can be successfully modeled using a system of arbitrary-order differential equations. However, finding analytical results for systems of ordinary differential equations involving nonlinear terms can be highly difficult, requiring approximation techniques to find numerical values.

Numerous numerical schemes have been created to obtain approximate results of non-integer order differential equations. Some of these methods include the fractional power series method,[33] exponential Galerkin method,[34] spectral collocation method, [35] and Galerkin finite element method.[36]

In the current study, we apply the Lagrange's interpolation approach based on the modified ABC derivative to approximate a novel model of excessive gambling problem.

The paper introduces a novel fractional-order model for gambling addiction problem using the modified ABC operator, addressing memory effects and non-local dynamics that classical models fail to capture. Key contributions include the derivation of the reproduction number $\mathcal{R}_0$ and stability analysis, sensitivity analysis to identify critical intervention parameters and optimal control strategies to manage gambling addiction. The study has some limitations, such as no demographic ageing and fixed parameters, no validation with real-world gambling addiction-reported data, and stochastic effects (e.g., sudden policy changes) may alter dynamics. The motivation for this study comes from the need to introduce a new mathematical model of gambling problems. The mABC operator was chosen for its non-singular kernel and ability to model memory effects, providing a more realistic representation of gambling addiction. The modified ABC fractional derivative operator is an acceptable option because of its ability to represent the dynamical systems effectively. Traditional models fail to capture memory effects and realistic transition dynamics. The modified ABC operator demonstrates superior performance in representing real-world problems. The study makes important contributions to the ongoing advancement of dynamical systems, particularly in gambling addiction.

The remainder of this paper is organized as follows. A definition of mABC fractional derivative and some of its basic properties are provided in Section 2. The fractional model for the problem of excessive gambling and its qualitative analysis are discussed in Section 3. The numerical scheme for approximating the proposed system is presented in Section 4. In Section 5, we discuss the results of numerical simulations. Finally, conclusion is presented.

2. Fundamental concepts

This section provides the most important definitions and properties applied throughout the study. It presents a clear overview of the modified ABC fractional calculus in the Caputo sense.

Definition 1. ([27–32]) *Let $m(t) \in L^1(0,T)$ be a function. The modified ABC derivative of $m(t)$ is defined as:*

$$ {}^{mABC}D^{\upsilon}_{0^+} m(t) = \frac{\mathcal{B}(\upsilon)}{1-\upsilon} \left[\begin{array}{c} m(t) - E_\upsilon(-\gamma_\upsilon t^\upsilon) m(0) \\ -\gamma_\upsilon \int_0^t (t-\eta)^{\upsilon-1} m(\eta) \\ \times E_{\upsilon,\,\upsilon}(-\gamma_\upsilon (t-\eta)^\upsilon)\, d\eta \end{array} \right]. \tag{1} $$

where $\gamma_\upsilon = \dfrac{\upsilon}{1-\upsilon}$, $\upsilon \in (0,\ 1)$ is the order of derivative, and E_υ is the Mittag-Leffler function defined by:

$$ E_\upsilon(\chi) = \sum_{i=0}^{\infty} \frac{\chi^i}{\Gamma(\upsilon i+1)}, \quad \chi \in \mathbb{C}, \tag{2} $$

and

$$ E_{\upsilon,\,\sigma}(\chi) = \sum_{i=0}^{\infty} \frac{\chi^i}{\Gamma(\upsilon i+\sigma)}, \quad \chi \in \mathbb{C}, \tag{3} $$

where $\upsilon \in (0,\ 1)$, $\mathcal{B}(\upsilon) = 1-\upsilon+\dfrac{\upsilon}{\Gamma(\upsilon)}$ characterized as $\psi(0) = \psi(1) = 1$. The $\mathcal{L}$-transform of equation (1) is defined as:

$$\mathcal{L}\left[{}_{0}^{mABC}D_{0}^{\upsilon}m(t)\right](\eta)$$
$$= \frac{\mathcal{B}(\upsilon)}{1-\upsilon}\left[\frac{\eta^{\upsilon}\mathcal{L}\{m(t)\}(\eta) - \eta^{\upsilon-1}m(0)}{\eta^{\upsilon}+\gamma_{\upsilon}}\right]. \tag{4}$$

Definition 2. ([27–32]) *Let $m(t) \in L^1(0,\ T)$ be a function. The modified ABC integral is then express as follows:*

$$^{mAB}I_{0}^{\upsilon}m(t) = \frac{\psi(1-\upsilon)}{\mathcal{B}(\upsilon)}\left[m(t)-m(0)\right] + \gamma_{\upsilon}\left[{}^{RL}I_{0}^{\upsilon}\left(m(t)-m(0)\right)\right]. \tag{5}$$

where, the Riemann-Liouville integral operator is given by

$$^{RL}I^{\upsilon}m(t) = \frac{1}{\Gamma(\upsilon)}\int_{0}^{t}\frac{s(\eta)}{(t-\eta)^{1-\upsilon}}d\eta. \tag{6}$$

3. Model formulation

In this section, we consider a newly created model of excessive gambling based on modified ABC fractional differential equations. We focus on the NAMPR model, which addresses the problem of gambling within a population. This model comprises the following five types.

- **N:** The number of individuals who know nothing about gambling (non-gamblers).
- **A:** The number of individuals who are aware of the risks of gambling and gamble without problems but are exposed to problem gambling (no problem gamblers).
- **M:** The number of individuals who have minor symptoms of problem gambling and are at risk of addiction (minor-risk gamblers).
- **P:** The number of individuals who are addicted to gambling and gamble permanently (permanent gamblers).
- **R:** The number of individuals who have stopped or recovered from problem gambling (recovered gamblers).

The total population is computed as follows:

$$T(t) = N(t) + A(t) + M(t) + P(t) + R(t).$$

All parameters in the model are assumed to have values greater than zero and are defined as follows: all new recruits, assumed to have no awareness of the risks of gambling and never having gambled before, are recruited at a rate of Λ. An individual in stage N is not introduced to gambling, but contacts from gamblers urge them to gamble with no problems, leading them to transition to stage A at time t. This contact is modeled using a parameter α_1, which is directly related to the proportion of non-problem gamblers, minor risk gamblers, and excessive problem gamblers already participating in gambling activities.

$$\frac{\vartheta_1}{\alpha_1} = \frac{\gamma_1 A + \gamma_2 M + \gamma_3 P}{T}.$$

Individuals in stage A may transfer to stage R by stopping any gambling activities before addiction, measured using a parameter $\wp$, or they may develop minor symptoms of problem gambling. This step could be induced by the influence rate, measured in terms of the proportion of already existing problematic gambling stages: $\frac{\gamma_2 M + \gamma_3 P}{T}$ multiplied by a proportionality parameter α_2. Therefore, the overall problem gambling rate is:

$$\frac{\vartheta_2}{\alpha_2} = \frac{\gamma_2 M + \gamma_3 P}{T}.$$

There are three possible transition directions from stage M. Individuals in stage M may return to stage A by minimizing the frequency and intensity of their gambling, measured using a parameter λ, transfer to stage R by stopping gambling activities, denoted by the stopping rate ς, or transition to the excessive gambling problem stage. The latter step can be introduced by the influence rate, measured in terms of the proportion in the excessive gambling stage: $\frac{\gamma_3 P}{T}$ multiplied by a constant α_3. Therefore, the overall problem of the excessive gambling rate is: $\vartheta_3 = \alpha_3\frac{\gamma_3 P}{T}$.

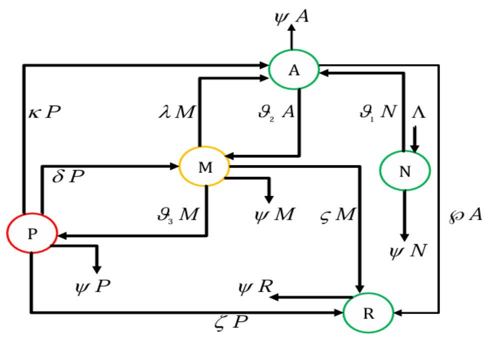

Figure 1. Problem gambling model flow diagram

There are also three possible transition directions from Stage P. Addicted gamblers may recover naturally or with the help of religion, social media awareness, school education, and professional help. When an addicted gambler returns to being a non-problem gambler, a transition to stage A occurs, modeled using κ. Individuals in the excessive problem gambling stage will either recover or stop all gambling activities, resulting in

a transition to stage R, modeled using the recovery parameter ζ. As the severity of the gambling problem decreases, an addicted problem gambler can transition to the minimum risk stage M at a rate δ. We assumed that recovered individuals would not return to the gambling activities and would never gamble again. The parameter ψ denotes the natural death rate. The model assumes no gambling problem-induced death and does not consider the age of gamblers. The approximate results of the gambling problem model using the modified ABC fractional operator are obtained using the Tofik-Atangana numerical scheme.
Figure 1 is a flow diagram of the gambling problem model, illustrating transitions between compartments and key parameters governing dynamics. In the modified ABC fractional operator, a model system of differential equations with positive initial conditions can be defined as follows:

$$\begin{cases} {}^{*}\mathcal{D}_t^{v} N(t) = \Phi_1(t) = \Lambda - \vartheta_1 N - \psi N, \\ {}^{*}\mathcal{D}_t^{v} A(t) = \Phi_2(t) = \vartheta_1 N + \lambda M + \kappa P \\ \qquad - (\wp + \vartheta_2 + \psi) A, \\ {}^{*}\mathcal{D}_t^{v} M(t) = \Phi_3(t) = \vartheta_2 A + \delta P \\ - (\lambda + \varsigma + \vartheta_3 + \psi) M, \\ {}^{*}\mathcal{D}_t^{v} P(t) = \Phi_4(t) = \vartheta_3 M - (\kappa + \delta + \zeta + \psi) P, \\ {}^{*}\mathcal{D}_t^{v} R(t) = \Phi_5(t) = \wp A + \varsigma M + \zeta P - \psi R. \end{cases} \tag{7}$$

where $*$ represents $mABC$. Equation (7) describes the rates of change for each compartment, incorporating fractional-order dynamics via the modified ABC operator. Parameters ϑ_1, ϑ_2 and ϑ_3 demonstrate model transitions influenced by problem gambler interactions.

3.1. Qualitative analysis

Here, we discuss the basic behaviors of model (7) such as solvability, existence and uniqueness, positivity, and boundedness of solutions. For $0 < t < T < +\infty$, system (7) can be written as follows:

$$^{mABC}D_0^{v} m(t) = \Phi(t,\ m(t)), \tag{8}$$

with starting solution

$$m(0) = m_0, \tag{9}$$

where, the vector-valued functions $m : [0,\ +\infty) \to \mathbb{R}^5$ and $\Phi : \mathbb{R}^5 \to \mathbb{R}^5$ such that

$$m(t) = \begin{pmatrix} N(t) \\ A(t) \\ M(t) \\ P(t) \\ R(t) \end{pmatrix},$$

and

$$\Phi(m(t)) = \begin{pmatrix} \Phi_1(t) \\ \Phi_2(t) \\ \Phi_3(t) \\ \Phi_4(t) \\ \Phi_5(t) \end{pmatrix}.$$

Theorem 1. *The function Φ in (8) is Lipschitz continuous in η.*

Proof. For the state variable N, we have

$$\|\Phi_1(t,\ N) - \Phi_1(t,\ N_1)\|$$

$$= \left\| \alpha_1 \frac{\gamma_1 A + \gamma_2 M + \gamma_3 P}{T} + \psi \right\| \|N_1 - N\|.$$

Using triangular inequality, we have

$$\|\Phi_1(t,\ N) - \Phi_1(t,\ N_1)\| \le c\,\|N_1 - N\|.$$

where $c = \alpha_1 \frac{\gamma_1 \varrho_2 + \gamma_2 \varrho_3 + \gamma_3 \varrho_3}{T} + \psi$.

$$\begin{cases} \|N(t)\| = \sup_{t\in T} |N(t)| = \varrho_1, \\ \|A(t)\| = \sup_{t\in T} |A(t)| = \varrho_2, \\ \|M(t)\| = \sup_{t\in T} |M(t)| = \varrho_3, \\ \|P(t)\| = \sup_{t\in T} |P(t)| = \varrho_4, \\ \|R(t)\| = \sup_{t\in T} |R(t)| = \varrho_5. \end{cases}$$

Thus, $\Phi_1(t,\ N)$ satisfies the Lipschitz condition with the Lipschitz constant c. Moreover, if $0 \le c < 1$, then $\Phi_1(t,\ N)$ is a contraction. Similarly, we can show the existence of Lipschitz constants and the contraction principle for $\Phi_2(t)$, $\Phi_3(t)$, $\Phi_4(t)$, and $\Phi_5(t)$. Consequently, we have:

$$\|\Phi(t,\ m(t)) - \Phi(t,\ m_1(t))\|_\infty \le \eta_\Phi \|m - m_1\|_\infty. \tag{10}$$

and hence Φ is a Lipschitz continuous function.

Theorem 2. ***(Solvability)*** *The mABC fractional gambling model has at least one solution $m(t) \in C([0,\ \tau],\ \mathbb{R}_+^5)$.*

Proof. The mABC system is equivalent to:

$$m(t) - m_0 = {}^{mAB} I_0^{v} \Phi(t,\ m(t)). \tag{11}$$

Equation (11) has the following form:

$$m(t)-m_0=\frac{1-\upsilon}{\mathcal{B}(\upsilon)}\Phi(t,\ m(t))$$
$$+\frac{\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon)}\int_0^t(t-s)^{\upsilon-1}\Phi(s,\ m((s))\,ds$$
$$-\frac{1-\upsilon}{\mathcal{B}(\upsilon)}\Phi(0,\ m(0))\left(1+\frac{\gamma_\upsilon}{\Gamma(\upsilon+1)}t^\upsilon\right). \tag{12}$$

Define $T:\mathbb{C}\left([0,\ \tau],\ \mathbb{R}^5\right)\to\mathbb{C}\left([0,\ \tau],\ \mathbb{R}^5\right)$ as:

$$T[m](t)=m_0+\frac{1-\upsilon}{\mathcal{B}(\upsilon)}\Phi(t,\ m(t))$$
$$+\frac{\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon)}\int_0^t(t-s)^{\upsilon-1}\Phi(s,\ m((s))\,ds$$
$$-\frac{1-\upsilon}{\mathcal{B}(\upsilon)}\Phi(0,\ m(0))\left(1+\frac{\gamma_\upsilon}{\Gamma(\upsilon+1)}t^\upsilon\right). \tag{13}$$

Since Φ is continuous, and the modified ABC integral preserves continuity $\Rightarrow$ T is continuous. Next, we need to show that all possible solutions to the equation $m=\delta T[m]$ for $\delta\in[0,\ 1]$ are uniformly bounded in the space $C\left([0,\ \tau],\ \mathbb{R}^5_+\right)$. For $\delta\in[0,\ 1]$, the equation $m=\delta T[m]$ expands to:

$$m(t)=\delta m_0+\delta\left[\frac{1-\upsilon}{\mathcal{B}(\upsilon)}\Phi(t,\ m(t))\right.$$
$$+\frac{\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon)}\int_0^t(t-s)^{\upsilon-1}\Phi(s,\ m((s))\,ds$$
$$\left.-\frac{1-\upsilon}{\mathcal{B}(\upsilon)}\Phi(0,\ m(0))\left(1+\frac{\gamma_\upsilon}{\Gamma(\upsilon+1)}t^\upsilon\right)\right].$$

Taking the supremum norm of both sides, we have:

$$||m||\le\delta||m_0||+\delta\left[\frac{1-\upsilon}{\mathcal{B}(\upsilon)}||\Phi(t,\ m(t))||\right.$$
$$+\frac{\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon)}\int_0^t(t-s)^{\upsilon-1}||\Phi(s,\ m((s))||ds$$
$$\left.+\frac{1-\upsilon}{\mathcal{B}(\upsilon)}\Phi(0,\ m(0))\left(1+\frac{\gamma_\upsilon}{\Gamma(\upsilon+1)}t^\upsilon\right)\right].$$

Simplify the integral:

$$||m||\le\delta||m_0||+\delta\left[\frac{1-\upsilon}{\mathcal{B}(\upsilon)}||\Phi||+\frac{\tau^\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon)}||\Phi||\right.$$
$$\left.+\frac{1-\upsilon}{\mathcal{B}(\upsilon)}\Phi_0\left(1+\frac{\gamma_\upsilon}{\Gamma(\upsilon+1)}\tau^\upsilon\right)\right].$$

Since $\delta\in(0,\ 1)$, we have:

$$||m||\le||m_0||+\frac{1-\upsilon}{\mathcal{B}(\upsilon)}\left[1+\frac{\tau^\upsilon}{(1-\upsilon)\Gamma(\upsilon)}\right]||\Phi||$$
$$+\frac{1-\upsilon}{\mathcal{B}(\upsilon)}\Phi_0\left(1+\frac{\gamma_\upsilon}{\Gamma(\upsilon+1)}\tau^\upsilon\right).$$

Assume that

$$||\Phi||\le C_\Phi(1+||m||).$$

Then,

$$||m||\le||m_0||+\frac{1-\upsilon}{\mathcal{B}(\upsilon)}\left(1+\frac{\gamma_\upsilon}{\Gamma(\upsilon+1)}\tau^\upsilon\right)$$
$$\times(\Phi_0+C_\Phi(1+||m||)).$$

Let

$$k=\frac{1-\upsilon}{\mathcal{B}(\upsilon)}\left(1+\frac{\gamma_\upsilon}{\Gamma(\upsilon+1)}\tau^\upsilon\right).$$

Then

$$||m||\le||m_0||+k(\Phi_0+C_\Phi(1+||m||)).$$

By rearranging this inequality, we obtain:

$$||m||\le\frac{||m_0||+k\Phi_0+kC_\Phi}{1-kC_\Phi}.$$

Therefore,

$$||m||\le M:=\frac{||m_0||+k\Phi_0+kC_\Phi}{1-kC_\Phi}.$$

This proves uniform boundedness.

Let $t_1,\ t_2\in[0,\ \tau]$. From the definition of T, we have:

$$T[m](t_2)-T[m](t_1)=$$
$$\frac{1-\upsilon}{\mathcal{B}(\upsilon)}(\Phi(t_2,\ m(t_2))-\Phi(t_1,\ m(t_1)))$$
$$+\frac{\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon)}(I_2-I_1) \tag{14}$$
$$+\frac{\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon+1)}\Phi_0\left[t_1^\upsilon-t_2^\upsilon\right],$$

where

$$I_2-I_1=\int_{t_1}^{t_2}(t_2-s)^{\upsilon-1}\Phi(s,\ m((s))\,ds$$
$$+\int_0^{t_1}\left((t_2-s)^{\upsilon-1}-(t_1-s)^{\upsilon-1}\right)\Phi(s,\ m((s))\,ds.$$

Since Φ is Lipschitz continuous in m and continuous in t, there exists η_Φ such that:

$$||\Phi(t_2,\ m(t_2))-\Phi(t_1,\ m(t_1))||$$
$$\le\eta_\Phi(|t_2-t_1|+||m(t_2-m(t_1)||). \tag{15}$$

Using mean value theorem, there exists $\chi\in(t_1,\ t_2)$ such that

$$|(t_2-s)^{\upsilon-1}-(t_1-s)^{\upsilon-1}|=(\upsilon-1)(\chi-s)^{\upsilon-2}|t_2-t_1|,$$

and we have:

$$||I_2 - I_1|| \le ||\Phi|| \left(\frac{(t_2 - t_1)^{\upsilon}}{\upsilon} + \tau^{\upsilon}(t_2 - t_1)^{\upsilon} \right). \tag{16}$$

From Equations (14) -(16), we have:

$$\begin{aligned} &||T[m]\,(t_2) - T[m]\,(t_1)\,|| \\ &\le \frac{1-\upsilon}{\psi(\upsilon)}\left(\eta_\Phi\left(|t_2 - t_1| + ||m(t_2 - m(t_1)||\right)\right) \\ &+\frac{\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon)}||\Phi||\left(\frac{(t_2 - t_1)^{\upsilon}}{\upsilon} + \tau^{\upsilon}(t_2 - t_1)^{\upsilon}\right) \\ &+\frac{\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon+1)}\Phi_0\left|t_2^{\upsilon} - t_1^{\upsilon}\right|. \end{aligned}$$

For $\upsilon \in (0,\ 1)$ and $t_2 > t_1 > 0$, we have:

$$t_2^{\upsilon} = (t_1 + (t_2 - t_1))^{\upsilon} \le t_1^{\upsilon} + (t_2 - t_1)^{\upsilon}$$

$$\Rightarrow |t_2^{\upsilon} - t_1^{\upsilon}| \le (t_2 - t_1)^{\upsilon}.$$

Thus,

$$||T[m]\,(t_2) - T[m]\,(t_1)\,|| \le c\,(t_2 - t_1)^{\upsilon},$$

where

$$c = \frac{1-\upsilon}{\psi(\upsilon)}\left(\eta_\Phi + \eta_\Phi k\right) + \frac{1}{\mathcal{B}(\upsilon)\Gamma(\upsilon)}\left(2 + \upsilon\tau^{\upsilon}\right)||\Phi||.$$

Given $\epsilon > 0$, choose $\delta = (\epsilon/c)^{1/\upsilon}$ such that

$$|t_2 - t_2| < \delta \Rightarrow ||T[m]\,(t_2) - T[m]\,(t_1)\,|| < \epsilon.$$

This holds uniformly for all m, proving equicontinuity. Since T is continuous, compact, and the set $m = \delta T[m]$ is bounded, there exists at least one fixed point $m = \delta T[m]$ which solves the system.

3.1.1. Existence and uniqueness of the solution

As shown in Theorem 2, T is continuous, $m = \delta T[m],\ \delta \in (0,\ 1)$ is bounded uniformly, and T is equicontinuous. By Schaefer's theorem, T has a fixed point $\hat{m} = T[\hat{m}]$, which is a solution.

Theorem 3. *Assuming that the condition of Equation* (15) *is satisfied, there is a solution of the mABC fractional system given by Equation* (8).

Proof. Equation (10) confirms the Lipschitz condition of $\Phi(t,\ m(t))$. For $m_1, m_2 \in C([0,\tau],\ \mathbb{R}^5,$ we have

$$\begin{aligned} ||T[m_2](t) - T[m_1](t)|| &\le \frac{1-\upsilon}{\mathcal{B}(\upsilon)}\eta_\Phi||m_2 - m_1|| \\ &+\frac{\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon)}\eta_\Phi||m_2 - m_1||\frac{\tau^{\upsilon}}{\upsilon} \\ &= \left(\frac{1-\upsilon}{\mathcal{B}(\upsilon)} + \frac{\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon+1)}\tau^{\upsilon}\right)\eta_\Phi||m_2 - m_1||. \end{aligned}$$

Define the contraction constant:

$$k(\tau) = \left(\frac{1-\upsilon}{\mathcal{B}(\upsilon)} + \frac{\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon+1)}\tau^{\upsilon}\right)\eta_\Phi < 1.$$

This is always possible since $\psi(\upsilon) = 1 - \upsilon + \frac{\upsilon}{\Gamma(\upsilon)} > 1 - \upsilon$:

$$\lim_{\tau\to 0^+} k(\tau) = \frac{1-\upsilon}{\mathcal{B}(\upsilon)}\eta_\Phi < 1$$

We assume that $\sup_{t\in[0,\ \tau]}|\Phi\,(t,\ m\,(0))| = \varrho$, $S_\upsilon = \left\{m\,(t) \in \mathbb{C}\left([0,\ \tau]\,,\ \mathbb{R}^5\right) : \|m\,(t)\|\right\} < \upsilon$. For $m\,(t) \in S_\upsilon$, and $t \in [0,\ \tau]$, we have

$$\|\Phi\,(t,\ m\,(t))\| = \|\Phi\,(t,\ m\,(t)) - \Phi_0 + \Phi_0\| \le \eta_\Phi\upsilon + \varrho. \tag{17}$$

Furthermore, for $m \in \left\{m \in \mathbb{C}\left([0,\ \tau]\,,\ \mathbb{R}^5\right)\right\}$, we have

$$\begin{aligned} \|m\,(t)\| = \max\|Tm\,(t)\| &\le m_0 + \frac{1-\upsilon}{\mathcal{B}(\upsilon)}\left(\eta_\Phi\,\|m\,(t)\| + \varrho\right) \\ &+\frac{\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon)}\left(\eta_\Phi\,\|m\,(t)\| + \varrho\right)t^{\upsilon} \\ &+\frac{1-\upsilon}{\mathcal{B}(\upsilon)}\Phi_0\left(1 + \frac{\gamma_\upsilon}{\Gamma(\upsilon+1)}t^{\upsilon}\right) \\ &= \pi_1 + \pi_2\|m\,(t)\,\|. \end{aligned}$$

We have that

$$\pi_1 = m_0 + \frac{1-\upsilon}{\mathcal{B}(\upsilon)}\Phi_0\left(1 + \frac{\gamma_\upsilon}{\Gamma(\upsilon+1)}t^{\upsilon}\right) + \left[\frac{1-\upsilon}{\mathcal{B}(\upsilon)} + \frac{\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon)}t^{\upsilon}\right]\varrho,$$

$$\pi_2 = \eta_\Phi\left[\frac{1-\upsilon}{\mathcal{B}(\upsilon)} + \frac{\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon)}t^{\upsilon}\right].$$

Thus,

$$\|m\,(t)\,\| \le \frac{\pi_1}{1-\pi_2}. \tag{18}$$

According to Leray-Schauder's alternative theorem, system (8) has a solution.

3.1.2. Positivity and boundedness of the result

Theorem 4. *The set of solutions of the system* (7) *with nonnegative initial conditions defined in the region* $\Omega = \{(N,\ A,\ M,\ P,\ R) \in \mathbb{R}_+^5 : 0 \le N(t) + A(t) + M(t) + P(t) + R(t) :\ T(t) \le \frac{\Lambda}{\psi}\}$ *is positively invariant.*

Proof. Differentiating the total population $T(t) = N(t) + A(t) + M(t) + P(t) + R(t)$ with respect to time and using equations in system (7), we obtain

$$ {}^{mABC}D_0^{\upsilon}T(t) \le \Lambda - \psi T(t). \tag{19} $$

Applying the Laplace transform on two sides of the above inequality, we obtain:

$$ \mathcal{L}\left\{{}^{mABC}D_0^{\upsilon}T(t);\ s\right\} \le \frac{\Lambda}{s} - \psi\mathcal{L}\{T(t);\ s\} $$

$$ \Rightarrow \frac{\mathcal{B}(\upsilon)s^{\upsilon}\mathcal{L}\{T(t);\ s\}}{\upsilon + (1-\upsilon)s^{\upsilon}} + \psi\mathcal{L}\{T(t);\ s\} \le \Lambda s^{\upsilon-(\upsilon+1)} + \frac{\mathcal{B}(\upsilon)T(0)s^{\upsilon-1}}{\upsilon+(1-\upsilon)s^{\upsilon}}. $$

Implies

$$ \mathcal{L}\{T(t);\ s\} \le \frac{\Lambda\upsilon}{\mathcal{B}(\upsilon)+(1-\upsilon)\psi} \times \frac{s^{\upsilon-(\upsilon+1)}}{s^{\upsilon}+\dfrac{\upsilon\psi}{\mathcal{B}(\upsilon)+(1-\upsilon)\psi}} + \left(\frac{\Lambda(1-\upsilon)}{\mathcal{B}(\upsilon)+(1-\upsilon)\psi} + \frac{\mathcal{B}(\upsilon)T(0)}{\mathcal{B}(\upsilon)+(1-\upsilon)\psi}\right) \times \frac{s^{\upsilon-1}}{s^{\upsilon}+\dfrac{\upsilon\psi}{\mathcal{B}(\upsilon)+(1-\upsilon)\psi}}. $$

Applying the inverse Laplace, we arrive at:

$$ T(t) \le \frac{\Lambda\upsilon}{\mathcal{B}(\upsilon)+(1-\upsilon)\psi}E_{\upsilon,\ \upsilon+1}(-\eta t^{\upsilon}) + \frac{1}{\mathcal{B}(\upsilon)+(1-\upsilon)\psi} \times (\Lambda(1-\upsilon)+\mathcal{B}(\upsilon)T(0))E_{\upsilon,\ 1}(-\eta t^{\upsilon}), \tag{20} $$

where $\eta = \dfrac{\upsilon\psi}{\mathcal{B}(\upsilon)+(1-\upsilon)\mu}$ and $E_{\alpha,\ \beta}(.)$ is the Mittag-Leffler function of two parameters $\alpha > 0$ and $\beta > 0$ defined as:

$$ E_{\alpha,\ \beta}(\chi) = \sum_{i=0}^{\infty}\frac{x^i}{\Gamma(\alpha i+\beta)}, $$

whose Laplace transform is

$$ \mathcal{L}\left\{\chi^{\beta-1}E_{\alpha,\ \beta}(\mp\eta\chi^{\alpha});\ s\right\} = \frac{s^{\alpha-\beta}}{s^{\alpha}\pm\eta} $$

provided that $s > |\eta|^{1/\alpha}$. The Mittag-Leffler function satisfies

$$ E_{\alpha,\ \beta}(\chi) = \frac{E_{\alpha,\ \beta-\alpha}(\chi)}{\chi} - \frac{1}{\chi\Gamma(\beta-\alpha)}, $$

and for the case $\alpha = \upsilon,\ \beta = \upsilon+1$ and $x = -\eta t^{\upsilon}$, we have

$$ E_{\upsilon,\ \upsilon+1}(-\eta t^{\upsilon}) = \frac{1}{\eta t^{\upsilon}}(1 - E_{\upsilon,\ 1}(-\eta t^{\upsilon})). \tag{21} $$

The Mittag-Leffeler function is bounded for all $t > 0$, possess an asymptotic behavior,[37] introducing the relation (21) in inequality (20), it is obvious that $T(t) \le \dfrac{\Lambda}{\psi}$ as $t \to \infty$. Thus, $N(t)$ and all other variables of the system (7) are bounded in the region Ω.

Theorem 5. *If the set of initial conditions* $\{N(0) \ge 0,\ A(0) \ge 0,\ M(0) \ge 0,\ P(0) \ge 0,\ R(0) \ge 0\} \in \mathbb{R}^5_+$ *then, the solutions of* $N(t),\ A(t),\ M(t),\ P(t)$, *and* $R(t)$ *are non-negative for all* $t \ge 0$.

Proof. Assume that all the state variables of the model are continuous. Consider the first equation of the system (7):

$$ {}^{mABC}D_0^{\upsilon}N(t) = \Lambda - \alpha_1 N\frac{\gamma_1 A+\gamma_2 M+\gamma_3 P}{T} - \psi N. \tag{22} $$

Since, all the solutions are bounded, let $\dfrac{\gamma_1 A+\gamma_2 M+\gamma_3 P}{T}$ is bounded by σ. Then,

$$ {}^{mABC}D_0^{\upsilon}N(t) \ge -aN, \tag{23} $$

where $a = \alpha_1\sigma + \psi$. Applying the Laplace transform in Eq. (23) and using Eq. (4), we have:

$$ \frac{\mathcal{B}(\upsilon)}{1-\upsilon}\left[\frac{s^{\upsilon}\mathcal{L}\{N(t);\ s\} - N(0)s^{\upsilon-1}}{s^{\upsilon}+\dfrac{\upsilon}{1-\upsilon}}\right] \ge -a\mathcal{L}\{N(t)\}. $$

Further simplification gives:

$$ \frac{\mathcal{B}(\upsilon)s^{\upsilon}\mathcal{L}\{N(t);\ s\}}{\upsilon+(1-\upsilon)s^{\upsilon}} - \frac{\mathcal{B}(\upsilon)N(0)s^{\upsilon-1}}{\upsilon+(1-\upsilon)s^{\upsilon}} \ge -a\mathcal{L}\{N(t)\}. $$

$$ [\mathcal{B}(\upsilon)s^{\upsilon} + a\upsilon + a(1-\upsilon)s^{\upsilon}]\mathcal{L}\{N(t);\ s\} \ge \mathcal{B}(\upsilon)s^{\upsilon-1}N(0) $$

$$ \mathcal{L}\{N(t);\ s\} \ge \frac{\mathcal{B}(\upsilon)N(0)}{\mathcal{B}(\upsilon)+a(1-\upsilon)}\frac{s^{\upsilon-1}}{s^{\upsilon}+\dfrac{a\upsilon}{\mathcal{B}(\upsilon)+a(1-\upsilon)}}. $$

Taking the inverse Laplace transform in the above inequality, we obtain:

$$ N(t) \ge \frac{\mathcal{B}(\upsilon)N(0)}{\mathcal{B}(\upsilon)+a(1-\upsilon)}E_{\upsilon,\ 1}\left(\frac{-a\upsilon t^{\upsilon}}{\mathcal{B}(\upsilon)+a(1-\upsilon)}\right). $$

Since $E_{\upsilon,\ 1}\left(-\dfrac{a\upsilon}{\mathcal{B}(\upsilon)+a(1-\upsilon)}t^{\upsilon}\right) > 0$ and $N(0) > 0$, then $N(t) \ge 0$. In the same way, it is easy to prove that the remaining state variables are non-negative for all $t > 0$. Therefore, the solutions of system (7) are non-negative for all $t \geq 0$.

3.2. Stability analysis of the model

In this section, the basic reproduction number and stability analyses of endemic and no problem gambling equilibrium points are discussed.

3.2.1. No problem gambling equilibrium point

To find the gambling problem-free equilibrium point of the model, we set the derivatives of all compartments to zero and assume that there are no individuals with gambling problem ($M = 0,\ P = 0$ and $R = 0$). The system reduces to:

$$\begin{cases} \Lambda - \alpha_1 N \dfrac{\gamma_1 A}{T} - \psi N = 0, \\ \\ \alpha_1 N \dfrac{\gamma_1 A}{T} - (\wp + \psi) A = 0. \end{cases} \tag{24}$$

Solving these equations yield:

$$E_0 = \left(s,\ \frac{\alpha_1\gamma_1 - \wp - \psi}{\wp + \psi} s,\ 0,\ 0,\ 0 \right).$$

where $s = \dfrac{\Lambda}{\alpha_1\gamma_1 - \wp}$ and $\alpha_1\gamma_1 > \wp$ to ensure positivity of N_0 and $\alpha_1\gamma_1 > \wp + \psi$ for positivity of A_0. This equilibrium represents a state where there are no individuals with gambling problems and the population consists only of non-gamblers (N) and non-problem gamblers (A).

3.2.2. The basic reproduction number

The basic reproduction number $\mathcal{R}_0$ is a critical threshold in epidemiological models, representing the average number of new "infections" (in this case, new problem gamblers) generated by a single addicted gambler in a fully susceptible population.

$$F = \begin{bmatrix} \alpha_2\gamma_2 q & \alpha_2\gamma_3 q \\ 0 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} u & -\delta \\ 0 & v \end{bmatrix}$$

where

$$q = \frac{\alpha_1\gamma_1 - \wp - \psi}{\alpha_1\gamma_1},\ u = \lambda + \varsigma + \psi,$$
$$v = \kappa + \delta + \zeta + \psi.$$

Using the next-generation matrix method, the basic reproduction number of the proposed model is the dominant eigenvalue of FV^{-1}. Therefore,

$$\mathcal{R}_0 = \alpha_2\gamma_2 \frac{\alpha_1\gamma_1 - \wp - \psi}{\alpha_1\gamma_1(\lambda + \varsigma + \psi)}.$$

Theorem 6. *The gambling problem-free equilibrium point E_0 is locally asymptotically stable when $\mathcal{R}_0 < 1$ and unstable when $\mathcal{R}_0 > 1$.*

Proof. To prove the local stability, we consider the Jacobian matrix of the system (7) at E_0 as follows:

$$J(E_0) = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & 0 \\ a_{21} & 0 & a_{23} & a_{24} & 0 \\ 0 & 0 & a_{33} & a_{34} & 0 \\ 0 & 0 & 0 & a_{44} & 0 \\ 0 & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix}, \tag{25}$$

where

$$a_{11} = -(\alpha_1\gamma_1 - \wp), \quad a_{12} = -(\wp + \psi)$$

$$a_{13} = -\gamma_2 \frac{\wp + \psi}{\gamma_1}, \quad a_{14} = -\gamma_3 \frac{\wp + \psi}{\gamma_1}$$

$$a_{21} = \alpha_1\gamma_1 - \wp - \psi,$$

$$a_{23} = \gamma_2 \frac{\wp + \psi}{\gamma_1} + \lambda - \alpha_2\gamma_2 \frac{\alpha_1\gamma_1 - \wp - \psi}{\alpha_1\gamma_1}$$

$$a_{24} = \gamma_3 \frac{\wp + \psi}{\gamma_1} + \kappa - \alpha_2\gamma_3 \frac{\alpha_1\gamma_1 - \wp - \psi}{\alpha_1\gamma_1},$$

$$a_{25} = 0, \quad a_{52} = \wp, \quad a_{53} = \varsigma, \quad a_{54} = \zeta$$

$$a_{33} = \left(\alpha_2\gamma_2 \frac{\alpha_1\gamma_1 - \wp - \psi}{\alpha_1\gamma_1} \right) - (\lambda + \varsigma + \psi)$$

$$a_{34} = \left(\alpha_2\gamma_3 \frac{\alpha_1\gamma_1 - \wp - \psi}{\alpha_1\gamma_1} \right) + \delta,$$

$$a_{44} = -(\kappa + \delta + \zeta + \psi), \quad a_{55} = -\psi$$

In this Jacobian matrix, three of the eigenvalues are negative, that is $m_1 = -\psi$, $m_2 = -(\kappa + \delta + \zeta + \psi)$ and $m_3 = -(\lambda + \varsigma + \psi)(1 - \mathcal{R}_0)$ for $\mathcal{R}_0 < 1$. The remaining eigenvalues can be obtained from the characteristic equation:

$$m^2 + d_1 m + d_2 = 0. \tag{26}$$

Since the coefficients $d_1 = \alpha_1\gamma_1 - \wp > 0$ for $\alpha_1\gamma_1 > \wp$ and $d_2 = (\wp + \psi)(\alpha_1\gamma_1 - \wp - \psi) > 0$ for $\alpha_1\gamma_1 > \wp + \psi$. The solutions of Equation (26) have negative real parts. Since the $Im(m_i = 0,\ \ i = 1,\ 2,\ 3,\ 4$, then clearly $|\arg(m_i)| = \pi > \dfrac{\upsilon\pi}{2}$, $\upsilon \in (0, 1)$. According to the Matignon criterion,[38] the equilibrium point E_0 is stable when $\mathcal{R}_0 < 1$.

3.2.3. Endemic equilibrium point

Adjusting the model equation to zero and solve simultaneously, we get the endemic equilibrium point $E^* = (N^*,\ A^*,\ M^*,\ P^*,\ R^*)$. where

$$M^* = \frac{c\Lambda}{\alpha_3\gamma_3\psi}, \quad \text{where } c = \kappa + \delta + \zeta + \psi$$

$$A^* = \frac{(d + \alpha_3\gamma_3 P^*)\dfrac{c\Lambda}{\alpha_3\gamma_3\psi} - \delta P^*}{\alpha_2\left(\dfrac{c\gamma_2}{\alpha_3\gamma_3} + \dfrac{\psi\gamma_3}{\Lambda}P^*\right)}, \quad d = \varsigma + \lambda + \psi$$

$$N^* = \frac{\Lambda}{\alpha_1\dfrac{\gamma_1 A^* + \gamma_2 M^* + \gamma_3 P^*}{T^*} + \psi}$$

$$R^* = \frac{\wp A^* + \varsigma M^* + \zeta P^*}{\psi}$$

and we have the following quadratic equation for P^*

$$a_1 P^{*2} - a_2\left(1 - \frac{1}{\mathcal{R}_0}\right)P^* = 0,$$

where $a_1 = \dfrac{\psi\alpha_2\gamma_3^2}{\Lambda\gamma_2(\kappa+\delta+\zeta+\psi)}$ and $b = \dfrac{(\varsigma+\lambda+\psi)\alpha_1\gamma_1}{\alpha_2\gamma_2}$. Since $P^* \neq 0$, we have $P^* = \dfrac{a_2}{a_1}\left(1 - \dfrac{1}{\mathcal{R}_0}\right)$ and P^* is positive when $\mathcal{R}_0 > 1$.

Theorem 7. *The gambling problem-free equilibrium point E_0 is globally asymptotically stable if $\mathcal{R}_0 \leq 1$ and unstable if $\mathcal{R}_0 > 1$.*

Proof. Assume the Lyapunov function:

$$L(t) = M(t) + P(t).$$

Taking the modified ABC derivative both sides and using Equation (7), we have:

$$\begin{aligned} {}^*\mathcal{D}_t^{\upsilon} L(t) &= \alpha_2\frac{\gamma_2 M + \gamma_3 P}{T}A + \delta P \\ &-(\lambda + \varsigma + \alpha_3\frac{\gamma_3 P}{T} + \psi)M \\ &+\alpha_3\frac{\gamma_3 P}{T}M - (\kappa+\delta+\zeta+\psi)P \end{aligned}.$$

At the gambling problem-free equilibrium point E_0, we have:

$$\begin{aligned} {}^*\mathcal{D}_t^{\upsilon} L(t) &= \alpha_2\gamma_2 M\frac{\alpha_1\gamma_1 - \wp - \psi}{\alpha_1\gamma_1} - (\lambda+\varsigma+\psi)M \\ &+\alpha_2\gamma_3 P\frac{\alpha_1\gamma_1 - \wp - \psi}{\alpha_1\gamma_1} - (\kappa+\zeta+\psi)P \\ &= (\Re 0 - 1)(\lambda+\varsigma+\psi)M \\ &+ \left[\alpha_2\gamma_3\frac{\alpha_1\gamma_1 - \wp - \psi}{\alpha_1\gamma_1} - (\kappa+\zeta+\psi)\right]P \end{aligned}.$$

All the model parameters are nonnegative; it follows that ${}^*\mathcal{D}_t^{\upsilon} L(t) \leq 0$ for $\mathcal{R}_0 < 1$ and $\alpha_2\gamma_3\dfrac{\alpha_1\gamma_1 - \wp - \psi}{\alpha_1\gamma_1} \leq (\kappa+\zeta+\psi)$. ${}^*\mathcal{D}_t^{\upsilon} L(t) = 0$ if and only if $M = P = 0$. Thus, by LaSalle's invariance principle, the gambling problem-free equilibrium point E_0 is globally asymptotically stable in a positively invariant region if $\mathcal{R}_0 \leq 1$.

4. Numerical method

In this section, we use an approximation technique for the system in (1). By applying the fundamental theorem of modified ABC fractional calculus, system (7) can be written as:

$$\begin{cases} m(t) - m(0) = \dfrac{1-\upsilon}{\mathcal{B}(\upsilon)}f(t,\ m(t)) \\ +\dfrac{\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon)}\int_0^t (t-\varsigma)^{\upsilon-1} f(\varsigma,\ m(\varsigma))\,d\varsigma \\ -\dfrac{1-\upsilon}{\mathcal{B}(\upsilon)}f(0,\ m(0))\left(1 + \dfrac{\gamma_\upsilon}{\Gamma(\upsilon+1)}t^{\upsilon}\right). \end{cases} \tag{27}$$

We are producing a numerical scheme for this system with the help of Lagrange's interpolation polynomials. Setting $t = t_{n+1},\ n = 0, 1,\ 2,\ \ldots,\ M$ with $h = \dfrac{T}{M}$, we have:

$$\begin{cases} m_{n+1}(t) = m(0) + \dfrac{1-\upsilon}{\mathcal{B}(\upsilon)}f(t_n,\ m(t_n)) \\ +\dfrac{\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon)}\int_0^{t_{n+1}} (t_{n+1}-\varsigma)^{\upsilon-1} f(\varsigma,\ m(\varsigma))\,d\varsigma \\ -\dfrac{1-\upsilon}{\mathcal{B}(\upsilon)}f(0,\ m(0))\left(1 + \dfrac{\gamma_\upsilon}{\Gamma(\upsilon+1)}t_n^{\upsilon}\right). \end{cases} \tag{28}$$

or equivalently,

$$\begin{cases} m_{n+1}(t) = m(0) + \dfrac{1-\upsilon}{\mathcal{B}(\upsilon)} f(t_n,\ m(t_n)) \\ + \dfrac{\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon)} \left(\begin{array}{l} \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} (t_{n+1} - \varsigma)^{\upsilon-1} \\ \times f(\varsigma,\ m(\varsigma))\, d\varsigma \end{array} \right) \\ - \dfrac{1-\upsilon}{\mathcal{B}(\upsilon)} f(0,\ m(0)) \left(1 + \dfrac{\gamma_\upsilon}{\Gamma(\upsilon+1)} t_n^{\upsilon} \right). \end{cases} \tag{29}$$

The function $f(\varsigma,\ m(\varsigma))$ can be approximated over $(t_i,\ t_{i+1})$ using the Lagrange's interpolation polynomial as:

$$f(\varsigma,\ m(\varsigma)) = \left[\begin{array}{l} \dfrac{f(t_i,\ m(t_i))}{h} (t - t_{i-1}) \\ -\dfrac{f(t_{i-1},\ m(t_{i-1}))}{h} (t - t_i) \end{array} \right]$$

Substituting it into Equation (29), we obtain:

$$\begin{cases} m_{n+1}(t) = m(0) + \dfrac{1-\upsilon}{\mathcal{B}(\upsilon)} f(t_n,\ m(t_n)) \\ + \dfrac{\upsilon}{\mathcal{B}(\upsilon)\Gamma(\upsilon)} \sum_{i=1}^{n} \left[\begin{array}{l} \dfrac{f(t_i,\ m(t_i))}{h} I_1 \\ -\dfrac{f(t_{i-1},\ m(t_{i-1}))}{h} I_2 \end{array} \right] \\ - \dfrac{1-\upsilon}{\mathcal{B}(\upsilon)} f(0,\ m(0)) \left(1 + \dfrac{\gamma_\upsilon}{\Gamma(\upsilon+1)} t_n^{\upsilon} \right). \end{cases} \tag{30}$$

where $I_1 = \int_{t_i}^{t_{i+1}} (t_{n+1} - \varsigma)^{\upsilon-1} (\varsigma - t_{i-1})\, d\varsigma$ and $I_2 = \int_{t_i}^{t_{i+1}} (t_{n+1} - \varsigma)^{\upsilon-1} (\varsigma - t_i)\, d\varsigma$. Computing these integrals, we finally get the approximate solution as:

$$\begin{cases} m_{n+1}(t) = m(0) + \dfrac{1-\upsilon}{\mathcal{B}(\upsilon)} f(t_n,\ m(t_n)) \\ + \dfrac{\upsilon}{\mathcal{B}(\upsilon)} \sum_{i=1}^{n} \left[\begin{array}{l} \dfrac{h^{\upsilon} f(t_i,\ m(t_i))}{\Gamma(\upsilon+2)} (A - B) \\ -\dfrac{h^{\upsilon} f(t_{i-1},\ m(t_{i-1}))}{\Gamma(\upsilon+2)} \\ (C - D) \end{array} \right] \\ - \dfrac{1-\upsilon}{\mathcal{B}(\upsilon)} f(0,\ m(0)) \left(1 + \dfrac{\gamma_\upsilon}{\Gamma(\upsilon+1)} (nh)^{\upsilon} \right). \end{cases} \tag{31}$$

where

$$\begin{cases} A = (n - i + 2 + \upsilon)(n + 1 - i)^{\upsilon} \\ B = (n - i + 2 + 2\upsilon)(n - i)^{\upsilon} \\ C = (n + 1 - i)^{\upsilon+1} \\ D = (n - i + 1 + \upsilon)(n - i)^{\upsilon} \end{cases}$$

5. Graphical results and discussion

The graphical results validate the theoretical processes. The choice of the fractional order υ in the modified ABC fractional derivative, which is in the range (0, 1], is primarily owing to its mathematical consistency with integer-order derivatives. In this section, we implement the proposed technique to solve system (7) with the initial solutions and model parameters provided in Table 1. The approximate results were obtained using the numerical scheme presented in Section 4. Due to the lack of organized data on excessive gambling, no real data have been provided for our model results. Therefore, the approximate values are not estimated based on real data, and the results are obtained using assumed parameter values and initial conditions. The gambling problem-free equilibrium point exists when $\alpha_1\gamma_1 > \wp + \psi$. The fractional order υ influences transient dynamics but not equilibrium point stability. The basic reproduction number is then computed to be $\mathcal{R}_0 = 1.4826$ and unstable gambling problem-free equilibrium point is $E_0 = (2.7583,\ 15.6306,\ 0,\ 0,\ 0)$. $\mathcal{R}_0 > 1$ means that on average each problem gambler spreads addiction to more than 1 person. Interventions must reduce $\mathcal{R}_0$ below one. Some key strategies are to reduce transmission α_2 &γ_2, increase recovery rates, λ, & ς and raise awareness.

5.1. Optimal control analysis

We consider the system (7) by adding the control variable $c(t)$, which ranges from 0 to 1, where zero corresponds to the absence of application of any control mechanism, and one refers to the case where there is a fully controlled scenario. The intermediate values $c(t) \in (0,\ 1)$ quantify the effect of applying the intervention mechanisms. The control directly reduces the progression to addiction by a factor of $1 - c$. The control variable:

- Represents interventions like counseling, self-exclusion programs, and early treatment
- Reduces the transition rate from M to P.

Table 1. Value of the model parameters for equation (7)

Parameter	Value	Source	Parameter	Value	Source
N_0	200	Assumed	A_0	75	Assumed
M_0	42	Assumed	P_0	14	Assumed
R_0	0	Assumed	Λ	0.33	Calculated
α_1	0.7	[10]	λ	0.07	[10]
γ_1	0.2	Assumed	$\wp$	0.02	Assumed
γ_2	0.3	Assumed	ς	0.015	Assumed
γ_3	0.4	Assumed	κ	0.34	[10]
α_2	0.5	Assumed	δ	0.014	Assumed
α_3	0.2	Assumed	ζ	0.01	Assumed
ψ	0.001	Assumed			

The modified fractional-order system with mABC derivative:

$$\left.\begin{aligned}
{}^{*}\mathcal{D}_t^{v}N(t) &= \Lambda - \alpha_1 N\tfrac{\gamma_1 A+\gamma_2 M+\gamma_3 P}{T} - \psi N\\
{}^{*}\mathcal{D}_t^{v}A(t) &= \alpha_1 N\tfrac{\gamma_1 A+\gamma_2 M+\gamma_3 P}{T} + \lambda M + \kappa P\\
&\quad - \left(\wp + \alpha_2\tfrac{\gamma_2 M+\gamma_3 P}{T} + \psi\right)A\\
{}^{*}\mathcal{D}_t^{v}M(t) &= \left(\alpha_2\tfrac{\gamma_2 M+\gamma_3 P}{T}\right)A + \delta P\\
&\quad - \left[\lambda + \varsigma + (1-c)\alpha_3\tfrac{\gamma_3 P}{T} + \psi\right]M\\
{}^{*}\mathcal{D}_t^{v}P(t) &= (1-c)\alpha_3\tfrac{\gamma_3 P}{T}M - (\kappa+\delta+\zeta+\psi)P\\
{}^{*}\mathcal{D}_t^{v}R(t) &= \wp A + \varsigma M + \zeta P - \psi R
\end{aligned}\right\}$$

The main objective is to find the optimal control unit $c(t)$ such that the following control objective function is minimized:

$$J(u) = \int_0^T \left[A_1 M(t) + A_2 P(t) + \frac{1}{2}Bc^2(t)\right]dt \tag{32}$$

where:

- $A_1, A_2 > 0$: Weights for at-risk and addicted populations.
- $B > 0$: Cost weight for control implementation.
- T: Final time

To solve the new system, we need to derive the necessary optimality conditions for the problem. To do this, we define the Hamiltonian function:

$$\begin{aligned}
H &= A_1 M(t) + A_2 P(t) + \frac{1}{2}Bc^2(t)\\
&\quad + d_N\left[\Lambda - \alpha_1 N\frac{\gamma_1 A+\gamma_2 M+\gamma_3 P}{T} - \psi N\right]\\
&\quad + d_A\left[\alpha_1 N\frac{\gamma_1 A+\gamma_2 M+\gamma_3 P}{T} + \lambda M + \kappa P\right.\\
&\quad \left. - \left(\wp + \alpha_2\frac{\gamma_2 M+\gamma_3 P}{T} + \psi\right)A\right]\\
&\quad + d_M\left[\alpha_2\frac{\gamma_2 M+\gamma_3 P}{T}A + \delta P\right.\\
&\quad \left. - \left[\lambda + \varsigma + (1-c)\alpha_3\frac{\gamma_3 P}{T} + \psi\right]M\right]\\
&\quad + d_P\left[(1-c)\alpha_3\frac{\gamma_3 P}{T}M - (\kappa+\delta+\zeta+\psi)P\right]\\
&\quad + d_R\left[\wp A + \varsigma M + \zeta P - \psi R\right]
\end{aligned} \tag{33}$$

where d_N, d_A, d_M, d_P and d_R are adjoint variables. To obtain the necessary optimality conditions:

$$\left.\begin{aligned}
{}_0^{*}D_t^{v}\Phi_i(t) &= \frac{\partial H}{\partial d_{\Phi_i}}(t)\\
{}_0^{*}D_t^{v}d_{\Phi_i}(t) &= -\frac{\partial H}{\partial \Phi_i}(t)\\
\frac{\partial H}{\partial c}(t) &= 0, \quad i = 1,\ 2,\ 3,\ 4,\ 5.
\end{aligned}\right\} \tag{34}$$

where $\Phi_1 = N(t)$, $\Phi_2 = A(t)$, $\Phi_3 = M(t)$, $\Phi_4 = P(t)$ and $\Phi_5 = R(t)$. The transversality conditions:

$$d_i(T) = 0, \quad i \in \{d_{\Phi_i}\}.$$

Accordingly, the optimal control $c*(t)$ of a new dynamic system, which minimizes the objective functional (32), is characterized by

$$c*(t) = \min\left(\max\left(0, \frac{\alpha_3\frac{\gamma_3 P(t)}{T}M(t)(d_P(t) - d_M(t))}{B}\right), 1\right). \tag{35}$$

5.2. Sensitivity analysis

Performing a sensitivity analysis on the gambling model helps to identify which model parameters most significantly influence the system's behavior. Next, we calculate the normalized sensitivity index for $\mathcal{R}_0$ with respect to the model parameter p as:

$$S_p = \frac{p}{\mathcal{R}_0} \times \frac{\partial \mathcal{R}_0}{\partial p}.$$

The sensitive parameters and their sensitivity index are shown in Table 2.

Table 2. Sensitivity indices of $\mathcal{R}_0$

Parameter	Sensitivity index
α_1	+0.1765
α_2	+1.0000
γ_2	+1.0000
ψ	−0.0200
$\wp$	−0.1681
λ	−0.8140
ς	−0.1744
γ_1	+0.1765

Table 2 discusses the sensitivity of the basic reproduction number $\mathcal{R}_0$. The parameters α_1, α_2, γ_1 and γ_2 have positive imapcts, which means decreasing these parameters leads to a reduction in the value of $\mathcal{R}_0$ α_1 has a positive but relatively weak influence on $\mathcal{R}_0$. To reduce $\mathcal{R}_0$, prioritize decreasing the parameters α_2 and γ_2. The parameters $\wp$, λ, ς and ψ have negative imapcts, which means increasing these parameters leads to a reduction in the value of $\mathcal{R}_0$. For instance, a 42.8571% decrease (from 0.7 to 0.4) in α_1 decreases $\mathcal{R}_0$ by approximately 13.2402%. A 40% decrease (from 0.5 to 0.3) in α_2 decreases $\mathcal{R}_0$ by approximately 40%. A 33.3334% decrease (from 0.3 to 0.2) in γ_2 decreases $\mathcal{R}_0$ by approximately 33.3334%. Similarly, a 28.5714% increase (from 0.07 to 0.09) in λ decreases $\mathcal{R}_0$ by approximately 18.8722%.

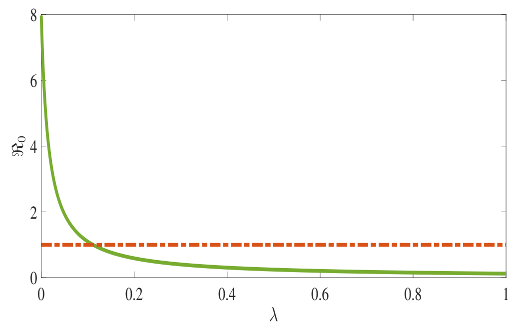

Figure 2. Effects of a parameter λ on the basic reproduction number $\mathcal{R}_0$

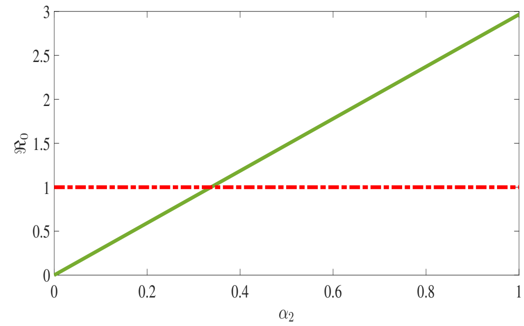

Figure 3. Effects of a parameter α_2 on the basic reproduction number $\mathcal{R}_0$

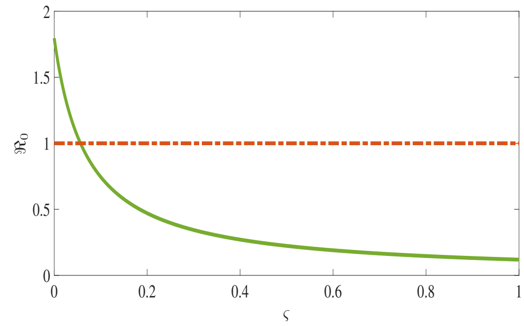

Figure 4. Effects of a parameter ς on the basic reproduction number $\mathcal{R}_0$

Figures 2-4 indicate the effects of senstive parameters on the basic reproduction number. To stabilize the gambling problem-free equilibrium point, limit interactions between aware individuals and problem gamblers and promote awareness of gambling risks.

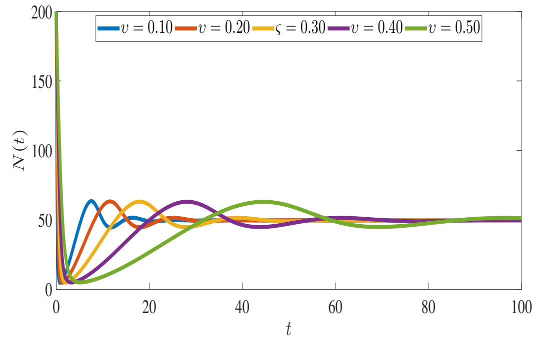

Figure 5. The approximate solutions of $N(t)$ with different fractional order values

In Figure 5, we plotted the approximate results of the system (7) using the numerical scheme presented in Section 4 for different fractional order values υ. The graph confirms that the order of fractional differential equations significantly affects the simulation of the system using the proposed scheme. A small change in the fractional order value can have a significant impact on the system results.

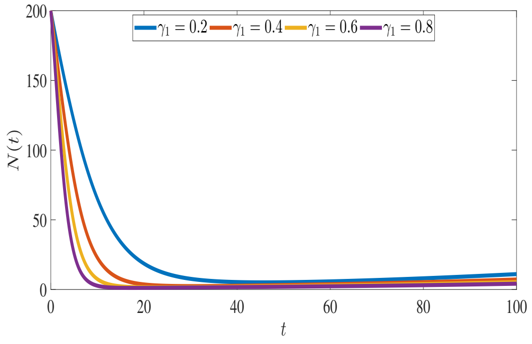

Figure 6. The approximate solutions of $N(t)$ with different values γ_1 when $\upsilon = 0.95$

Figure 6 examines how the parameter γ_1 (weight of non-problem gamblers A on no gamblers N) affects the solution $N(t)$ with time intervals. Higher γ_1 (e.g., $\gamma_1 = 0.8$) accelerates the decline of $N(t)$, as more individuals transition to start gambling. Conversely, lower γ_1 (e.g., $\gamma_1 = 0.2$) slows the decline, reducing recruitment into gambling activities. Public awareness campaigns (reducing γ_1) could slow gambling uptake.

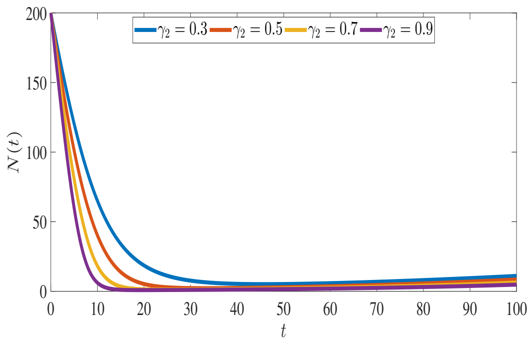

Figure 7. The approximate solutions of $N(t)$ with different values γ_2 when $\upsilon = 0.95$

Figure 7 tests the impact of γ_2, which is the influence of minor-risk gamblers. Early intervention for minor-risk gamblers may indirectly protect the new gamblers.

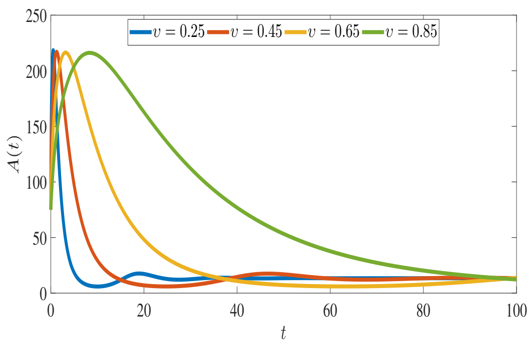

Figure 8. The approximate solutions of $A(t)$ with different fractional order values

Some non-problem gamblers may show interest in gambling frequently. For a few weeks, they will gamble with no problem but may show signs of addiction and will move to the addiction stage. Figures 8-9 confirm the behavior of no problem gamblers and minor-risk under varying fractional order values. Due to several factors, peaceful gambling can transition into excessive gambling activities.

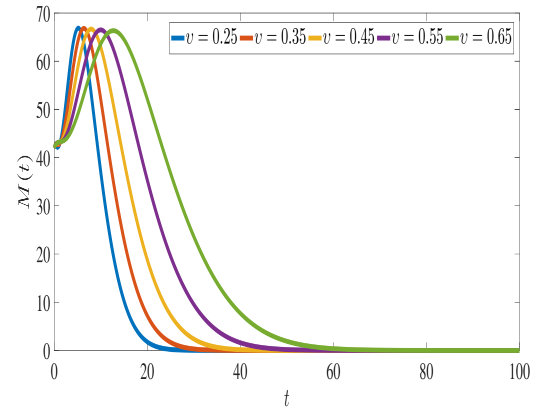

Figure 9. The approximate solutions of $M(t)$ with different fractional order values

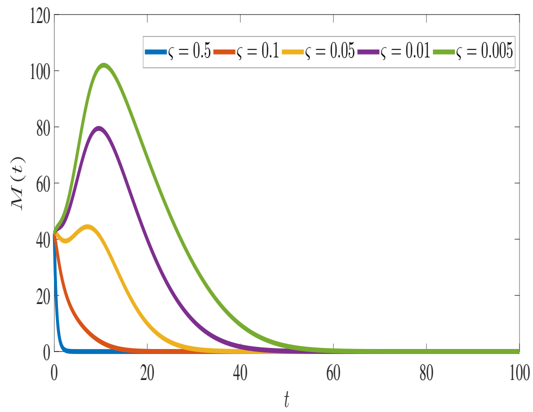

Figure 10. The approximate solutions of $M(t)$ with different values of recovery rate ς when $\upsilon = 0.5$

Figure 10 evaluates how the recovery rate ς (transition from M to R) affects $M(t)$. Higher recovery rate (e.g., $\varsigma = 0.5$) rapidly reduces $M(t)$, as more quit gambling. Lower recovery rate (e.g., $\varsigma = 0.005$) increases the risk of gambling in the population. Strengthening recovery programs (e.g., counseling, religion activities) reduces the risks of gamblers. Early decision to stop daily gambling is highly recommended, especially for young gamblers. Numerical results of the behavior of minor risk gamblers in relation to the recovery parameter ς are shown in Figure 10. It is clearly visible that as the recovery rate ς increases from 0.005 to 0.5, the number of individuals in the risk gambling stage decreases.

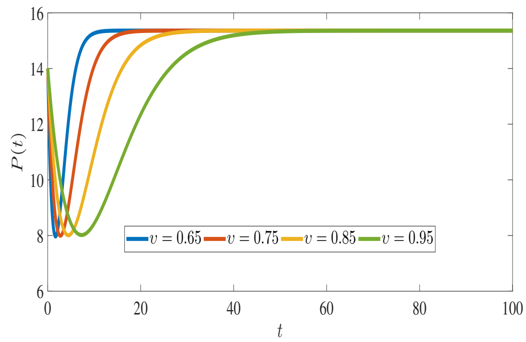

Figure 11. The approximate solutions of $P(t)$ with different fractional order values when $\mathcal{R}_0 > 1$

The large number of addicted gamblers can result from a daily focus on reward values. To reduce this number, individuals must focus on reducing their daily investment in gambling activities and return to their previous non-problematic stage. Figure 11 indicates the dynamics of addicted gamblers under different fractional order values. The graph is plotted when the basic reproduction number is greater than one.

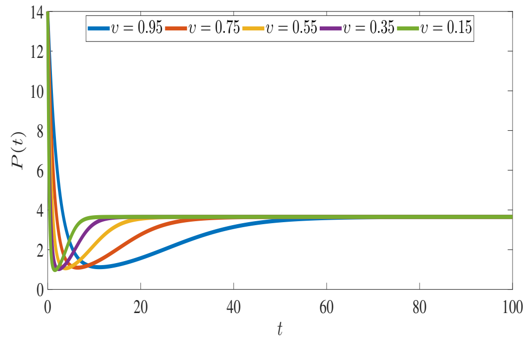

Figure 12. The approximate solutions of $P(t)$ with different fractional order values when $\mathcal{R}_0 < 1$

Figure 12 shows the behavior of addicted population with several fractional order υ when the basic reproduction number $\mathcal{R}_0 < 1$. Since $\mathcal{R}_0 < 1$, gambling problems will not persist (GPFE is stable).

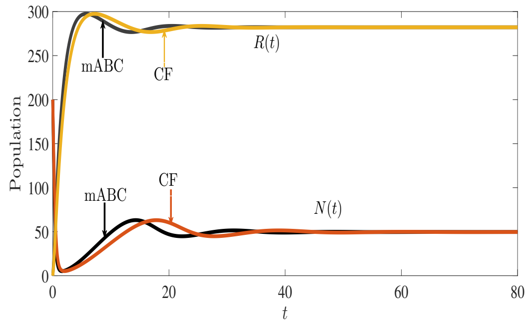

Figure 13. Solutions of the system (7) for Caputo–Fabrizio (CF) and mABC with $\upsilon = 0.35,\ \alpha_1 = 1$

Figure 13 depicts the solutions of $N(t)$ and $R(t)$ obtained by the CF and mABC approaches. The reader may see from these illustrations that our model solutions are found to be in good agreement under both fractional operators. Figure 14 shows the results of $A(t)$ and $M(t)$ obtained by the CF and mABC derivatives.

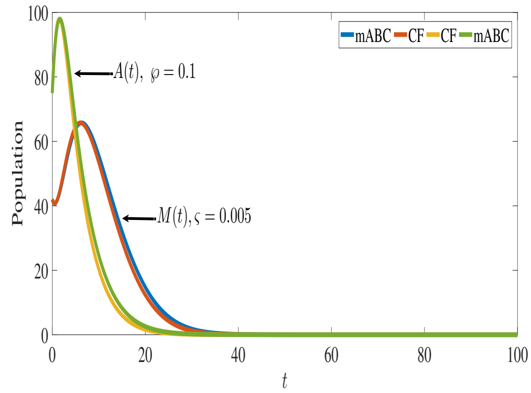

Figure 14. Solutions of the system 7 for C-F and mABC with $\upsilon = 0.70$

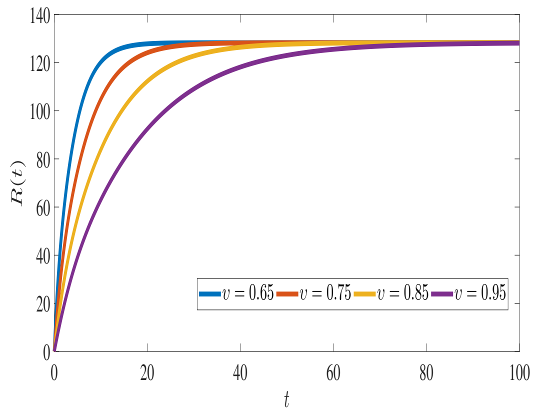

Figure 15. The approximate solutions of $R(t)$ with different fractional order values

Figure 15 demonstrates the dynamics of recovered individuals with different values of υ. Also, Figure 16 shows the effects of recovery rate ς.

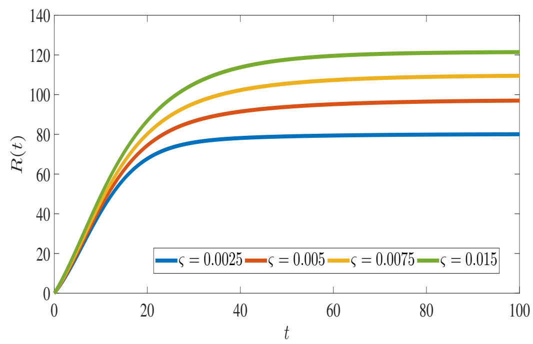

Figure 16. The approximate solutions of $R(t)$ with different values recovery rate ς when $\upsilon = 0.75$

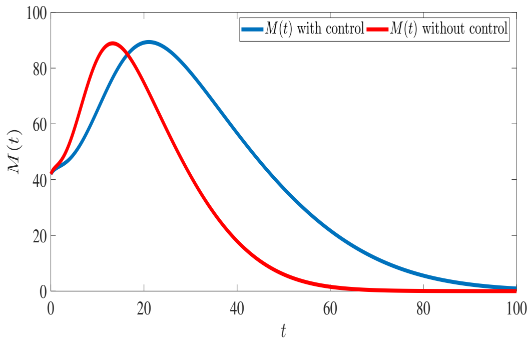

Figure 17. Minor risk infected population with control and without control

Figure 17 confirms the minor risk gamblers with and without the control measures, it can also be seen that when using the control measures, the minor risk population increases, this is due to the fact that the highly addicted population is reduced because of the control impact.

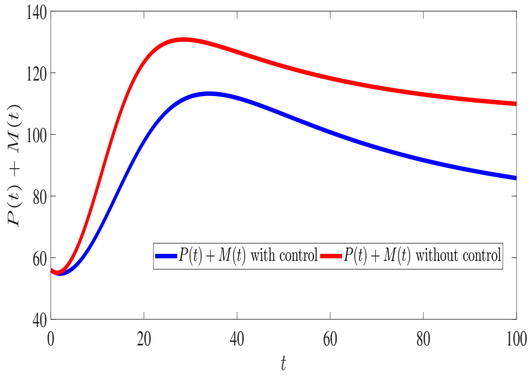

Figure 18. Total addicted population with control and without control

Figure 18 indicates the total addicted population with and without control strategy. From this figure, we confirmed that the total number of the addicted population is reduced due to the control strategy.

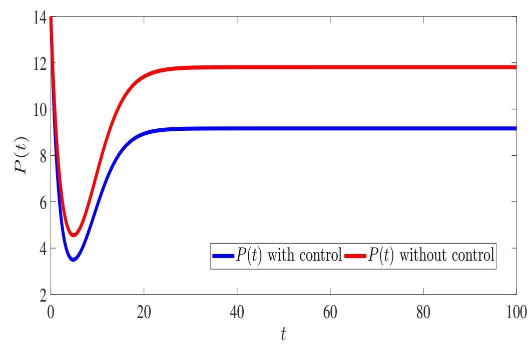

Figure 19. Permanent addicted population with control and without control

Figure 19 shows the highly addicted gamblers with and without the control measures, as we can see from the graph with control $c(t)$, the addicted population $P(t)$ reduces significantly. Interventions (e.g., counseling) reduce $P(t)$ by lowering $\mathcal{R}_0$ to less than unity.

6. Conclusion

In the present study, the problem of excessive gambling model has been investigated by one of the robust nonlocal fractional operator. The modified ABC fractional derivative operator is applied to present a system of differential equations, with approximate results achieved using numerical method. This study considers a deterministic problem gambling model under positive initial conditions, including non-negative model parameters. The fractional order is υ, and consideration was given to the dimensional consistency between the rest of the model parameters. As a result, several significant features of the proposed fractional version of the model have been documented, such as the model formulation, the existence/uniqueness of the solution, invariant region, stability analysis, and most importantly, the basic reproduction number. It should be noted that the fractional-type dynamical system under investigation comprehends the behavior of the model more correctly than the variant of the integer order. Figures 5-14 demonstrate how fractional order υ and sensitive parameters influence addiction dynamics, guiding targeted interventions. A comparative analysis between the fractional time derivative approach of mABC and CF models has been done. The comparisons are given in Figures 13-14. It is evident that these two operators are very effective for approximating fractional systems of differential equations, and the solutions are in good agreement under both derivatives. All the computations in the study were performed using MATLAB R2016a computational software. As a recommendation, the mathematical modeling of the problem of excessive gambling needs to be continued because of its significant social, economic, and political impacts. A better understanding of mathematical modeling research, including different factors of problem gambling, can inform policy-making and therapy for highly addicted individuals. Future work should incorporate stochastic effects and validate parameters based on confirmed real-world data, including different factors of gambling addiction.

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Conflict of interest

The authors declare they have no competing interests.

Author contributions

Conceptualization: Mulualem Aychluh, D.L. Suthar
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Investigation: Clemente Cesarano
Methodology: D.L. Suthar
Writing-original draft: Mulualem Aychluh, D.L. Suthar
Writing-review & editing: Clemente Cesarano, S.D. Purohit

Availability of data

Not applicable.

AI tools statement

All authors confirm that no AI tools were used in the preparation of this manuscript.

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