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RESEARCH ARTICLE

Predefined-time fractional-order terminal SMC for robot dynamics

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ABSTRACT

This study presents an investigation into fractional-order predefined-time terminal sliding mode control (FoPtSMC) for robotic manipulators, particularly focusing on addressing uncertainties and external disturbances. The study introduces a new predefined-time fractional-order SMC method to ensure guaranteed predefined-time convergence and superior tracking performance. This approach also aims to mitigate control input chattering, a common issue in such systems. The Lyapunov analysis is used, and the study establishes the predefined time stability of the proposed closed system. Furthermore, the effectiveness of the proposed FoPtSMC technique is validated through computer simulations applied to a robotic manipulator system.

1. Introduction

Precisely controlling robotic manipulators is crucial for automating tasks in various industries. However, it can be challenging due to their complex and unpredictable movements. Unknown disturbances and uncertainties can compromise precise motions and stable performance.[1] Traditional control methods rely on complex mathematical models to predict robot behavior and design control strategies. Real-world robots may behave differently due to manufacturing variations, resulting in unexpected behaviors or unmodeled dynamics that can significantly impact control performance, leading to errors, instability, or safety hazards.[2,3] This sensitivity to uncertainties underscores the increasing need for robust control approaches in robotics, which prioritize maintaining expected tracking performance and closed-loop stability even under unknown factors.[4]

The effectiveness of sliding mode control (SMC) in managing uncertainties and external disturbances has solidified its popularity in controlling complex systems, especially in the context of real-world robots with intricate dynamics that are difficult to model accurately.[5–9] Terminal Sliding Mode Control (TSM), an extension of SMC, provides the additional benefit of ensuring state convergence within a predetermined time frame, leading to enhanced precision and resilience for robots following a planned trajectory.[10,11] However, the literature suggests that TSM may demonstrate slower convergence compared to alternative methods.[12–14] Moreover, specific TSM implementations can present design complexities, necessitating careful attention.[15–18] Research endeavors have concentrated on enhancing the robustness and responsiveness of TSM.[19,20] One promising strategy is fractional-order SMC, which combines SMC with the fractional-order control scheme, offering several advantages.[21] Fractional-order SMC has the potential to enhance the system's response speed and path-following accuracy while reducing control jitters.[22] By having these advantages, fractional-order SMC emerges as a

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promising approach for precise and resilient control of robotic manipulators, particularly in scenarios involving uncertainties and disturbances.[23] Subsequent sections will examine further the theory of PtSMC, present simulation results demonstrating its effectiveness, and explore its implications for controlling robotic systems.

Finite-time control methods guarantee state convergence within a set timeframe, but the system's initial conditions can influence the speed of convergence.[24,25] Fixed-time SMC ensures convergence within a predefined time, independent of the initial state.[26] However, this bound may not always match real-world settling times, and achieving the desired performance can be challenging for certain systems due to design parameter dependencies. To address these limitations, predefined time control allows designers to specify a desired settling time bound in advance, offering greater flexibility and potentially faster convergence than fixed-time methods. Various predefined time control techniques have been developed for nonlinear systems, such as predefined time synchronization for chaotic systems using fast TSM controllers and predefined time parameters.[27] Another example is a unique sliding surface design based on a sigmoid function, ensuring robust predefined-time convergence for second-order nonlinear systems with matched disturbances.[28] This approach enables precise predefined-time contour tracking for robotic manipulators without requiring exact knowledge of the robot parameters.[29]

This study explores predefined-time convergence control for robotic manipulators in the presence of uncertainties and disturbances. To the best of the authors' knowledge, there is currently no literature on the fractional-order control with predefined-time SMC technique for robotic systems using the predefined-time lemma. Thus, we propose a predefined-time fractional-order sliding mode control (FoPtSMC) scheme, which offers the following:

(1) Enhanced trajectory tracking to obtain high performance by combining predefined-time SMC with a fractional-order control.
(2) Improved closed-loop system response through the use of fractional order.
(3) Uncertain dynamics are mitigated using the robust SMC scheme.
(4) Guarantee of predefined-time convergence of the closed-loop system through Lyapunov stability analysis.

This work is organized as follows: Section 2 consists of the preliminaries. Section 3 presents the robot model, control design of FoPtSMC, and stability investigations using Lyapunov theory. Section 4 includes simulations to validate our approach, and Section 5 discusses it. In the end, Section 6 provides a conclusion of this work.

2. Preliminaries

The important Lemmas are given in this section.

Lemma 1. For $P(t)$, the Lyapunov function with an initial value of $P(0)$, predefined stability is deduced by[29]

$$\dot{P}(t) \leq -d_1 P(t)^{1+\frac{h}{2}} - d_2 P(t)^{1-\frac{h}{2}}$$

where $d_i > 0$ and $0 < h < 1$. The predefined-time T can be obtained as

$$T = \frac{\pi}{h\sqrt{d_1 d_2}} \tag{1}$$

Lemma 2. We provide the significant inequalities for k_i as[30]

$$\begin{aligned} &\sum_{i=1}^{n} |k_i|^{1+h} \geq \left(\sum_{i=1}^{n} |k_i|^2\right)^{\frac{1+h}{2}}, \ when\ 0 < h < 1 \\ &\sum_{i=1}^{n} |k_i|^{h} \geq n^{1-h}\left(\sum_{i=1}^{n} |k_i|\right)^{h}, \ when\ h > 1. \end{aligned} \tag{2}$$

Definition 1: The fractional-order derivative with function $z(t) \in \Re$, we have

$$\mathcal{D}_t^{\varrho} z(t) = \frac{1}{\Gamma(1-\varrho)} \frac{d}{dt} \int_b^t \frac{z(\mathfrak{T})}{(t-\mathfrak{T})^{\varrho}} d\mathfrak{T}$$

where $0 \leq \varrho < 1$, we get the fractional derivative of signum function as[34]

$$\mathcal{D}_t^{\varrho} sign(z(t)) \begin{cases} > 0 & when\ \ z(t) > 0\ \ and\ \ t > 0 \\ < 0 & when\ \ z(t) < 0\ \ and\ \ t > 0\,. \end{cases}$$

3. Design of predefined-time control approach

The equation represents the robotic manipulator dynamics utilized in this study[30]:

$$m(x)\ddot{x} + c(x,\dot{x})\dot{x} + g(x) = v(t) + v_d + v_u \tag{3}$$

where $x, \dot{x}, \ddot{x} \in \Re^n$ is position, velocity, and acceleration, respectively. $m(x) \in \Re^{n\times n}$ is positive definite inertia matrix with the condition $0 < \bar{x}_1(m(x)) \leq \|m(x)\| \leq \bar{x}_2(m(x))$, where $\bar{x}_1$ and $\bar{x}_2$ expresses the min and max eigenvalues. $c(x,\dot{x}) \in \Re^{n\times n}$ denotes the coriolis and centripetal forces, $g(x) \in \Re^n$ represents the gravitational force vector. In addition, $v_u \in \Re^n$ are uncertain dynamics. $v(t) \in \Re^n$ denotes control torque, and $v_d \in \Re^n$ is bounded external disturbances.

Equation (3) is provided as follows:

$$\ddot{x} = m^{-1}(x)(v(t) + v_d + v_u - c(x,\dot{x})\dot{x} - g(x)) \tag{4}$$

The tracking error is given as:

$$\ddot{\varepsilon} = m^{-1}(x)(v(t) + v_d + v_u - c(x,\dot{x})\dot{x} - g(x)) - \ddot{x}_d \tag{5}$$

where $\varepsilon = x - x_d$ is tracking error, and x_d is desired input.

This section examines the development of an innovative control strategy for perturbed robotic manipulators in the existence of uncertainty. First, it analyzes the key characteristics of the proposed predefined-time fractional-order sliding mode control (FoPtSMC) scheme.

3.1. Predefined-time based fractional-order sliding surface

In light of SMC's rapid convergence and robustness, researchers have explored various sliding surfaces to enhance control performance. The demonstrated ability of the recommended sliding surface to deliver precise predefined-time control for n-degree-of-freedom (DOF) robotic manipulators makes it particularly advantageous. Consequently, the proposed sliding surface is formulated as

$$\begin{aligned}\sigma(t) &= a_1|\varepsilon|^{1+\frac{\eta}{2}} sign(\varepsilon) + a_2|\varepsilon|^{1-\frac{\eta}{2}} sign(\varepsilon) \\ &+ a_3\mathcal{D}^{\alpha} sign(\varepsilon) + \dot{\varepsilon}\end{aligned} \tag{6}$$

where sliding surface is represented by $\sigma(t) \in \Re^n$, $a_1, a_2, a_3 \in \Re^+$ are positive constants, and $0 < \eta < 1$.

Then, $\dot{\sigma}(t)$ can be obtained as

$$\begin{aligned}\dot{\sigma}(t) &= a_1(1+\tfrac{\eta}{2})|\varepsilon|^{\frac{\eta}{2}}\dot{\varepsilon} + a_2(1-\tfrac{\eta}{2})|\varepsilon|^{-\frac{\eta}{2}}\dot{\varepsilon} \\ &+ a_3\mathcal{D}^{\alpha+1} sign(\varepsilon) + \ddot{\varepsilon}\end{aligned} \tag{7}$$

From (5) into (7) yields

$$\begin{aligned}\dot{\sigma}(t) &= a_1(1+\tfrac{\eta}{2})|\varepsilon|^{\frac{\eta}{2}}\dot{\varepsilon} + a_2(1-\tfrac{\eta}{2})|\varepsilon|^{-\frac{\eta}{2}}\dot{\varepsilon} \\ &+ a_3\mathcal{D}^{\alpha+1} sign(\varepsilon) + m^{-1}(x)(v(t) + v_d + v_u \\ &- c(x,\dot{x})\dot{x} - g(x)) - \ddot{x}_d\end{aligned} \tag{8}$$

Equation (8) can be given as

$$\begin{aligned}\dot{\sigma}(t) &= a_1(1+\tfrac{\eta}{2})|\varepsilon|^{\frac{\eta}{2}}\dot{\varepsilon} + a_2(1-\tfrac{\eta}{2})|\varepsilon|^{-\frac{\eta}{2}}\dot{\varepsilon} \\ &+ a_3\mathcal{D}^{\alpha+1} sign(\varepsilon) + m^{-1}(x)(v(t) - c(x,\dot{x})\dot{x} - g(x)) \\ &+ \wp(x,\dot{x},\ddot{x}) - \ddot{x}_d\end{aligned} \tag{9}$$

where $\wp(x,\dot{x},\ddot{x}) = m^{-1}(x)(v_u + v_d)$, it is bounded by $\|\wp(x,\dot{x},\ddot{x})\| \le \wp_1 + \wp_2\|x\| + \wp_3\|\dot{x}\|^2$, and $\wp_1, \wp_2$ and, $\wp_3 > 0$.

Using the provided sliding surface, we will now develop the FoPtSMC method for n-DOF robotic manipulators. This control law is designed to be resilient to disturbances and uncertainties.

3.2. Design of FoPtSMC scheme

The control law FoPtSMC is designed to ensure the robust operation of perturbed robotic manipulators under bounded uncertain dynamics:

$$v(t) = m(x)\begin{pmatrix} m^{-1}(x)(c(x,\dot{x})\dot{x} + g(x)) + \ddot{x}_d \\ -(\wp_1 + \wp_2\|x\| + \wp_3\|\dot{x}\|^2) sign(\sigma(t)) \\ -a_1(1+\frac{\eta}{2})|\varepsilon|^{\frac{\eta}{2}}\dot{\varepsilon} - a_2(1+\frac{\eta}{2})|\varepsilon|^{-\frac{\eta}{2}}\dot{\varepsilon} \\ -a_3\mathcal{D}^{\alpha+1} sign(\varepsilon) \\ -b_1|\sigma(t)|^{1+\frac{\mu}{2}} sign(\sigma(t)) \\ -b_2|\sigma(t)|^{1-\frac{\mu}{2}} sign(\sigma(t)) \end{pmatrix} \tag{10}$$

where $|\varepsilon|^{-\frac{\eta}{2}} = 0$ if $\varepsilon = 0$, b_1, b_2 are positive constants, and $0 < \mu < 1$.

By substituting $v(t)$ into (9), $\dot{\sigma}(t)$ can be computed as

$$\begin{aligned}\dot{\sigma}(t) &= a_1(1+\tfrac{\eta}{2})|\varepsilon|^{\frac{\eta}{2}}\dot{\varepsilon} + a_2(1-\tfrac{\eta}{2})|\varepsilon|^{-\frac{\eta}{2}}\dot{\varepsilon} \\ &+ a_3\mathcal{D}^{\alpha+1} sign(\varepsilon) \\ &+ m^{-1}(x)\begin{pmatrix} m(x)\begin{pmatrix} m^{-1}(x)c(x,\dot{x})\dot{x} \\ +m^{-1}(x)g(x) \\ -(\wp_1 + \wp_2\|x\| + \wp_3\|\dot{x}\|^2) \\ \times sign(\sigma(t)) \\ -a_1(1+\frac{\eta}{2})|\varepsilon|^{\frac{\eta}{2}}\dot{\varepsilon} \\ -a_2(1+\frac{\eta}{2})|\varepsilon|^{-\frac{\eta}{2}}\dot{\varepsilon} \\ -a_3\mathcal{D}^{\alpha+1} sign(\varepsilon) + \ddot{x}_d \\ -b_1|\sigma(t)|^{1+\frac{\mu}{2}} sign(\sigma(t)) \\ -b_2|\sigma(t)|^{1-\frac{\mu}{2}} sign(\sigma(t)) \end{pmatrix} \\ -c(x,\dot{x})\dot{x} - g(x) \end{pmatrix} \\ &+ \wp(x,\dot{x},\ddot{x}) - \ddot{x}_d\end{aligned} \tag{11}$$

Simplifying (11), $\dot{\sigma}(t)$ is expressed as

$$\begin{aligned}\dot{\sigma}(t) &= -(\wp_1 + \wp_2\|x\| + \wp_3\|\dot{x}\|^2) sign(\sigma(t)) \\ &+ \wp(x,\dot{x},\ddot{x}) - b_1|\sigma(t)|^{1+\frac{\mu}{2}} sign(\sigma(t)) \\ &- b_2|\sigma(t)|^{1-\frac{\mu}{2}} sign(\sigma(t))\end{aligned} \tag{12}$$

We have finalized the design of the control scheme and sliding surface, and we are now ready to move forward with the stability analysis. The complete proposed diagram is given in Figure 1.

3.3. Stability analysis

In this section, we conduct an analysis of the closed-loop system's stability using the Lyapunov theorem. Equation (3) is utilized to represent the system dynamics.

If $\sigma(t) = 0$ in (6), one has

$$\begin{aligned}\dot{\varepsilon} &= -a_1|\varepsilon|^{1+\frac{\eta}{2}} sign(\varepsilon) - a_2|\varepsilon|^{1-\frac{\eta}{2}} sign(\varepsilon) \\ &- a_3 D^{\alpha} sign(\varepsilon)\end{aligned} \tag{13}$$

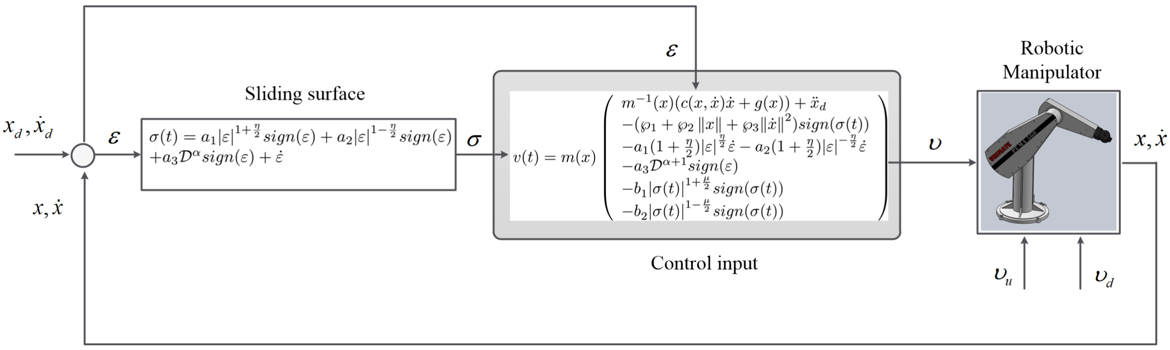

Figure 1. Proposed control model.[32]

We can obtain the stability of the tracking error by carefully selecting a Lyapunov function candidate:

$$P_1(t)=\frac{1}{2}\sum_{i=1}^{n}{\varepsilon_i}^2(t) \tag{14}$$

Then $\dot{P}_1(t)$ is calculated as

$$\dot{P}_1(t)=\sum_{i=1}^{n}\varepsilon_i(t)\dot{\varepsilon}_i(t) \tag{15}$$

Equation (13) substituted into (15), one obtains

$$\dot{P}_1(t)=\sum_{i=1}^{n}\varepsilon_i(t)\begin{pmatrix} -a_1|\varepsilon|^{1+\frac{\eta}{2}}sign(\varepsilon)\\ -a_2|\varepsilon|^{1-\frac{\eta}{2}}sign(\varepsilon)\\ -a_3\mathcal{D}^{\alpha}sign(\varepsilon)\end{pmatrix} \tag{16}$$

After simplification using Definition 1, the equation becomes:

$$\dot{P}_1(t)\le -a_1\sum_{i=1}^{n}\left(|\varepsilon_i(t)|^2\right)^{\frac{4+\eta}{4}}-a_2\sum_{i=1}^{n}\left(|\varepsilon_i(t)|^2\right)^{\frac{4-\eta}{4}} \tag{17}$$

By using Lemma 2, we can express the previous equation more concisely

$$\begin{aligned}\dot{P}_1(t)&\le -a_1 n^{-\frac{\eta}{4}}\left(\sum_{i=1}^{n}|\varepsilon_i(t)|^2\right)^{\frac{4+\eta}{4}}\\ &-a_2\left(\sum_{i=1}^{n}|\varepsilon_i(t)|^2\right)^{\frac{4-\eta}{4}}\end{aligned} \tag{18}$$

then Equation (18) can be expressed as

$$\begin{aligned}&\dot{P}_1(t)\le -a_1 2^{\frac{4+\eta}{4}}n^{-\frac{\eta}{4}}P_1(t)^{\frac{4+\eta}{4}}-a_2 2^{\frac{4-\eta}{4}}P_1(t)^{\frac{4-\eta}{4}}\\ &=-a_1 2^{1+\frac{\eta}{4}}n^{-\frac{\eta}{4}}P_1(t)^{1+\frac{\eta}{4}}-a_2 2^{1-\frac{\eta}{4}}P_1(t)^{1-\frac{\eta}{4}}\end{aligned} \tag{19}$$

Utilizing Lemma 1, we can prove that the sliding surface defined in Equation (6) converges to zero within a specific time frame

$$T_1=\frac{2\pi}{\eta\sqrt{2^{1+\frac{\eta}{4}}2^{1-\frac{\eta}{4}}n^{-\frac{\eta}{4}}a_1a_2}}=\frac{\pi}{\eta\sqrt{a_1a_2n^{-\frac{\eta}{4}}}} \tag{20}$$

Theorem 1: Using the n-DOF robotic manipulator model (3), designed sliding surface (6), and FoPtSMC control method (10) ensures the system trajectory converges to zero within a predefined time, under specified conditions for bounded uncertainties and disturbances.

Proof: We have selected the Lyapunov function candidate as shown below:

$$P_2(t)=\frac{1}{2}\sum_{i=1}^{n}\sigma_i^2(t) \tag{21}$$

The $\dot{P}_2(t)$ can be formulated as:

$$\dot{P}_2(t)=\sum_{i=1}^{n}\sigma_i(t)\dot{\sigma}_i(t) \tag{22}$$

Substituting Equation (12) into Equation (22), the following equation can be expressed as

$$\begin{aligned}&\dot{P}_2(t)=\sum_{i=1}^{n}\sigma_i(t)\\ &\times\begin{bmatrix} -(\wp_1+\wp_2\|x\|+\wp_3\|\dot{x}\|^2)sign(\sigma(t))\\ +\wp(x,\dot{x},\ddot{x})-b_1|\sigma(t)|^{1+\frac{\mu}{2}}sign(\sigma(t))\\ -b_2|\sigma(t)|^{1-\frac{\mu}{2}}sign(\sigma(t))\end{bmatrix}\end{aligned} \tag{23}$$

Solving Equation (23) using Assumption 1

$$\dot{P}_2(t)\le\sum_{i=1}^{n}\sigma_i(t)\begin{bmatrix} -b_1|\sigma(t)|^{1+\frac{\mu}{2}}sign(\sigma(t))\\ -b_2|\sigma(t)|^{1-\frac{\mu}{2}}sign(\sigma(t))\end{bmatrix} \tag{24}$$

It can be expressed as

$$\dot{P}_2(t)\le -b_1\sum_{i=1}^{n}\left(|\sigma_i(t)|^2\right)^{\frac{4+\mu}{4}}-b_2\sum_{i=1}^{n}\left(|\sigma_i(t)|^2\right)^{\frac{4-\mu}{4}} \tag{25}$$

By Lemma 2, the following equation can be given as

$$\begin{aligned}\dot{P}_2(t)&\le -b_2 n^{-\frac{\mu}{4}}\left(\sum_{i=1}^{n}|\sigma_i(t)|^2\right)^{\frac{4+\mu}{4}}\\ &-b_2\left(\sum_{i=1}^{n}|\sigma_i(t)|^2\right)^{\frac{4-\mu}{4}}\end{aligned} \tag{26}$$

Subsequently, we can derive Equation (26) as follows

$$\dot{P}_2(t)\le -b_1 n^{-\frac{\mu}{4}}2^{\frac{4+\mu}{4}}P_2(t)^{\frac{4+\mu}{4}}-b_2 2^{\frac{4-\mu}{4}}P_2(t)^{\frac{4-\mu}{4}} \tag{27}$$

This analysis confirms that the system trajectory converges to $\sigma(t)$ in predefined time, as provided by Lemma 1:

$$T_2 = \frac{2\pi}{\mu\sqrt{b_1 b_2 n^{-\frac{\mu}{4}} 2^{1+\frac{\mu}{4}} 2^{1-\frac{\mu}{4}}}} = \frac{\pi}{\mu\sqrt{n^{-\frac{\mu}{4}} b_1 b_2}} \tag{28}$$

The total settling time can be calculated by including T_1 and T_2, validating the predefined-time convergence of the proposed method in alignment with the FoPtSMC scheme.

4. Simulation results

A simulation study evaluates the effectiveness of the proposed FoPtSMC scheme applied to a PUMA 3-DOF robotic manipulator with external disturbances. The study describes the manipulator's model parameters, and details of the robot's dynamics are taken from.[33] To validate the performance of the proposed method, it has been compared with a fractional-order finite-time SMC scheme,[31] and the simulations have been performed on the MATLAB/Simulink software. The desired trajectories and uncertain dynamics are provided as follows:

$$x_d = \begin{bmatrix} sin(t\pi/3) - 1 + cos(t\pi/5) \\ cos(t\pi/5 + (\pi/2)) + sin(t\pi/3) \\ sin(t\pi/5 + (\pi/2)) - 1 \end{bmatrix},$$
$$v_u = \begin{bmatrix} 0.5\dot{x}_1 + sin(3x_1) \\ 1.3\dot{x}_2 - 1.8sin(2x_2) \\ -0.1\dot{x}_3 - 0.5sin(x_3) \end{bmatrix},$$
$$v_d = \begin{bmatrix} 0.5sin(\dot{x}_1) \\ 1.1sin(\dot{x}_2) \\ 0.15sin(\dot{x}_3) \end{bmatrix}.$$

In addition, the appropriate parameters for the developed FoPtSMC scheme are given in Table 1.

Table 1. Proposed control parameters

Parameter	Value
a_1	6
a_2	6
a_3	0.0002
b_1	30
b_2	30
η	0.3
μ	0.1
α	0.01
$x_1(0)$	-0.2
$x_2(0)$	-0.2
$x_3(0)$	-0.2

The performance of the proposed FoPtSMC scheme is evaluated comprehensively, with a particular focus on handling uncertain system dynamics under disturbances, as depicted in Figures 2 - 4.

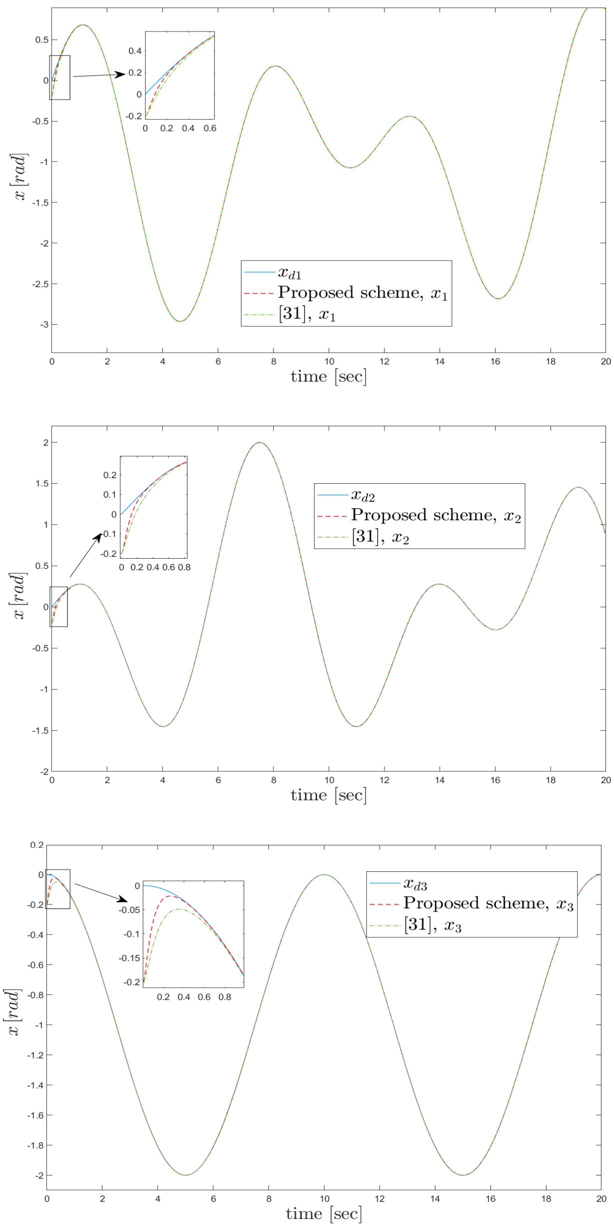

Figure 2. Position tracking under uncertain dynamics.

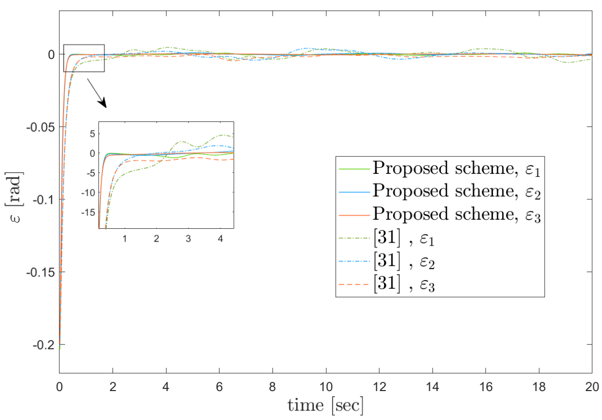

Figure 3. Tracking error under uncertain dynamics.

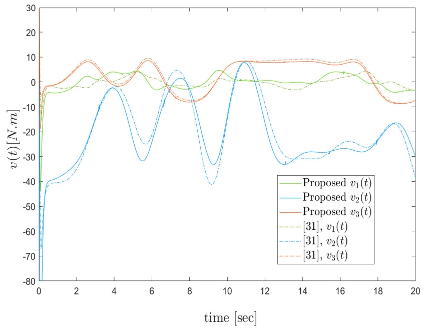

Figure 4. Control input under uncertain dynamics.

The expected performance of the FoPtSMC method under uncertain dynamics in comparison with[31] is effectively shown in Figures 2-4. Figure 2 showcases the proposed controller's precise ability to track the desired trajectory, while Figure 3 exhibits minimal tracking errors of the proposed scheme, affirming its effectiveness. Furthermore, Figure 4 illustrates the control inputs that guide the system state onto and maintain it on the desired surfaces to ensure the desired dynamics, indicating the proposed controller's robust performance in achieving accurate tracking. Additionally, the proposed method delivers smooth control torques. The comprehensive assessment presented in Figures 2-4 emphasizes the efficacy of the proposed FoPtSMC scheme in achieving precise position trajectory tracking under uncertain dynamics.

5. Discussion

The proposed FoPtSMC strategy has been designed to control nonlinear dynamical systems with bounded external disturbances. This research aims to improve tracking and transient characteristics while maintaining resilience in a closed-loop system. The FoPtSMC strategy is intended to ensure rapid sliding mode convergence within a specific time frame, and the Lyapunov technique was used to demonstrate stability.

The simulation findings show that the FoPtSMC technique effectively manages a second-order nonlinear dynamical robotic system under external perturbations. The graphical depictions in Figures 2-3 display the variables x_1, x_2, x_3, and the tracking error ε_1, ε_2, ε_3, demonstrating the successful control performance and the ability of the proposed system to reduce the tracking error to zero rapidly. Figure 4 depicts the control input, which has desirable qualities such as smoothness and acceptable tracking performance, successfully mitigating the effects of external disturbances. The simulation adequately illustrates and supports the theoretical analysis, emphasizing the FoPtSMC approach's advantages in increased responsiveness, lower tracking error, and greater control over nonlinear dynamics. This work thoroughly addresses the constraints of the proposed controller parameters and stability proofs. The FoPtSMC approach's proper parameters were carefully determined within designated ranges to obtain the overall system's stability and error convergence within a fixed time. The study underlines the need to select appropriate parameter values, such as $a_1, a_2, a_3 > 0$, $b_1, b_2 > 0$, $0 < \alpha, \mu, \eta < 1$, to achieve stability and convergence within a specific time period; thereby simplifying the process of picking acceptable values and increasing convergence speed.

In conclusion, the suggested FoPtSMC method has tremendous promise for increasing nonlinear systems' tracking capabilities and robustness, especially in the face of external disturbances. The work gives unique insights into the development and application of the FoPtSMC approach, with implications for a wide range of control engineering applications.

6. Conclusion

The research focuses on controlling the trajectory tracking of robotic manipulators in the presence of uncertainties and external disturbances. To address this issue, a predefined-time fractional-order sliding mode control (FoPtSMC) scheme is proposed. This scheme allows the system state to converge to the desired trajectory within a predefined time, thereby improving tracking performance. Simulations on a manipulator with uncertain dynamics confirm the effectiveness of FoPtSMC. It demonstrates faster response times, reduced tracking errors, and improved rejection of uncertainties and disturbances. Moreover, future work should look into unknown dynamics in nonlinear systems research, which will help advance our understanding and application of the FoPtSMC technique.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Author contributions

Conceptualization: All authors
Formal analysis: All authors
Investigation: All authors
Methodology: All authors
Writing – original draft: Saim Ahmed
Writing – review & editing: All authors

Availability of data

The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

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