Appendix for: 2.5 dimension soil seismic response to oblique incident waves based on exact free-field solution

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Appendix Governing equations of motion for 3D half-space

For the elastic, homogenous and isotropic half-space considered, the equation of motion can be expressed by the displacement $\mathbf{u}$ as:

$$(\lambda+\mu)\nabla\nabla\cdot\mathbf{u}+\mu\nabla^2\mathbf{u}+\rho\mathbf{f}=\rho\ddot{\mathbf{u}}, \text{(A1)}$$

where λ and μ are Lamé's constants, ρ and $\mathbf{f}$ the mass density and body force of the half-space, respectively, and $\mathbf{u}=(u,v,w)$ is the displacement consisting of three components along the horizontal (x), vertical (y) and longitudinal (z) directions. The purpose of putting forward the equations of motion above is to generate the partial differential equations of waves to follow. Here, by substituting the compression-wave potential $\Phi(\mathbf{x},t)$ and the shear-wave potential $\mathbf{\Psi}(\mathbf{x},t)$ into the Helmholtz potential, the displacement $\mathbf{u}$ can be written as:

$$\mathbf{u}(u,v,w)=\nabla\Phi(\mathbf{x},t)+\nabla\times\mathbf{\Psi}(\mathbf{x},t), \text{(A2a)}$$

$$\Psi_{xy}^{SH}=0, \text{(A2b)}$$

where $\mathbf{x}=(x,y,z)$, and $\mathbf{\Psi}=(\Psi_{xy}^{SV},\Psi_{xy}^{SH},\Psi_{yz}^{SV})$. With the out-of-plane (SH-wave-induced) motion neglected ($\Psi_{xy}^{SH}=0$), the three components of $\mathbf{u}$ are uniquely expressed in terms of the functions Φ, Ψ_{xy}^{SV} and Ψ_{yz}^{SV} for the shear waves on the x–y and y–z planes, respectively. Substituting Eq. (A2) into Eq. (A1) and ignoring the effect of body force, yield the wave equations for P and SV waves in partial differential form:

$$\nabla^2\Phi(\mathbf{x},t)=\frac{1}{c_P^2}\frac{\partial^2\Phi(\mathbf{x},t)}{\partial t^2}, \text{(A3a)}$$

$$\nabla^2 \mathbf{\Psi}(\mathbf{x}, t) = \frac{1}{c_S^2} \frac{\partial^2 \mathbf{\Psi}(\mathbf{x}, t)}{\partial t^2}, \text{ (A3b)}$$

where c_S and c_P are the P and SV wave velocities of the solid, defined as:

$$c_P = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \text{ (A4a)}$$

$$c_S = \sqrt{\frac{\mu}{\rho}}. \text{ (A4b)}$$

For the soil considered as a viscoelastic medium, the Lamé's constants, λ and μ, need to be multiplied by $(1 + 2i\beta)$ to account for hysteretic damping with ratio β [59].

By expanding Eq. (A2), the three displacement components (u, v, w) of each point in the half-space are:

$$u = \frac{\partial \Phi}{\partial x} + \frac{\partial \Psi_{xy}^{SV}}{\partial y}, \text{ (A5a)}$$

$$w = \frac{\partial \Phi}{\partial z} + \frac{\partial \Psi_{yz}^{SV}}{\partial y}, \text{ (A5b)}$$

$$v = \frac{\partial \Phi}{\partial y} - \frac{\partial \Psi_{yz}^{SV}}{\partial z} - \frac{\partial \Psi_{xy}^{SV}}{\partial x}. \text{ (A5c)}$$

Here, the setting of plus and minus signs for the displacement components of the SV waves contribution in Eq. (A5) can be explained by the fact that the horizontal (*x*) and longitudinal (*z*) displacements (u, w) generated by the SV waves on the *x*–*y* and *y*–*z* planes $(\Psi_{xy}^{SV}, \Psi_{yz}^{SV})$, respectively, have the same directions, so too the vertical displacement component (v) generated by the SV waves on the two planes. This condition ensures that the traveling direction along the *x*-axis (positive or negative) of the SV waves on the *x*–*y* plane is consistent with that along the *z*-axis of the SV waves on the *y*–*z* plane. Then, with the strain-displacement relations and Hooke's law applied to the displacement components in Eq. (A5), the half-space stress components can be also determined by Φ, Ψ_{xy}^{SV} and Ψ_{yz}^{SV}, expressed as:

$$\sigma_{xx} = \lambda \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Phi + 2\mu \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Psi_{xy}^{SV}}{\partial x \partial y} \right), \text{ (A6a)}$$

$$\sigma_{yy} = \lambda \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Phi + 2\mu \left(\frac{\partial^2 \Phi}{\partial y^2} - \frac{\partial^2 \Psi_{yz}^{SV}}{\partial y \partial z} - \frac{\partial^2 \Psi_{xy}^{SV}}{\partial x \partial y} \right), \text{ (A6b)}$$

$$\sigma_{zz} = \lambda \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Phi + 2\mu \left(\frac{\partial^2 \Phi}{\partial z^2} + \frac{\partial^2 \Psi_{yz}^{SV}}{\partial y \partial z} \right), \text{ (A6c)}$$

$$\tau_{xy} = \mu\left[2\frac{\partial^2 \Phi}{\partial x \partial y} - \frac{\partial^2 \Psi_{yz}^{SV}}{\partial x \partial z} + \left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2}\right)\Psi_{xy}^{SV}\right], \text{ (A6d)}$$

$$\tau_{yz} = \mu\left[2\frac{\partial^2 \Phi}{\partial y \partial z} - \frac{\partial^2 \Psi_{xy}^{SV}}{\partial x \partial z +}\middle| \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}\right]\Psi yzSV, \text{ (A6e)}$$

$$\tau_{xz} = \mu\left(2\frac{\partial^2 \Phi}{\partial x \partial z} + \frac{\partial^2 \Psi_{xy}^{SV}}{\partial y \partial z} + \frac{\partial^2 \Psi_{yz}^{SV}}{\partial x \partial y}\right). \text{ (A6f)}$$

The triple Fourier transform $\hat{\hat{H}}_n$ (represented by double "^") and its inverse used for the wave function H_n in this paper are:

$$\hat{\hat{H}}_n(k_x, y, k_z, \omega) = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H_n(x, y, z, t)\exp(ik_x x)\exp(ik_z z)\exp(-i\omega t)dxdzdt,$$

(A7a)

$$H_n(x, y, z, t) =$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{\hat{H}}_n(k_x, y, k_z, \omega)\exp(-ik_x x)\exp(-ik_z z)\exp(i\omega t)dk_x dk_z d\omega, \text{ (A7b)}$$

where k_x and k_z are the wave numbers along the *x*- and *z*-axes, respectively. Particularly, a minus sign is contained in both the terms $\exp(-ik_x x)$ and $\exp(-ik_z z)$ in Eq. (A7b), but not in Eq. (A7a), which ensures that the waves will propagate and attenuate both along the *x*- and *z*-axes in the positive direction. Then, by applying the triple Fourier transform, one can convert the wave equations in Eq. (A3) into the ordinary differential equations in terms of the vertical coordinate *y* as

$$\frac{\partial^2}{\partial y^2}\hat{\hat{\Phi}} + (-k_x^2 - k_z^2 + k_P^2)\hat{\hat{\Phi}} = 0, \text{ (A8a)}$$

$$\frac{\partial^2}{\partial y^2}\hat{\hat{\boldsymbol{\Psi}}} + (-k_x^2 - k_z^2 + k_P^2)\hat{\hat{\boldsymbol{\Psi}}} = 0, \text{ (A8b)}$$

where the wave numbers are: $k_P = \omega/c_P$ and $k_S = \omega/c_S$.

For P waves from Fig. 2(a), the reflected P and SV waves on the wave propagation plane are denoted by the angles θ_{PPr} and θ_{PSr}, respectively. For this, the incident and reflected waves have the same wave numbers along the intersection line (denoted with superscript Γ shown in Fig. 2) of the wave propagation plane with the *x*–*z* plane [41], namely,

$$k_{Pi}^{\Gamma} = k_{PPr}^{\Gamma} = k_{PSr}^{\Gamma}, \text{ (A9)}$$

where $k_{Pi}^{\Gamma} = \omega\sin\theta_{Piv}/c_P$, $k_{PPr}^{\Gamma} = \omega\sin\theta_{PPr}/c_P$, and $k_{PSr}^{\Gamma} = \omega\sin\theta_{PSr}/c_S$.

Besides, for SV waves from Fig. 2(b), the reflected P and SV waves on the wave propagation plane are denoted by the angles θ_{SPr} and θ_{SSr}, respectively. The relationship between the wave numbers along the horizontal directions of all waves on the wave propagation plane is [41]:

$$k_{Si}^{\Gamma} = k_{SPr}^{\Gamma} = k_{SSr}^{\Gamma}, \text{(A10)}$$

where $k_{Si}^{\Gamma} = \omega \sin\theta_{Siv}/c_S$, $k_{SPr}^{\Gamma} = \omega \sin\theta_{SPr}/c_P$ and $k_{SSr}^{\Gamma} = \omega \sin\theta_{SSr}/c_S$.