Appendices for:

Development of component stiffness equations for TOB connections to an enclosed RHS column under tension

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Appendix A. Formation of stiffness matrix equations for derivation of $\boldsymbol{k}_\mathrm{r}$

This appendix gives the derivation of the k_r coefficient for rotational stiffness contribution of RHS sidewalls or close-plates. The complex 3D RHS is replaced with a 2D plate model with rotational springs at four sides as shown in Fig. 7(a). For the derivation of k_r coefficient, the simplified two-beam model as shown in Fig. 7(b) is utilized. The similar approach was adopted by Park and Wang [24]. However, their work only involved two beams with the same moment of inertia and had some identifier mistakes in the matrix. The processes for deriving k_r of two beams with different moment of inertia are shown below. The k_r coefficient is calculated as the ratio of the applied moment M to the rotation θ, both at *Node C*. For the two-beam model, the stiffness matrix is as follows. Note that the node subscripts 1, 2, and 3 correspond to Nodes A, B, and C, and the beam subscripts 1 and 2 correspond to Beams AB and BC.

$$\begin{bmatrix} P_{1X} \\ P_{1Y} \\ M_1 \\ P_{2X} \\ P_{2Y} \\ M_2 \\ P_{3X} \\ P_{3Y} \\ M_3 \end{bmatrix} = \begin{bmatrix} \frac{EA_1}{L_1} & 0 & 0 & -\frac{EA_1}{L_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_1}{L_1^3} & \frac{6EI_1}{L_1^2} & 0 & -\frac{12EI_1}{L_1^3} & \frac{6EI_1}{L_1^2} & 0 & 0 & 0 \\ 0 & \frac{6EI_1}{L_1^2} & \frac{4EI_1}{L_1} & 0 & -\frac{6EI_1}{L_1^2} & \frac{2EI_1}{L_1} & 0 & 0 & 0 \\ -\frac{EA_1}{L_1} & 0 & 0 & \frac{EA_1}{L_1}+\frac{12EA_2}{L_2^3} & 0 & -\frac{6EA_2}{L_2^2} & -\frac{12EI_2}{L_2^3} & 0 & -\frac{6EI_2}{L_2^2} \\ 0 & -\frac{12EI_1}{L_1^3} & -\frac{6EI_1}{L_1^2} & 0 & \frac{12EI_1}{L_1^3}+\frac{EA_2}{L_2} & -\frac{6EI_1}{L_1^2} & 0 & -\frac{EA_2}{L_2} & 0 \\ 0 & \frac{6EI_1}{L_1^2} & \frac{2EI_1}{L_1} & -\frac{6EI_2}{L_2^2} & -\frac{6EI_1}{L_1^2} & \frac{4EI_1}{L_1}+\frac{4EI_2}{L_2} & \frac{6EI_2}{L_2^2} & 0 & \frac{2EI_2}{L_2} \\ 0 & 0 & 0 & -\frac{12EI_2}{L_2^3} & 0 & \frac{6EI_2}{L_2^2} & \frac{12EI_2}{L_2^3} & 0 & \frac{6EI_2}{L_2^2} \\ 0 & 0 & 0 & 0 & -\frac{EA_2}{L_2} & 0 & 0 & \frac{EA_2}{L_2} & 0 \\ 0 & 0 & 0 & -\frac{6EI_2}{L_2^2} & 0 & \frac{2EI_2}{L_2} & \frac{6EI_2}{L_2^2} & 0 & \frac{4EI_2}{L_2} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ \theta_1 \\ x_2 \\ y_2 \\ \theta_2 \\ x_3 \\ y_3 \\ \theta_3 \end{bmatrix} \quad \text{. (A1)}$$

Considering the boundary conditions $\left[x_1 = 0 \quad y_1 \quad \theta_1 = 0 \quad x_2 = 0 \quad y_2 = 0 \quad \theta_2 \quad x_3 = 0 \quad y_3 \quad \theta_3\right]$ and load conditions $M_2 = 0$, $P_{1Y} = 0$, we can get:

$$M_2 = \frac{2EI_1 \square 4d\left(\theta_2 w + 3y_1\right) + 2EI_2\left(2\theta_2 + \theta_3\right)}{w^2 d} = 0$$, (A2)

$$P_{1Y} = \frac{24EI_1\left(4y_1 + w\theta_2\right)}{w^3} = 0$$. (A3)

Solving for θ_2:

$$\theta_2 = -\frac{I_2 w\theta_3}{I_1 d + 2I_2 w}$$. (A4)

Putting the expression into M_3:

$$M_3 = \frac{2EI_2\left(\theta_2 + 2\theta_3\right)}{L_2} = \frac{2EI_2\left(-\frac{I_2 w\theta_3}{I_1 d + 2I_2 w} + 2\theta_3\right)}{d} = \frac{4EI_2\theta_3\left(I_1 d + 1.5I_2 w\right)}{d\left(I_1 d + 2I_2 w\right)}$$. (A5)

Dividing M_3 by θ_3 to get k_r:

$$k_r = \frac{M_3}{\theta_3} = \frac{4EI_2\left(I_1 d + 1.5I_2 w\right)}{d\left(I_1 d + 2I_2 w\right)}$$. (A6)