Front. Struct. Civ. Eng. 2020, 14(3): 586–598
https://doi.org/10.1007/s11709-020-0573-z

REVIEW

Computational modeling of fracture in concrete: A review

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ABSTRACT This paper presents a review of fracture modeling of concrete. The complex material, such as concrete, has been widely used in construction industries and become trending issue in the last decades. Based on comprehensive literature review, there are two main approaches considered to-date of concrete fracture modeling, such as macroscopic and micromechanical models. The purpose of this review is to provide insight comparison from different techniques in modeling of fracture in concrete which are available. In the first section, an overview of fracture modeling in general is highlighted. Two different approaches both of macroscopic and micromechanical models will be reviewed. As heterogeneity of concrete material is major concern in micromechanical-based concrete modeling, one section will discuss this approach. Finally, the summary from all of reviewed techniques will be pointed out before the future perspective is given.

KEYWORDS concrete fracture, macroscopic, micromechanical, heterogeneity

1 Introduction

The complex behavior of quasi-brittle material, such as concrete, has been used in many engineering structures due to its high strength and durability. The mechanical behavior of concrete is determined by its heterogeneity due to the presence microcracks, voids, aggregates, etc. The appearance of these microcracks can lead to the severe damage and causing to the strength degradation on any stages of concrete's service life. The prediction of fracture process in concrete material is significantly important and it has been trending research topic in the last past two decades. The underlying mechanical properties of concrete are depending on the composition of their microstructures and multi-phases scale from nano-, micro-, meso-to macro-level. At macro scale, concrete is treated as homogeneous material with nonlinear constitutive law. Whereas at the mesoscale, concrete is considered as a two or three phases material consists of aggregate, matrix and interface between them. Hence, understanding mechanical properties of concrete including its fracture phenomenon is critical and challenging issue in materials and engineering sciences. A huge effort has been made by researchers in the last two decades to develop novel and accurate methodology to model the complex fracture process in concrete, such as, the random particle model [1,2], the micromechanical model [3], the lattice model [4–6], the interface element technique [7,8], the augmented Lagrangian approach [9], the mesh-free methods [10–17], the remeshing [18,19], the screened-Poisson [20,21], the phase-field [22,23], the edge rotations [24–26], the cracking particle method [27–29], the dual-horizon peridynamics [30,31], the isogeometric [32–37], the multiscale approach [38–42], the XLME [43], the XFEM [44], the partition of unity [45], the injected elements [46], and the cohesive crack method [47,48] to name a few. In contrast to the technical papers, the review papers which explored the behavior of concrete materials are still infancy. Some published works with regard to the review of fracture analysis in concrete materials conducted by De Borst [49] and Murthy et al. [50] and some other works are related to the self-healing concrete [51–60] and reinforced concrete materials [61]. In relation to the aforementioned review papers, it is difficult to find the recent development which discussed about computational model of fracture in concrete structures. The one of well-known review paper about computational techniques was written by Rabczuk [62]. He made a comprehensive review about different fracture techniques

Article history: Received Sep 19, 2018; Accepted Dec 13, 2018

with continuum models applied in brittle and quasi-brittle materials. Considering the computational model as a valuable tool to support and provide an accurate insight prior to the experimental works, the review about recent development of computational fracture model in concrete structures is highly beneficial.

In this paper, the review on recent development of computational fracture model in concrete is carried out. First, the different techniques of fracture model will be highlighted, such as macroscopic model and micromechanical model. Subsequently, different approaches which are dealing with the heterogeneity of concrete microstructures are discussed. Finally, the paper ends with summary and future perspectives on research of computational fracture model in concrete materials. Since this review focus on fracture modeling techniques of concrete materials, the experimental works and the types of material other than concrete are excluded in this review.

2 Fracture modeling of concrete

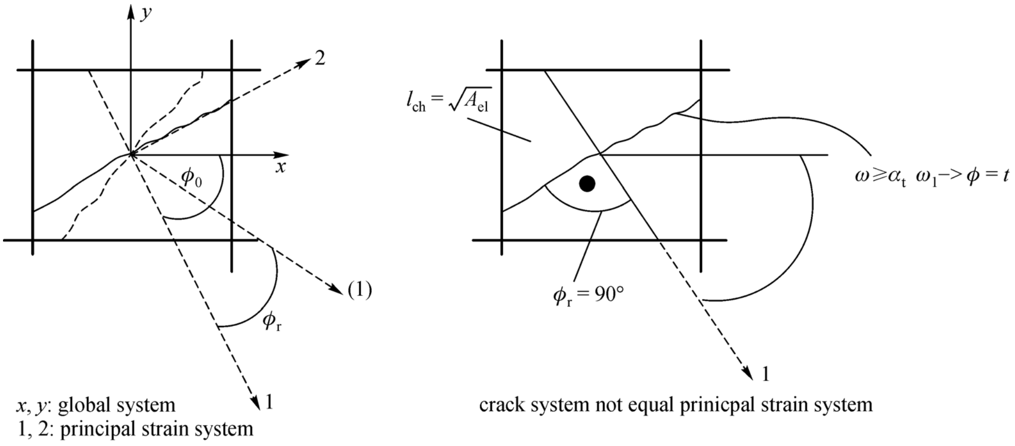
Modeling fracture process in concrete material is not an easy task. Even though there are a lot of researches have been made to explain this complex process, but until now there is no exact model able to simulate all nature's aspects of concrete fracture and describe it in detail. In these following sections, computational modeling of concrete fracture will divide into two categories, such as macroscopic and micromechanical models. The macroscopic models are based on phenomenological approach which are derived from theory plasticity and fracture mechanics, whereas micromechanical models are constructed by determining interaction between microstructures inside concrete material and its behavior in macroscopic scale [63,64].

2.1 Macroscopic models

The initiation and propagation of crack plays an important role in concrete structures. There are so many theories can be found in the literature discussed about crack models. In this review, two different techniques simulating the fracture process in concrete structures will be discussed, such as discrete and continuum approaches.

2.1.1 Discrete approaches

In the discrete method, the displacement field discontinuities obtained from fracture process are directly introduced into the numerical system. It is based on fracture mechanics theory and more sufficient to handle localization of the damage [65]. In the prior approach, usually the crack is modeled to be within an element as a "fictitious" crack and smeared crack models [66,67]. As an alternative to the smeared crack model, the discrete approach is introduced with a discontinuity. The application of discrete crack into the model can be carried out with element separation method along the boundaries [68] or propagate arbitrary within an element without remeshing [69–71]. Rabczuk et al. [72] developed a fictitious/smeared crack model for fracture in reinforced concrete structure. They combined fixed with rotating crack to simulate cracking process in the concrete as can be seen in Fig. 1. The combined fixed-rotating crack model will guarantee the deformation of material in arbitrary directions. The beam elements together with isotropic hardening was used to describe reinforcement in the model. Interaction characteristic between concrete and the reinforcement was captured by bond model to simulate both of failure mechanism, a pullout and splitting failure. The proposed model is applied to three prestressed concrete beams. The



(a) (b)

Fig. 1 The schematic model of fixed cracks and rotating cracks [72]. (a) Rotating crack; (b) fixed crack. (Reprinted from International Journal of Solids and Structures, 42(5–6), Rabczuk T, Akkermann J, Eibl J, A numerical model for reinforced concrete structures, 1327–1354, Copyright 2005, with permission from Elsevier.)

good results between numerical and experimental are obtained in term of crack patterns and load-displacement curves from three different cross sections and failure mechanisms.

The applicability of particle methods with Lagrange multipliers in modeling of static fracture in reinforced concrete structures was introduced by Rabczuk and Belytschko [27]. They used a cohesive crack particle method to simulate fracture process in the concrete which was proposed in dynamic case [73]. A linear rigid and bilinear non-rigid cohesive model was introduced at a particle when the stress in the region of particle exceeds a given limit as shown in Fig. 2.

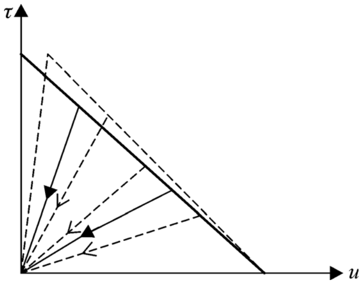


Fig. 2 Schematic of cohesive models applied in a particle method: rigid model (bold line) and non-rigid model (dash line).

The geometry of cracks is determined by a set of restricted discrete cracks which lie on the particular particles as shown in Fig. 3. With this technique, the direction of crack path is always pass through particles since geometry of the crack is not provided.

The 3D meshfree technique for modeling the arbitrary initiation and propagation of cracks in reinforced concrete structures was presented by Rabczuk et al. [47]. They used this method based on the partition of unity and formulated in nonlinear application. The cohesive zone model is introduced at post-crack initiation stage. The beam elements was used to model steel reinforcement which is connected by bond model into the surrounding concrete. The real behavior of bonding in the reinforcement depends on the surface of the bars. The adhesion and friction are the main principal in the bonding behavior for bars without ribs. Whereas for ribbed bars, the mechanism is more complex as it occurred in the effective concrete cover region, C_{eff}, as shown in Fig. 4.

The first lattice technique in the elasticity problems was introduced by Hrennikoff [74]. A lattice system was used by Bažant et al. [1] with random particle model to study the fracture behavior of aggregate using truss elements. In this lattice fracture model, a network of beam elements plays a role as continuum. Then the mapping of different microstructures into the beam elements can be done afterward using particular properties. The assigning particular properties depends on the type of material which beam elements are represented, such as aggregate and matrix. Some lattice models for fracture modeling on concrete are available in Refs. [75–79]. Schlangen and van Mier [75] used simple lattice model to simulate the fracture process in concrete materials. The effects of element type and the orientation of mesh on lattice model into the fracture behavior of concrete were also investigated [76]. Lilliu and van Mier [77] proposed a 3D beam lattice model to simulate fracture behavior on concrete material both of regular and random model. They assumed concrete as three phases materials consists of aggregate, matrix and its interface zone. For regular lattice model, the length of mesh elements are equal whereas in the random model, all elements have different length and stiffness. The nodes has been placed randomly inside the grid of regular cell size s, and then connected by Voronoi construction as can be seen in Fig. 5. The size ratio between sub-cell and main-cell of elements, *A/s*, is determined as randomness of the model. They conducted numerical model to simulate fracture in concrete with different particle density. The results showed that the peak load of the model decreases as particle density increases. When the interfacial strength equal to the mortar matrix, the particle density has no significant influence to the peak load and ductility. Recently, the Lattice Discrete Particle Model (LDPM) in the framework

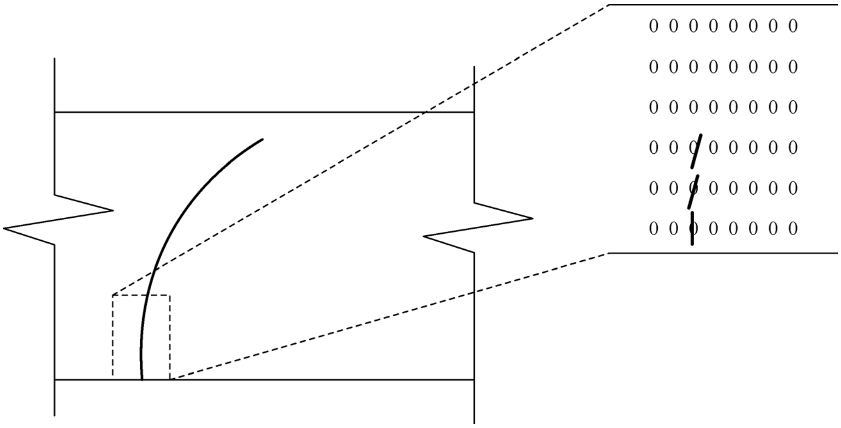
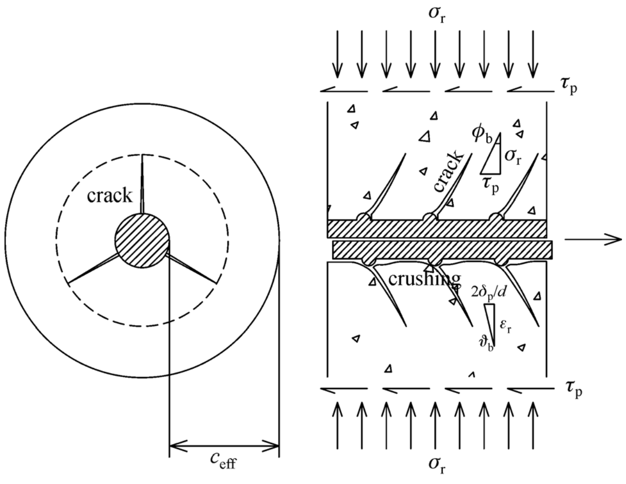


Fig. 3 The schematic of particle crack model for the crack.



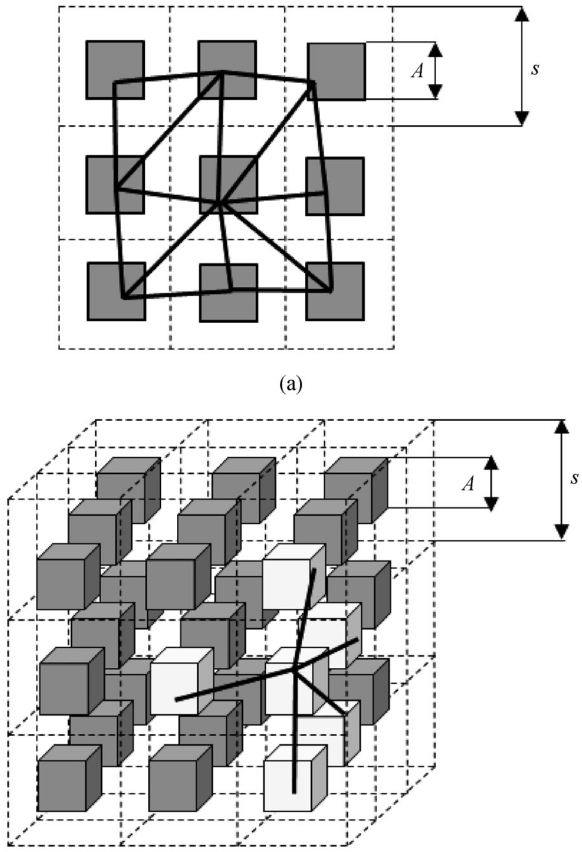
(a) (b)

Fig. 4 Bond model used in steel reinforcement. (a) Radial cracks are generated around the reinforcement; (b) the interaction between concrete and the rib, and the crushed of concrete caused by pulling out force in the reinforcement [47]. (Reprinted from Engineering Fracture Mechanics, 75(16), Rabczuk T, Zi G, Bordas S, Nguyen-Xuan H, A geometrically non-linear three-dimensional cohesive crack method for reinforced concrete structures, 4740–4758, Copyright 2008, with permission from Elsevier.)

of discrete models was formulated and proposed by Cusatis et al. [78,79]. This new lattice model was combination of formulation between the Confinement Shear Lattice (CSL) Model [80–82] and the Discrete Particle Model (DPM) [83]. The LDPM assessed the unknown displacement field established in a finite number of points characterized as center of aggregate particles. The contact interaction behavior between aggregates is determined by constitutive equations with strain-softening to simulate tensile fracture at mesoscale level.

2.1.2 Continuum approaches

In this continuum-based technique, the stress-strain relationship is defined at the macroscopic scale. Theoretically speaking, it is possible to define constitutive relationship between stress and strain at this scale to determine macroscopic behavior of the material. Fracture mechanism in this approach must be considered as dissipative process in the material level which treat cracks as micro-cracks and diffused into the whole representative elementary volume of the material. There are so many commonly used nonlinear constitutive model for concrete found in the literature, such as plasticity, damage mechanics, and combination between them. Stress-based plasticity approach are usually applied to characterize behavior of concrete under triaxial stress, since the yield surface of concrete at particular hardening region corresponds to the concrete strength [84–88]. To represent gradual reduction of the unloading stiffness which are detected in the experiments, strain-based isotropic damage mechanics is introduced [89–92]. Combination of plasticity and isotropic damage to explain special phenomenon that is existed in the experiment, such as irreversible deformation, has been commonly used by some researchers [93–100]. This model can be used to simulate both of tensile and compressive behavior of concrete and not limited to the low confined compression stress. Oliver et al. [101] introduced continuum strong discontinuity approach (CSDA) into the fracture model of concrete. They developed new algorithm based on heat conduction-like theory in order to track multiple cracks both in 2D and 3D cases. To avoid instabilities caused by multiple propagating cracks interaction, they also introduced a viscous perturbation on the crack surface. The novel continuum approach combined with simple discontinuities numerical modeling to model cracking process in concrete material was investigated by Tailhan et al. [102]. They applied statistical distribution of material properties to overcome heterogeneities of concrete in term of crack patterns and



(a)

(b)

Fig. 5 Schematic of Voronoi construction of lattice model with equal randomness, $A/s = 0.5$. (a) 3×3 2D mesh; (b) $3\times3\times3$ 3D mesh [77]. (Reprinted from Engineering Fracture Mechanics, 70 (7–8), Lilliu G, van Mier J G M, 3D lattice type fracture model for concrete, 927–941, Copyright 2003, with permission from Elsevier.)

opening. The 3D model of cracking in concrete material was also presented. Červenka and Papanikolaou [87] developed model for concrete which combined fracture and plasticity models. A fracture model based on smeared and crack band techniques was employed to describe tension behavior. The compression in concrete is handled by plasticity model based on the Mentrey-Willam fracture surface. Their model is integrated in the finite element software ATENA and validated by some experimental results found in the literature. Abu Al-Rub and Kim [103] investigated a coupled plasticity-damage technique based on continuum damage mechanics (CDM) to simulate fracture in plain concrete structure. They used both of isotropic and anisotropic damage model coupled with plasticity to predict plain concrete failure. For easiness implementation in numerical work, they adopted strain equivalence technique in the continuum framework such that strain in the undamaged state is equivalent to the damaged state. The developed algorithm was coded in UMAT subroutine and then implemented in commercial software Abaqus. Microplane technique developed by Bažant et al. [104–106] provides an alternative approach to simulate inelastic modeling of concrete behavior. The relationship between stress and strain tensor in the material is determined by various planes of orientations which are indicated as damage planes in micro scale and plays a role as contact surface between aggregate particles in concrete material as illustrated in Fig. 6.

Ožbolt et al. [107] introduced relaxed kinematic constraint into microplane model. Each plane in microplane consists of normal strain (ε_N) and shear strain (ε_T) components. The normal component is split into volumetric and deviatoric parts (ε_V, ε_D) whereas shear strain into perpendicular components (ε_M, ε_K) as given by

$$\vec{\varepsilon}_N = (\varepsilon_D + \varepsilon_V), \tag{1}$$

$$\vec{\varepsilon}_T = \left(\varepsilon_M \vec{m} + \varepsilon_K \vec{k}\right). \tag{2}$$

They introduced discontinuity function, ψ into Eqs. (1) and (2), except for volumetric part, as kinematic constraint to model discrete tensile cracking. This discontinuity function value ($0 \leqslant \psi \leqslant 1$) is defined by volumetric stress-strain relationship to indicate the discontinuity in each individual microplane and combined with macroscopic strain tensor, ε_{ij} to determine the effective microplane strains as

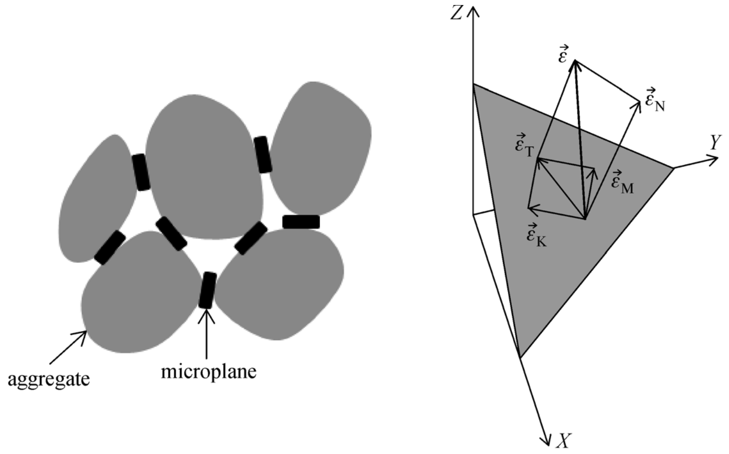
$$\varepsilon_V = \frac{\varepsilon_{kk}}{3},\ \varepsilon_D = (n_i n_j \varepsilon_{ij} - \varepsilon_V)\psi,\ \text{(normal components)} \tag{3}$$

and

$$\varepsilon_M = m_i n_j \varepsilon_{ij} \psi,\ \varepsilon_K = k_i n_j \varepsilon_{ij} \psi.\ \text{(shear components)} \tag{4}$$

2.2 Micromechanical models

In contrast with the aforementioned models, the micromechanical approach treats the heterogeneity of concrete microstructures as different phases to provide accurate characteristic of fracture behavior. At the mesoscale, concrete is represented by at least three phases materials, such as coarse aggregate, mortar matrix, and the interface between these phases. To characterize the heterogeneity in concrete materials, there are two basic approaches: the direct approach and the indirect approach [108,109] which will be discussed in the following sections.



(a) (b)

Fig. 6 Basic concept of microplane model. (a) Idealization of contact planes; (b) decomposition of strain tensor into microplane. Reproduced from Ref. [107]. (Reprinted from International Journal of Solids and Structures, 38(16), Ožbolt J, Li Y, Kožar I, Microplane model for concrete with relaxed kinematic constraint, 2683–2711, Copyright 2001, with permission from Elsevier.)

2.2.1 Direct approaches

The main idea of this approach is all microstructures in a concrete material such as mortar matrix, aggregate, and the interfaces between them are precisely modeled by finite elements. The material properties of each microstructures can be assigned directly afterward into the particular elements. Some researchers had been proposed direct approach to explicitly modeled the mortar matrix, coarse aggregates with random size and shape, and the interfaces between them in 2D model under tension and compression loading [7,8,110,111]. Caballero et al. [112] developed concrete mesoscale analysis in a 80 mm cube which consists of 14 and 64 aggregates embedded in the mortar matrix. They modeled only larger aggregates as a particle array and discretized using finite element. The cracks were represented by zero thickness elements whereas the continuum elements were assumed to be elastic. The probability of cracking inside matrix has been investigated by Trias et al. [113]. They developed two scale methodology of cracking matrix formation perpendicular to the cross section of fibers in unidirectional fiber-reinforced composite materials. The statistical representative volume element (SRVE) combined with two scale methodology is applied to capture transformation from microscale to macroscale. At microscale, the position of fibers are considered to be random, whereas the elastic and failure properties are assumed to be constant. The material is considered to be homogeneous at the macroscale, only the failure properties is assumed to be random. The schematic of this methodology can be seen in Fig. 7.

A similar approach to characterize random distribution of fibers based on Voronoi cells and Delaunay triangulation was also investigated by Al-Ostaz et al. [114]. The recent technology of imaging make realization of particle size and location at the mesoscale are more accurate such as X-ray computed tomography technique which can produce aggregate and void particles both of 2D and 3D model [115,116]. Figure 8(a) shows the cross section an array of pixel from 2D square slice. The quadrilateral elements in FE mesh can be generated directly from each pixel, and the interface between phases are represented by zigzagged red line as shown in Fig. 8(b). Furthermore, this zigzagged interface boundaries smoothed by converting corner elements into two triangles and the new phase values are adjusted as can be seen in Fig. 8(c).

The numerical modeling of concrete failure behavior at mesoscale level was conducted by Du et al. [117]. The concrete material was considered as three phases consist of mortar matrix, aggregate, and the interfacial zone between them as also can be found in the application of capsule-based self-healing concrete [59]. For simplicity, they used

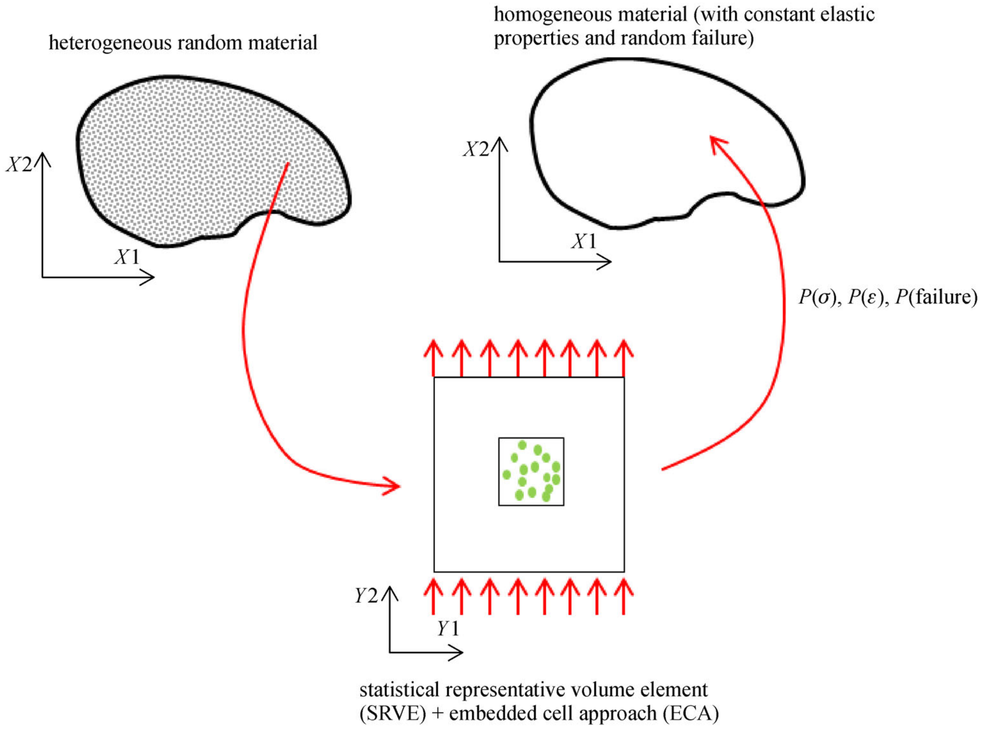


Fig. 7 The schematic of two-scale methodology for probabilistic-based model of unidirectional fiber-reinforced composite [113]. (Reprinted from Composites Science and Technology, 66(11), Trias D, Costa J, Fiedler B, Hobbiebrunken T, Hurtado J E, A two scale method for matrix cracking probability in fiber-reinforced composites based on a statistical representative volume element, 1766–1777, Copyright 2006, with permission from Elsevier.)

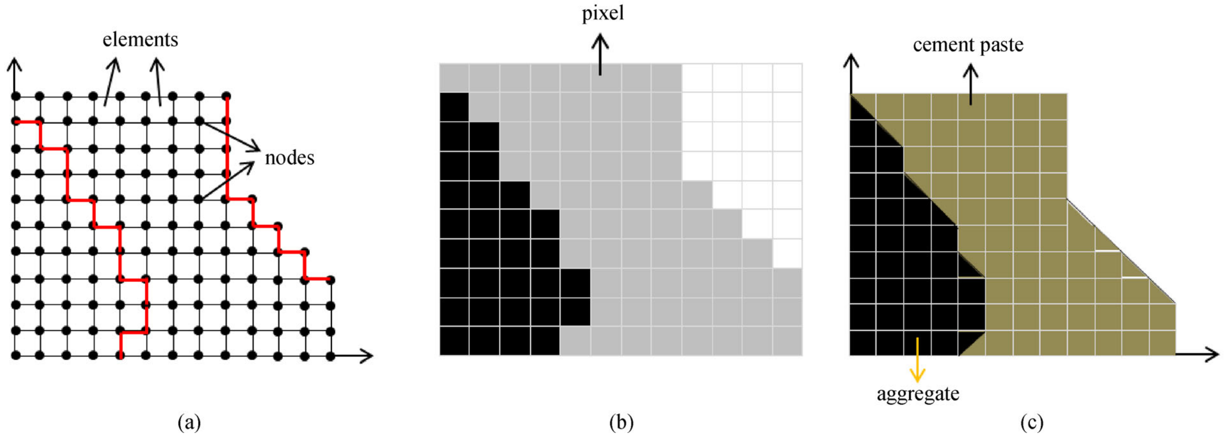


Fig. 8 X-ray transformation form. (a) Pixel-based image; (b) simple grid mesh; (c) smoothed mesh. Reproduced from Ref. [115]. (Reprinted from Engineering Fracture Mechanics, 133, Ren W, Yang Z, Sharma R, Zhang C, Withers P J, Two-dimensional X-ray CT image based mesoscale fracture modelling of concrete, 24–39, Copyright 2015, with permission from Elsevier.)

circular shape as aggregate particles and thin layers role as interfacial transition zone between mortar matrix and aggregates. To generate random coarse aggregate microstructures, they determined the distribution size of the aggregates to follow the Fuller's curve and then placed randomly into the mortar matrix one by one using the well known "take-and-place" technique [6,118,119] without overlapping between particles occurred. They generated 2D mesoscale concrete with volume fraction of aggregate is 46.9%. The dimension of specimen is 150 mm × 150 mm which includes six pieces of medium aggregate with diameter 30 mm and 56 pieces of the small one with diameter 12 mm. The interfacial transition zone is exist between aggregate and mortar matrix. The different colors show different materials with particular mechanical properties. The schematic of generated random specimen can be seen in Fig. 9.

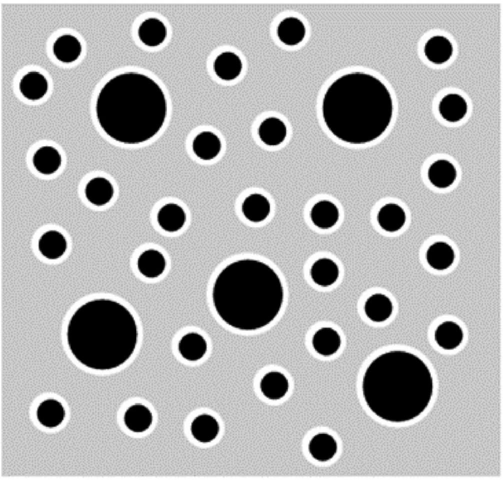


Fig. 9 Generated 2D random circular aggregate particles (black) inside mortar matrix (gray) along with the interfacial layers (white).

The specimen is loaded under vertical displacement at the upper edge of, whereas the lower edge is left free. All corresponding nodes are free from the horizontal displacement, except the left corner of lower edge of specimen. The numerical results showed that the random distribution of aggregates are not significantly influence the mechanical properties on concrete, but strongly dependent not only on the aggregate shape and size, but also the interfacial strength of transition zone between aggregate and mortar matrix.

Monte Carlo simulation of concrete fracture at mesoscale level was investigated by Wang et al. [120–122]. Combination analysis between numerical and statistical approach of heterogeneous concrete at mesoscale was conducted by Wang and Jivkov [123]. They developed both of mesoscale heterogeneous concrete with random elliptical aggregates and circular voids (2D) and ellipsoidal aggregates and spherical voids (3D). To represent potential cracks, zero thickness elements are inserted inside mortar, aggregates, and the interfaces between them. The statistical evaluation are employed to the all outputs started with standard deviation s from n samples as given by

$$s^2 = \frac{1}{n-1}\sum\nolimits_{i=1}^{n}(x_i - \bar{x})^2, \tag{5}$$

where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ is the average result from a series of samples and x_i is the result of sample i. Later on, the coefficient of variation is used to differentiate the output from different number of samples as following:

$$\varepsilon = \frac{s}{\bar{x}}. \tag{6}$$

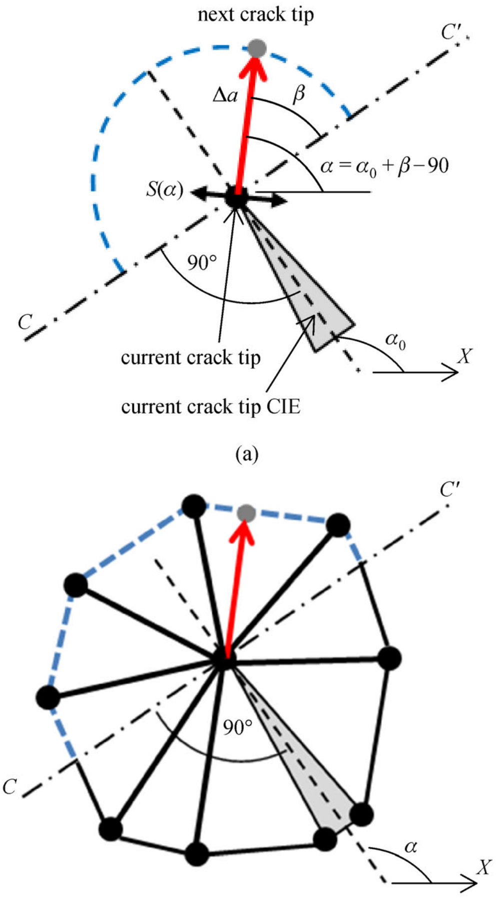
Equation (6) defines variation of measured output relative to its mean value. In the cohesive zone model, for establishing displacement continuity across the interface, the initial stiffness of tension is critical to choose before the tensile strength is achieved. They used the following criterion to predict initial stiffness:

$$k_{n0} = k_{s0} = k_{t0} = \frac{c(1-v)}{b(1+v)(1-2v)}E, \tag{7}$$

where E and v are Young's modulus and Poisson's ratio, respectively, b is the characteristic length, and c is dimensionless value ranging between 10 and 100.

2.2.2 Indirect approaches

In contrary to the direct approach, the indirect approach considered the different phases of concrete microstructures in implicit way. Variation of spatial random field with particular correlation is often used to assign specific material properties in the domain of interest such as tensile strength and fracture energy. Many new techniques have been introduced by some researchers to generate some random field of material properties [124–126], but most of those approaches have not been applied into fracture modeling. Application of Weibull random field into material properties to study propagation of the cracks in concrete beams were investigated by Most [127], Bruggi et al. [128], and Yang and Xu [129]. Heterogeneous cohesive (HC) crack model to predict macroscopic behavior based on Weibull random field of fracture properties were developed by Yang and Xu [129]. They used new stress-based criterion to define direction in which a crack propagates. Since the fracture strength is spatially random generated, it forced the propagation of crack to the direction where the fracture strength is low because of weak interfacial zone or voids as can be seen in Fig. 10.



(a)

(b)

Fig. 10 Schematic of crack propagation criterion based on heterogeneous fracture strength. (a) Idealization model. (b) Typical used in the model [129]. (Reprinted from Computer Methods in Applied Mechanics and Engineering, 197(45–48), Yang Z, Xu X F, A heterogeneous cohesive model for quasi-brittle materials considering spatially varying random fracture properties, 4027–4039, Copyright 2008, with permission from Elsevier.)

The current crack tip and the incremental length Δa forms an angle α_0 with the global axis X. The angle between the current and next crack tip should be less than 90° . The upcoming crack will be placed at radius Δa from half circle to the centered of current tip as illustrated in Fig. 10(a). Since the heterogeneity of fracture properties is main focus, they introduced a tendency indicator as direction function from crack propagation α

$$C(\Delta a,\alpha,\omega) = \frac{S(\alpha,\omega)}{f_t(X,\omega)} = \frac{S(\alpha,\omega)}{f_t(\Delta a,\alpha,\omega)}, \tag{8}$$

where $S(\alpha)$ is the normal stress exist on α plane for the current crack tip and only positive value which will be considered. Equation (8) defines tendency α direction at which the crack will propagate. The crack will tend to propagate into the point with lowest f_t. In other words, with this model, the crack will propagate into the direction which has the highest tendency indicator defined by

$$\frac{\partial C(\alpha,\omega)}{\partial \alpha} = 0 \text{ and } \frac{\partial^2 C(\alpha,\omega)}{\partial \alpha^2} \leqslant 0, \tag{9}$$

with

$$C(\alpha,\omega) > 0 \text{ and } \alpha_0 - 90° \leqslant \alpha \leqslant \alpha_0 + 90°. \tag{10}$$

Equation (9) can be solved by computing C in every 1° which leads to the maximum C as the direction of crack propagation. The next tip of the crack is always existed in the neighboring edges of current tip as described by blue dashed straight lines where the usual mesh is exist before remeshing (see Fig. 10(b)). Grassl et al. [130] developed mesoscale concrete fracture modeling with focused on the size and boundary conditions. They considered only large aggregates embedded in meso-structure and separated by interfacial zone. The discrete lattice approach is applied to simulate the mechanical response from those three phases. The spatial distribution of dissipated energy randomly applied in the mesoscale analysis to predict fracture process zone of concrete materials.

3 Future perspectives

This paper presented review of computational modeling approaches for simulating fracture in concrete materials. There are two different techniques found in the literatures, such as macroscopic and micromechanical models. The macroscopic models are basically extracted from plasticity theory and damage mechanics concept, whereas micromechanical models are obtained from interaction based between microstructures in concrete material to predict its macroscopic behavior. From those approaches, it is difficult to conclude which technique is the most suitable one since each approach has its own merit. In this case, the general point of view about the reliability of the models would be given.

The partition of unity techniques has proved to be one of the effective and reliable method to simulate static fracture in concrete materials where cracks occurred in moderate numbers. This method has focused on many application in single crack propagation without branching cracks while the other techniques have been developed to handle this branch phenomena. The reliable criterion for branching cracks in practical finite element modeling still needs to be improved. In addition, the coupled problems analysis is more simple compared to XFEM method with the enrichment that could be complicated. It would be ideal when the advantage of partition of unity method is able to combine with the other methods that can develop the new reliable and effective technique to capture complex fractures.

The complex fracture prediction in such heterogeneous material like concrete is really difficult due to stochastic in nature. The micro-structures inside concrete material, such as aggregates, voids, microcracks, etc., play significant role for determining onset of crack nucleation and propagation. Most of novel computational techniques for fracture in concrete as discussed above are based on deterministic methods. There are less efforts on statistical computational methods for fracture found in literatures. For example, in concrete [59] and polymeric nanocomposites (PNC) materials based on polynomial chaos expansions [131], Bayesian update [132], and artificial neural network [133]. The crucial challenges of computational techniques in the future is developing a new reliable and efficient methods based on stochastic approaches.

The main objective of computational methods is their efficiency in the “real-world” application. It includes choosing the suitable method for the particular application with regard to their reliability, accuracy, and robustness. However, the efforts to assess the quality of these methods based on a number of uncertain input parameters are far less contributions. For example, sensitivity analysis to quantify the influence of uncertain input parameters on uncertain outputs has been carried out by some researchers based on a surrogate model [134,135]. Screening methods based on Standardized Regression Coefficient and Regionalized Sensitivity Analysis are applied before the quantitative methods, such as Sobol, EFAST, and PAWN are used [135].

In addition, solid understanding about how concrete material will fail is a major research in materials science for designing new concrete materials. Since the fracture in concrete is determined by fine scale structures (nano or micro-scale), it is essential to consider these features in the fracture process. Hence, another future direction of research is the establishment of fracture models in multiscale methods. The main challenges of multiscale methods in the future will be the development of techniques to transfer length scales when damage occurs, choosing the suitable discretization and model based on error estimation, and to bridge different time scale. Another challenge that still remain is to overcome the high computational cost, especially in the application of computational materials design.

Acknowledgements This work was supported and financed by RISTEK-DIKTI (Directorate General of Resources for Science, Technology and Higher Education. Ministry of Research, Technology and Higher Education of Indonesia) under funding agreement No. 153.39/E4.4/2014, and the German Academic Exchange Program (DAAD). The supports are gratefully acknowledged.

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