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RESEARCH ARTICLE

Measurement-based entanglement purification for entangled coherent states

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The entangled coherent states (ECSs) have been widely used to realize quantum information processing tasks. However, the ECSs may suffer from photon loss and decoherence due to the inherent noise in quantum channel, which may degrade the fidelity of ECSs. To overcome these obstacles, we present a measurement-based entanglement purification protocol (MBEPP) for ECSs to distill some high-quality ECSs from a large number of low-quality copies. We first show the principle of this MBEPP without considering the photon loss. After that, we prove that this MBEPP is feasible to correct the error resulted from the photon loss. Additionally, this MBEPP only requires to operate the Bell state measurement without performing local two-qubit gates on the noisy pairs and the purified high-quality ECSs can be preserved for other applications. This MBEPP may have application potential in the implementation of long-distance quantum communication.

Keywords measurement-based entanglement purification, entangled coherent state, photon loss, decoherence

1 Introduction

Quantum entanglement is a counterintuitive phenomenon which leads the quantum mechanic to be different from classical one. There are two methods to encode the qubit such as discrete variables (DVs) and continuous variables (CVs). The DVs, i.e., polarization, time-bin and spatial modes, have been widely employed in quantum information processing, such as quantum dense coding [1, 2], quantum teleportation (QT) [3–6], quantum key distribution (QKD) [7–11], quantum secure direct communication (QSDC) [12–24], and some other important quantum information processing protocols [25, 26]. The CVs, i.e., coherent states, are quite important in quantum computation [27–30]. Entangled coherent state (ECS), which is the superposition of coherent states for different modes [31, 32], also plays a key role in the application of QT [33–37], QKD [38–43], and quantum metrology [44]. However, the ECSs are sensitive to photon loss in the noisy quantum channel, which makes the maximally entangled state become less-entangled state or even mixed state, thereby largely decreasing the security and distance of quantum communication.

To address this issue, the quantum repeater with ECSs [45] was proposed to establish entangled quantum channel with high fidelity. The research findings show that the photon-number-resolving detectors with extremely high efficiency are essential to perform entanglement swapping with high fidelity [45]. However, the realization of this kind of detector is beyond current technology. Thus, it is necessary for one to explore other methods to generate high-quality ECSs. For instance, Kuang *et al.* utilized technique of electromagnetically induced transparency to prepare ECSs between two distant atomic Bose–Einstein condensates, which is sensitive to detection inefficiency [46]. The scheme in Ref. [47] is robust to photon loss with inefficient detectors. In Ref. [48], the researchers presented a method to generate ECSs in a two-dimensional anisotropic trap. In 2019, Xiong *et al.* prepared a scheme to generate entangled cat states in a hybrid optomechanical system [49], which shows that scheme is feasible even in the presence of noise. Recently, Tian *et al.* demonstrated experimentally the simultaneous generation and detection of two types of continuous variable nonclassical states from one type-0 phase-matching opti-

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cal parametric amplification (OPA) and subsequent two ring filter cavities (RFCs) [50].

Additionally, the entanglement concentration protocols (ECPs) [51–58] and entanglement purification protocols (EPPs) [59–75] can be separately employed to distill maximal entangled states and high-fidelity entangled states from corrupted copies. For example, the hyperentanglement concentration for polarization-spatial-time-bin was investigated in multi-photon systems in Ref. [54]. In Ref. [56], the entanglement concentration protocols for cluster-type ECSs utilizing single-mode and two-mode coherent states were proposed to obtain the maximal cluster-type ECSs. In 2002, Jeong *et al.* first proposed the entanglement purification for ECSs and it is applicable to Werner-type ECSs [74]. In Ref. [75], the authors considered the imperfect detection in the generation process of ECSs and employed a random mode of entanglement purification without the manual intervention on purifying lossy errors. After that, Andersen *et al.* experimentally demonstrated an EPP for coherent states in linear optics assisted with an ancillary vacuum state [76]. In 2013, Sheng *et al.* used linear optical elements to purify mixed hybrid entangled states. The research findings showed that the error caused by the dissipation can be transformed to bit-flip error and then be purified [64].

In this paper, we investigate a measurement-based entanglement purification protocol (MBEPP) for ECSs in linear optics, which bases on the original work in Ref. [65], to resist to the effects of noisy quantum channel on ECSs and pave the route of measurement-based quantum repeater (MBQR) [77, 78]. We first illustrate the principle of MBEPP for ECSs under the ideal case without photon loss. We show that our MBEPP has higher efficiency than previous EPP for discrete variables. Then we consider a practical scenario with the photon loss caused by the noisy environment. Our findings show that the error resulted from the photon loss can be converted to bit-flip error and be purified by the same method in the next step. Moreover, the purified high-quality ECSs can be well preserved for other applications. Consequently, this work may have application potential in the implementation of long-distance quantum communication.

The paper is organized as follows. In Section 2, we first recall the BSM for ECSs and present the principle of MBEPP to correct the bit-flip error without the photon loss. In Section 3, we consider a practical scenario that the ECSs may encounter the dissipation simultaneously suffering from the bit-flip error. In Section 4, we present a discussion and make a conclusion.

2 MBEPP for ECSs without photon loss

For this MBEPP, the Bell state measurement (BSM) is an indispensable part. Thus, we start to briefly review the BSM for the ECSs [79]. As discussed in Ref. [27], the four quasi-Bell states can be given by

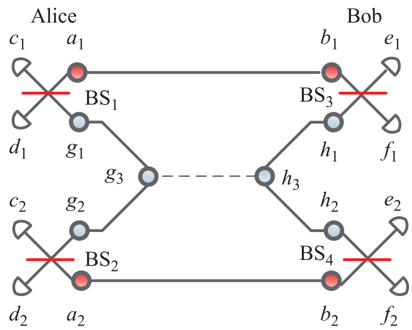

Fig. 1 The schematic diagram of our MBEPP for ECSs in linear optics. It needs two pairs of resource states entangled in modes $g_1g_2g_3$ and $h_1h_2h_3$, where the photons in modes g_1g_2 (h_1h_2) are input photons and that in g_3 (h_3) is the output photon. The photons in modes a_1 and g_1, a_2 and g_2, b_1 and h_1, b_2 and h_2 are directed to four 50:50 BSs (the red lines represent the BSs) to perform BSMs [79]. The photon detectors can distinguish the parity of photon number [79–81]. Here, we take BS$_1$ for example and the similar analysis can be carried out for the other BSs. If photon number detected in c_1 is even (odd) and no photon in d_1, we can deterministically distinguish $|\phi^+\rangle_{a_1g_1}$ ($\phi^-\rangle_{a_1g_1}$). Similarly, we can discriminate $|\psi^+\rangle_{a_1g_1}$ ($|\psi^-\rangle_{a_1g_1}$) provided that the photon detector in d_1 mode registers even (odd) clicks while that in c_1 mode registers no photon. The dotted line denotes the entangled photon pair in modes g_3 and h_3 after one round of this MBEPP.

$$
\begin{aligned}
|\phi^{\pm}\rangle_{ab} &= \frac{1}{N_{\pm}}(|\alpha\rangle_a|\alpha\rangle_b \pm |{-\alpha}\rangle_a|{-\alpha}\rangle_b),\\
|\psi^{\pm}\rangle_{ab} &= \frac{1}{N_{\pm}}(|\alpha\rangle_a|-\alpha\rangle_b \pm |-\alpha\rangle_a|\alpha\rangle_b),
\end{aligned}\tag{1}
$$

where $N_{\pm} = \sqrt{2 \pm 2\mathrm{e}^{-4\alpha^2}}$ and the subscripts a and b represent two parties Alice and Bob, respectively. $\pm\alpha$ represent the amplitudes of the coherent state $|\pm\alpha\rangle$, respectively. For simplicity, we consider α to be real throughout the paper. Obviously, the state $|\phi^+\rangle_{ab}$ and $|\psi^+\rangle_{ab}$ are nonorthogonal due to the fact that the inner product is $\langle\phi^+|\psi^+\rangle = 2(\mathrm{e}^{2\alpha^2} + \mathrm{e}^{-2\alpha^2})^{-1}$. However, if we choose a large α, i.e., $\alpha = 2$, the overlap $\langle\phi^+|\psi^+\rangle$ between them tends to 0. In this case, all the quasi-Bell states in Eq. (1) are nearly orthogonal. As shown in Fig. 1, the 50:50 beam-splitter (BS, take BS$_1$ as an example) makes the coherent state evolve to

$$
\begin{aligned}
&|\alpha\rangle_{a_1}|\alpha\rangle_{g_1} \to |\sqrt{2}\alpha\rangle_{c_1}|0\rangle_{d_1},\\
&|\alpha\rangle_{a_1}|-\alpha\rangle_{g_1} \to |0\rangle_{c_1}|\sqrt{2}\alpha\rangle_{d_1},\\
&|-\alpha\rangle_{a_1}|\alpha\rangle_{g_1} \to |0\rangle_{c_1}|-\sqrt{2}\alpha\rangle_{d_1},\\
&|-\alpha\rangle_{a_1}|-\alpha\rangle_{g_1} \to |-\sqrt{2}\alpha\rangle_{c_1}|0\rangle_{d_1}.
\end{aligned}\tag{2}
$$

After the photons in mode a_1 and mode g_1 passing through BS$_1$, the quasi-Bell states in Eq. (1) evolve to

$$|\phi^{+(-)}\rangle_{a_1g_1} \to |\text{even(odd)}\rangle_{c_1}|0\rangle_{d_1},$$

$$|\psi^{+(-)}\rangle_{a_1g_1} \to |0\rangle_{c_1}|\text{even(odd)}\rangle_{d_1}, \tag{3}$$

where

$$|\text{even}\rangle = N_e\{\sum_{n=0}^{\infty}\frac{[(\sqrt{2}\alpha)^n + (-\sqrt{2}\alpha)^n]}{\sqrt{n!}}\}|n\rangle,$$
$$|\text{odd}\rangle = N_o\{\sum_{n=0}^{\infty}\frac{[(\sqrt{2}\alpha)^n - (-\sqrt{2}\alpha)^n]}{\sqrt{n!}}\}|n\rangle. \tag{4}$$

The $|\text{even}\rangle$ and $|\text{odd}\rangle$ denote the even number and odd number of photons arriving at a photon detector, respectively. N_e and N_o are the normalization coefficients. Hence, we can unambiguously distinguish four quasi-Bell states according to the different responses of photon detectors. To be precise, if the photon detector in c_1 registers even (odd) number of photons and the photon detector in d_1 registers no photon, we can deterministically distinguish $|\phi^+\rangle_{a_1g_1}$ ($|\phi^-\rangle_{a_1g_1}$). Similarly, we can discriminate $|\psi^+\rangle_{a_1g_1}$ ($|\psi^-\rangle_{a_1g_1}$) if the photon detector in d_1 registers even (odd) photons while that in c_1 registers no photon. It needs to be pointed out that the state $|\sqrt{2}\alpha\rangle_{c_1}|0\rangle_{d_1}$ may be in $|0\rangle_{c_1}|0\rangle_{d_1}$ with the probability of $\mathrm{e}^{-2\alpha^2}$, which indicates a failure event for BSM. However, the failure probability tends to 0 when a large α is chosen. For example, if $\alpha = 2$, the success probability of the BSM is 0.9997.

Similar to the polarization degree of freedom, the ECSs also suffer from the channel noise during the entanglement distribution, which makes the maximal ECSs become less entangled states or even mixed states. In detail, we consider the initial state is $|\phi^-\rangle_{ab}$. If $|\phi^-\rangle_{ab}$ becomes $|\psi^-\rangle_{ab}$, we call it the bit-flip error. If the phase-flip error happens, $|\phi^-\rangle_{ab}$ will become $|\phi^+\rangle_{ab}$. And $|\phi^-\rangle_{ab}$ will evolve to $|\psi^+\rangle_{ab}$ when the bit-flip and phase-flip errors occur simultaneously. The phase-flip error cannot be purified directly. It is usually transformed to bit-flip error after the Hadamard operations. We can obtain the transformation as

$$|\phi^-\rangle \to |\phi^-\rangle,\ \ |\phi^+\rangle \to |\psi^+\rangle,$$
$$|\psi^-\rangle \to |\psi^-\rangle,\ \ |\psi^+\rangle \to |\phi^+\rangle, \tag{5}$$

with the assistance of operation $U_x(\pi/4)$ which means the rotation by $\pi/2$ around the x axis [27]. This operation can be given by

$$|\alpha\rangle \to \frac{\mathrm{e}^{-\mathrm{i}\pi/4}}{\sqrt{2}}(|\alpha\rangle + \mathrm{i}|-\alpha\rangle),$$
$$|-\alpha\rangle \to \frac{\mathrm{e}^{-\mathrm{i}\pi/4}}{\sqrt{2}}(\mathrm{i}|\alpha\rangle + |-\alpha\rangle). \tag{6}$$

Thus, we merely consider to purify the bit-flip error to describe the principle of this MBEPP. We assume that the mixed state can be written as

$$\rho_{ab} = F|\phi^-\rangle_{ab}\langle\phi^-| + (1-F)|\psi^-\rangle_{ab}\langle\psi^-|, \tag{7}$$

where $F = \langle\phi^-|\,\rho_{ab}\,|\phi^-\rangle$ is the initial fidelity of the mixed state. As a result, the whole system $\rho_{a_1b_1}\otimes\rho_{a_2b_2}$ can be described as the mixture of four pure states. To be specific, the whole system is in the state $|\phi^-\rangle_{a_1b_1}|\phi^-\rangle_{a_2b_2}$ with the probability of F^2. It is in the state $|\phi^-\rangle_{a_1b_1}|\psi^-\rangle_{a_2b_2}$ or $|\psi^-\rangle_{a_1b_1}|\phi^-\rangle_{a_2b_2}$ with an equal probability of $F(1-F)$. The whole system is in the state $|\psi^-\rangle_{a_1b_1}|\psi^-\rangle_{a_2b_2}$ with the probability of $(1-F)^2$. From Fig. 1, this MBEPP also needs two pairs of resource states which are entangled in modes $g_1g_2g_3$ and $h_1h_2h_3$, respectively. The resource state can be given by

$$|GHZ\rangle = \frac{1}{N_1}(|\alpha\rangle|\alpha\rangle|\alpha\rangle + |-\alpha\rangle|-\alpha\rangle|-\alpha\rangle), \tag{8}$$

where $N_1 = \sqrt{2+2\mathrm{e}^{-6\alpha^2}}$. The Ref. [82] discussed the generation of arbitrary concatenated Greenberger-Horne-Zeilinger (C-GHZ) state encoded in coherent states with linear optics. Thus, we let $N = 1$ and $M = 3$ where N and M respectively represent the number of blocks and the physical qubits in each block. This resource state as Eq. (8) can be prepared off-line in a probabilistic way.

Then, Alice and Bob make BSMs for the photons in modes a_1g_1, a_2g_2, b_1h_1, and b_2h_2, respectively. Whether the purification is successful or not is determined by the outcomes of BSMs [77]. The success cases and the corresponding operations performed on the photon in one of the output modes are listed in Table 1. In detail, with the probability of F^2, the state $\rho_{a_1b_1}\otimes\rho_{a_2b_2}\otimes|GHZ\rangle_{g_1g_2g_3}\otimes|GHZ\rangle_{h_1h_2h_3}$ is in

$$\begin{aligned}&|\phi^-\rangle_{a_1b_1}\otimes|\phi^-\rangle_{a_2b_2}\otimes|GHZ\rangle_{g_1g_2g_3}\otimes|GHZ\rangle_{h_1h_2h_3}\\ &\quad=\frac{1}{N_-^2N_1^2}(|\alpha,\alpha\rangle_{a_1b_1}-|-\alpha,-\alpha\rangle_{a_1b_1})\\ &\quad\otimes(|\alpha,\alpha\rangle_{a_2b_2}-|-\alpha,-\alpha\rangle_{a_2b_2})\\ &\quad\otimes(|\alpha,\alpha,\alpha\rangle_{g_1g_2g_3}+|-\alpha,-\alpha,-\alpha\rangle_{g_1g_2g_3})\\ &\quad\otimes(|\alpha,\alpha,\alpha\rangle_{h_1h_2h_3}+|-\alpha,-\alpha,-\alpha\rangle_{h_1h_2h_3}).\end{aligned} \tag{9}$$

From Eq. (9), the two modes in $|\phi^-\rangle$ have the same amplitudes. Therefore, if the results of BSMs on (g_1, a_1) and (h_1, b_1) are the same (different), the measurement outcomes on (g_2, a_2) and (h_2, b_2) must be the same (different) without considering the sign. Hence, the resultant state is $|\phi^+\rangle_{g_3h_3}$ or $|\phi^-\rangle_{g_3h_3}$. We define that when above measurement results are obtained, our MBEPP is successful. The state $|\psi^-\rangle_{a_1b_1}\otimes|\psi^-\rangle_{a_2b_2}\otimes|GHZ\rangle_{g_1g_2g_3}\otimes|GHZ\rangle_{h_1h_2h_3}$ can also lead to the successful measurement results. After the BSMs, $|\psi^-\rangle_{a_1b_1}\otimes|\psi^-\rangle_{a_2b_2}\otimes|GHZ\rangle_{g_1g_2g_3}\otimes|GHZ\rangle_{h_1h_2h_3}$ will evolve to $|\psi^+\rangle_{g_3h_3}$ or $|\psi^-\rangle_{g_3h_3}$.

On the other hand, we will show that the crossed combinations $|\phi^-\rangle_{a_1b_1}|\psi^-\rangle_{a_2b_2}$ and $|\psi^-\rangle_{a_1b_1}|\phi^-\rangle_{a_2b_2}$ can be eliminated automatically. For example,

$$\begin{aligned}&|\phi^-\rangle_{a_1b_1}\otimes|\psi^-\rangle_{a_2b_2}\otimes|GHZ\rangle_{g_1g_2g_3}\otimes|GHZ\rangle_{h_1h_2h_3}\\ &\quad=\frac{1}{N_-^2N_1^2}(|\alpha,\alpha\rangle_{a_1b_1}-|-\alpha,-\alpha\rangle_{a_1b_1})\\ &\quad\otimes(|\alpha,-\alpha\rangle_{a_2b_2}-|-\alpha,\alpha\rangle_{a_2b_2})\\ &\quad\otimes(|\alpha,\alpha,\alpha\rangle_{g_1g_2g_3}+|-\alpha,-\alpha,-\alpha\rangle_{g_1g_2g_3})\end{aligned}$$

$$\otimes (|\alpha,\alpha,\alpha\rangle_{h_1h_2h_3}+|-\alpha,-\alpha,-\alpha\rangle_{h_1h_2h_3}). \tag{10}$$

It can be found that the measurement outcomes on (g_1,a_1) and (h_1,b_1) are the same (different) while the results on (g_2,a_2) and (h_2,b_2) are different (same). The similar measurement results can be obtained for $|\psi^-\rangle_{a_1b_1}|\phi^-\rangle_{a_2b_2}$. In this way, the crossed combinations states cannot lead to the successful measurement results, so that they can be discarded automatically.

Consequently, if the number of sign "−" in the BSM outcomes is odd, i.e., $|\phi^+\rangle_{g_1a_1}|\psi^-\rangle_{g_2a_2}|\phi^+\rangle_{h_1b_1}|\psi^+\rangle_{h_2b_2}$, the output state can be described as

$$\rho_{g_3h_3}=F_1|\phi^-\rangle_{g_3h_3}\langle\phi^-|+(1-F_1)|\psi^-\rangle_{g_3h_3}\langle\psi^-|, \tag{11}$$

where $F_1=\frac{F^2}{F^2+(1-F)^2}$. The fidelity of the new mixed state is larger than that of the initial one when $F>0.5$ [59]. While if the number of sign "−" is even, such as $|\phi^+\rangle_{g_1a_1}|\phi^-\rangle_{g_2a_2}|\phi^-\rangle_{h_1b_1}|\phi^+\rangle_{h_2b_2}$, the resultant state is

$$\rho'_{g_3h_3}=F_1|\phi^+\rangle_{g_3h_3}\langle\phi^+|+(1-F_1)|\psi^+\rangle_{g_3h_3}\langle\psi^+|. \tag{12}$$

After operating σ_z on the photon in one of the output modes, we can transform the mixed state $\rho'_{g_3h_3}$ in Eq. (12) to $\rho_{g_3h_3}$ in Eq. (11) (see Appendix A). The operation σ_x corresponding to be a phase shift can be implemented by employing the giant Kerr nonlinearity [83] and the operation σ_z can be constructed by using a displacement operator $D(\mathrm{i}\pi/4\alpha\sqrt{1-T})$, where $T\to 1$ is the transmission coefficient of beam splitter. The success probability of this MBEPP is $(1-\mathrm{e}^{-2\alpha^2})^4[F^2+(1-F)^2]$.

From the analysis beforehand, the fidelity of this MBEPP for ECSs is the same as that of the MBEPPs for discrete variables such as polarization in linear optics [77, 78] while the success probability of this MBEPP is larger. The BSM plays a key role in MBEPP. The success probability of those MBEPPs for polarization entanglement in linear optics [73, 77, 78] is $[F^2+(1-F)^2]/16$, for only two of the four Bell states can be distinguished with the standard BSM in linear optics, so that the success probability for each BSM is only $\frac{1}{2}$ [79]. However, the four Bell states in Eq. (1) can be completely discriminated with the probability $1-\mathrm{e}^{-2\alpha^2}$, which results in the total success probability of this MBEPP being $(1-\mathrm{e}^{-2\alpha^2})^4[F^2+(1-F)^2]$. In this way, if $\alpha>\sqrt{\ln 2/2}$, the success probability of this MBEPP is larger than that of the MBEPPs in Refs. [73, 77, 78]. Moreover, with the growth α, the success probability will tend to $F^2+(1-F)^2$ [59]. This MBEPP has another attractive advantage that the purified ECSs can be well remained for further application, i.e., quantum communication [38, 39, 45].

3 MBEPP for ECSs with photon loss

So far, we have completely discussed the principle of our MBEPP without considering the photon loss. However, in the practical scenario, the coherent states may suffer from photon loss which will degrade the fidelity of entanglement and threaten the security of quantum communication. To address this issue, it is essential to correct the error of photon loss caused by dissipation. Here, we utilize BS with the transmission coefficient of η and the reflection coefficient of $1-\eta$ to describe the loss model. Let's denote $|\beta\rangle=|\sqrt{\eta}\alpha\rangle$ and $|\gamma\rangle=|\sqrt{1-\eta}\alpha\rangle$. In this case, the state $|\phi^-\rangle$ will not only face the bit-flip error but also suffer from the photon loss. Therefore, the mixed state ρ_{ab} in Eq. (7) will evolve to

$$\rho^l_{ab}=F|\phi^-_1\rangle_{ab}\langle\phi^-_1|+(1-F)|\psi^-_1\rangle_{ab}\langle\psi^-_1|, \tag{13}$$

where

$$\begin{aligned}|\phi^-_1\rangle_{ab}&=\frac{1}{N_-}(|\beta\rangle_a|\gamma\rangle_{E_a}|\beta\rangle_b|\gamma\rangle_{E_b}\\&\quad-|-\beta\rangle_a|-\gamma\rangle_{E_a}|-\beta\rangle_b|-\gamma\rangle_{E_b}),\\|\psi^-_1\rangle_{ab}&=\frac{1}{N_-}(|\beta\rangle_a|\gamma\rangle_{E_a}|-\beta\rangle_b|-\gamma\rangle_{E_b}\\&\quad-|-\beta\rangle_a|-\gamma\rangle_{E_a}|\beta\rangle_b|\gamma\rangle_{E_b}).\end{aligned} \tag{14}$$

The superscript l means the mixed state undergoing the photon loss and the subscripts E_a and E_b denote the environment modes at Alice and Bob. Thus, the whole system $\rho^l_{a_1b_1}\otimes\rho^l_{a_2b_2}$ can be described as follows. With the probability of F^2, the system is in the state $|\phi^-_1\rangle_{a_1b_1}|\phi^-_1\rangle_{a_2b_2}$. With an equal probability of $F(1-F)$, the system is in the state $|\phi^-_1\rangle_{a_1b_1}|\psi^-_1\rangle_{a_2b_2}$ or $|\psi^-_1\rangle_{a_1b_1}|\phi^-_1\rangle_{a_2b_2}$. It is in the state $|\psi^-_1\rangle_{a_1b_1}|\psi^-_1\rangle_{a_2b_2}$ with the probability of $(1-F)^2$. In addition, the resource state entangled in modes $g_1g_2g_3$ and $h_1h_2h_3$ can be rewritten as

$$|GHZ\rangle^n=\frac{1}{N_2}(|\beta\rangle|\beta\rangle|\beta\rangle+|-\beta\rangle|-\beta\rangle|-\beta\rangle), \tag{15}$$

where $N_2=\sqrt{2+2\mathrm{e}^{-6\eta\alpha^2}}$ and the superscript n means the new resource state regenerated corresponding to the

Table 1 The outcomes of BSMs and the corresponding operations performed on one mode of output state without photon loss. The first column denotes the measurement outcomes on g_1a_1, g_2a_2, h_1b_1 and h_2b_2. The second column represents the parity of number of "−", i.e., the number of "−" of $|\phi^-\rangle_{g_1a_1}|\phi^-\rangle_{g_2a_2}|\phi^-\rangle_{h_1b_1}|\phi^-\rangle_{h_2b_2}$ is even. The third column means the additional operations are needed to operate.

BSM outcomes	Number of "−"	Operation
$|\phi\rangle_{g_1a_1}|\phi\rangle_{g_2a_2}|\phi\rangle_{h_1b_1}|\phi\rangle_{h_2b_2}$	odd "−"	I
$|\phi\rangle_{g_1a_1}|\psi\rangle_{g_2a_2}|\phi\rangle_{h_1b_1}|\psi\rangle_{h_2b_2}$		
$|\psi\rangle_{g_1a_1}|\phi\rangle_{g_2a_2}|\psi\rangle_{h_1b_1}|\phi\rangle_{h_2b_2}$	even "−"	σ_z
$|\psi\rangle_{g_1a_1}|\psi\rangle_{g_2a_2}|\psi\rangle_{h_1b_1}|\psi\rangle_{h_2b_2}$		
$|\phi\rangle_{g_1a_1}|\phi\rangle_{g_2a_2}|\psi\rangle_{h_1b_1}|\psi\rangle_{h_2b_2}$	odd "−"	σ_x
$|\phi\rangle_{g_1a_1}|\psi\rangle_{g_2a_2}|\psi\rangle_{h_1b_1}|\phi\rangle_{h_2b_2}$		
$|\psi\rangle_{g_1a_1}|\phi\rangle_{g_2a_2}|\phi\rangle_{h_1b_1}|\psi\rangle_{h_2b_2}$	even "−"	$\sigma_x\sigma_z$
$|\psi\rangle_{g_1a_1}|\psi\rangle_{g_2a_2}|\phi\rangle_{h_1b_1}|\phi\rangle_{h_2b_2}$		

transmission coefficient η. Therefore, with the probability of F^2, the state $|GHZ\rangle^n_{g_1g_2g_3}\otimes|GHZ\rangle^n_{h_1h_2h_3}\otimes\rho^l_{a_1b_1}\otimes\rho^l_{a_2b_2}$ can be written as

$$\begin{aligned}
&|GHZ\rangle^n_{g_1g_2g_3}|GHZ\rangle^n_{h_1h_2h_3}|\phi_1^-\rangle_{a_1b_1}|\phi_1^-\rangle_{a_2b_2}\\
&=\frac{1}{N_-^2N_2^2}(|\beta\rangle_{g_1}|\beta\rangle_{g_2}|\beta\rangle_{g_3}+|-\beta\rangle_{g_1}|-\beta\rangle_{g_2}|-\beta\rangle_{g_3})\\
&\quad\otimes(|\beta\rangle_{h_1}|\beta\rangle_{h_2}|\beta\rangle_{h_3}+|-\beta\rangle_{h_1}|-\beta\rangle_{h_2}|-\beta\rangle_{h_3})\\
&\quad\otimes(|\beta\rangle_{a_1}|\beta\rangle_{b_1}|\gamma\rangle_{E_{a_1}}|\gamma\rangle_{E_{b_1}}-|-\beta\rangle_{a_1}|-\beta\rangle_{b_1}\\
&\quad\otimes|-\gamma\rangle_{E_{a_1}}|-\gamma\rangle_{E_{b_1}})(|\beta\rangle_{a_2}|\beta\rangle_{b_2}|\gamma\rangle_{E_{a_2}}|\gamma\rangle_{E_{b_2}}\\
&\quad-|-\beta\rangle_{a_2}|-\beta\rangle_{b_2}|-\gamma\rangle_{E_{a_2}}|-\gamma\rangle_{E_{b_2}}). \qquad (16)
\end{aligned}$$

Due to the existence of photon loss, only the items

$$\begin{aligned}
&|GHZ\rangle^n_{g_1g_2g_3}|GHZ\rangle^n_{h_1h_2h_3}(|-\beta\rangle_{a_1}|-\beta\rangle_{b_1}|-\gamma\rangle_{E_{a_1}}\\
&\otimes|-\gamma\rangle_{E_{b_1}}|-\beta\rangle_{a_2}|-\beta\rangle_{b_2}|-\gamma\rangle_{E_{a_2}}|-\gamma\rangle_{E_{b_2}}\\
&+|\beta\rangle_{a_1}|\beta\rangle_{b_1}|\gamma\rangle_{E_{a_1}}|\gamma\rangle_{E_{b_1}}|\beta\rangle_{a_2}|\beta\rangle_{b_2}|\gamma\rangle_{E_{a_2}}|\gamma\rangle_{E_{b_2}}), \qquad (17)
\end{aligned}$$

can be employed to distill high-quality entanglement.

Similarly, with the probability of $(1-F)^2$, the state $|GHZ\rangle^n_{g_1g_2g_3}\otimes|GHZ\rangle^n_{h_1h_2h_3}\otimes\rho^l_{a_1b_1}\otimes\rho^l_{a_2b_2}$ is in

$$\begin{aligned}
&|GHZ\rangle^n_{g_1g_2g_3}|GHZ\rangle^n_{h_1h_2h_3}|\psi_1^-\rangle_{a_1b_1}|\psi_1^-\rangle_{a_2b_2}\\
&=\frac{1}{N_-^2N_2^2}(|\beta\rangle_{g_1}|\beta\rangle_{g_2}|\beta\rangle_{g_3}+|-\beta\rangle_{g_1}|-\beta\rangle_{g_2}|-\beta\rangle_{g_3})\\
&\quad\otimes(|\beta\rangle_{h_1}|\beta\rangle_{h_2}|\beta\rangle_{h_3}+|-\beta\rangle_{h_1}|-\beta\rangle_{h_2}|-\beta\rangle_{h_3})\\
&\quad\otimes(|\beta\rangle_{a_1}|-\beta\rangle_{b_1}|\gamma\rangle_{E_{a_1}}|-\gamma\rangle_{E_{b_1}}-|-\beta\rangle_{a_1}|\beta\rangle_{b_1}\\
&\quad\otimes|-\gamma\rangle_{E_{a_1}}|\gamma\rangle_{E_{b_1}})(|\beta\rangle_{a_2}|-\beta\rangle_{b_2}|\gamma\rangle_{E_{a_2}}|-\gamma\rangle_{E_{b_2}}\\
&\quad-|-\beta\rangle_{a_2}|\beta\rangle_{b_2}|-\gamma\rangle_{E_{a_2}}|\gamma\rangle_{E_{b_2}}), \qquad (18)
\end{aligned}$$

in which only the components

$$\begin{aligned}
&|GHZ\rangle^n_{g_1g_2g_3}|GHZ\rangle^n_{h_1h_2h_3}(|\beta\rangle_{a_1}|-\beta\rangle_{b_1}|\gamma\rangle_{E_{a_1}}\\
&\otimes|-\gamma\rangle_{E_{b_1}}|\beta\rangle_{a_2}|-\beta\rangle_{b_2}|\gamma\rangle_{E_{a_2}}|-\gamma\rangle_{E_{b_2}}+|-\beta\rangle_{a_1}\\
&\otimes|\beta\rangle_{b_1}|-\gamma\rangle_{E_{a_1}}|\gamma\rangle_{E_{b_1}}|-\beta\rangle_{a_2}|\beta\rangle_{b_2}|-\gamma\rangle_{E_{a_2}}|\gamma\rangle_{E_{b_2}}), \qquad (19)
\end{aligned}$$

make contributions to this MBEPP.

Moreover, with the similar principle as described in Section 2, the crossed combinations $|\phi_1^-\rangle_{a_1b_1}|\psi_1^-\rangle_{a_2b_2}$ and $|\psi_1^-\rangle_{a_1b_1}|\phi_1^-\rangle_{a_2b_2}$ can be eliminated automatically. All the cases corresponding to the successful purification and the corresponding operations are listed in Table 2 in detail.

In this way, when the MBEPP is successful, we can obtain the resultant state as (see Appendix B)

$$\rho^l_{g_3h_3}=F_1|\phi_1^-\rangle_{g_3h_3}\langle\phi_1^-|+(1-F_1)|\psi_1^-\rangle_{g_3h_3}\langle\psi_1^-|, \qquad (20)$$

where $F_1=\frac{F^2}{F^2+(1-F)^2}$ and

$$\begin{aligned}
|\phi_1^-\rangle_{g_3h_3}&=\frac{1}{N_3}(|\beta\rangle_{g_3}|\beta\rangle_{h_3}|\gamma\rangle_{E_{a_1}}|\gamma\rangle_{E_{a_2}}|\gamma\rangle_{E_{b_1}}|\gamma\rangle_{E_{b_2}}\\
&\quad-|-\beta\rangle_{g_3}|-\beta\rangle_{h_3}|-\gamma\rangle_{E_{a_1}}|-\gamma\rangle_{E_{a_2}}|-\gamma\rangle_{E_{b_1}}\\
&\quad|-\gamma\rangle_{E_{b_2}}), \qquad (21)\\
|\psi_1^-\rangle_{g_3h_3}&=\frac{1}{N_3}(|\beta\rangle_{g_3}|-\beta\rangle_{h_3}|\gamma\rangle_{E_{a_1}}|\gamma\rangle_{E_{a_2}}|-\gamma\rangle_{E_{b_1}}|-\gamma\rangle_{E_{b_2}}\\
&\quad-|-\beta\rangle_{g_3}|\beta\rangle_{h_3}|-\gamma\rangle_{E_{a_1}}|-\gamma\rangle_{E_{a_2}}|\gamma\rangle_{E_{b_1}}|\gamma\rangle_{E_{b_2}}), \qquad (22)
\end{aligned}$$

Table 2 The outcomes of BSMs and corresponding operations performed on the photon in one output mode with the photon loss. The first column denotes the measurement outcomes on g_1a_1, g_2a_2, h_1b_1 and h_2b_2. The second column represents the parity of number of "−". The third column means the additional operations are required to operate.

BSM outcomes	Number of "−"	Operation
$\|\phi\rangle_{g_1a_1}\|\phi\rangle_{g_2a_2}\|\phi\rangle_{h_1b_1}\|\phi\rangle_{h_2b_2}$	odd "−"	I
$\|\psi\rangle_{g_1a_1}\|\psi\rangle_{g_2a_2}\|\psi\rangle_{h_1b_1}\|\psi\rangle_{h_2b_2}$	even "−"	σ_z
$\|\phi\rangle_{g_1a_1}\|\phi\rangle_{g_2a_2}\|\psi\rangle_{h_1b_1}\|\psi\rangle_{h_2b_2}$	odd "−"	σ_x
$\|\psi\rangle_{g_1a_1}\|\psi\rangle_{g_2a_2}\|\phi\rangle_{h_1b_1}\|\phi\rangle_{h_2b_2}$	even "−"	$\sigma_x\sigma_z$

where $1/N_3$ is a normalization coefficient. For the whole system, the loss modes $|\pm\gamma\rangle_{E_{a_1}}|\pm\gamma\rangle_{E_{a_2}}$ and $|\pm\gamma\rangle_{E_{b_1}}|\pm\gamma\rangle_{E_{b_2}}$ can be respectively viewed as a large environment as $|\pm\sqrt{2}\gamma\rangle_{E_a}$ and $|\pm\sqrt{2}\gamma\rangle_{E_b}$ for Alice and Bob. As a result, the Eqs. (21) and (22) can be rewritten as

$$\begin{aligned}
|\phi_2^-\rangle_{g_3h_3}&=\frac{1}{N_3}(|\beta\rangle_{g_3}|\beta\rangle_{h_3}|\sqrt{2}\gamma\rangle_{E_a}|\sqrt{2}\gamma\rangle_{E_b}\\
&\quad-|-\beta\rangle_{g_3}|-\beta\rangle_{h_3}|-\sqrt{2}\gamma\rangle_{E_a}|-\sqrt{2}\gamma\rangle_{E_b}), \qquad (23)
\end{aligned}$$

and

$$\begin{aligned}
|\psi_2^-\rangle_{g_3h_3}&=\frac{1}{N_3}(|\beta\rangle_{g_3}|-\beta\rangle_{h_3}|\sqrt{2}\gamma\rangle_{E_a}|-\sqrt{2}\gamma\rangle_{E_b}\\
&\quad-|-\beta\rangle_{g_3}|\beta\rangle_{h_3}|-\sqrt{2}\gamma\rangle_{E_a}|\sqrt{2}\gamma\rangle_{E_b}). \qquad (24)
\end{aligned}$$

With the method described in Ref. [64], we use an orthogonal two dimensional basis $\{|u\rangle,|v\rangle\}$ to represent $|\pm\sqrt{2}\gamma\rangle_y$ where $y=E_a$ or E_b, yielding

$$\begin{aligned}
&|\sqrt{2}\gamma\rangle_y=\mu_y|u\rangle_y+\nu_y|v\rangle_y,\\
&|-\sqrt{2}\gamma\rangle_y=\mu_y|u\rangle_y-\nu_y|v\rangle_y, \qquad (25)
\end{aligned}$$

in which $\mu_y^2=\frac{1+e^{-4(1-\eta)\alpha^2}}{2}$ and $\nu_y^2=\frac{1-e^{-4(1-\eta)\alpha^2}}{2}$.

Then, when we trace out the loss modes, the states $|\phi_2^-\rangle_{g_3h_3}$ and $|\psi_2^-\rangle_{g_3h_3}$ will become

$$\begin{aligned}
\rho_{l1}&=N_4^{-1}[N_{1-}^2(1+e^{-8(1-\eta)\alpha^2})|\phi_3^-\rangle_{g_3h_3}\langle\phi_3^-|\\
&\quad+N_{1+}^2(1-e^{-8(1-\eta)\alpha^2})|\psi_3^+\rangle_{g_3h_3}\langle\psi_3^+|], \qquad (26)
\end{aligned}$$

and

$$\begin{aligned}
\rho_{l2}&=N_4^{-1}[N_{1-}^2(1+e^{-8(1-\eta)\alpha^2})|\psi_3^-\rangle_{g_3h_3}\langle\psi_3^-|\\
&\quad+N_{1+}^2(1-e^{-8(1-\eta)\alpha^2})|\phi_3^+\rangle_{g_3h_3}\langle\phi_3^+|], \qquad (27)
\end{aligned}$$

where N_4^{-1} is a normalization factor and

$$|\phi_3^{\pm}\rangle_{g_3h_3} = \frac{1}{N_{1\pm}}(|\beta\rangle_{g_3}|\beta\rangle_{h_3} \pm |-\beta\rangle_{g_3}|-\beta\rangle_{h_3}),$$
$$|\psi_3^{\pm}\rangle_{g_3h_3} = \frac{1}{N_{1\pm}}(|\beta\rangle_{g_3}|-\beta\rangle_{h_3} \pm |-\beta\rangle_{g_3}|\beta\rangle_{h_3}). \tag{28}$$

In Eq. (28), $N_{1\pm} = \sqrt{2 \pm 2\mathrm{e}^{-4\eta\alpha^2}}$. Subsequently, we can rewrite Eq. (20) as

$$\begin{aligned}\rho_1^l = {}& N_5[F_1F_2|\phi_3^-\rangle_{g_3h_3}\langle\phi_3^-| + F_1(1-F_2) \\ &\times |\psi_3^+\rangle_{g_3h_3}\langle\psi_3^+| + (1-F_1)F_2|\psi_3^-\rangle_{g_3h_3}\langle\psi_3^-| \\ &+ (1-F_1)(1-F_2)|\phi_3^+\rangle_{g_3h_3}\langle\phi_3^+|],\end{aligned} \tag{29}$$

where N_5 is the normalization factor and

$$F_2 = \frac{N_{1-}^2[1+\mathrm{e}^{-8(1-\eta)\alpha^2}]}{N_{1-}^2[1+\mathrm{e}^{-8(1-\eta)\alpha^2}] + N_{1+}^2[1-\mathrm{e}^{-8(1-\eta)\alpha^2}]}. \tag{30}$$

It is clear for one to observe from Eq. (29) that the photon loss resulted from the noisy environment can be transformed to the bit-flip error. Thus, the mixed state given by Eq. (20) can be rewritten as the general case containing both bit-flip error and phase-flip error. In this case, the high-quality ECSs can be obtained from Eq. (29) if $F_1F_2 > \frac{1}{2}$. As a result, after performing one round of purification, one can obtain the new mixed state as

$$\begin{aligned}\rho_n^l = {}& A|\phi_3^-\rangle_{g_3h_3}\langle\phi_3^-| + B|\psi_3^+\rangle_{g_3h_3}\langle\psi_3^+| \\ &+ C|\psi_3^-\rangle_{g_3h_3}\langle\psi_3^-| + D|\phi_3^+\rangle_{g_3h_3}\langle\phi_3^+|,\end{aligned} \tag{31}$$

with

$$A = \frac{[F_1^2+(1-F_1)^2]F_2^2}{F_2^2+(1-F_2)^2}, \quad B = \frac{[F_1^2+(1-F_1)^2](1-F_2)^2}{F_2^2+(1-F_2)^2},$$
$$C = \frac{2F_1(1-F_1)F_2^2}{F_2^2+(1-F_2)^2}, \quad D = \frac{2F_1(1-F_1)(1-F_2)^2}{F_2^2+(1-F_2)^2}. \tag{32}$$

The fidelity of ρ_n^l is $\langle\phi_3^-|\rho_n^l|\phi_3^-\rangle = A$. To ensure that the fidelity of Eq. (31) is larger than that of Eq. (13) after tracing out the loss modes, we require

$$\frac{[F_1^2+(1-F_1)^2]F_2^2}{[F_2^2+(1-F_2)^2]F} > \frac{(1-\mathrm{e}^{-4\eta\alpha^2})(1+\mathrm{e}^{-4(1-\eta)\alpha^2})}{2(1-\mathrm{e}^{-4\alpha^2})}. \tag{33}$$

Further, it is necessary for one to perform the operation $U_x(\pi/4)$ to transform phase-flip error to bit-flip error, i.e., $|\phi_3^+\rangle \Leftrightarrow |\psi_3^+\rangle$ before each round of purification which is similar to that in Ref. [60]. Thus, the fidelity of the state $|\phi^-\rangle_{g_3h_3}$ increases with an increasing round of purification, accordingly. In addition, we assume that F_1 tends to unity after purifying the mixed state as the form of Eq. (20) after a large number of purification rounds. Under this circumstance, we can obtain

$$N_{1-}^2[1+\mathrm{e}^{-8(1-\eta)\alpha^2}] > N_{1+}^2[1-\mathrm{e}^{-8(1-\eta)\alpha^2}], \tag{34}$$

when $\eta > \frac{2}{3}$.

It is clear to observe from Eq. (34) that the basic requirement of this MBEPP just relies on the transmission efficiency without depending on the photon number. So far, we have completely carried out the analysis for the MBEPP under the dissipation combined with the bit-flip error. Interestingly, the error caused by dissipation can be transformed to bit-flip error which is similar as Ref. [64].

4 Discussion and conclusion

We have discussed the MBEPP for ECSs under the photon loss combined with the bit-flip error. The phase-flip error can be transformed to bit-flip error assisted by the operation $U_x(\frac{\pi}{4})$ [27] and can be purified in the subsequent purification step. It is interesting for us to compare this MBEPP with the conventional EPP using controlled-not (CNOT) gate [59], in which one pair is considered to be a control and the other pair as a target. After passing through the CNOT gate, the target pair is measured by z-basis and the source pair is retained if the measurement results are the same. In this MBEPP, let us denote $|\phi\rangle \equiv |0\rangle$ and $|\psi\rangle \equiv |1\rangle$. For Alice's side, the outcomes of BSMs on g_1a_1 and g_2a_2 have four cases, i.e., $|\phi\rangle_{g_1a_1}|\phi\rangle_{g_2a_2}$, $|\phi\rangle_{g_1a_1}|\psi\rangle_{g_2a_2}$, $|\psi\rangle_{g_1a_1}|\phi\rangle_{g_2a_2}$ and $|\psi\rangle_{g_1a_1}|\psi\rangle_{g_2a_2}$, which correspond to $|0\rangle|0\rangle$, $|0\rangle|1\rangle$, $|1\rangle|0\rangle$ and $|1\rangle|1\rangle$. Subsequently, we can obtain $|0\rangle$ and $|1\rangle$ by applying additional modulo 2 to these two measurement outcomes. The same case happens at Bob's side. We retain the states entangled in modes g_3 and h_3 when the results of addition modulo 2 at Alice's and Bob's sides are the same. In addition, the corresponding operations should be performed on one of the output modes based on the number of sign "$-$" among four outcomes of BSMs. In this view, it plays a similar role as a CNOT gate. Moreover, this MBEPP is able to tolerate photon loss while the MBEPPs in Refs. [73, 77, 78] become invalid once any photon loses. In this MBEPP, the photon-number-resolving detector is pivotal. Fortunately, the photon counting detector working at visible and near-infrared wavelengths have been constructed [80], which employs superconducting transition-edge sensors. The efficiency of this detector is 95% at 1556 nmwavelengths and it can be further improved by utilizing some appropriate methods.

In conclusion, we propose the first MBEPP for ECSs to correct the bit-flip error. Subsequently, we consider the practical scenario that the coherent states suffer from the dissipation and the bit-flip error. Surprisingly, we show that the error resulted from the dissipation can be converted to bit-flip error and then be corrected with the same method for correcting the bit-flip error. Additionally, if one combines our MBEPP with the entanglement generation [46–49] and entanglement swapping [45] as well as quantum memory [84] for ECSs, the long-distance quantum communication based on ECSs may be realized in the future.

Appendix A

In this appendix, we show the MBEPP without considering the photon loss. As shown in Fig. 1 of the main text, we need two noisy copies, the whole mixed state is

$$\begin{aligned}\rho_t = F^2 & |\phi^-\rangle_{a_1b_1}\langle\phi^-| \otimes |\phi^-\rangle_{a_2b_2}\langle\phi^-| \\ &+ F(1-F)\times|\phi^-\rangle_{a_1b_1}\langle\phi^-|\otimes|\psi^-\rangle_{a_2b_2}\langle\psi^-| \\ &+ F(1-F)\times|\psi^-\rangle_{a_1b_1}\langle\psi^-|\otimes|\phi^-\rangle_{a_2b_2}\langle\phi^-| \\ &+ (1-F)^2\times|\psi^-\rangle_{a_1b_1}\langle\psi^-|\otimes|\psi^-\rangle_{a_2b_2}\langle\psi^-|. \end{aligned} \tag{35}$$

Here, we only carry out the analysis for the item $|\phi^-\rangle_{a_1b_1}|\phi^-\rangle_{a_2b_2}$. The discussion for the other items can be done with the same principle. Thus, with the probability of F^2, the system $\rho_{a_1b_1}\otimes\rho_{a_2b_2}\otimes|GHZ\rangle_{g_1g_2g_3}\otimes|GHZ\rangle_{h_1h_2h_3}$ is in the state

$$\begin{aligned}&|\phi^-\rangle_{a_1b_1}\otimes|\phi^-\rangle_{a_2b_2}\otimes|GHZ\rangle_{g_1g_2g_3}\otimes|GHZ\rangle_{h_1h_2h_3}\\ &= \frac{1}{N_-^2N_1^2}(|\alpha,\alpha\rangle_{a_1b_1}-|{-\alpha},-\alpha\rangle_{a_1b_1})\otimes(|\alpha,\alpha\rangle_{a_2b_2}\\ &\quad -|{-\alpha},-\alpha\rangle_{a_2b_2})\otimes(|\alpha,\alpha,\alpha\rangle_{g_1g_2g_3}+|{-\alpha},-\alpha,-\alpha\rangle_{g_1g_2g_3})\\ &\quad \otimes(|\alpha,\alpha,\alpha\rangle_{h_1h_2h_3}+|{-\alpha},-\alpha,-\alpha\rangle_{h_1h_2h_3}). \end{aligned}\tag{36}$$

Subsequently, if we reorganize the order of the photons, the state in Eq. (36) can be expanded as

$$\begin{aligned}&|\phi^-\rangle_{a_1b_1}\otimes|\phi^-\rangle_{a_2b_2}\otimes|GHZ\rangle_{g_1g_2g_3}\otimes|GHZ\rangle_{h_1h_2h_3}\\ &=|\alpha,\alpha\rangle_{g_3h_3}[|\alpha,\alpha\rangle|\alpha,\alpha\rangle|\alpha,\alpha\rangle|\alpha,\alpha\rangle-|\alpha,\alpha\rangle|\alpha,-\alpha\rangle\\ &\quad|\alpha,\alpha\rangle|\alpha,-\alpha\rangle-|\alpha,-\alpha\rangle|\alpha,\alpha\rangle|\alpha,-\alpha\rangle|\alpha,\alpha\rangle+|\alpha,-\alpha\rangle\\ &\quad|\alpha,-\alpha\rangle|\alpha,-\alpha\rangle|\alpha,-\alpha\rangle]+|\alpha,-\alpha\rangle_{g_3h_3}[|\alpha,\alpha\rangle|\alpha,\alpha\rangle\\ &\quad|-\alpha,\alpha\rangle|-\alpha,\alpha\rangle-|\alpha,\alpha\rangle|\alpha,-\alpha\rangle|-\alpha,\alpha\rangle|-\alpha,-\alpha\rangle\\ &\quad-|\alpha,-\alpha\rangle|\alpha,\alpha\rangle|-\alpha,-\alpha\rangle|-\alpha,\alpha\rangle+|\alpha,-\alpha\rangle|\alpha,-\alpha\rangle\\ &\quad|-\alpha,-\alpha\rangle|-\alpha,-\alpha\rangle]+|-\alpha,\alpha\rangle_{g_3h_3}[|-\alpha,\alpha\rangle|-\alpha,\alpha\rangle\\ &\quad|\alpha,\alpha\rangle|\alpha,\alpha\rangle-|-\alpha,\alpha\rangle|-\alpha,-\alpha\rangle|\alpha,\alpha\rangle|\alpha,-\alpha\rangle\\ &\quad-|-\alpha,-\alpha\rangle|-\alpha,\alpha\rangle|\alpha,-\alpha\rangle|\alpha,\alpha\rangle+|-\alpha,-\alpha\rangle\\ &\quad|-\alpha,-\alpha\rangle|\alpha,-\alpha\rangle|\alpha,-\alpha\rangle]+|-\alpha,-\alpha\rangle_{g_3h_3}[|-\alpha,\alpha\rangle\\ &\quad|-\alpha,\alpha\rangle|-\alpha,\alpha\rangle|-\alpha,\alpha\rangle-|-\alpha,\alpha\rangle|-\alpha,-\alpha\rangle\\ &\quad|-\alpha,\alpha\rangle|-\alpha,-\alpha\rangle-|-\alpha,-\alpha\rangle|-\alpha,\alpha\rangle|-\alpha,-\alpha\rangle\\ &\quad|-\alpha,\alpha\rangle+|-\alpha,-\alpha\rangle|-\alpha,-\alpha\rangle|-\alpha,-\alpha\rangle\\ &\quad|-\alpha,-\alpha\rangle], \end{aligned}\tag{37}$$

where we neglect the normalization factor. The photon pairs in the brackets are in the order of g_1a_1, g_2a_2, h_1b_1 and h_2b_2. Here, we merely consider the items Eq. (38) of Eq. (37) and the similar analysis can be carried out for the other remaining items

$$\begin{aligned}&|\alpha,\alpha\rangle_{g_3h_3}|\alpha,\alpha\rangle_{g_1a_1}|\alpha,\alpha\rangle_{g_2a_2}|\alpha,\alpha\rangle_{h_1b_1}|\alpha,\alpha\rangle_{h_2b_2}\\ &\quad+|-\alpha,-\alpha\rangle_{g_3h_3}|-\alpha,-\alpha\rangle_{g_1a_1}|-\alpha,-\alpha\rangle_{g_2a_2}\\ &\quad|-\alpha,-\alpha\rangle_{h_1b_1}|-\alpha,-\alpha\rangle_{h_2b_2}. \end{aligned}\tag{38}$$

Then, we replace $\pm\alpha$ of Eq. (38) with the quasi-Bell basis $\{\phi^\pm,\psi^\pm\}$, yielding

$$\begin{aligned}&(|\alpha,\alpha\rangle_{g_3h_3}+|-\alpha,-\alpha\rangle_{g_3h_3})(N_+^4|\phi^+\rangle|\phi^+\rangle|\phi^+\rangle|\phi^+\rangle\\ &\quad+N_+^2N_-^2|\phi^+\rangle|\phi^+\rangle|\phi^-\rangle|\phi^-\rangle+N_+^2N_-^2|\phi^+\rangle|\phi^-\rangle|\phi^+\rangle|\phi^-\rangle\\ &\quad+N_+^2N_-^2|\phi^+\rangle|\phi^-\rangle|\phi^-\rangle|\phi^+\rangle+N_+^2N_-^2|\phi^-\rangle|\phi^+\rangle|\phi^+\rangle|\phi^-\rangle\\ &\quad+N_+^2N_-^2|\phi^-\rangle|\phi^+\rangle|\phi^-\rangle|\phi^+\rangle+N_+^2N_-^2|\phi^-\rangle|\phi^-\rangle|\phi^+\rangle|\phi^+\rangle\\ &\quad+N_-^4|\phi^-\rangle|\phi^-\rangle|\phi^-\rangle|\phi^-\rangle)\\ &\quad+(|\alpha,\alpha\rangle_{g_3h_3}-|-\alpha,-\alpha\rangle_{g_3h_3})(N_+^3N_-|\phi^+\rangle|\phi^+\rangle|\phi^+\rangle|\phi^-\rangle\\ &\quad+N_+^3N_-|\phi^+\rangle|\phi^+\rangle|\phi^-\rangle|\phi^+\rangle+N_+^3N_-|\phi^+\rangle|\phi^-\rangle|\phi^+\rangle|\phi^+\rangle\\ &\quad+N_+N_-^3|\phi^+\rangle|\phi^-\rangle|\phi^-\rangle|\phi^-\rangle+N_+^3N_-|\phi^-\rangle|\phi^+\rangle|\phi^+\rangle|\phi^+\rangle\\ &\quad+N_+N_-^3|\phi^-\rangle|\phi^+\rangle|\phi^-\rangle|\phi^-\rangle+N_+N_-^3|\phi^-\rangle|\phi^-\rangle|\phi^+\rangle|\phi^-\rangle\\ &\quad+N_+N_-^3|\phi^-\rangle|\phi^-\rangle|\phi^-\rangle|\phi^+\rangle). \end{aligned}\tag{39}$$

It is clear for one to see that if we get odd number of $|\phi^-\rangle$, the state will collapse to

$$|\phi^-\rangle_{g_3h_3}=\frac{1}{N_-}(|\alpha,\alpha\rangle_{g_3h_3}-|-\alpha,-\alpha\rangle_{g_3h_3}).\tag{40}$$

On the contrary, an even number of $|\phi^-\rangle$ makes the state change to

$$|\phi^+\rangle_{g_3h_3}=\frac{1}{N_+}(|\alpha,\alpha\rangle_{g_3h_3}+|-\alpha,-\alpha\rangle_{g_3h_3}).\tag{41}$$

Thus, an additional operation σ_z is performed on the coherent state which will realize the transformation from $|\phi^+\rangle_{g_3h_3}$ to $|\phi^-\rangle_{g_3h_3}$. The same analysis can be carried out for the other remaining items of the state in Eq. (37). Similarly, the discussion on the state $|\phi^-\rangle_{a_1b_1}\otimes|\psi^-\rangle_{a_2b_2}\otimes|GHZ\rangle_{g_1g_2g_3}\otimes|GHZ\rangle_{h_1h_2h_3}$ and $|\psi^-\rangle_{a_1b_1}\otimes|\phi^-\rangle_{a_2b_2}\otimes|GHZ\rangle_{g_1g_2g_3}\otimes|GHZ\rangle_{h_1h_2h_3}$ as well as $|\psi^-\rangle_{a_1b_1}\otimes|\psi^-\rangle_{a_2b_2}\otimes|GHZ\rangle_{g_1g_2g_3}\otimes|GHZ\rangle_{h_1h_2h_3}$ can be done with the same principle. Finally, we can readily obtain the new mixed state with a higher-fidelity with or without operations, which is given by Table 1 in the main text.

Appendix B

This appendix presents the detailed discussion on the MBEPP with the photon loss. Similar to the ideal case, the total system encountering from the photon loss can be described as

$$\begin{aligned}\rho_t^l = F^2&|\phi_1^-\rangle_{a_1b_1}\langle\phi_1^-|\otimes|\phi_1^-\rangle_{a_2b_2}\langle\phi_1^-|\\ &+F(1-F)\times|\phi_1^-\rangle_{a_1b_1}\langle\phi_1^-|\otimes|\psi_1^-\rangle_{a_2b_2}\langle\psi_1^-|\\ &+F(1-F)\times|\psi_1^-\rangle_{a_1b_1}\langle\psi_1^-|\otimes|\phi_1^-\rangle_{a_2b_2}\langle\phi_1^-|\\ &+(1-F)^2\times|\psi_1^-\rangle_{a_1b_1}\langle\psi_1^-|\otimes|\psi_1^-\rangle_{a_2b_2}\langle\psi_1^-|, \end{aligned}\tag{42}$$

where $|\phi_1^-\rangle$ and $|\psi_1^-\rangle$ are given by Eqs. (21) and Eq. (22), respectively. Here, we merely consider the item $|\psi_1^-\rangle_{a_1b_1}$

$|\psi_1^-\rangle_{a_2b_2}$. As a result, the state $|\psi_1^-\rangle_{a_1b_1}$ $|\psi_1^-\rangle_{a_2b_2}$ combines with two pairs of resource states $|GHZ\rangle^n_{g_1g_2g_3}$ $|GHZ\rangle^n_{h_1h_2h_3}$ can be written as

$$\begin{aligned}
&|GHZ\rangle^n_{g_1g_2g_3}|GHZ\rangle^n_{h_1h_2h_3}|\psi_1^-\rangle_{a_1b_1}|\psi_1^-\rangle_{a_2b_2}\\
&=\frac{1}{N_-^2N_2^2}(|\beta\rangle_{g_1}|\beta\rangle_{g_2}|\beta\rangle_{g_3}+|-\beta\rangle_{g_1}|-\beta\rangle_{g_2}|-\beta\rangle_{g_3})\\
&\quad\otimes(|\beta\rangle_{h_1}|\beta\rangle_{h_2}|\beta\rangle_{h_3}+|-\beta\rangle_{h_1}|-\beta\rangle_{h_2}|-\beta\rangle_{h_3})\\
&\quad\otimes(|\beta\rangle_{a_1}|-\beta\rangle_{b_1}|\gamma\rangle_{E_{a_1}}|-\gamma\rangle_{E_{b_1}}-|-\beta\rangle_{a_1}|\beta\rangle_{b_1}\\
&\quad\otimes|-\gamma\rangle_{E_{a_1}}|\gamma\rangle_{E_{b_1}})(|\beta\rangle_{a_2}|-\beta\rangle_{b_2}|\gamma\rangle_{E_{a_2}}|-\gamma\rangle_{E_{b_2}}\\
&\quad-|-\beta\rangle_{a_2}|\beta\rangle_{b_2}|-\gamma\rangle_{E_{a_2}}|\gamma\rangle_{E_{b_2}}).
\end{aligned}\tag{43}$$

Due to the photon loss, only some components of Eq. (43) can make contributions to this MBEPP, which are described as

$$\begin{aligned}
&|\beta,\beta\rangle_{g_3h_3}[|\beta,\beta\rangle_{g_1a_1}|\beta,\beta\rangle_{g_2a_2}|\beta,-\beta\rangle_{h_1b_1}|\beta,-\beta\rangle_{h_2b_2}\\
&\quad\otimes|\sqrt{2}\gamma\rangle_{E_a}|-\sqrt{2}\gamma\rangle_{E_b}+|\beta,-\beta\rangle_{g_1a_1}|\beta,-\beta\rangle_{g_2a_2}\\
&\quad\otimes|\beta,\beta\rangle_{h_1b_1}|\beta,\beta\rangle_{h_2b_2}|-\sqrt{2}\gamma\rangle_{E_a}|\sqrt{2}\gamma\rangle_{E_b}]+|\beta,-\beta\rangle_{g_3h_3}\\
&\quad\otimes[|\beta,\beta\rangle_{g_1a_1}|\beta,\beta\rangle_{g_2a_2}|-\beta,-\beta\rangle_{h_1b_1}|-\beta,-\beta\rangle_{h_2b_2}\\
&\quad\otimes|\sqrt{2}\gamma\rangle_{E_a}|-\sqrt{2}\gamma\rangle_{E_b}+|\beta,-\beta\rangle_{g_1a_1}|\beta,-\beta\rangle_{g_2a_2}\\
&\quad\otimes|-\beta,\beta\rangle_{h_1b_1}|-\beta,\beta\rangle_{h_2b_2}|\sqrt{2}\gamma\rangle_{E_a}|-\sqrt{2}\gamma\rangle_{E_b}]\\
&\quad+|-\beta,\beta\rangle_{g_3h_3}[|-\beta,\beta\rangle_{g_1a_1}|-\beta,\beta\rangle_{g_2a_2}|\beta,-\beta\rangle_{h_1b_1}\\
&\quad\otimes|\beta,-\beta\rangle_{h_2b_2}|\sqrt{2}\gamma\rangle_{E_a}|-\sqrt{2}\gamma\rangle_{E_b}+|-\beta,-\beta\rangle_{g_1a_1}\\
&\quad\otimes|-\beta,-\beta\rangle_{g_2a_2}|\beta,\beta\rangle_{h_1b_1}|\beta,\beta\rangle_{h_2b_2}|\sqrt{2}\gamma\rangle_{E_a}|-\sqrt{2}\gamma\rangle_{E_b}]\\
&\quad+|-\beta,-\beta\rangle_{g_3h_3}[|-\beta,\beta\rangle_{g_1a_1}|-\beta,\beta\rangle_{g_2a_2}|-\beta,-\beta\rangle_{h_1b_1}\\
&\quad\otimes|-\beta,-\beta\rangle_{h_2b_2}|\sqrt{2}\gamma\rangle_{E_a}|-\sqrt{2}\gamma\rangle_{E_b}+|-\beta,-\beta\rangle_{g_1a_1}\\
&\quad\otimes|-\beta,-\beta\rangle_{g_2a_2}|-\beta,\beta\rangle_{h_1b_1}|-\beta,\beta\rangle_{h_2b_2}\\
&\quad\otimes|\sqrt{2}\gamma\rangle_{E_a}|-\sqrt{2}\gamma\rangle_{E_b}].
\end{aligned}\tag{44}$$

Here, we neglect the normalization factor. Similarly to Eq. (39), we only discuss the components in Eq. (45) (With the same principle, the analysis for the other components in Eq. (44) can be done.).

$$\begin{aligned}
&|\beta,-\beta\rangle_{g_3h_3}|\beta,\beta\rangle_{g_1a_1}|\beta,\beta\rangle_{g_2a_2}|-\beta,-\beta\rangle_{h_1b_1}\\
&\quad\otimes|-\beta,-\beta\rangle_{h_2b_2}|\sqrt{2}\gamma\rangle_{E_a}|-\sqrt{2}\gamma\rangle_{E_b}+|-\beta,\beta\rangle_{g_3h_3}\\
&\quad\otimes|-\beta,-\beta\rangle_{g_1a_1}|-\beta,-\beta\rangle_{g_2a_2}|\beta,\beta\rangle_{h_1b_1}|\beta,\beta\rangle_{h_2b_2}\\
&\quad\otimes|\sqrt{2}\gamma\rangle_{E_a}|-\sqrt{2}\gamma\rangle_{E_b}.
\end{aligned}\tag{45}$$

Let us rewrite Eq. (45) with the quasi-Bell basis $\{\phi^\pm,\psi^\pm\}$, we can obtain

$$\begin{aligned}
&(|\beta,-\beta\rangle_{g_3h_3}|\sqrt{2}\gamma\rangle_{E_a}|-\sqrt{2}\gamma\rangle_{E_b}+|-\beta,\beta\rangle_{g_3h_3}|\sqrt{2}\gamma\rangle_{E_a}\\
&\quad\otimes|-\sqrt{2}\gamma\rangle_{E_b})(N_{1+}^4|\phi_3^+\rangle|\phi_3^+\rangle|\phi_3^+\rangle|\phi_3^+\rangle\\
&\quad+N_{1+}^2N_{1-}^2|\phi_3^+\rangle|\phi_3^+\rangle|\phi_3^-\rangle|\phi_3^-\rangle\\
&\quad-N_{1+}^2N_{1-}^2|\phi_3^+\rangle|\phi_3^-\rangle|\phi_3^+\rangle|\phi_3^-\rangle\\
&\quad-N_{1+}^2N_{1-}^2|\phi_3^+\rangle|\phi_3^-\rangle|\phi_3^-\rangle|\phi_3^+\rangle\\
&\quad-N_{1+}^2N_{1-}^2|\phi_3^-\rangle|\phi_3^+\rangle|\phi_3^+\rangle|\phi_3^-\rangle\\
&\quad-N_{1+}^2N_{1-}^2|\phi_3^-\rangle|\phi_3^+\rangle|\phi_3^-\rangle|\phi_3^+\rangle\\
&\quad+N_{1+}^2N_{1-}^2|\phi_3^-\rangle|\phi_3^-\rangle|\phi_3^+\rangle|\phi_3^+\rangle\\
&\quad+N_{1-}^4|\phi_3^-\rangle|\phi_3^-\rangle|\phi_3^-\rangle|\phi_3^-\rangle)\\
&\quad-(|\beta,-\beta\rangle_{g_3h_3}|\sqrt{2}\gamma\rangle_{E_a}|-\sqrt{2}\gamma\rangle_{E_b}\\
&\quad-|-\beta,\beta\rangle_{g_3h_3}|\sqrt{2}\gamma\rangle_{E_a}\\
&\quad\otimes|-\sqrt{2}\gamma\rangle_{E_b})(N_{1+}^3N_{1-}|\phi_3^+\rangle|\phi_3^+\rangle|\phi_3^+\rangle|\phi_3^-\rangle\\
&\quad+N_{1+}^3N_{1-}|\phi_3^+\rangle|\phi_3^+\rangle|\phi_3^-\rangle|\phi_3^+\rangle\\
&\quad-N_{1+}^3N_{1-}|\phi_3^+\rangle|\phi_3^-\rangle|\phi_3^+\rangle|\phi_3^+\rangle\\
&\quad-N_{1+}N_{1-}^3|\phi_3^+\rangle|\phi_3^-\rangle|\phi_3^-\rangle|\phi_3^-\rangle\\
&\quad-N_{1+}^3N_{1-}|\phi_3^-\rangle|\phi_3^+\rangle|\phi_3^+\rangle|\phi_3^+\rangle\\
&\quad-N_{1+}N_{1-}^3|\phi_3^-\rangle|\phi_3^+\rangle|\phi_3^-\rangle|\phi_3^-\rangle\\
&\quad+N_{1+}N_{1-}^3|\phi_3^-\rangle|\phi_3^-\rangle|\phi_3^+\rangle|\phi_3^-\rangle\\
&\quad+N_{1+}N_{1-}^3|\phi_3^-\rangle|\phi_3^-\rangle|\phi_3^-\rangle|\phi_3^+\rangle).
\end{aligned}\tag{46}$$

From Eq. (46), we can obtain

$$\begin{aligned}
|\psi_2^-\rangle_{g_3h_3}=\frac{1}{N_3}(&|\beta,-\beta\rangle_{g_3h_3}|\sqrt{2}\gamma\rangle_{E_a}|-\sqrt{2}\gamma\rangle_{E_b}\\
&-|-\beta,\beta\rangle_{g_3h_3}|\sqrt{2}\gamma\rangle_{E_a}|-\sqrt{2}\gamma\rangle_{E_b}),
\end{aligned}\tag{47}$$

when the number of $|\psi^-\rangle$ is odd. While if it is even, we can get

$$\begin{aligned}
|\psi_2^+\rangle_{g_3h_3}=\frac{1}{N_3}(&|\beta,-\beta\rangle_{g_3h_3}|\sqrt{2}\gamma\rangle_{E_a}|-\sqrt{2}\gamma\rangle_{E_b}\\
&+|-\beta,\beta\rangle_{g_3h_3}|\sqrt{2}\gamma\rangle_{E_a}|-\sqrt{2}\gamma\rangle_{E_b}),
\end{aligned}\tag{48}$$

which can be evolved to $|\psi_2^-\rangle_{g_3h_3}$ with the assistance of σ_z. With the similar principle, the analysis for the other items of $\rho^l_{a_1b_1}\otimes\rho^l_{a_2b_2}\otimes|GHZ\rangle^n_{g_1g_2g_3}\otimes|GHZ\rangle^n_{h_1h_2h_3}$ can be done. Consequently, the state $|\phi_1^-\rangle_{a_1b_1}$ $|\psi_1^-\rangle_{a_1b_2}$ $|GHZ\rangle^n_{g_1g_2g_3}$ $|GHZ\rangle^n_{h_1h_2h_3}$ and $|\psi_1^-\rangle_{a_1b_1}$ $|\phi_1^-\rangle_{a_1b_2}$ $|GHZ\rangle^n_{g_1g_2g_3}$ $|GHZ\rangle^n_{h_1h_2h_3}$ can be eliminated according to the measurement outcomes. Thus, the new mixed state can be written as Eq. (20).

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References

1. C. H. Bennett and S. J. Wiesner, Communication via one- and two-particle operators on Einstein–Podolsky–Rosen states, *Phys. Rev. Lett.* 69(20), 2881 (1992)
2. T. Das, R. Prabhu, A. Sen(De), and U. Sen, Distributed quantum dense coding with two receivers in noisy environments, *Phys. Rev. A* 92(5), 052330 (2015)
3. C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Teleporting an unknown quantum state via dual classical and Einstein–Podolsky–Rosen channels, *Phys. Rev. Lett.* 70(13), 1895 (1993)

4. B. G. Taketani, F. de Melo, and R. L. de Matos Filho, Optimal teleportation with a noisy source, *Phys. Rev. A* 85(2), 020301(R) (2012)
5. X. M. Hu, C. Zhang, C. J. Zhang, B. H. Liu, Y. F. Huang, Y. J. Han, C. F. Li, and G. C. Guo, Experimental certification for nonclassical teleportation, *Quant. Engineering* 1(2), e3 (2019)
6. Z. H. Yan, J. L. Qin, Z. Z. Qin, X. L. Su, X. J. Jia, C. D. Xie, and K. C. Peng, Generation of non-classical states of light and their application in deterministic quantum teleportation, *Fundamental Res.* 1(1), 43 (2021)
7. A. K. Ekert, Quantum cryptography based on Bells theorem, *Phys. Rev. Lett.* 67(6), 661 (1991)
8. H. K. Lo, M. Curty, and B. Qi, Measurement-device independent quantum key distribution, *Phys. Rev. Lett.* 108(13), 130503 (2012)
9. F. H. Xu, X. F. Ma, Q. Zhang, H. K. Lo, and J. W. Pan, Secure quantum key distribution with realistic devices, *Rev. Mod. Phys.* 92(2), 025002 (2020)
10. Z. X. Cui, W. Zhong, L. Zhou, and Y. B. Sheng, Measurement-device-independent quantum key distribution with hyper-encoding, *Sci. China Phys. Mech. Astron.* 62(11), 110311 (2019)
11. Y. F. Yan, L. Zhou, W. Zhong, and Y. B. Sheng, Measurement-device-independent quantum key distribution of multiple degrees of freedom of a single photon, *Front. Phys.* 16(1), 11501 (2021)
12. G. L. Long and X. S. Liu, Theoretically efficient high capacity quantum-key-distribution scheme, *Phys. Rev. A* 65(3), 032302 (2002)
13. F. G. Deng, G. L. Long, and X. S. Liu, Two-step quantum direct communication protocol using the Einstein–Podolsky–Rosen pair block, *Phys. Rev. A* 68(4), 042317 (2003)
14. F. G. Deng and G. L. Long, Secure direct communication with a quantum one-time pad, *Phys. Rev. A* 69(5), 052319 (2004)
15. W. Zhang, D. S. Ding, Y. B. Sheng, L. Zhou, B. S. Shi, and G. C. Guo, Quantum secure direct communication with quantum memory, *Phys. Rev. Lett.* 118(22), 220501 (2017)
16. F. Z. Wu, G. J. Yang, H. B. Wang, J. Xiong, F. Alzahran, A. Hobiny, and F. G. Deng, High-capacity quantum secure direct communication with two-photon six-qubit hyperentangled states, *Sci. China Phys. Mech. Astron.* 60(12), 120313 (2017)
17. S. S. Chen, L. Zhou, W. Zhong, and Y. B. Sheng, Three step three-party quantum secure direct communication, *Sci. China Phys. Mech. Astron.* 61(9), 90312 (2018)
18. L. Zhou, Y. B. Sheng, and G. L. Long, Device independent quantum secure direct communication against collective attacks, *Sci. Bull.* 65(1), 12 (2020)
19. Z. R. Zhou, Y. B. Sheng, P. H. Niu, L. G. Yin, G. L. Long, and L. Hanzo, Measurement-device-independent quantum secure direct communication, *Sci. China Phys. Mech. Astron.* 63(3), 230362 (2020)
20. T. Li, Z. K. Gao, and Z. H. Li, Measurement-device independent quantum secure direct communication: Direct quantum communication with imperfect measurement device and untrusted operator, *EPL* 131(6), 60001 (2020)
21. T. Li and G. L. Long, Quantum secure direct communication based on single-photon Bell-state measurement, *New J. Phys.* 22(6), 063017 (2020)
22. D. Pan, Z. S. Lin, J. W. Wu, H. R. Zhang, Z. Sun, D. Ruan, L. G. Yin, and G. L. Long, Experimental free-space quantum secure direct communication and its security analysis, *Photon. Res.* 8(9), 1522 (2020)
23. C. Wang, Quantum secure direct communication: Intersection of communication and cryptography, *Fundamental Res.* 1(1), 91 (2021)
24. Z. D. Ye, D. Pan, Z. Sun, C. G. Du, L. G. Yin, and G. L. Long, Generic security analysis framework for quantum secure direct communication, *Front. Phys.* 16(2), 21503 (2021)
25. J. Y. Quan, Q. Li, C. D. Liu, J. J. Shi, and Y. Peng, A simplified verifiable blind quantum computing protocol with quantum input verification, *Quant. Engineering* 3(1), e58 (2021)
26. D. X. Li, C. Yang, and X. Q. Shao, Dissipative engineering of a tripartite Greenberger–Horne–Zeilinger state for neutral atoms, *Quant. Engineering* 3(2), e66 (2021)
27. H. Jeong and M. S. Kim, Efficient quantum computation using coherent states, *Phys. Rev. A* 65(4), 042305 (2002)
28. M. Paternostro, M. S. Kim, and P. L. Knight, Vibrational coherent quantum computation, *Phys. Rev. A* 71(2), 022311 (2005)
29. A. P. Lund, T. C. Ralph, and H. L. Haselgrove, Fault tolerant linear optical quantum computing with small amplitude coherent states, *Phys. Rev. Lett.* 100(3), 030503 (2008)
30. L. M. Zhang, T. Gao, and F. L. Yan, Transformations of multilevel coherent states under coherence-preserving operations, *Sci. China Phys. Mech. Astron.* 64(6), 260312 (2021)
31. A. Mecozzi and P. Tombesi, Distinguishable quantum states generated via nonlinear birefringence, *Phys. Rev. Lett.* 58(11), 1055 (1987)
32. B. C. Sanders, Entangled coherent states, *Phys. Rev. A* 45(9), 6811 (1992)
33. H. Jeong, M. S. Kim, and J. Lee, Quantum-information processing for a coherent superposition state via a mixed entangled coherent channel, *Phys. Rev. A* 64(5), 052308 (2001)
34. X. G. Wang, Quantum teleportation of entangled coherent states, *Phys. Rev. A* 64(2), 022302 (2001)
35. S. J. van Enk and O. Hirota, Entangled coherent states: Teleportation and decoherence, *Phys. Rev. A* 64(2), 022313 (2001)
36. J. Joo and E. Ginossar, Efficient scheme for hybrid teleportation via entangled coherent states in circuit quantum electrodynamics, *Sci. Rep.* 6(1), 26338 (2016)
37. K. Park and H. Jeong, Entangled coherent states versus entangled photon pairs for practical quantum information processing, *Phys. Rev. A* 82(6), 062325 (2010)

38. D. S. Simon, G. Jaeger, and A. V. Sergienko, Entangled coherent-state quantum key distribution with entanglement witnessing, *Phys. Rev. A* 89(1), 012315 (2014)
39. S. L. Zhang, Improving long-distance distribution of entangled coherent state with the method of twin-field quantum key distribution, *Opt. Express* 27(25), 37087 (2019)
40. X. F. Ma, P. Zeng, and H. Y. Zhou, Phase-matching quantum key distribution, *Phys. Rev. X* 8(3), 031043 (2018)
41. X. D. Wu, Y. J. Wang, H. Zhong, Q. Liao, and Y. Guo, Plug-and-play dual-phase-modulated continuous variable quantum key distribution with photon subtraction, *Front. Phys.* 14(4), 41501 (2019)
42. J. J. Ma, Y. Zhou, X. Yuan, and X. F. Ma, Operational interpretation of coherence in quantum key distribution, *Phys. Rev. A* 99(6), 062325 (2019)
43. X. D. Wu, Y. J. Wang, D. Huang, and Y. Guo, Simultaneous measurement-device-independent continuous variable quantum key distribution with realistic detector compensation, *Front. Phys.* 15(3), 31601 (2020)
44. S. Y. Lee, Y. S. Ihn, and Z. Kim, Optimal entangled coherent states in lossy quantum-enhanced metrology, *Phys. Rev. A* 101(1), 012332 (2020)
45. N. Sangouard, C. Simon, N. Gisin, J. Laurat, R. Tualle-Brouri, and P. Grangier, Quantum repeaters with entangled coherent states, *J. Opt. Soc. Am. B* 27(6), A137 (2010)
46. L. M. Kuang, Z. B. Chen, and J. W. Pan, Generation of entangled coherent states for distant Bose–Einstein condensates via electromagnetically induced transparency, *Phys. Rev. A* 76(5), 052324 (2007)
47. A. P. Lund, T. C. Ralph, and H. Jeong, Generation of distributed entangled coherent states over a lossy environment with inefficient detectors, *Phys. Rev. A* 88(5), 052335 (2013)
48. Z. R. Zhong, X. J. Huang, Z. B. Yang, L. T. Shen, and S. B. Zheng, Generation and stabilization of entangled coherent states for the vibrational modes of a trapped ion, *Phys. Rev. A* 98(3), 032311 (2018)
49. B. Xiong, X. Li, S. L. Chao, Z. Yang, W. Z. Zhang, and L. Zhou, Generation of entangled Schrödinger cat state of two macroscopic mirrors, *Opt. Express* 27(9), 13547 (2019)
50. L. Tian, S. P. Shi, Y. H. Tian, Y. J. Wang, Y. H. Zheng, and K. C. Peng, Resource reduction for simultaneous generation of two types of continuous variable nonclassical states, *Front. Phys.* 16(2), 21502 (2021)
51. Y. B. Sheng, L. Zhou, S. M. Zhao, and B. Y. Zheng, Efficient single-photon-assisted entanglement concentration for partially entangled photon pairs, *Phys. Rev. A* 85(1), 012307 (2012)
52. Y. B. Sheng, L. Zhou, and S. M. Zhao, Efficient two-step entanglement concentration for arbitrary Wstates, *Phys. Rev. A* 85(4), 042302 (2012)
53. B. C. Ren and G. L. Long, General hyperentanglement concentration for photon systems assisted by quantum-dot spins inside optical microcavities, *Opt. Express* 22(6), 6547 (2014)
54. H. Wang, B. C. Ren, A. H. Wang, A. Alsaedi, T. Hayat, and F. G. Deng, General hyperentanglement concentration for polarization-spatial-time-bin multi-photon systems with linear optics, *Front. Phys.* 13(5), 130315 (2018)
55. H. Wang, B. C. Ren, F. Alzahrani, A. Hobiny, and F. G. Deng, Hyper entanglement concentration for polarization-spatial-time-bin hyperentangled photon systems with linear optics, *Quantum Inform. Process.* 16(10), 237 (2017)
56. M. Sisodia, C. Shukla, and G. L. Long, Linear optics based entanglement concentration protocols for cluster type entangled coherent state, *Quantum Inform. Process.* 18(8), 253 (2019)
57. J. Liu, L. Zhou, W. Zhong, and Y. B. Sheng, Logic Bell state concentration with parity check measurement, *Front. Phys.* 14(2), 21601 (2019)
58. L. Zhou, J. Liu, Z. K. Liu, W. Zhong, and Y. B. Sheng, Logic W-state concentration with parity check, *Quant. Engineering* 3(2), e63 (2021)
59. C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin, and W. K. Wootters, Purification of noise entanglement and faithful teleportation via noisy channels, *Phys. Rev. Lett.* 76(5), 722 (1996)
60. D. Deutsch, A. Ekert, R. Jozsa, C. Macchiavello, S. Popescu, and A. Sanpera, Quantum privacy amplification and the security of quantum cryptography over noisy channels, *Phys. Rev. Lett.* 77(13), 2818 (1996)
61. M. Murao, M. B. Plenio, S. Popescu, V. Vedral, and P. L. Knight, Multiparticle entanglement purification protocols, *Phys. Rev. A* 57(6), R4075 (1998)
62. J. W. Pan, C. Simon, C. Brukner, and A. Zeilinger, Entanglement purification for quantum communication, *Nature* 410(6832), 1067 (2001)
63. C. Simon and J. W. Pan, Polarization entanglement purification using spatial entanglement, *Phys. Rev. Lett.* 89(25), 257901 (2002)
64. Y. B. Sheng, L. Zhou, and G. L. Long, Hybrid entanglement purification for quantum repeaters, *Phys. Rev. A* 88(2), 022302 (2013)
65. M. Zwerger, H. J. Briegel, and W. Dür, Universal and optimal error thresholds for measurement-based entanglement purification, *Phys. Rev. Lett.* 110(26), 260503 (2013)
66. M. Zwerger, H. J. Briegel, and W. Dür, Robustness of hashing protocols for entanglement purification, *Phys. Rev. A* 90(1), 012314 (2014)
67. G. Y. Wang, T. Li, Q. Ai, A. Alsaedi, T. Hayat, and F. G. Deng, Faithful entanglement purification for high capacity quantum communication with two-photon four qubit systems, *Phys. Rev. Appl.* 10(5), 054058 (2018)
68. L. Zhou, W. Zhong, and Y. B. Sheng, Purification of the residual entanglement, *Opt. Express* 28(2), 2291 (2020)
69. M. Y. Wang, F. L. Yan, and T. Gao, Entanglement purification of two-photon systems in multiple degrees of freedom, *Quantum Inform. Process.* 19(7), 206 (2020)
70. D. Y. Chen, Z. Lin, M. Yang, Q. Yang, X. P. Zang, and Z. L. Cao, Distillation of lossy hyperentangled states, *Phys. Rev. A* 102(2), 022425 (2020)
71. X. M. Hu, C. X. Huang, Y. B. Sheng, L. Zhou, B. H. Liu, Y. Guo, C. Zhang, W. B. Xing, Y. F. Huang, C. F. Li, and G. C. Guo, Long-distance entanglement purification for quantum communication, *Phys. Rev. Lett.* 126(1), 010503 (2021)

72. P. S. Yan, L. Zhou, W. Zhong, and Y. B. Sheng, Feasible time-bin entanglement purification based on sum frequency generation, *Opt. Express* 29(2), 571 (2021)

73. P. S. Yan, L. Zhou, W. Zhong, and Y. B. Sheng, Feasible measurement-based entanglement purification in linear optics, *Opt. Express* 29(6), 9363 (2021)

74. H. Jeong and M. S. Kim, Purification of entangled coherent states, *Quantum Inf. Comput.* 2(3), 208 (2002)

75. J. Clausen, L. Knöll, and D. G. Welsch, Lossy purification and detection of entangled coherent states, *Phys. Rev. A* 66(6), 062303 (2002)

76. U. L. Andersen, R. Filip, J. Fiurášek, V. Josse, and G. Leuchs, Experimental purification of coherent states, *Phys. Rev. A* 72(6), 060301(R) (2005)

77. M. Zwerger, W. Dür, and H. J. Briegel, Measurement based quantum repeaters, *Phys. Rev. A* 85(6), 062326 (2012)

78. M. Zwerger, H. J. Briegel, and W. Dür, Measurement based quantum communication, *Appl. Phys. B* 122(3), 50 (2016)

79. S. W. Lee and H. Jeong, Near-deterministic quantum teleportation and resource-efficient quantum computation using linear optics and hybrid qubits, *Phys. Rev. A* 87(2), 022326 (2013)

80. A. E. Lita, A. J. Miller, and S. W. Nam, Counting near infrared single-photons with 95% efficiency, *Opt. Express* 16(5), 3032 (2008)

81. M. Jönsson, M. Swillo, S. Gyger, V. Zwiller, and G. Björk, Temporal array with superconducting nanowire single photon detectors for photon-number-resolution, *Phys. Rev. A* 102(5), 052616 (2020)

82. R. Guo, L. Zhou, S. P. Gu, X. F. Wang, and Y. B. Sheng, Generation of concatenated Greenberger–Horne–Zeilinger-type entangled coherent state based on linear optics, *Quantum Inform. Process.* 16(3), 68 (2017)

83. L. V. Hau, S. E. Harris, Z. Dutton, and C. H. Behroozi, Light speed reduction to 17 metres per second in an ultracold atomic gas, *Nature* 397(6720), 594 (1999)

84. K. Jensen, W. Wasilewski, H. Krauter, T. Fernholz, B. M. Nielsen, M. Owari, M. B. Plenio, A. Serafini, M. M. Wolf, and E. S. Polzik, Quantum memory for entangled continuous-variable states, *Nat. Phys.* 7(1), 13 (2011)