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RESEARCH ARTICLE

Unidirectional propagation of water waves near ancient Luoyang Bridge

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Supporting Information

Section 1: Theoretical analysis

1 Corresponding relationship between electromagnetic wave equation for TM mode and general water wave equation

In our previous work, we found the corresponding relationship between electromagnetic wave for TM mode (Eq. (S1)) and shallow water wave equation ($\lambda \gg h$) (Eq. (S2)):

$$\nabla \cdot \left(\frac{\overleftrightarrow{\varepsilon}}{\det(\overleftrightarrow{\varepsilon})\,\varepsilon_0} \nabla H_z \right) + \mu\mu_0\omega^2 H_z = 0, \tag{S1}$$

where $\overleftrightarrow{\varepsilon} = \begin{bmatrix} \varepsilon_x & 0 \\ 0 & \varepsilon_y \end{bmatrix}$, $\frac{\overleftrightarrow{\varepsilon}}{\det(\overleftrightarrow{\varepsilon})} = \begin{bmatrix} 1/\varepsilon_y & 0 \\ 0 & 1/\varepsilon_x \end{bmatrix}$, ε_0 is the dielectric constant, $\overleftrightarrow{\varepsilon}$ is the relative dielectric constant tensor, μ_0 is permeability constant, μ is the relative permeability, ω is the angular frequency;

$$\nabla \cdot (\overleftrightarrow{h} \nabla p) + \frac{\omega^2}{g} p = 0, \tag{S2}$$

where $\overleftrightarrow{h}$ is a tensor with $\overleftrightarrow{h} = \begin{bmatrix} h_x & 0 \\ 0 & h_y \end{bmatrix}$, p is the hydrostatic pressure of water surface ($p = \rho g \eta$), ρ is the fluid density, g is the gravitational acceleration, η is the vertical displacement of the water wave (as shown in the lower right corner of Fig. 1(b) in the main text), ω is the angular frequency.

Then we compare Eqs. (S1) and (S2), we find that

$$H_z \leftrightarrow p, \frac{1}{\varepsilon_y \varepsilon_0} \leftrightarrow \frac{h_x}{\rho}, \frac{1}{\varepsilon_x \varepsilon_0} \leftrightarrow \frac{h_y}{\rho}, \frac{1}{\mu\mu_0} \leftrightarrow \rho g\,. \tag{S3}$$

Through Eq. (S3), we can manipulate the movement of water waves by adjusting water depth h and gravitational acceleration g, just as we manipulate electromagnetic waves by adjusting the dielectric constant ε and permeability μ. Let us now see whether this corresponding relationship still exist in water waves with a general water depth ($\lambda \sim h$). We write the general water equation in anisotropic water layers [1]:

$$\nabla \cdot \left(\frac{\tanh(k\overleftrightarrow{h})}{k} \cdot \nabla p \right) + \frac{\omega^2}{g} \eta = 0. \tag{S4}$$

The water wave equation is governed by nonlinear dispersion $\omega = \sqrt{g\tanh(kh)}k$. k is propagation wave number of water wave.

Through Eq. (S4), we can find that when the water depth does not meet the condition of shallow water ($\lambda \gg h$), the control equation of water wave becomes nonlinear, which makes it difficult to correspond with the electromagnetic waves.

Here we introduce a new variable u (equivalent depth), $u = \tanh(kh)/k$, so $\overleftrightarrow{u} = \tanh(k\overleftrightarrow{h})/k \Rightarrow \begin{bmatrix} u_x & 0 \\ 0 & u_y \end{bmatrix} = \begin{bmatrix} \tanh(kh_x)/k & 0 \\ 0 & \tanh(kh_y)/k \end{bmatrix}$. In this way, we can make a simplification of Eq. (S2):

$$\nabla \cdot (\overleftrightarrow{u} \cdot \nabla p) + \frac{\omega^2}{g}\eta = 0. \qquad (S5)$$

And the dispersion becomes $\omega = \sqrt{guk}$.

Then we get a new corresponding relationship:

$$H_z \leftrightarrow p, \frac{1}{\varepsilon_y \varepsilon_0} \leftrightarrow \frac{u_x}{\rho}, \frac{1}{\varepsilon_x \varepsilon_0} \leftrightarrow \frac{u_y}{\rho}, \frac{1}{\mu\mu_0} \leftrightarrow \rho g. \qquad (S6)$$

Therefore, the movement of water wave with a general water depth can still be manipulated by adjusting water depth h and gravitational acceleration g, even in the existence of nonlinearity, yet the water depth h needs some adjustment according to the wave number k.

With these corresponding relationships, we can solve some manipulation parameters of metagratings for water waves.

2 Dispersion relationship of the surface mode of SPRG (corresponding to Eq. (7) in the main text)

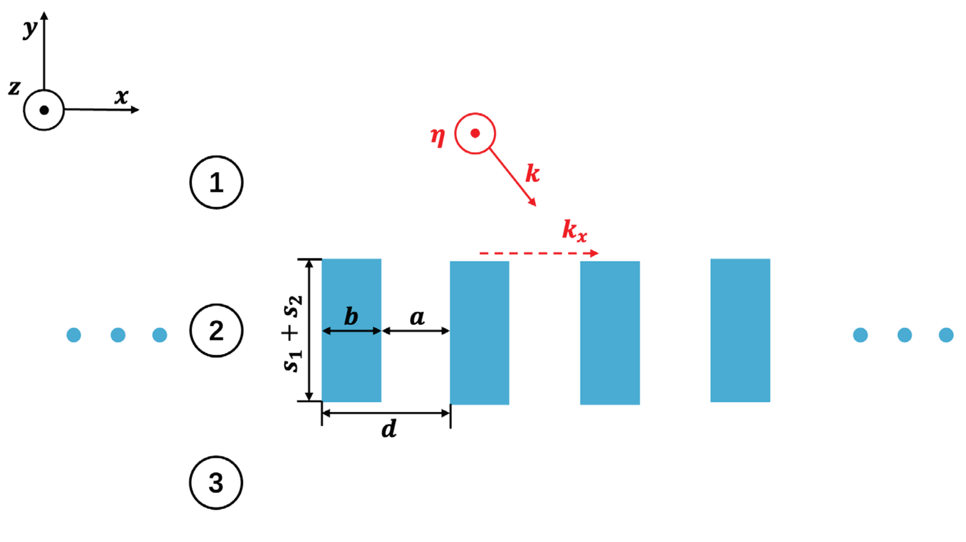

Fig. S1 SPRG structure diagram. The length of a single cycle unit is $s_1 + s_2$, the width is b, the interval between two cycle units is a and the cycle period length is d. The incident wave is water wave, propagates along the x direction, the vertical displacement of water wave η is in the z direction, and propagation direction of water wave k is in the x− y plane.

Because the location of the SPRG does not affect its transmission and reflection coefficient, for the convenience of solving, we put the upper bound of SPRG at $y = 0$. In Region 1 of Fig. S1, the total vertical displacement of water wave can be written as the superposition of the vertical displacement of incident water wave and the vertical displacement of water wave reflected of all diffraction orders [2, 3]:

$$\eta_1 = e^{ik_x x} e^{-ik_y y} + \sum_{-\infty}^{\infty} R_n \, e^{ik_x^n x} e^{ik_y^n y}, \qquad (S7)$$

where $k_x^n = k_x + 2\pi n/d$ and $k_y^n = \sqrt{k_0^2 - (k_x^n)^2}$, R_n is the reflection coefficient of the nth diffraction order. As the period length d of the array is much smaller than the incident wavelength, all diffraction orders can be ignored except the specular reflection order. Therefore, the vertical displacement of water wave of Region 1 can be written as:

$$\eta_1 = e^{ik_x x}e^{-ik_y y} + Re^{ik_x x}e^{ik_y y}, \quad (S8)$$

where R is the specular reflection coefficient.

In Region 2, since the interval between two cycle units a is much smaller than the incident wavelength, there is only the fundamental mode of the waveguide in the slit ($|x - x_j| < a/2$,where x_j is the middle coordinate of each period), so the total vertical displacement of water wave in the slit of Region 2 is [2]

$$\eta_2 = c_1 e^{-ik_0 y} + c_2 e^{ik_0 y} \quad \left(|x - x_j| < \frac{a}{2}\right), \quad (S9)$$

where c_1 is the amplitude coefficient of the forward propagating water wave (along the negative y-axis) and c_2 is the amplitude coefficient of the backward propagating water wave (along the positive y-axis).

In Region 3, the vertical displacement of water wave can be written in the form of transmitted waves, so the vertical displacement of water wave can be written as [3]

$$\eta_3 = Te^{ik_x x}e^{-ik_y y}, \quad (S10)$$

where T is the transmission coefficient.

We assume the flow of water wave is $\vec{\zeta}$, then $\vec{\zeta} = \frac{igh}{\omega}(\nabla\eta) \Rightarrow \zeta_y = \frac{igh}{\omega}\frac{\partial\eta}{\partial y}$ [4], where h is the water depth, g is the gravitational acceleration. The background water depth and gravitational acceleration in Fig. S1 is h_0 and g_0, then we can get the expression for the total flow of the water wave in the y direction in Region 1 as

$$\zeta_{1y} = \left(\frac{-ig_0 h_0 k_y}{\omega}\right) * \left(e^{ik_x x}e^{-ik_y y} - Re^{ik_x x}e^{ik_y y}\right). \quad (S11)$$

In the Region 2, when $|x - x_j| < a/2$ (where x_j is the middle coordinate of each period), the flow of water wave in the y direction can pass through it, so we can get

$$\zeta_{2y} = \left(\frac{-ig_0 h_0 k_0}{\omega}\right) * \left(c_1 e^{-ik_0 y} - c_2 e^{ik_0 y}\right). \quad (S12)$$

When $a/2 < |x - x_j| < d/2$, the SPRG is a rigid body, and the flow of water wave in the y direction cannot pass through it. Therefore, $\zeta_{2y} = \frac{ig_0 h_0}{\omega}\frac{\partial\eta_2}{\partial y} = 0$, while the total flow of the water wave in the y direction of Region 2 is

$$\zeta_{2y} = \begin{cases} \left(\frac{-ig_0 h_0 k_0}{\omega}\right) * \left(be^{-ik_0 y} - ce^{ik_0 y}\right) & (|x - x_j| < a/2) \\ 0 & (a/2 < |x - x_j| < d/2) \end{cases}. \quad (S13)$$

The flow of the water wave in the y direction of Region 3 is

$$\eta_3 = \left(\frac{-ig_0 h_0 k_y}{\omega}\right) * Te^{ik_x x}e^{-ik_y y}. \quad (S14)$$

At $y = 0$,

$$\eta_1 = (1 + R)e^{ik_x x}, \quad (S15.1)$$

$$\eta_2 = c_1 + c_2 \quad \left(|x - x_j| < \frac{a}{2}\right), \quad (S15.2)$$

$$\zeta_{1y} = \left(\frac{-ig_0 h_0 k_y}{\omega}\right) * (1 - R)e^{ik_x x}, \quad (S15.3)$$

$$\zeta_{2y}=\begin{cases}\left(\frac{-ig_0h_0k_0}{\omega}\right)*(c_1-c_2) & (|x-x_j|<a/2)\\ 0 & (a/2<|x-x_j|<d/2)\end{cases}. \quad (S15.4)$$

At $y=-(s_1+s_2)$,

$$\eta_2=c_1e^{ik_0(s_1+s_2)}+c_2e^{-ik_0(s_1+s_2)}\quad \left(|x-x_j|<\frac{a}{2}\right), \quad (S16.1)$$

$$\eta_3=Te^{ik_y(s_1+s_2)}e^{ik_xx}, \quad (S16.2)$$

$$\zeta_{2y}=\begin{cases}\left(\frac{-ig_0h_0k_0}{\omega}\right)*\left(be^{ik_0(s_1+s_2)}-ce^{-ik_0(s_1+s_2)}\right) & (|x-x_j|<a/2)\\ 0 & (a/2<|x-x_j|<d/2)\end{cases}, \quad (S16.3)$$

$$\zeta_{3y}=\left(\frac{-ig_0h_0k_y}{\omega}\right)*Te^{ik_y(s_1+s_2)}e^{ik_xx}. \quad (S16.4)$$

At $y=0$, when $|x-x_j|<a/2$, surface hydrostatic pressure p ($p=\rho g\eta$, ρ is fluid density) should be continuous (for the detailed derivation process, please refer to the Supplementary Materials of Ref. [1]), so $p_1=p_2$. When $|x-x_j|<d/2$, ζ_y should be continuous, so $\zeta_{1y}=\zeta_{2y}$, namely,

$$(1+R)e^{ik_xx}=(c_1+c_2)\quad \left(|x-x_j|<\frac{a}{2}\right), \quad (S17)$$

$$k_y*(1-R)e^{ik_xx}=\begin{cases}k_0*(c_1-c_2) & \left(|x-x_j|<\frac{a}{2}\right)\\ 0 & \left(\frac{a}{2}<|x-x_j|<\frac{d}{2}\right)\end{cases}. \quad (S18)$$

At $y=-(s_1+s_2)$, when $|x-x_j|<a/2$, surface hydrostatic pressure p should be continuous, so $p_2=p_3$. When $|x-x_j|<d/2$, ζ_y should be continuous, so $\zeta_{2y}=\zeta_{3y}$, namely,

$$Te^{ik_y(s_1+s_2)}e^{ik_xx}=\left(c_1e^{ik_0(s_1+s_2)}+c_2e^{-ik_0(s_1+s_2)}\right)\quad \left(|x-x_j|<\frac{a}{2}\right), \quad (S19)$$

$$k_y*Te^{ik_y(s_1+s_2)}e^{ik_xx}=\begin{cases}k_0*\left(c_1e^{ik_0(s_1+s_2)}-c_2e^{-ik_0(s_1+s_2)}\right) & \left(|x-x_j|<\frac{a}{2}\right)\\ 0 & \left(\frac{a}{2}<|x-x_j|<\frac{d}{2}\right)\end{cases}. \quad (S20)$$

Integrate (S11, S13) on $|x-x_j|<a/2$ and (S19, S20) on $a/2<|x-x_j|<d/2$ to obtain:

$$a(1+R)=(b+c)s_0, \quad (S21)$$

$$d\frac{k_y}{k_0}(1-R)=(c_1-c_2)s_0, \quad (S22)$$

$$aTe^{ik_y(s_1+s_2)}=\left(c_1e^{ik_0(s_1+s_2)}+c_2e^{-ik_0(s_1+s_2)}\right)s_0, \quad (S23)$$

$$d\frac{k_y}{k_0}Te^{ik_y(s_1+s_2)}=\left(c_1e^{ik_0(s_1+s_2)}-c_2e^{-ik_0(s_1+s_2)}\right)s_0, \quad (S24)$$

where $s_0 = \int_{x_j-a/2}^{x_j+a/2} e^{-ik_x x} dx = e^{-ik_x x_j} \frac{a*sin(k_x a/2)}{k_x a/2} = e^{-ik_x x_j} sinc(k_x a/2)$.

From Eqs. (S21)–(S24) we obtain

$$R = \frac{\left(e^{i2k_0(s_1+s_2)} - 1\right)\left(a^2k_0^2 - d^2k_y^2\right)}{\left(ak_0 + dk_y\right)^2 - \left(ak_0 e^{ik_0(s_1+s_2)} - dk_y e^{ik_0(s_1+s_2)}\right)^2}. \quad (S25)$$

Then by extending Eq.(S25) to $k_x > k_0(k_y = i\sqrt{k_x^2 - k_0^2})$, and calculating the zero point of the denominator [5], we obtain the dispersion relationship of the surface mode of SPRG:

$$k_x = \frac{\sqrt{d^2(e^{ik_0(s_1+s_2)} + 1)^2 - a^2(e^{ik_0(s_1+s_2)} - 1)^2}}{d(1 + e^{ik_0(s_1+s_2)})} k_0, \quad (S26)$$

which is Eq. (5) in the main text.

3 Reflection coefficient and dispersion relationship of the surface mode of the equivalent anisotropic water layer (corresponding to Eq. (8) in the main text)

We first write the revised water equation [1] in anisotropic water layers:

$$\nabla \cdot \left(\frac{\tanh(k\overleftrightarrow{h})}{k} \cdot \nabla p \right) + \frac{\omega^2}{g} \eta = 0, \quad (S27)$$

where $\overleftrightarrow{h}$ is a tensor with $\overleftrightarrow{h} = \begin{bmatrix} h_x & 0 \\ 0 & h_y \end{bmatrix}$, p is the hydrostatic pressure of water surface ($p = \rho g \eta$), ρ is the fluid density, g is the gravitational acceleration, η is the vertical displacement of the water wave (as shown in the lower right corner of Fig. 1(b) in the main text), ω is the angular frequency, the surface wave equation above the water wave is governed by nonlinear dispersion $\omega = \sqrt{g\tanh(kh)k}$. k is propagation wave number of water wave.

We define a new variable u (reduced water depth), $u = \tanh(kh)/k$, so $\overleftrightarrow{u} = \tan k(k\overleftrightarrow{h})/k = [\begin{smallmatrix} u_x & 0 \\ 0 & u_y \end{smallmatrix}]$. In this way, we can make a simplification of Eq. (S27):

$$\nabla \cdot (\overleftrightarrow{u} \cdot \nabla p) + \frac{\omega^2}{g} \eta = 0. \quad (S28)$$

And the dispersion becomes $\omega = \sqrt{guk}$.

With Eq. (S28), we can write the equivalent water layer with an anisotropic depth and a specific gravity of the SPRG, as shown in Fig. S2. The parameters of the equivalent water layer are shown below:

$$u_{2x} = 0,\ \ u_{2y} = a/d * u_0,\ \ g_2 = d/a * g_0 \quad (S29)$$

where $u_0 = \tanh(kh_0)/k$, $h_0$is the water depth of Region 1 and Region 3, g_0 is the gravitational acceleration of Region 1 and Region 3 in Fig. S2.

We assume that the reduced water depth and gravitational acceleration of Region 1 is $u_1 = u_0, g_1 = g_0$, the water depth and gravitational acceleration of Region 2 is $\overleftrightarrow{u_2} = \begin{bmatrix} u_{2x} & 0 \\ 0 & u_{2y} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & a/d * u_0 \end{bmatrix}, g_2 = d/a * g_0$, the water depth and gravitational acceleration of Region 3 is

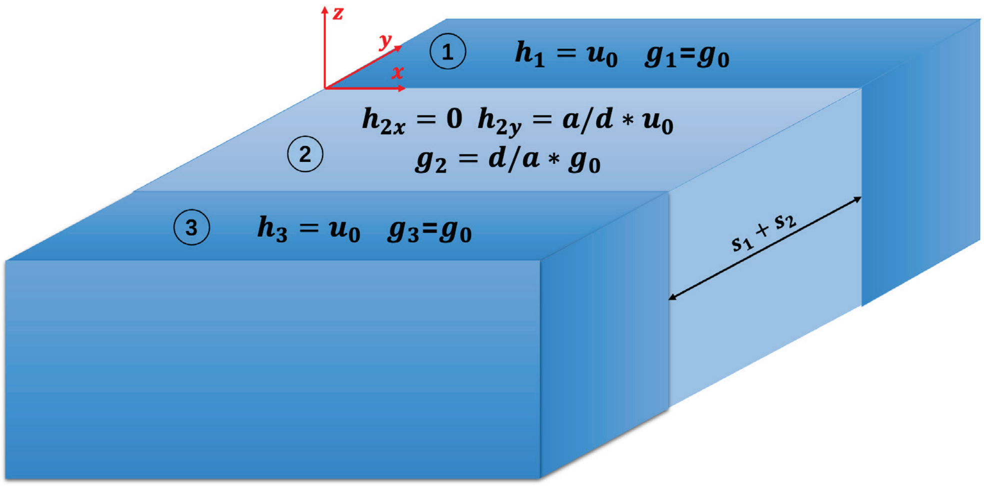

Fig. S2 Anisotropic water layer equivalent model.

$u_3 = u_0, g_3 = g_0$, θ_1 is the angle between the direction of the incident water wave and the normal direction of the interface, θ_2 is the angle between the direction of the refraction water wave and the normal direction of the interface as shown in Fig. S3.

When the water wave goes from Region 1 to Region 2, the angular frequency ω remains the same, so $\omega_2 = \omega_1 = \sqrt{g_0 u_0} k_0$, k_0 is water wave propagation wavenumber of Region 1.

Through Eq. (S28), we can find the relationship between the wave vector components in each direction in the equivalent anisotropic water layer (Region 2):

$$\frac{g_2 u_{2x}}{g_0 u_0} k_{2x}^2 + \frac{g_2 u_{2y}}{g_0 u_0} k_{2y}^2 = k_0^2. \tag{S30}$$

Then we can get

$$k_{2x} = \frac{k_0 sin\theta_2}{\sqrt{g_2 u_{2x}/(g_0 h_0)}}, \tag{S31}$$

$$k_{2y} = \frac{k_0 cos\theta_2}{\sqrt{g_2 u_{2y}/(g_0 h_0)}}, \tag{S32}$$

where θ_1 is the angle between the direction of the incident water wave and the normal direction of the interface, θ_2 is the angle between the direction of the refraction water wave and the normal direction of the interface as shown in Fig. S3.

Conservation of the parallel wave vector components at the interface of Region 1 and Region 2 leads to $k_{1x} = k_{2x}$, then we can get

$$k_0 sin\theta_1 = \frac{k_0 sin\theta_2}{\sqrt{g_2 u_{2x}/(g_0 u_0)}}, \tag{S33}$$

which leads to

$$k_{2y} = \frac{k_0 cos\theta_2}{\sqrt{g_2 u_{2y}/(g_0 u_0)}}. \tag{S34}$$

We bring $u_{2x} = 0, u_{2y} = a/d * u_0, g_2 = d/a * g_0$ into $k_0 sin\theta_1 = \frac{k_0 sin\theta_2}{\sqrt{g_2 u_{2y}/(g_0 u_0)}}$ to get $\theta_2 = arcsin(\sqrt{g_2 u_{2y}/(g_0 u_0)} * sin\theta_1) \approx 0$, so $k_{2y} = \frac{k_0 cos\theta_2}{\sqrt{g_2 u_{2y}/(g_0 u_0)}} \approx k_0$.

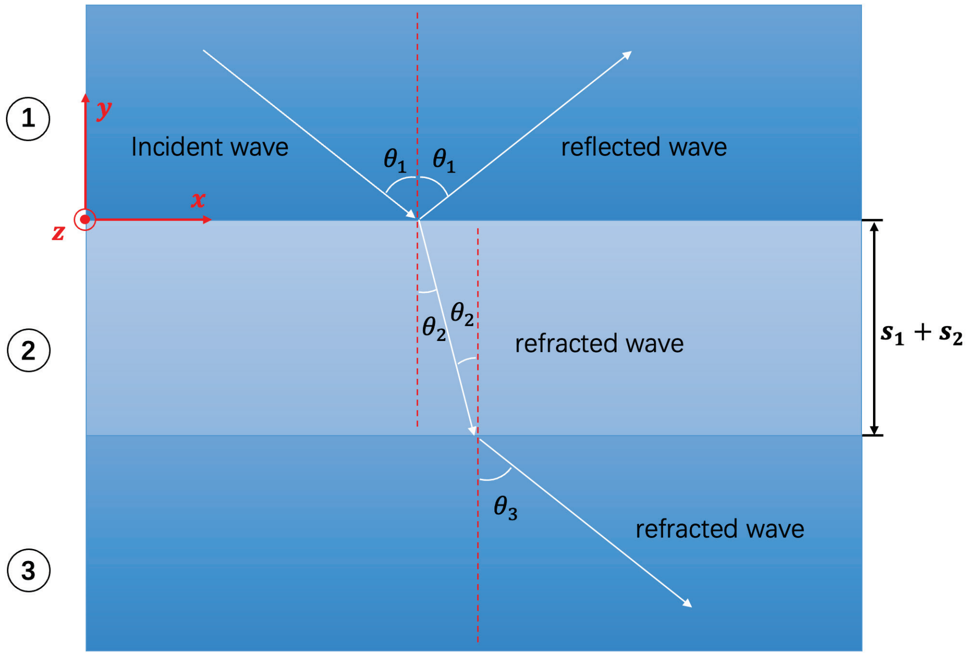

Fig. S3 Propagation of water waves in different regions.

Therefore, no matter what angle the incident water wave enters into the anisotropic equivalent water layer, the refraction angle will become 0 degree, and the wave number propagating in the y direction will become k_0.

At the same time, $k_0 sin\theta_1 = \frac{k_0 sin\theta_2}{\sqrt{g_2 u_{2x}/(g_0 h_0)}} = \frac{k_0 sin\theta_3}{\sqrt{g_3 u_3/(g_0 h_0)}}$, because the water depth and gravity are the same in Region 1 and Region 3, so $k_0 \sin\theta_1 = k_0 \sin\theta_3$, i.e., $\theta_1 = \theta_3$, $k_{3y} = k_{1y}$. Then we can use the transmission matrix method [6] to calculate the reflection coefficient of the surface of Region 1.

From the transmission matrix theory, we can obtain the relationship between the amplitude of the incident wave and the outgoing wave as

$$\begin{bmatrix} b_1 \\ c_1 \end{bmatrix} = D_{1\to2} P_{2\to3} D_{2\to3} \begin{bmatrix} b_3 \\ c_3 \end{bmatrix} = M \begin{bmatrix} b_3 \\ c_3 \end{bmatrix}, \quad (S35)$$

where b_n is the amplitude coefficient of the forward propagating water wave (along the negative y-axis) in the Region n and c_n is the amplitude coefficient of the backward propagating water wave (along the positive y-axis) in the Region n $(n = 1,2,3)$.

$$D_{1\to2} = \frac{1}{2} \begin{bmatrix} 1+\rho_1 & 1-\rho_1 \\ 1-\rho_1 & 1+\rho_1 \end{bmatrix}, P_{2\to3} = \begin{bmatrix} e^{-ik_{2y}(s_1+s_2)} & 0 \\ 0 & e^{ik_{2y}(s_1+s_2)} \end{bmatrix},$$

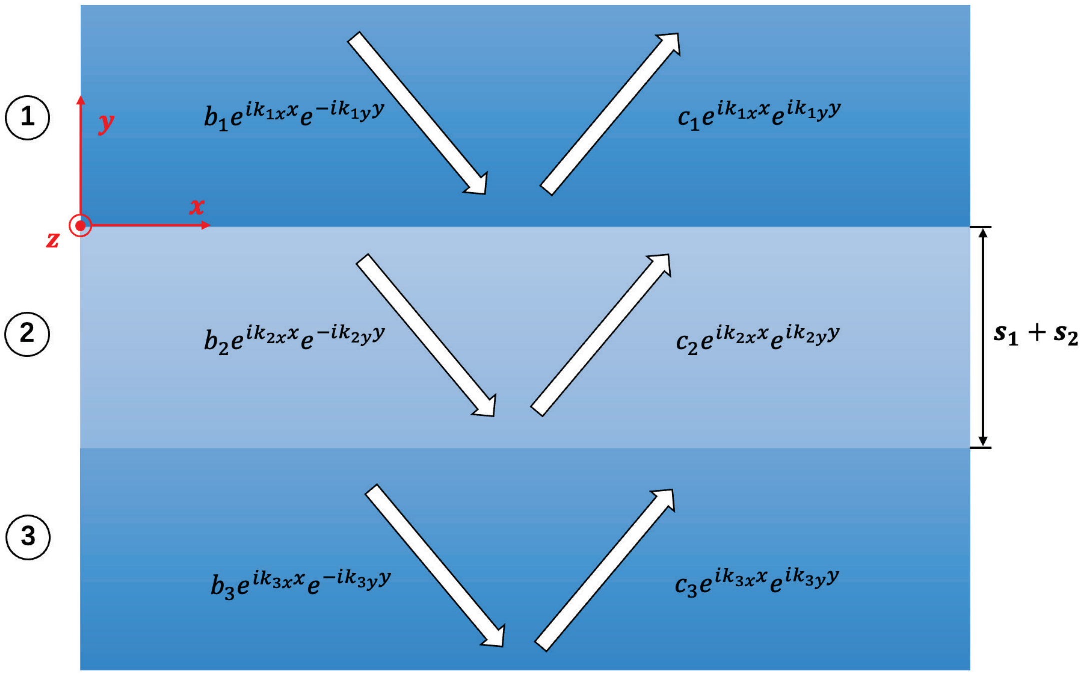

Fig. S4 Schematic diagram of water wave propagation in the equivalent water layer. The incident wave is water wave, which propagates obliquely along the x direction. The vertical displacement of water wave η is in the z direction, and the propagation direction of water wave k is in the $x-y$ plane.

$D_{2\to3} = \frac{1}{2}\begin{bmatrix} 1+\rho_2 & 1-\rho_2 \\ 1-\rho_2 & 1+\rho_2 \end{bmatrix}$, $\rho_1 = \frac{k_{2y}u_{2y}}{k_{1y}u_{1y}}, \rho_2 = \frac{k_{3y}u_{3y}}{k_{2y}u_{2y}}$. Then bringing $h_{3y} = h_{1y} = h_0, h_{2y} = a/d * h_0, k_{3y} = k_{1y}, k_{2y} \approx k_0$ into Eq. (S35) to get

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1+\dfrac{ak_0}{dk_{1y}} & 1-\dfrac{ak_0}{dk_{1y}} \\ 1-\dfrac{ak_0}{dk_{1y}} & 1+\dfrac{ak_0\varepsilon_{1x}}{dk_{1y}\varepsilon_{2x}} \end{bmatrix}\begin{bmatrix} e^{-ik_0(s_1+s_2)} & 0 \\ 0 & e^{ik_0(s_1+s_2)} \end{bmatrix}\begin{bmatrix} 1+\dfrac{dk_{1y}}{ak_0} & 1-\dfrac{dk_{1y}}{ak_0} \\ 1-\dfrac{dk_{1y}}{ak_0} & 1+\dfrac{dk_{1y}}{ak_0} \end{bmatrix}. \quad (S36)$$

Then we obtain

$$M_{11} = \frac{1}{4}\left[\left(2+\frac{dk_{1y}}{ak_0}+\frac{ak_0}{dk_{1y}}\right)e^{-ik_0(s_1+s_2)} + \left(2-\frac{dk_{1y}}{ak_0}-\frac{ak_0}{dk_{1y}}\right)e^{ik_0(s_1+s_2)}\right],$$

$$M_{12} = \frac{1}{4}\left[\left(\frac{ak_0}{dk_{1y}}-\frac{dk_{1y}}{ak_0}\right)e^{-ik_0(s_1+s_2)} + \left(\frac{dk_{1y}}{ak_0}-\frac{ak_0}{dk_{1y}}\right)e^{ik_0(s_1+s_2)}\right],$$

$$M_{21} = \frac{1}{4}\left[\left(\frac{dk_{1y}}{ak_0}-\frac{ak_0}{dk_{1y}}\right)e^{-ik_0(s_1+s_2)} + \left(\frac{ak_0}{dk_{1y}}-\frac{dk_{1y}}{ak_0}\right)e^{ik_0(s_1+s_2)}\right],$$

$$M_{22} = \frac{1}{4}\left[\left(2-\frac{dk_{1y}}{ak_0}-\frac{ak_0}{dk_{1y}}\right)e^{-ik_0(s_1+s_2)} + \left(2+\frac{dk_{1y}}{ak_0}+\frac{ak_0}{dk_{1y}}\right)e^{ik_0(s_1+s_2)}\right].$$

Eventually, we can get the reflection coefficient R of Region 1 through the M matrix.

$$R = \frac{M_{21}}{M_{11}} = \frac{\left(\dfrac{dk_{1y}}{ak_0}-\dfrac{ak_0}{dk_{1y}}\right)e^{-ik_0(s_1+s_2)} + \left(\dfrac{ak_0}{dk_{1y}}-\dfrac{dk_{1y}}{ak_0}\right)e^{ik_0(s_1+s_2)}}{\left(2+\dfrac{dk_{1y}}{ak_0}+\dfrac{ak_0}{dk_{1y}}\right)e^{-ik_0(s_1+s_2)} + \left(2-\dfrac{dk_{1y}}{ak_0}-\dfrac{ak_0}{dk_{1y}}\right)e^{ik_0(s_1+s_2)}}. \quad (S37)$$

Then by extending Eq. (S37) to $k_x > k_0(k_y = i\sqrt{k_x^2 - k_0^2})$, and calculating the zero point of the denominator [5], we obtain the dispersion relationship of the surface mode of equivalent anisotropic water layer:

$$k_x = \frac{\sqrt{d^2(e^{ik_0(s_1+s_2)}+1)^2 - a^2(e^{ik_0(s_1+s_2)}-1)^2}}{d(1+e^{ik_0(s_1+s_2)})}k_0. \quad (S38)$$

By comparing Eqs. (S26) and (S38), we can find that the dispersion relationship of the surface mode of the equivalent anisotropic water layer and the SPRG are exactly the same, which proves that this equivalence is feasible. This is Eq. (6) in the main text.

4 Analytical solution of field pattern in Fig. 2(c) in the main text

The total vertical displacement of water wave with angular momentum is: $\eta = H_1(k_0 r)e^{i\theta}$, where H_1 is the Hankel function of the first kind, k_0 is wave vector. $k_0 = 2\pi/\lambda$, λ is the wavelength. r and θ are cylindrical coordinate systems. Because $H_1(k_0 r)e^{i\theta}$ is in the form of cylindrical waves. The equivalent anisotropic water layer is rectangular, so it is difficult to directly solve the expression of the field patterns excited by $H_1(k_0 r)e^{i\theta}$. Therefore, we first convert $H_1(k_0 r)e^{i\theta}$ into the form of plane wave by using the method of spatial Fourier transform [7], as shown in Eq. (S32):

$$\eta = H_1(k_0 r)e^{i\theta} = \int_{-\infty}^{\infty} \psi(k_x)\, e^{ik_x x} e^{ik_y|y-y_{source}|} dk_x, \quad (S39)$$

where $\psi(k_x) = \begin{cases} \psi_1(k_x) = \frac{-i}{\pi k_0}\left[\frac{k_x + ik_y}{k_y}\right], y > y_{source} \\ \psi_2(k_x) = \frac{-i}{\pi k_0}\left[\frac{k_x - ik_y}{k_y}\right], y < y_{source} \end{cases}$, y_{source} is the location of the excited source, k_x is the wave number in the x direction, $k_y = \sqrt{k_0^2 - k_x^2}$ is the wave number in the y direction.

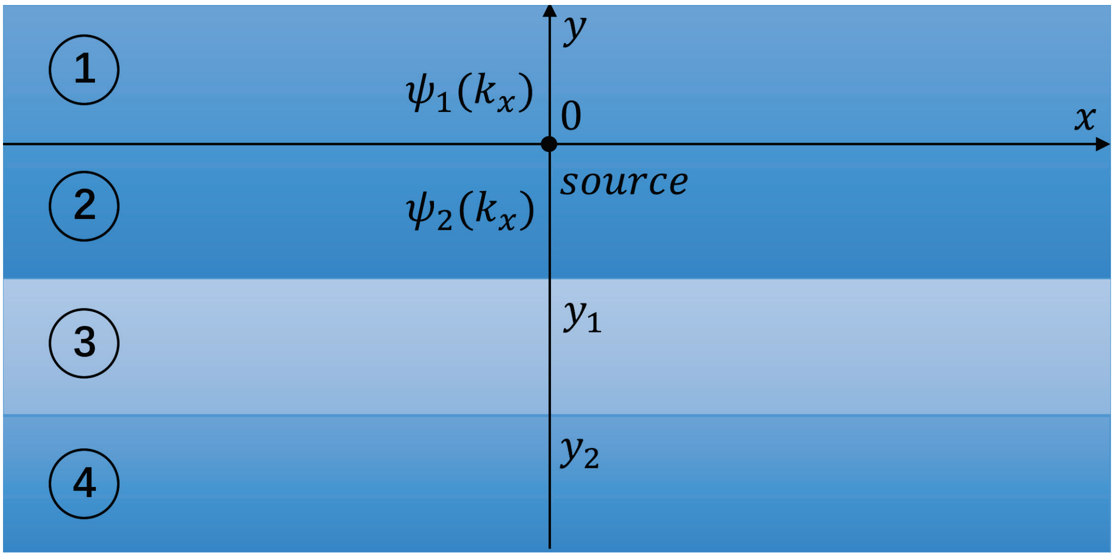

Fig. S5 Water waves with angular momentum are incident on anisotropic water layers.

For the convenience of calculation, we put the wave source at the origin, namely, $y_{source} = 0$. In this way, Fig. S5 is divided into four Regions, its Regions 1 and 2 correspond to Region 1 in Fig. S3, and its Region 3 and 4 correspond to Regions 2 and 3 in Fig. S3, so the reduced water depth of the Regions 1,2,4 is u_0, the gravitational acceleration of the Regions 1,2,4 is g_0. The relationship between the propagation wave numbers in the x and y directions of Region 1, 2, 4 is: $k_x^2 + k_{ny}^2 = k_0^2 (n = 1,2,4)$, the reduced water depth of Region 3 is $u_{3x} = 0, u_{3y} = a/d * u_0$, the gravitational acceleration of Region 3 is $g_3 = d/a * g_0$. The propagation wave number in the x direction of the Region 3 does not change, and the wave number propagating in the y direction of the Region 3 is

$k_{3y} = k_0$. The calculation process of transmission and reflection has been written in details in the previous calculation sections. Here we briefly describe the calculation process and write the transmittance and reflection coefficient directly.

Using continuity of boundary conditions, we can solve for the transmission and reflection coefficients R_{23} and T_{23} at the junction of Regions 2 and 3 and the transmission and reflection coefficients R_{34} and T_{34} at the junction of regions 2 and 3 as

$$R_{23} = \frac{1-\alpha}{1+\alpha}, T_{23} = \frac{2}{1+\alpha}, \quad (S40)$$

$$R_{34} = \frac{\alpha-1}{\alpha+1}, T_{34} = \frac{2\alpha}{1+\alpha}, \quad (S41)$$

where $\alpha = ak_0/(d\sqrt{k_0^2 - k_x^2})$.

Then we can write the total transmission and reflection coefficient expression and water wave amplitude expression of each Region as follows [8]: The total water wave in Region 1 ($y > 0$) is formed by the superposition of the water wave incident from the origin of the wave source upward to Region 1 and the water wave reflected from Region 2, so the total vertical displacement of water wave of Region 1 is

$$\eta_1 = \int_{-\infty}^{\infty} \left(\psi_1(k_x)e^{ik_{1y}y} + \psi_2(k_x)Re^{ik_{2y}2y_1}e^{ik_{1y}y}\right)e^{ik_x x}dk_x. \quad (S42)$$

The total water wave in Region 2 ($y_1 < y < 0$) is formed by the superposition of the water wave incident from the origin of the wave source downward to Region 2 and the water wave reflected from the junction of Regions 2 and 3, so the total vertical displacement of water wave of Region 2 is as follows:

$$\eta_2 = \int_{-\infty}^{\infty} \psi_2(k_x)\left(e^{-ik_{2y}y} + Re^{-ik_{2y}(2y_1-y)}\right)e^{ik_x x}dk_x. \quad (S43)$$

The total water wave in Region 3 ($y_2 < y < y_1$) is formed by the superposition of the water wave transmitted from Region 2 to Region 3 and the water wave reflected from the junction of Regions 3 and 4, so the total vertical displacement of water wave of Region 3 is as follows:

$$\eta_3 = \int_{-\infty}^{\infty} \psi_2(k_x)Ae^{-ik_{2y}y_1}\left(e^{-ik_0(y-y_1)} + R_{34}e^{-ik_0(2y_2-y-y_1)}\right)e^{ik_x x}dk_x. \quad (S44)$$

The total water wave in Region 4 ($y < y_2$) is formed by the water wave transmitted from Region 3 to Region 4, so the total vertical displacement of water wave of Region 4 is as follows:

$$\eta_4 = \int_{-\infty}^{\infty} \psi_2(k_x)Te^{-ik_{2y}y_1}e^{-ik_0(y_2-y_1)}e^{-ik_{4y}(y-y_2)}e^{ik_x}dk_x, \quad (S45)$$

where

$$k_{ny}^2 = \sqrt{k_0^2 - k_x^2}\ (n = 1,2,4), \quad (S46)$$

$$R = P\left[R_{23} + R_{34}e^{-i2k_0(y_2-y_1)}\right], \quad (S47)$$

$$A = PT_{23}, \quad T = PT_{23}T_{34}, \quad (S48)$$

$$P = \left[1 + R_{23}R_{34}e^{-i2k_0(y_2-y_1)}\right]^{-1}. \quad (S49)$$

Bringing (S40,S41) and (S46–S49) into (S42–S45) and then integrating them, we can plot the field pattern as shown in Fig. 2(c) in the main text.

5 Approximate water depth and gravitational acceleration of the equivalent gradient anisotropic water layer of the Luoyang Bridge (corresponding to Eq. (6) in the main text)

Because the single circle structure of Luoyang Bridge can be divided into an isosceles triangle and a rectangle, the equivalent gradient anisotropic water layer of Luoyang Bridge should also be divided into two regions. Here, by taking Fig. 2(f) in the main text as an example, we assume that the reduced water depth and gravitational acceleration of background region ($y > -6$ m and $y < -17$ m) is u_0 and g_0, the rectangular part (-17 m $< y < -11$ m) under Luoyang Bridge has exactly the same structure as SPRG, so the equivalent water layer depth of the lower part is also $u_x = 0, u_y = a/d * u_0, g = d/a * g_0$. For the equivalent anisotropic gradient water layer in the upper isosceles triangular area (-11 m $< y < -6$ m), we can use the layered method to calculate. We divide the triangular area into many layers. As the number of layers increases, the shape of each layer becomes closer to a rectangle. When there are enough layers, each layer can be approximately regarded as a SPRG without thickness, and its equivalent parameters can also be obtained through the previous method, at this time we can find that when $y = -11$ m, $u_x = 0, u_y = a/d * u_0, g = d/a * g_0$, when $y = -6$ m, $u_x = 0, u_y = u_0, g = g_0$, and the parameter change of each layer is continuous, because the distance between the isosceles triangle parts of the circular structure of the two Luoyang bridges changes according to a linear function. Therefore, the water depth in y direction of its equivalent water layer should also change according to a linear function. Therefore, the water depth in y direction of its equivalent water layer should also change according to a linear function. We can then write down an approximate reduced water depth expression $\overleftrightarrow{u}$ and gravitational acceleration expression g of the equivalent anisotropic gradient water layer of the whole Luoyang Bridge. When -11 m $< y < -6$ m, $u_y = \frac{(d+y+s_1+1)}{d} * u_0$, and because the propagation wave number of the water wave in the y direction in the equivalent gradient anisotropic water layer should be equal to k_0, that is, $\sqrt{gu_y} = 1$, so the gravitational acceleration in this area is $g = \frac{d}{(d+y+s_1+1)} * g_0$. And in the -11 m $< y < -6$ m, as long as the width of isosceles triangle parts of the circular structure is not equal to 0, the flow of water can only propagate along the y direction, and cannot pass through the x direction in the Luoyang Bridge structure. Therefore, the water depth in the x direction is always equal to 0. When -11 m $< y < -6$ m, $u_x = 0$. Finally, we can write down the whole approximate reduced water depth expression $\overleftrightarrow{u}$ and gravitational acceleration expression g of the equivalent gradient anisotropic water layer of the Luoyang Bridge as follows:

$$\overleftrightarrow{u}, g = \begin{cases} u_x = 0, u_y = \dfrac{(d+y+s_1+1)}{d} * u_0, g = \dfrac{d}{(d+y+s_1+1)} * g_0 & (-11m < y < -6m) \\ u_x = 0, u_y = \dfrac{a}{d} * u_0, g = \dfrac{d}{a} * g_0 & (-17m < y < -11m) \end{cases} . \quad (S50)$$

This is Eq. (10) in the main text.

Section 2: Experimental realization

1 Experimental set up

The experimental set up is shown in Fig. S6(a). The experiment is carried out in a transparent water tank. The water tank is 124 cm long and 73 cm wide. The point source with angular momentum is generated by the middle motor and propeller. The wavelength of the water wave can be changed by changing the rotation period of the motor. The orange part below the motor is the reduced metagrating model of Luoyang Bridge mentioned in the main text. It is fabricated with a 3D printer. The material is PLA plastic. It is an impermeable rigid body and can completely reflect water waves.

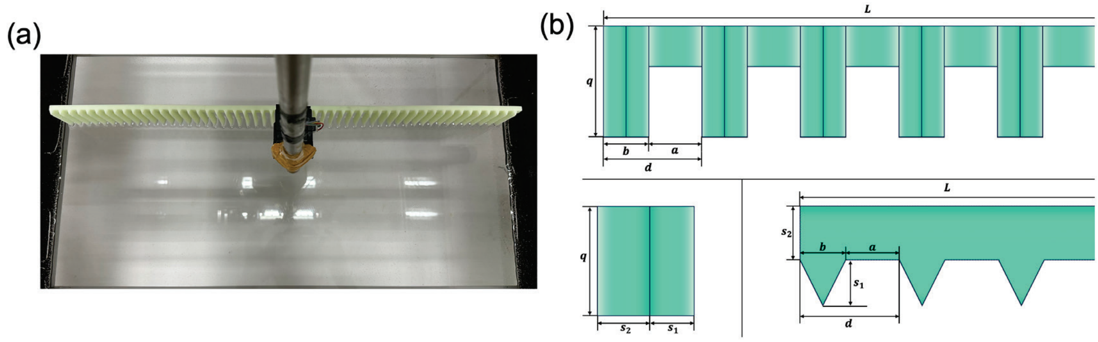

Fig. S6 Experimental equipment structure diagram. **(a)** Experimental set up diagram. **(b)** Schematic diagram of the reduced structure of Luoyang Bridge.

The enlarged view of the metagrating is shown in Fig. S6(c), and the size is $L = 64$ cm, $q = 5$ cm, $d = 1.1$ cm, $a = 0.5$ cm, $b = 0.6$ cm, $s_1 = 0.5$ cm, $s_2 = 0.6$ cm.

Considering the shallow water approximation of water wave equation, the water intake depth h is 7 mm in our experiment. The black parts on the left and right sides are the wave elimination device, which can greatly reduce the reflection of water waves of outer boundaries. The specific structure of the wave eliminator is shown in Fig. S7.

2 Wave elimination device

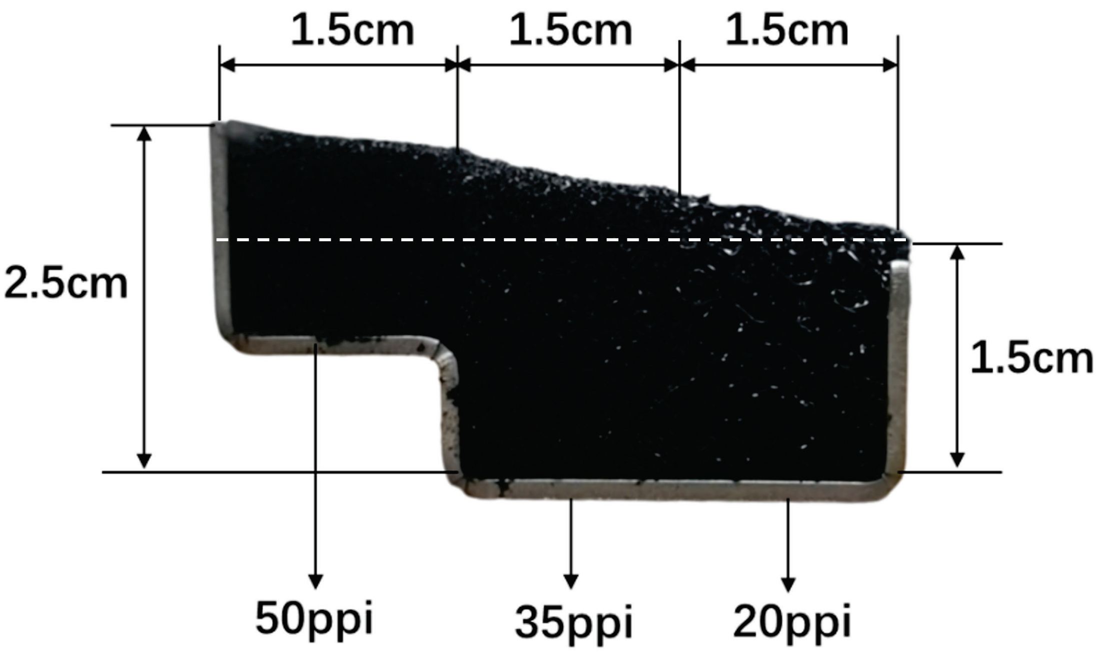

Fig. S7 Cross sectional view of the water elimination device. It is divided into three layers, and the pore size of the sponge increases from left to right [1].

In this experiment, we use the same wave elimination device as before [1], which is an inclined plane composed of three layers of sponge. Both inclined plane and the pores in the sponge can help reduce the energy of the water wave, thereby reducing or even eliminating the reflection of the water wave at the boundary, making the experimental results more accurate. Its specific structure and size are shown in Fig. S7. The water wave elimination device can be divided into two areas, one is a triangular area with a slope above the white dotted line, the height is 1 cm, bottom edge length is 4.5 cm, the other is a square area below the white dotted line, the height is 1.5 cm, the length is 4.5 cm, and the thickness of each layer of sponge is 1.5 cm. The density of pores in each layer of sponge is different, from left to right are 50 ppi, 35 ppi and 20 ppi. Because a small piece of glass is added at the boundary of the water cylinder for our experiment to enhance the air tightness, there is a square notch on the left of Fig. S7, which can make the wave elimination device fit better with the water cylinder.

3 Indicating float ball

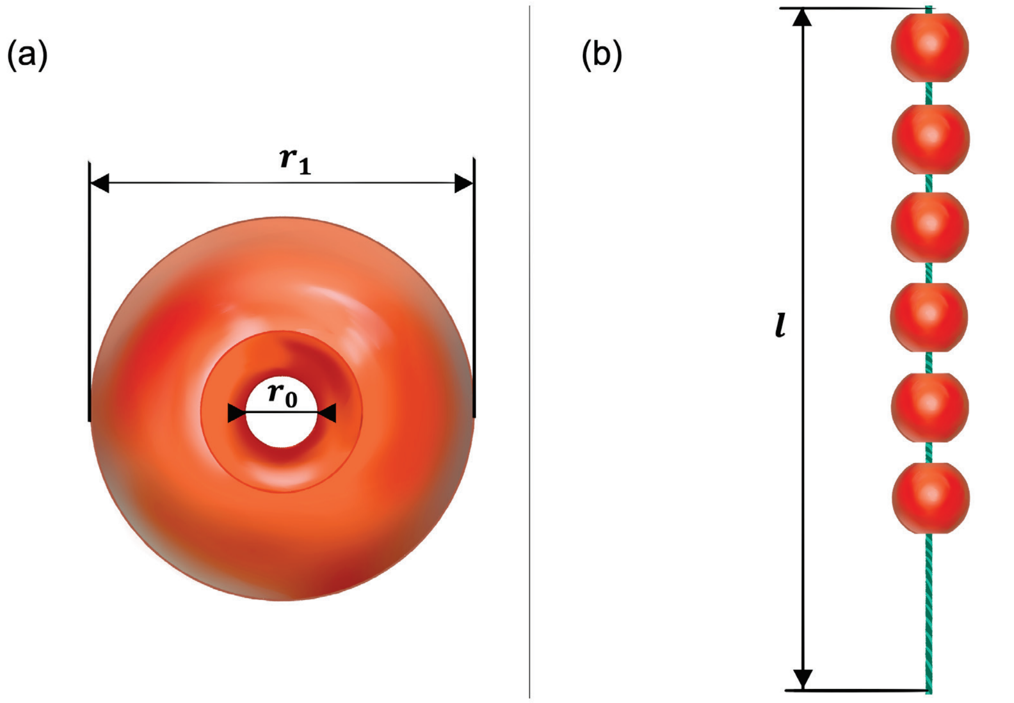

Fig. S8 Schematic diagram of the buoy consisting of float ball. **(a)** Structure diagram of single float ball. **(b)** Structure diagram of the buoy consisting of float ball.

The float ball is in orange and is made of ABS plastic. The specific dimensions are shown in Fig. S8(a), the outer diameter of the float ball r_1 is 200 mm and the inner diameter r_0 is 20 mm, which can provide 4 kg buoyancy at most. Then we connect six float balls in series with nylon rope to form a buoy consisting of float balls. The interval between each float ball is 50 cm. The total length of the buoy composed of six float balls and nylon rope l is 7 m. As shown in Fig. S8(b), the buoy is hung at the apex of the triangular part of Luoyang Bridge, as for the symmetrical distribution of the wave source center, as shown in Fig. 3(d) of the main text, the red dot indicates the hanging position of the buoy consisting of float balls, and the red arrow indicates the position of the buoy consisting of float balls.

4 Video demonstration of experimental results

Figure S9(a) (corresponding to Fig. 3(e) in the main text) shows the clockwise rotating vortex wave excites a unidirectional water surface wave propagating to left near the metagrating. Figure S9(b) (corresponding to Fig. 3(f) in the main text) shows the counterclockwise rotating vortex wave excites a unidirectional water surface wave propagating to right near the metagrating. Both Figs. S9(a) and (b) have the corresponding video demonstrations, where unidirectional water surface waves can be clearly seen. Figure S9(c) (corresponding to Fig. 4(a) in the main text) shows clockwise rotating vortex wave excites a unidirectional water surface wave propagating to the left near the real Luoyang Bridge, Fig. S9(d) (corresponding to Fig. 4(b) in the main text) shows that using buoys consisting of float ball to make the unidirectional water surface waves (propagating to left) excited by Luoyang Bridge look more intuitive.

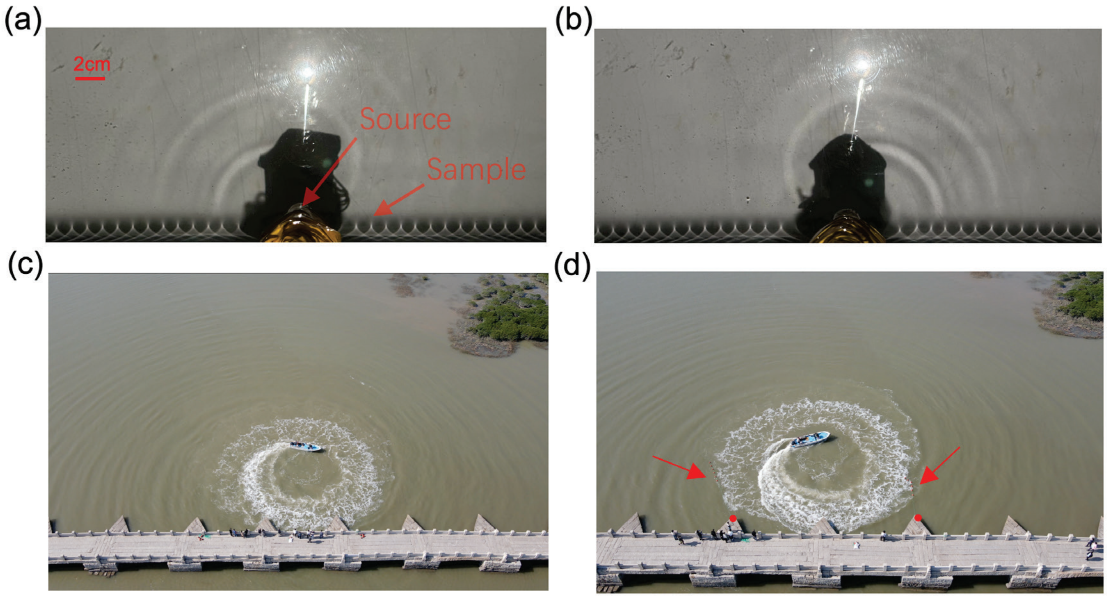

Fig. S9 Demonstration of experimental results. **(a)** Schematic diagram of Video 1, unidirectional water wave propagating to the left excited by the reduced structure of Luoyang Bridge. **(b)** Schematic diagram of Video 2, unidirectional water wave propagating to the right excited by the reduced structure of Luoyang Bridge. **(c)** Schematic diagram of Video 3, unidirectional water wave propagating to the left excited by the real Luoyang Bridge. **(d)** Schematic diagram of Video 4, unidirectional water wave propagating to the left (indicated by red buoys) excited by the real Luoyang Bridge.

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