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Quantum vortices get stretched

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Since its discovery, superfluidity [1, 2] remains one of the most remarkable phenomena in quantum physics. The onset of the quantum regime in liquid helium is visually striking, when the fluid flows without dissipation or detaches from a rotating bucket, remaining still upon soft rotations. The same phenomenon is present in the context of ultracold bosonic quantum gases where, most of the time, superfluidity walks hand-in-hand with the transition to Bose–Einstein condensation [3].

In these systems, the theoretical framework is much simpler than in liquid Helium, as bosonic quantum gases are very accurately described by Gross–Pitaevskii equation [Eq. (1)], where the bosonic wavefunction Ψ is a manybody wavefunction evolving at zero temperature with the two-body interactions taken as a mean-field energy contribution,

$$\mathrm{i}\hbar\frac{\partial\Psi(\boldsymbol{r},t)}{\partial t}=\left[-\frac{\hbar^2}{2m}\nabla^2+V_{ext}(\boldsymbol{r},t)+g\left|\Psi(\boldsymbol{r},t)\right|^2\right]\Psi(\boldsymbol{r},t), \tag{1}$$

where $g=\frac{4\pi a\hbar^2}{m}$ is the mean-field interaction parameter and a is the s-wave scatering length.

The solution of Eq. (1) from the hydrodynamic point of view explicitly highlights the long-range phase coherence of the quantum fluid and reads

$$\Psi(\boldsymbol{r},t)=\sqrt{n(\boldsymbol{r},t)}\exp(\mathrm{i}\phi(\boldsymbol{r},t)). \tag{2}$$

Indeed, this solution allows one to show that the velocity field of a bosonic quantum fluid is given by the gradient of the phase of the order parameter, $\boldsymbol{v}=\nabla\phi$, with the direct consequence that the superfluid is irrotational as $\nabla\times\nabla\phi\equiv 0$, as it has been demonstrated for liquid Helium [3].

In fact, upon sufficiently large rotation, the irrotational superfluid starts to develop quantized vortices, voids of density around which the phase of the order parameter winds up [4, 5]. The condition that the phase must be continuous restricts the winding of the phase to multiples of 2π and the vortex to carry angular momentum as multiples of $\hbar$. The appearance of quantized vortices in superfluids under rotation is such a "natural" phenomenon that the detection of vortices in a given configuration is the undisputable smoking gun of superfluidity of that system [6].

Since ultracold quantum gases became routinely available in several laboratories, the study of vortices has been a constant. From the dynamics of single [4], double and several vortices in a sample, to the observation of vortex lattices [7–9] and also multiply quantized vortices [10], there were numerous studies on this topic. Also, the dynamics of vortex dipoles [11], where vortices of opposite rotation coexist in the same sample were studied. Vortices also appear in the context of quantum turbulence, both in ultracold quantum fluids [12, 13] and in superfluid liquid helium [14] and the dynamics of these vortices in the superfluid is taken as universal [15].

In any context where quantized vortices have been studied so far they can be accurately modeled as point-like objects. When vortices' dimensions are relevant, each individual vortex can be tought as a cyllinder with round cross-section [3] where the radius is given by the interplay between interatomic interactions and kinectic energy, known as the healing length. Even when instabilities are taken into account, like Kelvin waves [20] in the processes of decay, recconection or interaction of vortices, the local cross-section is always taken as circular. Indeed, there were no reasons to question these fully symmetrical models for quantum vortices, since all relevant energy contributions to the system are spherically symmetric.

That scenario changed a few years ago with the observation of ultracold quantum gases of dipolar atoms [16–18]. After the seminal works with chromium atoms [16], there came Bose–Einstein condensates of Erbium [17] and Dysprosium [18], the latter being the atomic species with the largest magnetic moment. Dipolar quantum fluids add dipole–dipole interactions to Eq. (1) which are anisotropic and long-range, allowing the observation of a myriad of new phenomena [19]. The dipole–dipole interaction energy for polarized dipoles reads

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$$V_{ddi} = \frac{C_{dd}}{4\pi}\frac{1-3\cos^2\theta}{r^2}, \quad (3)$$

where θ is the angle between the line connecting two dipoles and the polarization direction, r is the distance between dipoles and $C_{dd} = \mu_0\mu^2$ ($C_{dd} = \frac{d^2}{\epsilon_0}$) for magnetic (electric) dipoles with magnetic (electric) moment μ (d) and the usual magnetic (electric) constant μ_0 (ϵ_0). Such interaction potential explicitly shows the anisotropic character of the dipole–dipole interaction and opens the possibility to new phenomena and anisotropic structures wherever isotropic arrangements were previously predicted.

Not long after the availability of such dipolar quantum fluids, it has been realized that the mean-field Gross–Pitaevskii equation [Eq. (1)] together with dipole-dipole term [Eq. (3)] was not enough to describe the experimental findings. Indeed, it was necessary to add energy contributions that take into account quantum fluctuations, beyond the mean-field interpretation of quantum fluids [21, 22]. For quantum fluids with dipole–dipole interactions the interaction energy reads

$$E = \frac{g}{2}\frac{N_0^2}{V}\left[1 + \frac{128}{15\sqrt{\pi}}\sqrt{n_0 a^3}Q_5(\epsilon_{dd})\right], \quad (4)$$

where the first term is the usual mean-field energy, the second term is the first order beyond mean-field correction, also known as the Lee–Huang–Yang correction, $Q_5(x) = \int \mathrm{d}\theta \sin\theta[1 + x(3\cos^2\theta - 1)]^{5/2}$ and $\epsilon_{dd} = \frac{C_{dd}}{3g}$ sets the relative strengh of the dipole–dipole and contact interaction, that can be tuned via Feshbach resonances.

These quantum fluctuations allow a dipolar quantum fluid to display the long-sought but never observed before supersolid phase [23]. Indeed, supersolids have been observed almost simultaneously by three research groups [24–26] and soon after triggered a frenzy of experimental and theoretical investigations [19]. In a simple picture, as the dipolar interaction is changed through a change in ϵ_{dd} the continuous Bose–Einstein condensate breaks its translational symmetry in superfluid islands or droplets. Such droplets are self-trapped. They do not expand when the external confinement is removed and although are individually superfluid, they do not hold a long-range phase coherence between the individual droplets [27, 28]. Only in a tiny window of the parameter space long-range phase coherence of the system is preserved by an underlying superfluid sea. This former phase is known as individual droplets phase and the latter is the supersolid phase.

The natural question that arises is: since either those phases, namely the individual droplet phase and the supersolid phase, are examples of superfluid samples, what is the behavior of quantum vortices imprinted in such droplets and/or in a supersolid? So far, vortices have already been observed in a dipolar BEC [29] and very recently it has been reported the observation of vortices in a dipolar supersolid [30]. Although the observation of vortices in individual superfluid droplets remains elusive, it should follow very soon.

On the theoretical side, Cidrim *et al.* [31] investigated a single vortex in a single droplet when the vortex has its axis parallel to the polarization direction of the atoms along the vortex and has found a very unstable system, with the vortex decaying quickly.

However, in a recent publication, Li and collaborators [32] have demonstrated theoretically that anisotropic vortex quantum droplets are fully stable in a 2D dipolar quantum fluid in the droplet regime, with rotation imposed perpendicular to the plane of the droplet itself and also to the polarization direction. In that specific configuration, stability is achieved because the density void of the vortex occupies an area around the vortex rotation direction, where the dipole–dipole interaction energy contribution is mainly repulsive, thus lowering the total energy of the system. Interestingly, such configuration is not the same as the one reported for the vortex observation in a supersolid [30] where the polarization direction is tilted compared both to the quantum fluid plane and also vortex core direction, which means there is quite some space for further investigations of the stability phase space of such vortex states in droplets and/or supersolids.

Besides predicting the stable vortex droplet Li *et al.* [32] have also studied the main properties of such individual vortex and one of the main findings is that the vortex is not round as usually predicted and observed for non-dipolar quantum vortices but displays a very large elongation along the polarization direction. The ratio between vortex dimensions parallel and perpendicular to the polarization direction ranges from 4 to 7 as a function of the number of atoms and interaction strength, demonstrating vortices that are extremely stretched along the polarization of the dipoles.

Additionally, authors show that collisions between two of such vortex droplets allow the formation of bound states of vortex–antivortex–vortex which may lead and open avenues of investigation on exotic superfluid bound states.

This work [32] together with the observation of vortices in a supersolid [30] opens exciting perspectives for the study of quantum vortices in such exotic dipolar superfluids. While the authors of Ref. [32] propose the creation of vortex droplet via phase-imprinting a dark soliton in a dipolar droplet and let it decay in a vortex, which is a established experimental technique, the vortices in a supersolid were produced by magnetostirring [29, 30]. Either way, the experimental stage is set and the observation of vortices in the droplet phase should be a reality very soon.

It will be exciting to observe such very stretched vortex states and how such states respond to external torque and the ensuing dynamics, with oscillation modes, both of center of mass and of vortex shape. Also, it should be interesting to observe how the vortex stability and elongation follows an additional rotation of the polarization direction and how energy is rearranged when and if the vortex core is subjected to stretch-compression-stretch protocols. Also, the evolution of such vortices as the BEC is changed from a quantum fluid to a supersolid through the droplet phase might reveal an interesting dynamics. Eventually, the formation of vortex lattices of elongated vortex dipoles may display very different geometries allowing the observation of unusual vortex crystals. In the same way the field of vortices in quantum gases has been a prolific field of investigation, the subset of vortices in dipolar quantum fluids, either in the supersolid, BEC or droplet phase should be a fruitful stage of theoretical and experimental investigation in the near future.

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