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Genuine tripartite entanglement and geometric quantum discord in entangled three-body Unruh–DeWitt detector system

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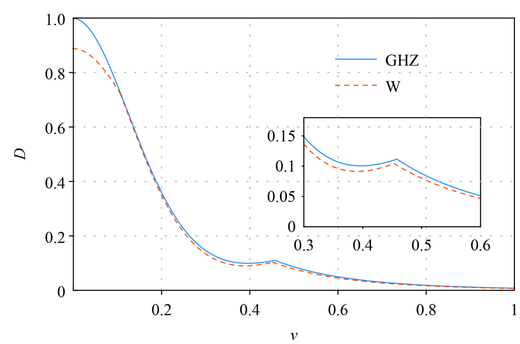
ABSTRACT

We studied the quantum correlations of a three-body Unruh–DeWitt detector system using genuine tripartite entanglement (GTE) and geometric quantum discord (GQD). We considered two representative three-body initial entangled states, namely the GHZ state and the W state. We demonstrated that the quantum correlations of the tripartite system are completely destroyed at the limit of infinite acceleration. In particular, it is found that the GQD of the two initial states exhibits "sudden change" behavior with increasing acceleration. It is shown that the quantum correlations of the W state are more sensitive than those of the GHZ state under the effect of Unruh thermal noise. The GQD is a more robust quantum resource than the GTE, and we can achieve robustness in discord-type quantum correlations by selecting the smaller energy gap in the detector. These findings provide guidance for selecting appropriate quantum states and resources for quantum information processing tasks in a relativistic setting.



Keywords Unruh–DeWitt detector, Unruh effect, relativistic quantum information, geometric quantum discord

1 Introduction

The Unruh–DeWitt detector was originally famous for a thought experiment by Unruh [1] to simulate an accelerating qubit in a vacuum and was later extended by DeWitt [2]. It is a point-like two-level quantum system coupled with fluctuating vacuum quantum fields that is widely used to study fundamental physics problems within the framework of quantum field theory in curved spacetime [3, 4]. Such a system provides an operational method to prove that the notion of particles is observer-dependent, which is a milestone result of quantum field theory. The detector model is semiclassical because it possesses a classical world line while its internal degree of freedom obeys the rules of quantum mechanics. The pioneering work of Reznik *et al.* [5] has shown that, the final state of the initially nonentangled Unruh–DeWitt detectors violates the Bell's inequality, implying the existence of quantum entanglement in the vacuum. The entangled detector model has been adopted to study the behavior of quantum resource [6–9], and quantum information processing tasks [10, 11] under the influence of detector-field interaction in the perspective of relativistic quantum information [12–22]. Recently, Lorek *et al.* [23] demonstrated the extraction of tripartite entanglement from the vacuum of a periodic cavity field. Mendez-Avalos *et al.* [24] discovered that even if one detector is at a considerable distance from the other two, the capture of tripartite entanglement can still be achieved. It was also shown that even in cases of instantaneous

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interaction with the field, the accelerated detectors can attain Greenberger–Horne–Zeilinger (GHZ) type entanglement [25]. Most recently, Membrere *et al.* [26] studied tripartite entanglement harvesting in the vicinity of a black hole.

Quantum entanglement [27] is the most fundamental concept that can be rigorously quantified and characterized by the frameworks of quantum resource theory. The study of tripartite entanglement is very interesting because there are types of bipartite entanglement in three-body systems, but tripartite entanglement cannot be reduced to any combination of all bipartite entanglements [28]. There is bipartite entanglement between a qubit and the remaining two qubits called one-tangle. And there is bipartite entanglement between two qubits named two-tangle between the reduced bipartite systems. Based on the concepts of one-tangle and two-tangle, one can define a measure of tripartite entanglement, i.e., the residual entanglement [29, 30]. Then the genuine tripartite entanglement (GTE) is defined as the minimally residual entanglement. As a measure of tripartite entanglement, the GTE is regarded as a very important quantum resource that is essential for many quantum tasks such as quantum error correction [31], quantum metrology [32] and quantum teleportation [33]. It is well known that entanglement plays a prominent role in the understanding of the thermodynamics and information loss problems of black holes [34–36], as well as the nature of causality in quantum theory [37]. Therefore, a crucial research area pertains to the study of tripartite entangled states in a relativistic setting [38] and, more specifically, examining how the Unruh effect affects the relationship between the GTE and acceleration [39–41].

However, as we all know, quantum entanglement is not the only measure of quantum correlations in a quantum system, and there exist quantum tasks that display quantum advantages without entanglement [42–44]. Quantum discord [45, 46] was introduced to measure quantum correlations in two-body systems. Nevertheless, due to the difficult optimization process involved, only some special two-qubit states [47–49] can be used to obtain fully analytical expressions of quantum discord. To overcome such difficulties, Dakić *et al.* [50] introduced the geometric quantum discord (GQD) as a measure of quantum correlations. Unlike the quantum discord defining via the conditional entropy, the GQD is related to the relative entropy. Such a measure offers a geometric vantage point for quantifying quantum correlations [51]. Subsequently, Zhou *et al.* [52] extended this method from the two-qubit states to the three-body states. It has been proven that multipartite quantum correlations are essential for successful quantum computing and quantum communication tasks [53]. Therefore, studying the dynamics of multipartite quantum correlations in a relativistic setting is of particular importance.

In this paper, we analyze the dynamics of the quantum correlations of the three-body relativistic system when one detector is accelerated. We find that GQD exhibits "sudden change" behavior as a function of acceleration compared to the monotonically decaying variation trend of the GTE. The quantum correlations for the W state are more sensitive than those of the GHZ state in the face of Unruh thermal noise. Compared to GTE, GQD is a more robust quantum resource, and we can choose detectors with smaller energy gaps to obtain more robust discord-type quantum correlations.

The outline of the paper is as follows. In Section 2, we introduce the quantum information description of the entangled Unruh–DeWitt detectors and the evolution of the prepared states in the case of one detector accelerated. In Section 3, we briefly introduce the measurements of tripartite quantum correlations, i.e., the GTE and GQD. In Section 4, the behaviors of tripartite quantum entanglement and discord in the relativistic quantum system with initial GHZ state, and W state are discussed in detail. We summarize our results in Section 5 and provide an Appendix for the details of our calculations.

2 Evolution of the tripartite relativistic quantum system

In Ref. [11], the authors discussed the dynamics of bipartite entanglement between a pair of initially entangled Unruh–DeWitt detectors. In this section, we take one step further by generalizing the systems from two-body to three-body. In addition, to get a multiple-perspective study on the behavior of quantum correlations in the three-body Unruh–DeWitt detector system, two types of initial states, namely the GHZ and W states, are considered,

$$|\Psi^{(1)}_{ABC}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle), \tag{1}$$

$$|\Psi^{(2)}_{ABC}\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle), \tag{2}$$

where each particle in the system is named from left to right (A, B, C). It is assumed that the third detector carried by Charlie is uniformly accelerated for a finite amount of proper time Δ. The world line of Charlie's detector is given by

$$\begin{aligned} &t(\tau) = a^{-1}\sinh a\tau, \quad x(\tau) = a^{-1}\cosh a\tau, \\ &y(\tau) = z(\tau) = 0, \end{aligned} \tag{3}$$

where τ and a are the Charlie's proper time and proper acceleration, respectively, and (t, x, y, z) are the usual Cartesian coordinates of Minkowski spacetime.

We assume that the initial state of a complete system consisting of the detectors and the external scalar field has the form

$$|\Psi_{-\infty}^{ABC\phi}\rangle = |\Psi_{ABC}^{(i)}\rangle \otimes |0_M\rangle, \tag{4}$$

where $|\Psi_{ABC}^{(i)}\rangle$ are the initial states given in Eqs. (1) and (2), and $|0_M\rangle$ refers to Minkowski scalar-field vacuum.

Assuming that the Charlie's detector interacts with the real massless scalar field $\phi(x)$, the interaction Hamiltonian $H_{int}^{C\phi}$ is [6, 7, 11]

$$H_{int}^{C\phi}(\tau) = \epsilon(\tau)\int_{\Sigma_\tau} \mathrm{d}^3\boldsymbol{x}\sqrt{-g}\phi(x)[\psi(\boldsymbol{x})C + \overline{\psi}(\boldsymbol{x})C^\dagger], \tag{5}$$

where C and $C^\dagger$ represent the creation and annihilation operators of the Charlie's particle detector, respectively. The coupling constant $\epsilon(\tau)$ is introduced to ensure that the detector remains switched on for the duration Δ, and switched off outside that interval. $\sum = \{\tau = const\}$ denotes that the integration is over the global spacelike Cauchy surface in Minkowski spacetime. If the detector is assumed be localized, $\psi(\boldsymbol{x}) = (\kappa\sqrt{2\pi})^{-3}\exp(-\boldsymbol{x}^2/(2\kappa^2))$ is a Gaussian coupling function with variance $\kappa = const$, which describes that the detector only interacts with the neighbor field. Therefore, we can get the total Hamiltonian of the entire tripartite system is

$$H_{3\phi} = H_A + H_B + H_C + H_{KG} + H_{int}^{C\phi}. \tag{6}$$

The H_{KG} stands for the Hamiltonian of the massless scalar field. $H_P = \Omega P^\dagger P$ $(P = A, B, C)$ represents the free Hamiltonian of each particle detectors. $P^\dagger$ and P represent the creation and annihilation operators for the particle detector, namely $P^\dagger|1\rangle = P|0\rangle = 0$, $P|1\rangle = |0\rangle$, and $P^\dagger|0\rangle = |1\rangle$, $|1\rangle$ and $|0\rangle$ are the excited and unexcited energy eigenstates. Ω is the energy gap of the the two-level atoms.

It is widely acknowledged that the evolution of a total system comprising a detector and an external field can be described by the Schrödinger equation. By utilizing the interaction picture and considering the first perturbation order, the final state of the detector-field system can be determined [1, 11],

$$\left|\Psi_{\infty}^{ABC\phi}\right\rangle = \left(I + a_{RI}^\dagger(\lambda)C - a_{RI}(\bar{\lambda})C^\dagger\right)\left|\Psi_{-\infty}^{ABC\phi}\right\rangle, \tag{7}$$

where $\left|\Psi_{\infty}^{ABC\phi}\right\rangle$ and $\left|\Psi_{-\infty}^{ABC\phi}\right\rangle$ are the final and the initial states of the whole system, respectively. The a_{RI} and $a_{RI}^\dagger$ are Rindler annihilation and creation operators in region I of the Rindler spacetime. The modes λ satisfy $\lambda = -KEf$ with a compact support complex function $f \equiv \epsilon(t)\mathrm{e}^{-\mathrm{i}\Omega t}\psi(\boldsymbol{x})$ in terms of Minkowski coordinates. The K operator takes the positive-frequency part of the solutions of the Klein–Gordon equation $\nabla_a\nabla^a\phi(x) = 0$ with respect to the timelike isometry, and Ef is defined as

$$Ef = \int \mathrm{d}^4x'\sqrt{-g(x')}\left[G^{\mathrm{adv}}(x,x') - G^{\mathrm{ret}}(x,x')\right]f(x'),$$

where G^{adv} and G^{ret} are the advanced and retarded Green's functions, respectively. And E is the difference between these two Green functions.

Substituting $\Psi_{-\infty}^{ABC\phi}$ of Eq. (7) with the initial state Eq. (4), we can obtain the final reduced density matrix of the detector by tracing out the field degrees of freedom. When initial state is GHZ state, we can get: $\rho_{\infty(G)}^{ABC} = [G_{ij}]\,(i,j = 1,2,\ldots,8)$, where the nonzero elements G_{ij} are

$$G_{11} = G_{18} = G_{81} = G_{88} = S_0, G_{22} = S_1, G_{77} = S_2. \tag{8}$$

While $\rho_{\infty(W)}^{ABC} = [W_{ij}]\,(i,j = 1,2,\ldots,8)$, where the nonzero elements W_{ij} are

$$W_{11} = P_0, \tag{9}$$

$$\begin{aligned} W_{22} &= W_{23} = W_{25} = W_{32} = W_{33} = W_{35} = W_{52} \\ &= W_{53} = W_{55} = P_1, \end{aligned} \tag{10}$$

$$W_{44} = W_{46} = W_{64} = W_{66} = P_2. \tag{11}$$

These elements are (see Appendix for detail derivation)

$$\begin{aligned} S_0 &= \frac{1-q}{\nu^2(1+q)+2(1-q)}, \quad P_0 = \frac{\nu^2}{\nu^2(1+2q)+3(1-q)}, \\ S_1 &= \frac{\nu^2 q}{\nu^2(1+q)+2(1-q)}, \quad P_1 = \frac{1-q}{\nu^2(1+2q)+3(1-q)}, \\ S_2 &= \frac{\nu^2}{\nu^2(1+q)+2(1-q)}, \quad P_2 = \frac{\nu^2 q}{\nu^2(1+2q)+3(1-q)}, \end{aligned} \tag{12}$$

where $q \equiv \mathrm{e}^{-2\pi\Omega/a}$ is the parameterized acceleration and ν is a effective coupling parameter, which is $\nu^2 \equiv \|\lambda\|^2 = \frac{\epsilon^2\Omega\Delta}{2\pi}\mathrm{e}^{-\Omega^2\kappa^2}$ [6, 11]. For the relations above to be valid, it requires $\epsilon \ll \Omega^{-1} \ll \Delta$. q as a monotonic function of acceleration a, when $q \to 0$ means zero acceleration, while $q \to 1$ means the asymptotic limit of infinite acceleration.

3 Measurements of tripartite quantum correlations

In a tripartite system, the negativity is the presence of an observer measuring its entanglement between the other two parties, which is the one-tangle [29, 30]

$$N_{A(BC)} = \left\|\rho_{ABC}^{T_A}\right\| - 1. \tag{13}$$

And the two-tangle measures the entanglement of the observer party and their other partner $N_{AB} = \left\|\rho_{AB}^{T_A}\right\| - 1$, where T_A represents the partial transpose of ρ_{ABC} and ρ_{AB} with respect to the observer A. The trace norm $\|R\|$ is given by $\|R\| = Tr\sqrt{RR^\dagger}$ [54].

Note that a CKW-inequality-like monogamy inequality

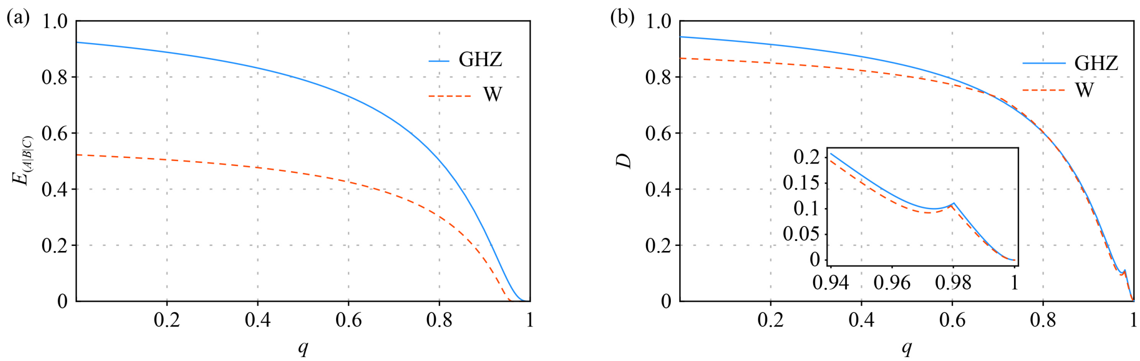

Fig. 1 The GTE and GQD of GHZ and W states as a function of the acceleration q. We set the fixed effective coupling parameter $v = 0.2$.

$$\mathcal{N}_{AB}^2 + \mathcal{N}_{AC}^2 \leqslant \mathcal{N}_{A(BC)}^2 \tag{14}$$

is always valid. We define the genuine tripartite entanglement (GTE) [29, 30] as the minimally residual tripartite entanglement. The latter is the minimum of each non-negative difference between the two sides of inequality (14) in a subsystem

$$E_{(A|B|C)} = \min_{(A,B,C)} \left(N_{A(BC)}^2 - N_{AB}^2 - N_{AC}^2 \right), \tag{15}$$

where (A, B, C) shows all the permutations of the three mode indices.

On the other hand, quantum discord [45, 46], which can quantify all quantum correlations, including entanglement in bipartite systems, is defined as the difference between total correlations and classical correlations. Recently, many efforts to generalization of quantum discord to multipartite systems have been made [55, 56]. In Ref. [52], Zhou *et al.* proposed an exact formula of geometric quantum discord (GQD) for an arbitrary three-body state. The GQD of a tripartite quantum state in Hilbert space $H_A \otimes H_B \otimes H_C$ is defined as [50, 57]

$$D(\rho) = \min_{\rho_c \in \Omega_0} \|\rho - \rho_c\|^2, \tag{16}$$

where Ω_0 denotes the set of zero-discord states and $\|\cdot\|$ is the usual Hilbert–Schmidt norm. For an arbitrary three-body state, the density matrix is described as

$$\rho = \frac{1}{8} \left(\sum_{i,j,k=0}^{3} c_{ijk} \sigma_i \otimes \sigma_j \otimes \sigma_k \right), \tag{17}$$

where σ_0 is the identity operator and the others are Pauli operators, and $c_{ijk} = Tr(\rho\sigma_i \otimes \sigma_j \otimes \sigma_k)$. Then the geometric quantifier of quantum correlations in bipartite cut $A|BC$ is

$$D_A(\rho) = \frac{1}{8} \left(\sum_i k_i - \max_i k_i \right), \tag{18}$$

where k_i are the eigenvalues of 3×3 matrix $\mathrm{xx}^t + TT^t$, $\mathrm{x} = (c_{m00})^t$ $(m = 1, 2, 3)$ is the column vector, and the matrix $T = (t_{mn\{j,k\}}) = (c_{mjk})$ is a 3×15 matrix. Using the same definition method as above, one can define $D_B(\rho)$ and $D_A(\rho)$ in the bipartite cut $B|AC$ and $C|AB$, respectively. Finally, the tripartite GQD of three-body states is given as [52]

$$D(\rho) = \min \{D_A(\rho), D_B(\rho), D_C(\rho)\}. \tag{19}$$

4 Behaviors of tripartite quantum correlations under the influence of Unruh thermal noise

The GHZ state and the W state are two distinct entangled states for a three-body system that cannot be transformed into each other through local operations and classical communication [58]. Here we discuss the dynamical evolution of the quantum correlations of the three-body Unruh–DeWitt detector system. After some caculations, we obtain the GTE and GQD of the three-body system for the GHZ initial state case

$$E_{(A|B|C)}^{\mathrm{GHZ}} = (\sqrt{S_0^2 + S_1^2} + \sqrt{S_0^2 + S_2^2} - S_1 - S_2)^2, \tag{20}$$

$$\begin{aligned} D_{ABC}^{\mathrm{GHZ}} = {} & 6S_0^2 + S_1^2 + S_2^2 - 2S_0(S_1 + S_2) \\ & - \frac{1}{4} \max \{8S_0^2, 4(2S_0^2 + S_1^2 + S_2^2 \\ & - 2S_0(S_1 + S_2))\}. \end{aligned} \tag{21}$$

The GTE $E_{(A|B|C)}^{\mathrm{W}}$ and D_{ABC}^{W} of the W state can be calculated in the similar way.

In Fig. 1, we plot the GTE and GQD for different initial states as a function of the acceleration q. It is shown that the GTE of the two initial states have similar monotonic decreasing tendencies. It is worth noting that

$$\lim_{q \to 1} E_{(A|B|C)}^{\mathrm{GHZ}} = 0, \quad \lim_{q \to 1} E_{(A|B|C)}^{\mathrm{W}} = 0, \tag{22}$$

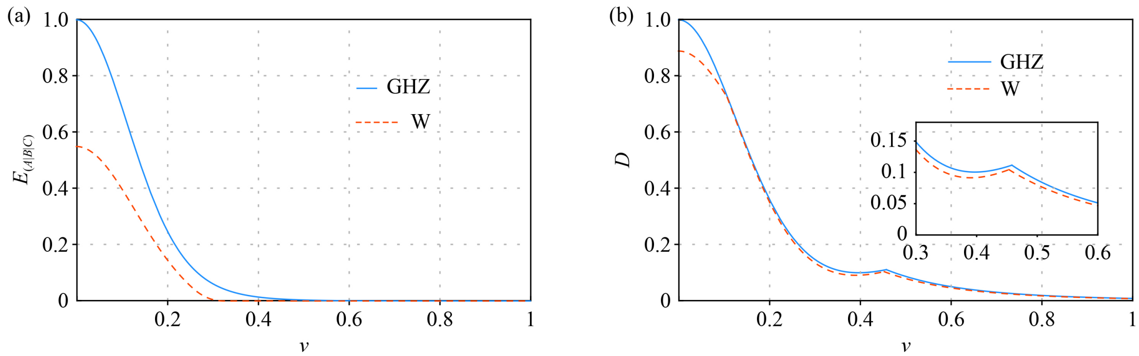

Fig. 2 The GTE and GQD of GHZ and W state as a function of the effective coupling parameter v with the fixed acceleration $q = 0.9$.

and

$$\lim_{q\to 1} D_{ABC}^{\mathrm{GHZ}} = 0, \quad \lim_{q\to 1} D_{ABC}^{\mathrm{W}} = 0. \tag{23}$$

This indicates that, as the acceleration tends to infinity, the thermal noise induced by Unruh radiation can completely destroy the quantum correlations among the detectors. In this limit, all the initial correlations are transformed between the detectors and external fields. At higher accelerations, the Unruh thermal bath contains more particles that can interact with the detector, resulting in a greater loss of quantum correlations. It is also shown that, compared to the W state, the quantum correlations of the GHZ state are more robust against the Unruh effect. This suggests that the quantum correlations of the GHZ state are more effective in resisting the Unruh effect and are more suitable for handling relativistic quantum information tasks in three-body systems. It is worth mentioning that the dynamics of the GQD exhibits a "sudden change" behavior. Before the change point, the GQD decreases with increasing acceleration q. After the change point, GQD decreases to zero. The sudden change results from the maximization procedure. Substitute $\nu = 0.2$ and solve the following equation,

$$8S_0^2 = 4\left[2S_0^2 + S_1^2 + S_2^2 - 2S_0(S_1 + S_2)\right], \tag{24}$$

we find that the change critical point of GHZ state is $q = 0.980197$. For the W state, the change critical point is found to be $q = 0.979235$. For larger values of ν, the sudden change point moves to the left on the q axis.

Figure 2 show the behaviors of GTE and GQD as a function of the effective coupling parameter ν. It is found that the GQD also shows a "sudden change" behavior and an increase in the effective coupling reduces the quantum correlations among the detectors. In addition, the larger the value of acceleration, the closer the sudden-change point is to the origin. As the effective coupling parameter increases, the GTE decays to zero before the GQD, which indicates that the GQD is a more robust quantum resource than GTE. In addition, we find that the W state of the detector is more sensitive than the GHZ state when the detector interacts with the external field.

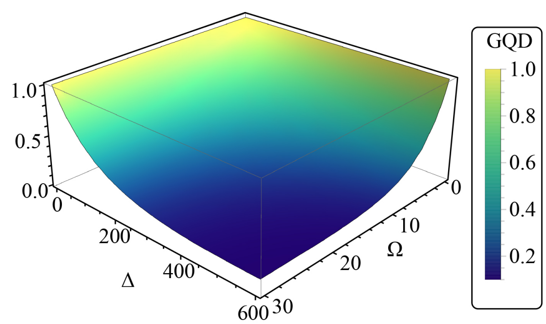

Fig. 3 The graph exhibits the GQD of the tripartite system for the GHZ state as functions of the interaction time duration Δ and the energy gap Ω, with $\varepsilon^2 = 8\pi^2 \cdot 10^{-6}$, $\kappa = 0.02$ and $q = 0.9$.

Then we further investigate how the interaction between the accelerated detector and the external scalar field influences the GQD. Figure 3 shows the GQD of the tripartite system as a function of the energy gap Ω and interaction time Δ for the GHZ state. It is shown that the discord-type correlation decreases as the energy gap of the accelerating detector increases. This means that the smaller the energy gap, the more robust the GQD will be over the interaction time. Therefore, one can better perform relativistic quantum information by preparing suitable detectors with some artificial two-level atoms with appropriate energy gaps.

5 Conclusions

In this paper, we have studied the influence of the Unruh effect on the quantum correlations of the three-detector system when one detector is moving with uniform acceleration. The results show that the thermal noise of Unruh radiation can completely disrupt the quantum correlations among the detectors. This is because the Unruh effect predicts that the information formed in some regions in Rindler space is leaked into the causally disconnected region due to the acceleration of one party. It is worth mentioning that the discord-type quantum correlations happen "sudden change" behavior, which is quite different form the behavior of entanglement. In addition, the quantum correlations of the W state are more sensitive than those of the GHZ state. The behaviors of quantum correlations are quite different from those of quantum coherence because it was found in Ref. [59] that the quantum coherence of the W state is more robust than the GHZ state against Unruh thermal bath. It is also shown that the GQD is more robust than GTE against the decoherence induced by the Unruh effect and we can achieve robustness in discord-type quantum correlations by choosing the shortest interaction time and some small energy gaps.

Declarations The authors declare that they have no competing interests and there are no conflicts.

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Appendix: The derivation of Eq. (12)

Here, we present the derivation of the ultimate evolution of the tripartite Unruh–DeWitt detector system. We assume that only the detector carried by Charlie accelerates uniformly, while the other two detectors remain stationary. When the initial state of the three-body system is the GHZ state, by combining the initial state Eq. (4) with Eq. (7), the final state of the entire system can be expressed as

$$\left|\Psi_{\infty}^{(1)ABC\phi}\right\rangle = \left|\Psi_{-\infty}^{(1)ABC\phi}\right\rangle + \frac{1}{\sqrt{2}}\Big[|110\rangle\otimes\left(a_{RI}^{\dagger}(\lambda)\,|0_M\rangle\right) + |001\rangle\otimes\left(a_{RI}(\bar{\lambda})\,|0_M\rangle\right)\Big], \tag{A1}$$

where the creation and annihilation operators $a_{RI}^{\dagger}(\lambda)$ and $a_{RI}(\lambda)$ are defined in the Rindler region I, and $|0_M\rangle$ is the Minkowski vacuum. According to Refs. [10, 11], the Bogoliubov transformations between the Rindler operators and the Minkowski operators are

$$a_{RI}(\overline{\lambda}) = \frac{a_M(\overline{F_{1\Omega}}) + \mathrm{e}^{-\pi\Omega/a}a_M^{\dagger}(F_{2\Omega})}{(1-\mathrm{e}^{-2\pi\Omega/a})^{1/2}}, \tag{A2}$$

$$a_{RI}^{\dagger}(\lambda) = \frac{a_M^{\dagger}(F_{1\Omega}) + \mathrm{e}^{-\pi\Omega/a}a_M(\overline{F_{2\Omega}})}{(1-\mathrm{e}^{-2\pi\Omega/a})^{1/2}}, \tag{A3}$$

where $F_{1\Omega} = \frac{\lambda+\mathrm{e}^{-\pi\Omega/a}\lambda\circ w}{\left(1-\mathrm{e}^{-2\pi\Omega/a}\right)^{1/2}}$, and $F_{2\Omega} = \frac{\overline{\lambda\circ w}+\mathrm{e}^{-\pi\Omega/a}\bar{\lambda}}{\left(1-\mathrm{e}^{-2\pi\Omega/a}\right)^{1/2}}$. In $F_{1\Omega}$ and $F_{1\Omega}$, $w(t,x,y,z)=(-t,-x,y,z)$ represents the wedge reflection isometry that makes a reflection from λ in Rindler region I to $\lambda\circ w$ in Rindler region II [10, 11], where the symbol $\circ$ represents the multiplication of mappings, in other words, the composite mapping of two mappings.

By using the Bogliubov transformations given in Eqs. (A2) and (A3), Eq. (A1) can be rewritten as

$$\left|\Psi_{\infty}^{(1)ABC\phi}\right\rangle = \left|\Psi_{-\infty}^{(1)ABC\phi}\right\rangle + \frac{1}{\sqrt{2}}\nu\left[\frac{|110\rangle\otimes|1_{\tilde{F}_{1\Omega}}\rangle}{(1-\mathrm{e}^{-2\pi\Omega/a})^{1/2}} + \mathrm{e}^{-\pi\Omega/a}\frac{|001\rangle\ \otimes|1_{\tilde{F}_{2\Omega}}\rangle}{(1-\mathrm{e}^{-2\pi\Omega/a})^{1/2}}\right], \tag{A4}$$

where $\tilde{F}_{\mathrm{i}\Omega} = F_{\mathrm{i}\Omega}/\nu$.

We are interested in the evolution of the detectors' state after interacting with the field, by tracing out the scalar field degrees of freedom, one can obtain

$$\rho_{\infty(G)}^{ABC} = ||\Psi_{\infty}^{(1)ABC\phi}||^{-2}\mathrm{tr}_{\phi}|\Psi_{\infty}^{(1)ABC\phi}\rangle\langle\Psi_{\infty}^{(1)ABC\phi}|, \tag{A5}$$

where

$$||\Psi_{\infty}^{(1)ABC\phi}||^2 = 1 + \frac{\nu^2(1+\mathrm{e}^{-2\pi\Omega/a})}{2(1-\mathrm{e}^{-2\pi\Omega/a})}$$

normalizes the final density matrix, i.e., $\mathrm{tr}\rho_{\infty(G)}^{ABC}=1$. Therefore, for the case of initial GHZ state, the final state of detectors is found to be

$$\rho_{\infty(G)}^{ABC} = \begin{pmatrix} S_0 & 0 & 0 & 0 & 0 & 0 & 0 & S_0 \\ 0 & S_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_2 & 0 \\ S_0 & 0 & 0 & 0 & 0 & 0 & 0 & S_0 \end{pmatrix}, \tag{A6}$$

where the parameters S_0, S_1, S_2 are given by

$$S_0 = \frac{1-\mathrm{e}^{-2\pi\Omega/a}}{\nu^2\left(1+\mathrm{e}^{-2\pi\Omega/a}\right)+2(1-\mathrm{e}^{-2\pi\Omega/a})},$$
$$S_1 = \frac{\nu^2\mathrm{e}^{-2\pi\Omega/a}}{\nu^2\left(1+\mathrm{e}^{-2\pi\Omega/a}\right)+2(1-\mathrm{e}^{-2\pi\Omega/a})},$$
$$S_2 = \frac{\nu^2}{\nu^2\left(1+\mathrm{e}^{-2\pi\Omega/a}\right)+2(1-\mathrm{e}^{-2\pi\Omega/a})}.$$

Similarly, in the case of the W initial state described by Eq. (2), by combining Eq. (4) with Eq. (7), and utilizing the Bogliubov transformations provided in Eqs. (A2) and (A3), the final state of the detector-field system can be rewritten as

$$\left|\Psi_{\infty}^{(2)ABC\phi}\right\rangle = \left|\Psi_{-\infty}^{(2)ABC\phi}\right\rangle + \frac{1}{\sqrt{3}}\nu\left[\frac{|000\rangle\otimes|1_{\tilde{F}_{1\Omega}}\rangle}{(1-\mathrm{e}^{-2\pi\Omega/a})^{1/2}} + \mathrm{e}^{-\pi\Omega/a}\frac{(|101\rangle+|011\rangle)\otimes|1_{\tilde{F}_{2\Omega}}\rangle}{(1-\mathrm{e}^{-2\pi\Omega/a})^{1/2}}\right]. \tag{A7}$$

Tracing out the degree of freedom of the external field, namely, $\rho^{ABC}_{\infty(W)}=||\Psi^{(2)ABC\phi}_{\infty}||^{-2}\mathrm{tr}_{\phi}|\Psi^{(2)ABC\phi}_{\infty}\rangle\langle\Psi^{(2)ABC\phi}_{\infty}|$, here $||\Psi^{(2)ABC\phi}_{\infty}||^{2}=\frac{\nu^2\left(1+2\mathrm{e}^{-2\pi\Omega/a}\right)+3(1-\mathrm{e}^{-2\pi\Omega/a})}{3(1-\mathrm{e}^{-2\pi\Omega/a})}$ normalizes the final density matric, we can obtain the final state of the detectors

$$\rho^{ABC}_{\infty(W)} = \begin{pmatrix} P_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & P_1 & P_1 & 0 & P_1 & 0 & 0 & 0 \\ 0 & P_1 & P_1 & 0 & P_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & P_2 & 0 & P_2 & 0 & 0 \\ 0 & P_1 & P_1 & 0 & P_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & P_2 & 0 & P_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A8}$$

with

$$P_0 = \frac{\nu^2}{\nu^2\left(1+2\mathrm{e}^{-2\pi\Omega/a}\right)+3(1-\mathrm{e}^{-2\pi\Omega/a})},$$
$$P_1 = \frac{1-\mathrm{e}^{-2\pi\Omega/a}}{\nu^2\left(1+2\mathrm{e}^{-2\pi\Omega/a}\right)+3(1-\mathrm{e}^{-2\pi\Omega/a})},$$
$$P_2 = \frac{\nu^2\mathrm{e}^{-2\pi\Omega/a}}{\nu^2\left(1+2\mathrm{e}^{-2\pi\Omega/a}\right)+3(1-\mathrm{e}^{-2\pi\Omega/a})}.$$

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