FRONTIERS OF PHYSICS

VIEW & PERSPECTIVE
Volume 19 / Issue 2 / 24303 / 2024

Simple collective model for nuclear chiral mode

R. V. Jolos[1,2,†], E. A. Kolganova[1,2], D. R. Khamitova[1,2]

1 Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia
2 Dubna State University, 141982 Dubna, Moscow Region, Russia
Corresponding author. E-mail: †jolos@theor.jinr.ru
Received April 1, 2023; accepted September 27, 2023

ABSTRACT

A simple semi-analytical collective model that takes into account the limitations of the variation interval of the collective variable is suggested to describe the chiral dynamics in triaxial odd–odd nuclei with a fixed particle–hole configuration. The collective Hamiltonian is constructed with the potential energy obtained using the postulated ansatz for the wave function symmetric with respect to chiral transformation. By diagonalizing the collective Hamiltonian the wave functions of the lowest states are obtained and the evolution of the energy splitting of the chiral doublets in transition from chiral vibration to chiral rotation regime is demonstrated.

Keywords collective, model, nuclear, chiral

1 Introduction

Since the chiral mode was proposed by Frauendorf and Meng [1] it has become one of the hot topics in nuclear structure physics. The experimental indications on the chiral bands were observed in many nuclei and the experimental data and theoretical results, including reviews, were presented in numerous publications [2–6].

In the deformed triaxial odd–odd nuclei there are three mutually nearly orthogonal angular momenta: the collective angular momentum and momenta of the groups of the high-j valence particles and holes. They can be arranged to form two systems with opposite handedness, namely, the right-handed and the left-handed system. These two systems are transformed into each other by the chiral operator which combines time reversal and spatial rotation of 180°.

If the total angular momentum is not too high it can oscillate between left- and right-handed configuration. To describe dynamics of the considered system it is necessary to find a dynamical variable which describes transition from the left-handed to the right-handed system. Of course in the reality there are several degrees of freedom which affect the dynamics of the system under consideration. However, in the case in question, only one of them describes the collective motion of a large amplitude, namely that which describes transition from the left-handed to the right-handed system.

To characterize transition from the left-handed to the right-handed system it was suggested in [7] to use the orientation operator $\hat{\sigma}$ introduced in Ref. [8] which in case of a system with one proton particle and one neutron hole can be presented as

$$\hat{\sigma} = \left(\hat{\boldsymbol{j}}_\pi \times \hat{\boldsymbol{j}}_\nu\right) \cdot \hat{\boldsymbol{R}}, \tag{1}$$

where $\hat{\boldsymbol{R}}$ is the operator of the collective rotation. In the Tilted axis cranking approach [1] the orientation operator is written as

$$\hat{\sigma} = |\hat{\boldsymbol{j}}_\pi| \cdot |\hat{\boldsymbol{j}}_\nu| \cdot |\hat{\boldsymbol{R}}| \sin\theta_{PN} \cdot \sin\theta \cdot \sin\phi, \tag{2}$$

where θ_{PN} denote the angle between of $\hat{\boldsymbol{j}}_\pi$ and $\hat{\boldsymbol{j}}_\nu$, and the angles θ and ϕ parameterize the tilt of the total angular momentum. For a given θ the angle ϕ which varies from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ describes a transition from the left-handed to the right-handed system. It was shown by calculations in Refs. [9, 10] that the potential energy is more soft in the ϕ direction compare to the θ direction. For this reason, the angle ϕ is used below as a collective variable.

Depending on the height of the barrier of the collective potential separating the localizations of the right-handed

*Special Topic: Nuclear Chirality over the Past 25 Years (Eds. Qibo Chen & Pengwei Zhao).

Higher Education Press

https://doi.org/10.1007/s11467-023-1356-5

and left-handed configurations by chiral angle ϕ, the excitation spectrum of the states related to the chirality mode evolves from typically vibrational to typically rotational ones. In the first limit, the spectrum consists in the states of the alternating parity with respect to change in the sign of the angle ϕ. In the second limit the spectrum consists in weakly splitted doublets of symmetric and antisymmetric states mentioned above.

To describe the evolution of the spectrum of the states related to chiral mode a collective Hamiltonian should be constructed. The aim of the present paper is to formulate a simple, partly analytical model to describe the physical picture introduced above basing on the results obtained in Ref. [7].

2 Description of the model

Starting from the Tilted-axis cranking model a system of the $h_{11/2}$ proton particle and $h_{11/2}$ neutron hole coupled to a triaxial rigid rotor is considered in Ref. [7]. The collective Hamiltonian including one chiral degree of freedom has been constructed. The potential energy of the Hamiltonian as a function of the angle ϕ, describing a transition from the left-handed to the right-handed configurations, has been determined by minimizing the total Routhian surface for a given value of the variable ϕ. It is shown that this potential is evolving from a nearly flat-bottomed potential at a small rotational frequencies to a two-minimum potential with large barrier between them at high rotational frequencies. The collective potential is an even function of the angle ϕ and its two minima are located symmetrically at positive and negative values of ϕ. The angle ϕ varies from $-\pi/2$ to $\pi/2$. The saddle point at $\phi = 0$ (located in the plane of the short and long axis of the $\gamma = \pi/6$ triaxial rotor) allows the angular momentum oscillate between right-handed and left-handed systems. The physical picture looks similar to that which is realized in description of the octupole alternating parity bands [11].

The inertia coefficient was determined based on the cranking-like model. It was found that the inertia coefficient being approximately a constant within the most part of the potential, then increases dramatically when ϕ approaches to $\pm\pi/2$. This is due to the limited range of possible values of ϕ. For this reason a wall approximation can be adopted for inertia coefficient.

Based on these characteristics of the potential and the inertia coefficient we formulate below a collective Hamiltonian, containing a small number of parameters having a clear physical meaning, which can serve as a ground to reveal a universal features in behavior of the chiral bands:

$$H_{coll} = -\frac{\hbar^2}{2B}\frac{\mathrm{d}^2}{\mathrm{d}y^2} + V(y), \tag{3}$$

where y is a dynamical variable proportional to the angle ϕ. The assumption that the inertia coefficient is a constant inside the region of variation of the chiral angle ϕ is based on the results obtained in Ref. [7].

We determine an analytical form of the collective potential based on the procedure used in supersymmetric quantum mechanics [12, 13]. The saddle point of the collective potential corresponds to the situation when three angular momenta: of the group of the high-j valence particles, of the group of the high-j valence holes and the collective angular momentum lies in one plane. This value of ϕ separates right-handed and left-handed configurations. If the potential barrier at the saddle point is not high enough total angular momentum can fluctuate from the right-handed to the left-handed system and back. If the barrier is sufficiently high then the case of the spontaneous symmetry breaking in the intrinsic frame is realized. However, since the total Hamiltonian is symmetric with respect to chiral transformation [14, 15] the total wave function will consists in symmetric or antisymmetric superposition of the left-handed and right-handed configurations. Both limits of the barrier height are described by the following wave function:

$$\Psi^{(+)} \sim (1-y^2)^a \left[\mathrm{e}^{-\frac{\xi^2}{2x_m^2}(y+x_m)^2} + \mathrm{e}^{-\frac{\xi^2}{2x_m^2}(y-x_m)^2}\right], \tag{4}$$

where $y = \phi/(\frac{\pi}{2})$.

This wave function is constructed by analogy with the wave function describing alternating parity bands associated with octupole deformation [11]. The parameter x_m, where $|x_m| \leq 1$ determines the positions of the right-handed and left-handed configurations. In contrast to the situation with the octupole alternating parity bands we should take into account the limitation of the area of variation of the y variable. With this aim, a multiplier $(1-y^2)^a$ is introduced into definition of the wave function (4). Coefficient a is taken to be very small, so as not to affect the wave function within the allowed interval of its variation. It is seen from (4) that with increase of ξ the wave function evolves from chiral vibration to chiral rotation cases. This means that ξ increases with increase of rotational frequency.

The ansatz (4) for the wave function yields the following expression for the collective potential:

$$V_\xi = \frac{\hbar^2}{2B}\frac{\mathrm{d}^2\Psi^{(+)}/\mathrm{d}y^2}{\Psi^{(+)}} + E_0, \tag{5}$$

where E_0 is the energy of the lowest state. Substituting (4) into (5) and introducing a new variable $x = y/x_m$, we obtain

Table 1 Energy splitting of the doublets consisting in the first excited and lowest states ΔE_{01}, and of the third and second excited states ΔE_{23} of the chiral Hamiltonian. Energies are given in arbitrary units. Calculations are performed with $x_m = 0.6$ and $c = 2.2$.

ξ	0.7	0.9	1.1	1.3	1.5	1.6	1.7	1.8	1.9	2.0
ΔE_{01}	1.307	0.701	0.371	0.166	0.064	0.038	0.022	0.013	0.008	0.004
ΔE_{23}					0.490	0.365	0.259	0.175	0.112	0.069

$$
\begin{aligned}
V_\xi =&\hbar\omega\Bigg[\frac{1}{2}(\xi^2-1)+\frac{1}{2}\xi^2x^2-\xi^2x\tanh(\xi^2x)\\
&+\frac{a}{\frac{1}{x_m^2}-x^2}\left(2x^2-\frac{1}{\xi^2}\right)+\frac{a(a-1)}{\frac{1}{x_m^2}-x^2}\frac{2}{\xi^2}x^2\\
&-2\frac{a}{\frac{1}{x_m^2}-x^2}x\tanh(\xi^2x)\Bigg]+E_0\\
\equiv&\hbar\omega v_\xi(x)+E_0,
\end{aligned}
\tag{6}
$$

where

$$
\hbar\omega\equiv\frac{\hbar^2}{B}\frac{\xi^2}{x_m^2}. \tag{7}
$$

The new variable x varies in the limits $-\frac{1}{x_m}\le x\le\frac{1}{x_m}$. In the following consideration we omit the constant E_0 in the expression for V_ξ.

We have used the procedure described above to build the collective potential that meets our goals, relying on the ansatz for the wave function of the lowest state. This potential give us the viewable physical picture and at the same time has a fairly simple form. However, such a consideration establishes a rigid relationship between dimensional parameters characterizing kinetic and potential energy terms. This is necessary if we want to build a partner potential in a supersymmetric scheme. But we will not follow this way. For this reason we are free to use below an additional scaling parameter in the expression for the kinetic energy:

$$
\hat{T}=-\frac{\hbar^2}{2Bc}\frac{\mathrm{d}^2}{\mathrm{d}y^2}=-\hbar\omega\frac{1}{2\xi^2c}\frac{\mathrm{d}^2}{\mathrm{d}x^2}, \tag{8}
$$

where c is the scaling parameter for the inertia coefficient. Proportionality of the inertia coefficient to ξ indicates on increase of inertia coefficient with rotational frequency. Thus, the total Hamiltonian $\hat{H}$ looks as

$$
\hat{H}=\hbar\omega\left[\frac{1}{2\xi^2c}\frac{\mathrm{d}^2}{\mathrm{d}x^2}+v_\xi(x)\right]. \tag{9}
$$

Schrödinger equation with this Hamiltonian can be solved analytically in the limit of $\xi\ll1$, when the potential is practically reduced to the rectangular potential, and in the limit of $\xi\gg1$, when we have two separated minima symmetrically located around x=0.

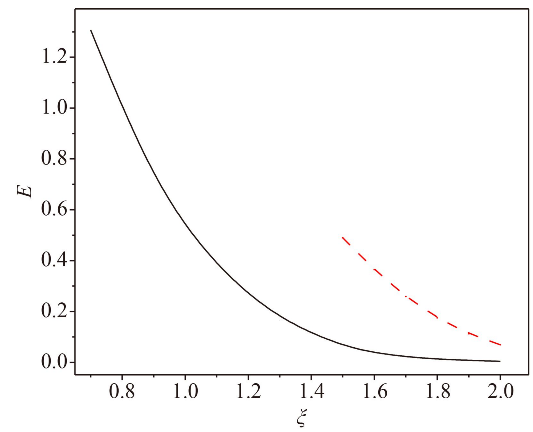

Fig. 1 Energy splitting of the doublet consisting in the first excited and the lowest states ΔE_{01} (solid black line), and the doublet consisting in the third and the second excited states ΔE_{23} (dashed red line) of the chiral Hamiltonian. Energy is given in arbitrary units. Calculations are performed with $x_m = 0.6$ and $c = 2.2$.

3 Results

In the investigation of the evolution of the chiral excitation spectra the very interesting quantity is the difference between the energies of the first excited and the lowest states ΔE_{01}. The value of ΔE_{01} characterizes a splitting of a doublet formed by the states which are antisymmetric and symmetric combinations of the components located in the minima corresponding to the left-handed and right-handed configurations. The parameter ξ is related to the total angular momentum I and increases with I. The results of calculation of ΔE_{01} are presented in Table 1 and Fig. 1.

Since the numerical results are given in order to illustrate an evolution of the chiral mode dynamics from vibrational motion to rotational one rather than to describe experimental data, the values of the x_m and c parameters were fixed relatively arbitrarily.

A similar dependence on ξ demonstrates the energy splitting of the third and second excited states of the Hamiltonian (9) ΔE_{23}. Although in this case an absolute value of the splitting is larger compare to ΔE_{01}, as it is seen in Fig. 1 and Table 1.

The results presented in Fig. 1 show that the energy splitting of the states forming doublet decreases very quickly with angular momentum increase. This splitting

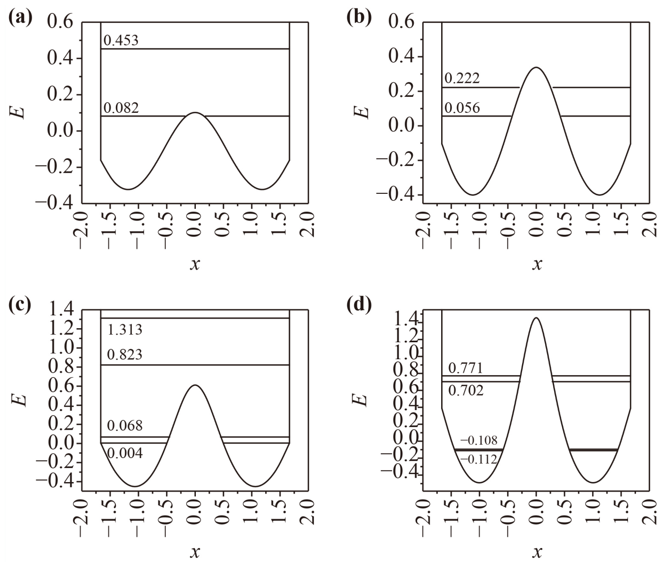

Fig. 2 Collective potential (5) for x_m=0.6 and different values of ξ: **(a)** $\xi = 1.1$, **(b)** $\xi = 1.3$, **(c)** $\xi = 1.5$, **(d)** $\xi = 2.0$. Several low lying eigenstates are indicated by horizontal lines.

can be parameterized by the smooth function of ξ with a very small number of the parameters similar to the situation with description of the parity splitting in the case of the alternating parity bands.

In Fig. 2 is shown the collective potential of the Hamiltonian (9) together with the indicated positions of the ground and excited states for several values of the parameter ξ. It is seen that at relatively low ξ, i.e., at relatively low total angular momentum, even the lowest state is located, practically, at the barrier at $\phi = 0$. However, with increase of ξ the energies of the states decrease and appears below the barrier. For instance, at $\xi = 2.0$ already two doublets are located in energy below the barrier.

In Fig. 3 are shown the wave functions of the four lowest states at $\xi = 1.9$. As it is seen from Fig. 3 the position and the width of the maximum of the wave functions of the ground and the first excited states practically coincide at $x \le 0$ (for $x \ge 0$ this is true for the maximum and the minimum). This means that the wave functions of these states can be presented as a sum and a difference of two Gaussians similar to the wave function given in (4).

In principle, the orientation of the rotational axis in triaxial nuclei should be parameterized by both, the azimuth angle ϕ and the polar angle θ [1]. However, above in this paper and in Ref. [7] only dynamics in ϕ direction was considered. It was shown in Ref. [16] that with increase of the rotational frequency considerable softness is observed also in θ-angle. This indicates an importance of the θ dynamical variable at high rotational frequencies.

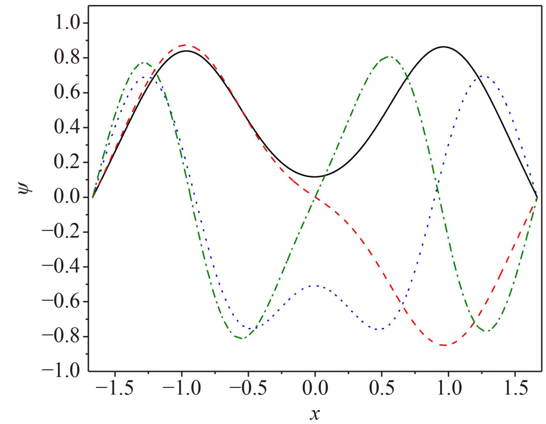

Fig. 3 Wave functions of the four lowest states at x_m = 0.6, c = 1.2 and $\xi = 1.9$: solid line (black) – ground state, dashed line (red) – first excited state, dotted line (blue) – second excited state and dash-dotted line (green) – third excited state.

4 Summary

In summary, a semi-analytical collective model which is able to describe the chiral vibration and rotation is proposed to describe a system of two nucleons (particles or holes) coupled to triaxial rigid rotor. In fact, it includes quantum fluctuations in the chiral mode and restores the chiral symmetry in the laboratory frame. The energies and the wave functions of the collective states corresponding to the motion along the chiral degree of freedom are obtained by diagonalizing the collective Hamiltonian. Although the model is very simple, it indicates a possibility of a simple parameterization of the dependence of the energy splitting of chiral doublets on angular momentum, and a possibility of approximation of the wave functions of the lowest energy states with simple analytical expressions. It is demonstrated that with the increase of the barrier separating left-handed and right-handed configurations, which corresponds to an increase of the rotational frequency, the chiral partner states become more degenerate.

Declarations The authors declare that they have no competing interests and there are no conflicts.

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