Frontiers of Physics
https://doi.org/10.1007/s11467-020-1028-7
Front. Phys.
15(6), 61502 (2020)

VIEW & PERSPECTIVE

# Compact Greenberger–Horne–Zeilinger state generation via frequency combs and graph theory

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Quantum entanglement is one of the most mind-boggling phenomena predicted by quantum physics. It says that two particles can influence each other, even though they are separated over large distances. In disbelieve, Einstein called this prediction *spooky action at a distance* [1]. In the 1960s, Bell has demonstrated that quantum mechanics indeed seems to conflict with our classical worldview, which is local (information can be transmitted maximally with the speed of light) and realistic (properties exist prior to and independent of their measurement) [2]. Over the course of the next 50 years, experimentalists have confirmed the predictions of quantum mechanics and rejected the local-realistic worldview in ever more stringent measurements [3–8].

In the late 1980s, Greenberger, Horne, and Zeilinger (appreviated as GHZ) asked the question what might happen if instead of two, now three particles are involved in the entanglement. Remarkably they found that the rejection of local-realistic theories is not merely a statistical prediction, but can be observed in an all-versus-nothing measurement. This result is now known as the GHZ-theorem and is in some sense the strongest possible rejection of classical theories [9, 10].

The GHZ state, $|\Psi\rangle = 1/\sqrt{2}\ (|0;0;0\rangle + |1;1;1\rangle)$, which encodes this type of strong non-classical correlation, has been observed for the first time in the late 1990 [11, 12]. Interestingly, GHZ states, which were first investigated purely out of curiosity of fundamental questions, have developed as a cornerstone in many applications in quantum information such as quantum computing. Increasing the number of particles involved in a GHZ state is crucial both for foundational studies as well as for practical applications. For that reason, a huge effort has been made by several experimental groups around the world to push the size of GHZ states.

One of the main players in this game is photonic technology. A systematic scheme to produce $2n$-photonic GHZ states in polarisation uses $n$ probabilistic photon pair sources (such as nonlinear crystals). These crystals are combined with a number of polarisation beam splitters to erase the *which-crystal* information. Impressive experimental results have been reported over the years [13–17], culminating in the ability of entangling 12 photons such that they are in a coherent superposition of either all photons being horizontally polarized, or all photons being vertically polarized [18] (imagine how fascinating this really is!).

If one wants to go to larger GHZ states, the experimental hardware increases proportionally. This is actually not surprising, it seems intuitive that one needs a larger number of photon pair sources if one wants to use more photon pairs, as indicated in Fig. 1(a).

This is exactly where the result of Zhu *et al.* comes in Ref. [20]. They proposed a setup to increase the size of GHZ states without increasing the number of optical elements in the experimental setup. The photons are entangled in polarisation, but instead of distinguishing them in their spatial path [21, 22], the photons are distinguished by frequency. The key element in the proposal is a micro-ring resonator (MRR), which allows frequency combs with potentially 100s of sharp lines precisely spaced apart and establish correlations between a large number of frequency bins [19, 23, 24]. In this impressive new technology, perfectly frequency-correlated photon pairs centred around the spectral mode of pump laser are created because of the conservation of energy in the spontaneous four-wave-mixing process inside the micro-ring resonators, as shown in Fig. 1(b).

The authors used this technology and combined it with another new, theoretical concept: A mapping between photonic quantum experiments and graph theory that we (together with Erhard and Zeilinger)

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*Received October 30, 2020. This article can also be found at http://journal.hep.com.cn/fop/EN/10.1007/s11467-020-1028-7.

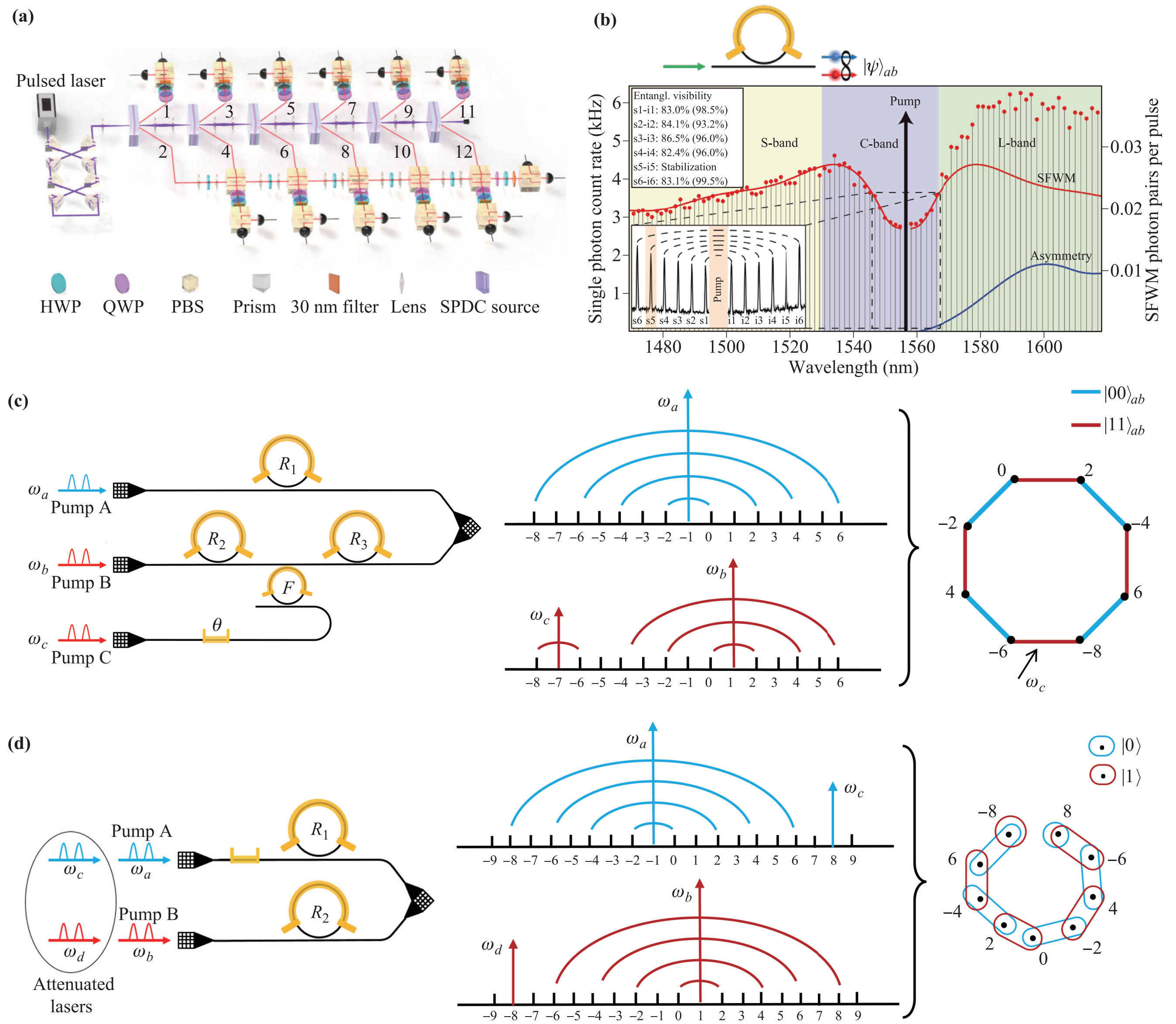

**Fig. 1** Systematic approaches for generating GHZ states. **(a)** Six entanglement sources pumped by laser pulses are combined with several polarisation beam splitters for producing a 12-photon GHZ entangled state, figure adapted with permission from Ref. [18], Copyright © 2018 APS. **(b)** A micro-ring resonator (MRR) can be used to create entanglement shared among several frequency modes from spontaneous four-wave mixing. The frequency differences between resonances, namely the free-spectral-range (FSR), allow certain frequency modes hosted in the frequency comb, figure reprinted with permission from Ref. [19], Copyright © 2016 Science Advance. **(c)** With just three MRRs, one could create GHZ states of arbitrary size with a mapping to graph theory. An 8-photon GHZ state can be described as a circle graph of edges with alternating blue and red color, where blue is $|0\rangle$ and red is $|1\rangle$ ($|0\rangle$ : horizontal polarisation; $|1\rangle$ : vertical polarisation). The first MRR $R_1$ contributes a perfect matching in blue while the MRRs $R_2$ and $R_3$ contribute to a perfect matching in red. The post-selected quantum state is given as the coherent superposition of perfect matchings in the graph, which is an 8-photon GHZ state. **(d)** Generalization of experimental setups to hypergraphs. For example, a combination of weak coherent lasers and MRR sources can be used for creating 9-photon GHZ states. (c, d) reproduced from Ref. [20].

introduced three years ago [25, 26], and that has become a handy tool to design and analyse photonic quantum entanglement and new quantum optical hardware [27–29].

In the mapping, every vertex stands for a distinguishable photonic mode, usually the path of the photon, but in the proposal by Zhu *et al.*, it stands for a discrete frequency mode. The edges between two vertices stand for correlated photon pairs. The color of the edge stands, in this case, for the polarisation of the photons (blue mean horizontal polarisation, i.e., $|0\rangle$, while red stands for vertical polarisation, i.e., $|1\rangle$ ). The quantum state (conditioned on a photon in every frequency mode) can be calculated as the coherent superposition of all *perfect matchings* in the graph. A perfect matching of a graph is a subset of edges such that every vertex is contained exactly once. In this setting, a GHZ is a simple cycle graph with an even number of vertices and edges with alternating colors (of blue and red).

The clever insight of the authors now is the following: They have observed that the graph of a GHZ state of arbitrary size can be created with just three MRRs. The first two create a linear graph of edges with alternating color. The final MRR then connects the two ends of the linear graph. The size of the graph, and therefore the size of the GHZ, now only depends on the number of correlated modes from the first two MRRs, as described in Fig. 1(c). The authors also show, how a combination of weak coherent lasers and probabilistic sources can be used together to create GHZ states based on our generalization of the experimental bridge to hypergraphs [30]. Again, in their proposal, the number of photons in the GHZ state can be increased without increasing the complexity of the experimental setup, as seen in Fig. 1(d).

That being said, the proposal is based on probabilistic photon sources, thus for creating a $2n$ photonic GHZ states, we need $n$ pair-creation events. The scaling to higher number is difficult: If the probability to create one photon pair is $p$, then the probability to create $n$ pairs is $p^n$. The probability $p$ cannot be arbitrarily large, because multi-photonic events could contribute as noise. Low-loss optical elements, highly efficient photon detectors or photon-number resolving detectors could help to increase $n$ to high values.

To sum up, the proposal by Zhu *et al.* is surprising and interesting, and gives a new view on experimental techniques that are inspired using a bridge to graph theory. There are several questions that come to mind that will be interesting for future investigations. First and foremost, are the proposal feasible to create in actual experiments? In particular, can the three MRRs be combined in a coherent way, and what is the influence of non-equal production probability in different frequency bins? Second, so far the authors have demonstrated how GHZ states can be component-efficiently generated. It would be very useful to understand how far their ideas can be stretched towards finding component-efficient implementations of more general graphs. Can they be generalized to high-dimensional quantum systems [31–33]? This is also particularly important as weighted colored graphs have recently been used for the articial intelligent-based design of new quantum experiments [34]. There, much more general experimental solutions are proposed in the form of graphs. Generalizations of the ideas proposed here could therefore have wide practical applications. Certainly, we expect many exciting developments in the near future.

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