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RESEARCH HIGHLIGHT

Log-periodic quantum oscillations in topological or Dirac materials

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As a manifestation of the underlying physical nature, quantum oscillations with the applied magnetic field (B) are one of the most important topics in condensed matter physics. The research history can be tracked to 1930 when Lev Shubnikov and W. J. de Haas observed Shubnikov-de Haas (SdH) oscillations in the magnetoresistance (MR) of bismuth crystals [1]. Since then, researchers have observed quantum oscillations in diverse materials, including metals, metallic compounds, semimetals, semiconductors and even insulators, as well as in artificial mesoscopic microstructures [2–12]. Nowadays, quantum oscillation detected by magnetotransport investigation has been a powerful tool to detect the physical properties in solid-state systems.

Two classes of quantum oscillations have been discovered in solids up to now, and both of them can be demonstrated by the change in MR. The first type is quantum oscillations with a periodicity in the inverse magnetic field ($1/B$). The typical example is the SdH effect, which denotes the resistance of a material in a magnetic field oscillates periodically with $1/B$. As a paradigm of Landau quantization of the energy levels, the SdH effect can be usually observed at low temperatures and high magnetic fields for a clean single-crystalline material [2]. Analyses on the SdH oscillations can obtain important information of a sample, such as the carrier density, the effective mass, the Dingle temperature, the scattering time, etc. In particular, plotting the Landau fan diagram of SdH oscillations has been a commonly used method to identify the nonzero Berry phase of a topological material. Thus, the SdH oscillations in MR have paved way to map the Fermi surface of materials. It is noted that the corresponding oscillations in magnetization is called de Haas–van Alphen effect, which also behaves as a periodic waveform when plotted as a function of $1/B$ [4]. The second type of quantum oscillations display a periodicity when plotted as a function of B. For instance, the Aharonov–Bohm effect arising from the quasiparticle quantum interference can induce MR oscillations periodic in B in mesoscopic systems [5]. The Aharonov–Bohm effect is important because it implies that the electromagnetic vector potential has real physical meaning rather than being a mathematical reformulation. The potential can affect the quantum behavior of a charged particle even when it travels through a region of space in which both electric and magnetic fields are zero. Looking back at the history, it can be found that the discovery of each type of quantum oscillations generally reveals significant physics.

Recently, we have reported a new type of quantum oscillations with the peculiar periodicity in the logarithmic magnetic field ($\log B$) [13]. The research samples are high quality $\mathrm{ZrTe_5}$ single crystals, a three-dimensional layered topological material [14–17]. With the very low carrier densities and the strong anisotropic property, the crystals can reach the quantum limit (QL) in a small magnetic field, i.e., all the carriers enter the lowest Landau level. By performing the high magnetic field measurements, log-periodic oscillations involving up to five oscillating cycles (5 peaks and 5 dips) were observed in the MR of ultra-quantum $\mathrm{ZrTe_5}$ [Figs. 1(a) and (b)]. The remarkable log-periodicity of the magneto-oscillations is clearly demonstrated by the index plot with a semi-log scale [Fig. 1(c)] and the Fast Fourier Transform (FFT) results [Fig. 1(d)]. Besides, the novel phenomenon is confirmed by the magnetotransport results from different samples and different facilities with the maximum magnetic field up to 58 Tesla (Fig. 2).

The discovery presents the third type of quantum oscillations in condensed matter physics, which opens a new chapter in the nearly 90-year history of quantum oscillations. Firstly, the observed $\log B$ periodicity are appreciably different from all previously known quantum oscillations. By plotting B_n vs. n, we find that the oscillations are not periodic in $1/B$ or B. Furthermore, detailed analyses show that the origin of log-periodic oscillations cannot be attributed to the physical framework for conventional quantum oscillations, which are associated with either the Landau level physics or the quantum interference effect.

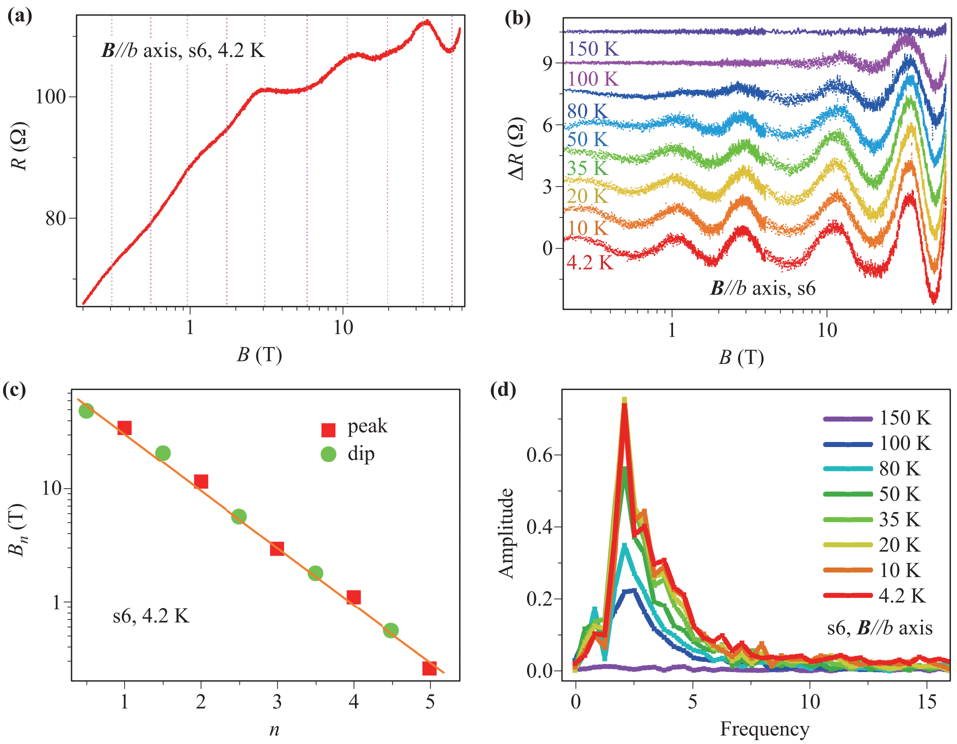

Fig. 1 Log-periodic MR oscillations in ZrTe$_5$. **(a)** MR behavior of sample #6 in an ultrahigh magnetic field up to 58 T at 4.2 K. Dotted lines serve as guides to the eye. **(b)** MR oscillations in s6 after subtracting a smooth background from the raw data. **(c)** Index plot for the log-periodic oscillations. **(d)** FFT results for the MR oscillations at different temperatures in the form of ΔR versus $\log(B/B')$ with $B' = 1$ T. Reproduced from Ref. [13].

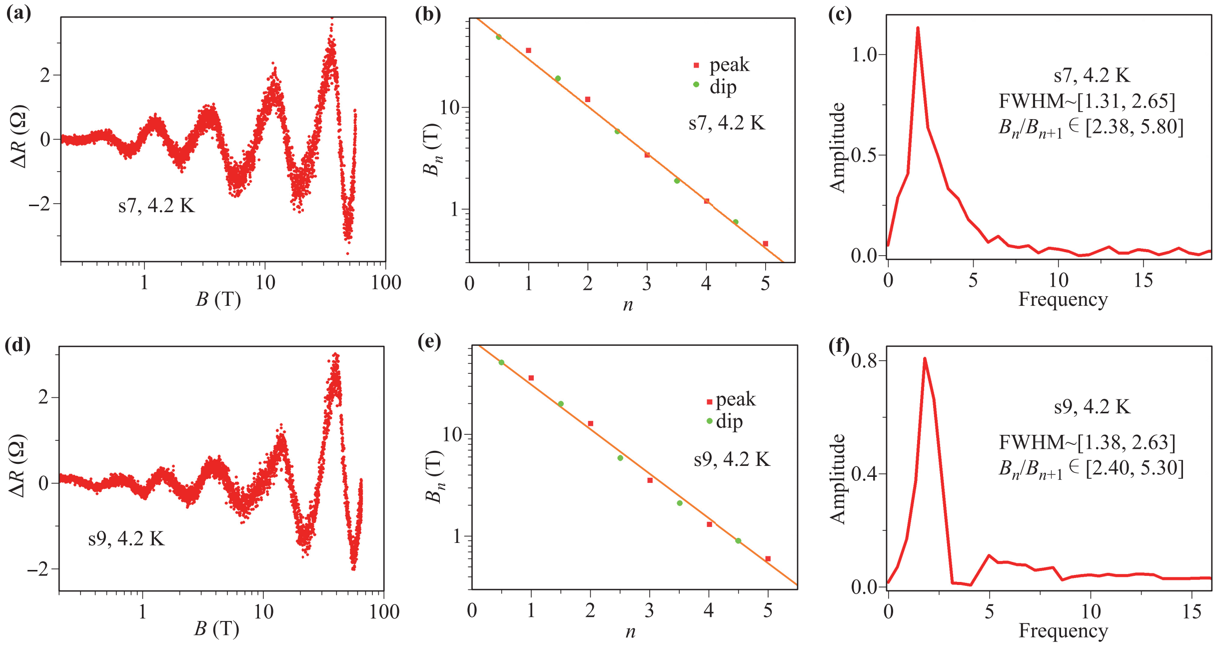

Fig. 2 Reproducible log-periodic oscillations in different ZrTe$_5$ samples. **(a–c)** log-periodic oscillations, index plot, FFT results for sample #7 at 4.2 K. **(d–f)** log-periodic oscillations, index plot, FFT results for sample #9 at 4.2 K. Reproduced from Ref. [13].

In particular, the SdH effect, even considering the superposition of oscillations with different SdH periods of multiple carriers, cannot explain the observed oscillations [13]. Meanwhile, the mechanism of the SdH oscillations with the consideration of Zeeman splitting can also be excluded for the exotic log-periodic oscillations since the results do not satisfy features of the SdH oscillations with spin splitting [13]. This indicates that the log-periodic quantum oscillation is a new member of the quantum oscillation family.

In addition, the log-periodic oscillations do not agree with the behavior of previously known physical mechanisms beyond the QL of a three-dimensional (3D) electronic system, such as the fractional Hall effect (FQHE), Wigner crystal, and density wave transition [18]. Firstly, considering the fractional indexes and the surviving high temperature for the phenomenon, it is impossible to assign the observation of the MR oscillations beyond the QL in the FQHE diagram. We measured the resistance of $\mathrm{ZrTe_5}$ sample versus sweeping bias at 25 mK under a selectively fixed magnetic field and the results do not support the mechanism of a typical FQHE state or a Winger crystal state [13]. The deformation and/or reconstruction of the Fermi surface by wave transition or magnetic breakdown cannot be responsible for the observed oscillations, either. Commonly, the deformation and/or reconstruction of the Fermi surface largely influence the carrier density and a remarkably sharp transition can be expected in the magneto-resistance [19, 20]. However, in our observations, the magneto-resistance does not show any sharp transition. Moreover, such reconstruction generally occurs at only one critical magnetic field strength, while the magneto-resistance of our samples shows five oscillating cycles with a particular oscillatory frequency. Thus, our experimental results are qualitatively different from the field-induced Fermi surface deformation/reconstruction scenario. Very importantly, these known mechanisms do not have the peculiar log periodicity, while it is a remarkable feature of our experimental results. This discovery potentially points to a new quantum state beyond the QL.

The log-periodicity of the new type of quantum oscillations in the MR data is reminiscent of the discrete scale invariance (DSI) behavior, which originates from log-periodic corrections to scaling [21]. A system with (continuous) scale invariance means that it can reproduce itself on different temporal or spatial scales. DSI is a partial breaking of continuous scale invariance, and consequently, observables of the system obey the scale invariance only for a certain geometrical set of characteristic length scales. The DSI featuring intriguing log-periodicity exists in animals, growth processes, financial crisis, earthquakes, turbulence, and so on [21]. For a classical physical system, the DSI exists in the fractal structures induced by non-linear equations [21]. For a quantum system, the prerequisite for the appearance of DSI is harsh [22] and thus the DSI can be hardly observed in experiments except the Efimov trimers in cold atoms [23–27].

The Dirac material provides a promising platform to search for this important DSI phenomenon in quantum physics. In Dirac materials, the quasi-particles (transport carriers) obey the relativistic equation, and the value of effective fine-structure constant $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar v_F}$ is much larger than the value in vacuum. Taking the $\mathrm{ZrTe_5}$ crystal as an example, the light holes obey the massless Dirac equation and their Fermi velocity is 450 km/s, much smaller than the speed of light in vacuum [15, 16]. Thus, the effective fine-structure constant of the system $\alpha > 1$. The relativistic quantum mechanics predicts that when the charge of a superheavy atomic nucleus Z exceeds a certain value satisfying $Z\alpha > 1$ (here α is the fine structure constant and $\alpha \sim 1/137$), the resulting strong Coulomb attraction causes an unusual supercritical collapse condition [28, 29]. Whether there are stable high-atomic-number elements is selected as one of the 125 big questions that face scientific inquiry over the next quarter-century [30]. However, this fundamental problem of nuclear physics has not been directly manifested in experiments. Due to the large value of effective fine-structure constant, the topological $\mathrm{ZrTe_5}$ system can satisfy the supercritical collapse condition which gives rise to the formation of two-body quasi-bound states obeying the DSI feature in both energy spectrum and radius [13]. In the presence of magnetic fields, the energy of quasi-bound states successively approaches the Fermi surface at the corresponding magnetic field value obeying the geometric progression. The elastic scattering between the mobile carriers and the quasi-bound states around the Fermi energy strongly influences the transport properties and gives rise to the log-periodic quantum oscillations in MR. Therefore, the discovery of the new type of quantum oscillations with the logB periodicity could virtually illustrate the intriguing DSI feature in topological materials [13, 31, 32].

On the basis of this finding, we further justify the universality of the log-periodic quantum magneto-oscillations and DSI phenomenon in Dirac or topological materials with Coulomb attraction by magneto-transport results of $\mathrm{HfTe_5}$ crystals [33]. Firstly, the log-periodic quantum magneto-oscillations showing discrete scale invariance (DSI) are observed in the longitudinal magnetoresistance of topological $\mathrm{HfTe_5}$ crystals, independent on the minor differences of the sample quality. Besides, for the first time the DSI feature with log-periodicity is also discovered in the Hall traces of a solid material, which indicates an overall effect of the DSI on the transport properties.

In summary, we discovered a new type of quantum oscillations with the log-periodicity, which are manifestation of DSI in Dirac or topological materials. This is the first magnetotransport display of the distinctive DSI in a condensed matter system, which is an extremely rare phenomenon in quantum physics. The underlying physical nature can be attributed to the two-body quasi-bound states and relates to the longstanding prediction of atomic collapse phenomenon. Thus, the discovery provides a new perspective on the ground state of topological materials and represents a breakthrough beyond Landau level physics.

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