Frontiers of Physics
https://doi.org/10.1007/s11467-018-0861-4
Front. Phys.
14(1), 12601 (2019)

RESEARCH ARTICLE

Photon-phonon squeezing and entanglement in a cavity optomechanical system with a flying atom

Jun-Hao Liu, Yu-Bao Zhang, Ya-Fei Yu, Zhi-Ming Zhang[†]

Guangdong Provincial Key Laboratory of Nanophotonic Functional Materials & Devices (SIPSE), and Guangdong Provincial Key Laboratory of Quantum Engineering & Quantum Materials, South China Normal University, Guangzhou 510006, China
Corresponding author. E-mail: [†]*zhangzhiming@m.scnu.edu.cn*
Received May 6, 2018; accepted August 26, 2018

We study the quadrature squeezing and entanglement in a cavity optomechanical system (COMS). In our model, a flying atom sequentially passes through and interacts with the COMS and a Ramsey pulse zone, and subsequently the atomic state is detected. In this way, the photon-phonon squeezing and entanglement can be generated. The dynamic evolution of the squeezing and entanglement in the presence of losses are examined by using the master equation method.

Keywords optomechanics, squeezing, entanglement

1 Introduction

In recent years, cavity optomechanics [1, 2] has become an exciting research field for exploring quantum effects in an extremely wide range (from microscale to macroscale). In a typical cavity optomechanical system (COMS) a cavity field couples with a movable mirror due to the radiation pressure. This system has many potential applications in quantum science and technology, for example, quantum computation [3], quantum metrology [4], and so on. With the deepening of theoretical and experimental research, rich physical effects have been discovered in COMSs, for example, the observations of the normal-mode splitting [5], Kerr nonlinearity [6], bistability [7], optomechanically induced transparency (OMIT) [8–10], the quantum ground state cooling of the nanomechanical resonators [11, 12], and so on. In addition, various kinds of nonclassical effects such as macroscopic superposition [16], quantum entanglement [17], the squeezing effect [18], and so on, which are generated in COMS [13–15], have also attracted considerable attention.

Quantum entanglement [19] is a typical nonclassical effect. On the one hand, quantum entanglement plays an important role in the study on the fundamental issues of quantum physics, for example, it can help us to reveal the boundary and the transition between classical world and quantum world. On the other hand, quantum entanglement has a lot of potential applications in quantum computation and quantum communication. Many proposals have been brought forward to generate quantum entanglement in various COMSs [20–23]. The quantum squeezing [24] is another typical nonclassical effect and a key resource for many applications, such as gravitational wave detection and ultrahigh precision measurements [25]. Many schemes for creating quantum squeezing have also been proposed in COMSs [26–28]. Experimental squeezing of a nanomechanical object has been achieved through a nonlinear Duffing resonator [29].

Recently, some works studying the entangling and squeezing properties of a hybrid COMS at single-excitation level have been published. Ge and Zubairy put forward a scheme to generate the deterministic entanglement between two movable mirrors using a flying atom in superposition state [30]. They also propose a scheme to generate macroscopic superposition via periodic qubit flipping [31]. Liao et al. proposed a method to generate entanglement between two macroscopic mechanical resonators in a two-cavity optomechancial system [32]. Inspiring by the above works, we want to study the photon-phonon entanglement and squeezing in a hybrid cavity optomechanical system, in which a two-level atom is flying through the cavity. We find that, in the large-detuning case, the photon-phonon entanglement and squeezing will only be generated when we apply a $\pi/2$ Ramsey pulse to the atom and detect the atomic state.

This paper is organized as follows. In Section 2 we introduce the theoretical model and derive the effective Hamiltonian under the condition of large detuning. In Section 3 we calculate the dynamic evolution under ideal conditions and discuss the generation of the entanglement and photon-phonon squeezing in the COMS. In Section 4 we use the master equation method to study the entangling and squeezing properties of the COMS in the presence of losses. Finally, in Section 5 we discuss the experimental feasibility and provide a brief summary.

2 Model and Hamiltonian

Our proposed scheme is shown in Fig. 1. A two-level atom (with the ground state $|g\rangle$ and the excited state $|e\rangle$) sequentially passes through and interacts with the COMS and the Ramsey pulse zone, and then its state is detected [33]. The Hamiltonian of the system reads ($\hbar = 1$)

$$H = \frac{\omega_a}{2}\sigma_z + \omega_c a^\dagger a + \omega_m b^\dagger b \\ -\mathrm{i}G(a\sigma_+ - a^\dagger\sigma_-) - g a^\dagger a(b^\dagger + b), \tag{1}$$

where $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$, $\sigma_+ = |e\rangle\langle g|$, and $\sigma_- = |g\rangle\langle e|$ are the Pauli operators of the atom with frequency ω_a, $a(a^\dagger)$ and $b(b^\dagger)$ are the annihilation (creation) operators of the cavity mode and the mechanical mode with frequency ω_c and ω_m, respectively. G describes the Jaynes-Cummings coupling strength between the atom and the cavity field, and g is the single-photon optomechanical coupling strength between the cavity field and the mechanical mode.

In the interaction picture, we have

$$H_I = -\mathrm{i}G(a\sigma_+ \mathrm{e}^{\mathrm{i}\Delta t} - a^+\sigma_- \mathrm{e}^{-\mathrm{i}\Delta t}) \\ -g a^\dagger a(b^\dagger \mathrm{e}^{\mathrm{i}\omega_m t} + b\mathrm{e}^{-\mathrm{i}\omega_m t}), \tag{2}$$

where $\Delta = \omega_a - \omega_c$ is the frequency detuning between the atom and the cavity field. When $\Delta = \omega_m >> G, g$, the effect of rapidly oscillating terms is neglected, and the effective Hamiltonian can be derived as [34]

$$H_{eff} = \alpha[(a^\dagger a + 1)\sigma_{ee} - a^\dagger a \sigma_{gg}] \\ -\eta(a^\dagger a)^2 + \mathrm{i}\beta(ab\sigma_+ - a^\dagger b^\dagger \sigma_-), \tag{3}$$

where $\alpha = G^2/\Delta$, $\eta = g^2/\omega_m$, $\beta = Gg/\omega_m$, $\sigma_{ee} = |e\rangle\langle e|$, $\sigma_{gg} = |g\rangle\langle g|$. In the expression of Eq. (3), the first term describes the photon-number dependent Stark shifts, and the second is the Kerr nonlinear term, the last terms describe the tripartite interaction (with the atom makes a transition from the excited state to the ground state, a photon and a phonon are created simultaneously; and vice versa), which plays a crucial role in generating the entangled states.

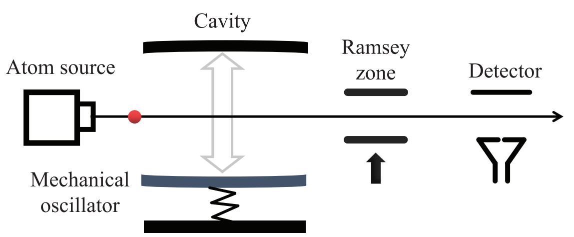

Fig. 1 Schematic diagram of the system. A two-level atom sequentially passes through the COMS and the Ramsey pulse zone, and then its state is detected.

The state of the system can be expressed as $|\Psi\rangle = \sum_{Aij} C_{Aij}|A\rangle \otimes |i\rangle_c \otimes |j\rangle_m \equiv \sum_{Aij} C_{Aij}|Aij\rangle$, where $|A\rangle = |e\rangle(|g\rangle)$ describes the excited (ground) state of the two-level atom, $|i\rangle_c$ and $|j\rangle_m$ represent the Fock states of the cavity field and the mechanical oscillator, respectively. C_{Aij} is the complex probability amplitude corresponding to the states $|Aij\rangle$.

3 Ideal situation

First we discuss the situation in which the environmental effects are not taken into account. As shown in Fig. 2, before the atom enters into the COMS, we prepare the system in the state $|\Psi(t_0)\rangle = |e00\rangle$, i.e., the atom is sent into the COMS in the exited state, the cavity field is in the vacuum state, and the mechanical resonator is cooled to its ground state. When the atom is in the COMS, the governing Hamiltonian is $H_1 = H_{eff}$. The state of the system under the action of the Hamiltonian H_1 will evolve to

$$|\Psi(t)\rangle = M(t)|e00\rangle + N(t)|g11\rangle, \tag{4}$$

where

$$M(t) = \frac{\mathrm{e}^{\mathrm{i}\eta t/2}}{R}\left[R\cos\left(\frac{Rt}{2}\right) - \mathrm{i}(2\alpha+\eta)\sin\left(\frac{Rt}{2}\right)\right], \tag{5}$$

$$N(t) = -\frac{2\beta \mathrm{e}^{\mathrm{i}\eta t/2}}{R}\sin\left(\frac{Rt}{2}\right), \tag{6}$$

and the system will oscillate between $|e00\rangle$ and $|g11\rangle$ with the frequency $R = \sqrt{4\alpha^2 + 4\beta^2 + 4\alpha\eta + \eta^2}$.

At moment t_1, the atom leaves the COMS, and the state of the system is $|\Psi(t_1)\rangle$, which also acts as the initial state of the next evolution process (from t_1 to t_2). Then the governing Hamiltonian becomes $H_2 = -\eta(a^\dagger a)^2$ (H_{eff} with $G = 0$). The state of the system at moment t' ($t_1 \leq t' \leq t_2$) can be written as

$$|\Psi(t')\rangle = M(t_1)|e00\rangle + N(t_1)\mathrm{e}^{\mathrm{i}\eta\tau}|g11\rangle, \tag{7}$$

where $\tau = t' - t_1$. It can be seen that the effect of Hamiltonian H_2 is only a change of the state's phase.

Now we consider following four cases:

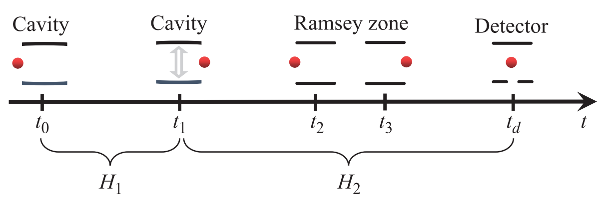

Fig. 2 The temporal sequence of our scheme. H_1 and H_2 are the governing Hamiltonians of the system for each period defined in the text. The insets above time points represent the position of the flying atom.

Case I. A $\pi/2$ Ramsey pulse is applied to the atom *and* the atomic states are detected. At moment t_2, the atom enters the Ramsey pulse zone, we apply a $\pi/2$ Ramsey pulse to the atom, the $\pi/2$ Ramsey pulse transforms the atomic states as $|g\rangle \to (|g\rangle - |e\rangle)/\sqrt{2}$, $|e\rangle \to (|g\rangle + |e\rangle)/\sqrt{2}$, and the atom leaves the Ramsey pulse zone at moment t_3, then the atom keeps on flying and enters the detector at moment t_d. During this period (from t_2 to t_d), the governing Hamiltonian is still H_2, the system will continue to evolve according to the Hamiltonian H_2, Hence the state of the system at moment t_d can be expressed as

$$|\Psi(t_d)\rangle = \frac{1}{\sqrt{2}}(|e\rangle|\varphi_-\rangle + |g\rangle|\varphi_+\rangle), \tag{8}$$

after we detect the atomic states, the system will collapse to

$$|\varphi_\pm\rangle = M(t_1)|00\rangle \pm N(t_1)\mathrm{e}^{\mathrm{i}\eta(t_d-t_1)}|11\rangle, \tag{9}$$

where the $+(-)$ sign corresponds to the detected atomic states $|g\rangle(|e\rangle)$, respectively. We can see that if we apply a $\pi/2$ Ramsey pulse to the atom *and* detect the atomic state we can obtain the entangled state of the photon-phonon subsystem.

Case II. A $\pi/2$ Ramsey pulse is applied to the atom *but* the atomic states are not detected. One can obtain the reduced density operator of the photon-phonon subsystem

$$\rho_{nd} = |M(t_1)|^2|00\rangle\langle 00| + |N(t_1)|^2|11\rangle\langle 11|, \tag{10}$$

this is a mixed state of the photon-phonon subsystem.

Case III. We do not apply the $\pi/2$ Ramsey pulse *but* the atomic states are detected. The states of the photon-phonon subsystem will be $|00\rangle$ (if the atomic state is $|e\rangle$) or $|11\rangle$ (if the atomic state is $|g\rangle$).

Case IV. We do not apply the $\pi/2$ Ramsey pulse *nor* detect the atomic states. the reduced density operator of the photon-phonon subsystem will be $\rho_{nrd} = \rho_{nd}$.

From cases II, III, and IV, we can see that if we do not apply a $\pi/2$ Ramsey pulse to the atom *or* do not detect the atomic state we can not obtain the entangled state of the photon-phonon subsystem.

We can also discuss the entangling properties of the states $|\varphi_\pm\rangle$ and $\rho_{nd}(\rho_{nrd})$ from the point of view of the concurrence [35]. The concurrence can be expressed as $C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$, and the λ_i are the eigenvalues, in decreasing order, of the Hermitian matrix $R = \sqrt{\sqrt{\rho}\widetilde{\rho}\sqrt{\rho}}$, here $\widetilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$, we have

$$C(\rho_{nd}) = 0, \quad C(\varphi_\pm) = 2|M(t_1)N(t_1)|. \tag{11}$$

For the case II and case IV, the density operator $\rho_{nd}(\rho_{nrd})$ only contains the diagonal elements, and there is no interference effect, the degree of entanglement between the photon mode and phonon mode is zero. For the case I, we can obtain a bipartite entangled pure state $|\varphi_\pm\rangle$ between the photon mode and phonon mode, its density operator $|\varphi_\pm\rangle\langle\varphi_\pm|$ contains non-diagonal elements $M(t_1)N^*(t_1)\mathrm{e}^{-\mathrm{i}\eta(t_d-t_1)}|00\rangle\langle 11|$ and $M^*(t_1)N(t_1)\mathrm{e}^{\mathrm{i}\eta(t_d-t_1)}|00\rangle\langle 11|$, which lead to the generation of entanglement.

Then we discuss the squeezing properties of the states $|\varphi_\pm\rangle$ and ρ_{nd}. The two-mode quadrature component operators in the interaction picture are obtained as

$$X_1 = \frac{1}{2^{3/2}}(\widetilde{a} + \widetilde{a}^\dagger + \widetilde{b} + \widetilde{b}^\dagger), \tag{12}$$

$$X_2 = \frac{1}{\mathrm{i}2^{3/2}}(\widetilde{a} - \widetilde{a}^\dagger + \widetilde{b} - \widetilde{b}^\dagger), \tag{13}$$

where $\widetilde{a} = a\mathrm{e}^{-\mathrm{i}\omega_c t}$, $\widetilde{b} = b\mathrm{e}^{-\mathrm{i}\omega_m t}$, and $[X_1, X_2] = \mathrm{i}/2$.

For the state $\rho_{nd}(\rho_{nrd})$, the standard deviations of the photon-phonon quadrature components are $\Delta X_1^{\rho_{nd}} = \Delta X_2^{\rho_{nd}} = \frac{1}{2}\sqrt{1+2|N(t_1)|^2}$, and we have $\Delta X_1^{\rho_{nd}}, \Delta X_2^{\rho_{nd}} \geq 1/2$. This shows that for the case II and case IV, there is no the photon-phonon squeezing.

For the state $|\varphi_-\rangle$ of case I, we have

$$\Delta X_1^{\varphi_-} = \frac{1}{2}\sqrt{1+2|N(t_1)|^2 - 2Re[sV]}, \tag{14}$$

$$\Delta X_2^{\varphi_-} = \frac{1}{2}\sqrt{1+2|N(t_1)|^2 + 2Re[sV]}, \tag{15}$$

where $s = \mathrm{e}^{\mathrm{i}\omega_a(t_d-t_0)}$, which is derived from the effect of the free evolution associated with the free Hamiltonian, and $V = M(t_1)N^*(t_1)\mathrm{e}^{\mathrm{i}\eta(t_d-t_1)}$. In this case, since the existence of the non-diagonal elements ($V \neq 0$), $\Delta X_1^{\varphi_-}$ or $\Delta X_2^{\varphi_-}$ can be smaller than 1/2. In other words, the photon-phonon subsystem can be in a two-mode quadrature squeezed state. Moreover, we have $\Delta X_2^{\varphi_+} = \Delta X_1^{\varphi_-}$, $\Delta X_1^{\varphi_+} = \Delta X_2^{\varphi_-}$.

4 Dissipative situation

Now we consider the dynamic evolution of the system in the presence of losses. We assume that the cavity mode, the mechanical mode, and the atom are coupled to their bathes at zero temperature. Then the time evolution of the density operator $\rho(t)$ of the system can be described by the following quantum master equation

$$\dot{\rho}(t) = -\mathrm{i}[H, \rho] + \gamma_c L[a]\rho + \gamma_a L[\sigma_-]\rho + \gamma_m L[b]\rho, \tag{16}$$

where the Liouville operator $L[o]\rho = o\rho o^\dagger - (o^\dagger o\rho + \rho o^\dagger o)/2$ $(o = a, b, \sigma_-)$. γ_c, γ_a and γ_m are the loss rates of the cavity field, the atom and the mechanical resonator, respectively. In general, the density operator of the system can be expressed as

$$\rho(t) = \sum_{AijA'i'j'} W_{AijA'i'j'}|Aij\rangle\langle A'i'j'|, \tag{17}$$

where $W_{AijA'i'j'} = C_{Aij}C^*_{A'i'j'}$.

When the atom is in the COMS, the governing Hamiltonian is $H_1 = H_{eff}$. From the initial state, the Hamiltonian

H_1, and the master equation (16), we get to know that the five base vectors $\{|e00\rangle, |g11\rangle, |g10\rangle, |g01\rangle, |g00\rangle\}$ constitute a closed subspace, the state of the system, when the atom is in the COMS, can be written as

$$\begin{aligned}\rho_1(t) = & A_1(t)|e00\rangle\langle e00| + B_1(t)|g11\rangle\langle g11| \\ & +C_1(t)|e00\rangle\langle g11| + D_1(t)|g11\rangle\langle e00| \\ & +X_1(t)|g00\rangle\langle g00| + Y_1(t)|g01\rangle\langle g01| \\ & +Z_1(t)|g10\rangle\langle g10|. \end{aligned} \tag{18}$$

By inserting the density operator (18) and the Hamiltonian H_1 into the master equation (16), we can obtain a set of coupled differential equations for the coefficients

$$\dot{A}_1(t) = \beta[D_1(t) + C_1(t)] - \gamma_a A_1(t), \tag{19}$$
$$\dot{B}_1(t) = -\beta[D_1(t) + C_1(t)] - (\gamma_c + \gamma_m)B_1(t), \tag{20}$$
$$\dot{C}_1(t) = \beta[B_1(t) - A_1(t)] - (\mathrm{i}\zeta + \Gamma)C_1(t), \tag{21}$$
$$\dot{D}_1(t) = \beta[B_1(t) - A_1(t)] + (\mathrm{i}\zeta - \Gamma)D_1(t), \tag{22}$$
$$\dot{X}_1(t) = \gamma_a A_1(t) + \gamma_c Z_1(t) + \gamma_m Y_1(t), \tag{23}$$
$$\dot{Y}_1(t) = \gamma_c B_1(t) - \gamma_m Y_1(t), \tag{24}$$
$$\dot{Z}_1(t) = \gamma_m B_1(t) - \gamma_c Z_1(t), \tag{25}$$

where $\Gamma = (\gamma_a + \gamma_c + \gamma_m)/2$, $\zeta = (2G^2 + g^2)/\omega_m$.

The atom leaves the COMS at moment t_1, and the state of the system is $\rho_1(t_1)$, which is the initial state of the next evolution process (from t_1 to t_2). The governing Hamiltonian becomes H_2. The density operator of the system at moment t' ($t_1 \le t' \le t_2$) can be written as

$$\begin{aligned}\rho_2(t') = & A_2(t')|e00\rangle\langle e00| + B_2(t')|g11\rangle\langle g11| \\ & +C_2(t')|e00\rangle\langle g11| + D_2(t')|g11\rangle\langle e00| \\ & +X_2(t')|g00\rangle\langle g00| + Y_2(t')|g01\rangle\langle g01| \\ & +Z_2(t')|g10\rangle\langle g10|. \end{aligned} \tag{26}$$

By inserting the density operator (26) into the master equation (16) with the governing Hamiltonian H_2 and solving the obtained differential equations, we can get the formal solutions for the coefficients

$$A_2(t') = A_1(t_1)\mathrm{e}^{-\gamma_a\tau}, B_2(t') = B_1(t_1)\mathrm{e}^{-(\gamma_c+\gamma_m)\tau}, \tag{27}$$
$$C_2(t') = C_1(t_1)\mathrm{e}^{-(\mathrm{i}\eta+\Gamma)\tau}, D_2(t') = D_1(t_1)\mathrm{e}^{(\mathrm{i}\eta-\Gamma)\tau}, \tag{28}$$
$$\begin{aligned}X_2(t') = & (1 - \mathrm{e}^{-\gamma_m\tau})Y_1(t_1) + Z_1(t_1)(1 - \mathrm{e}^{-\gamma_c\tau}) \\ & +\mathrm{e}^{-(\gamma_c+\gamma_m)\tau}(\mathrm{e}^{\gamma_c\tau} - 1) \\ & \times(\mathrm{e}^{\gamma_m\tau} - 1)B_1(t_1) + X_1(t_1) \\ & +(1 - \mathrm{e}^{-\gamma_a\tau})A_1(t_1), \end{aligned} \tag{29}$$
$$Y_2(t') = \mathrm{e}^{-\gamma_m\tau}[Y_1(t_1) + B_1(t_1)(1 - \mathrm{e}^{-\gamma_c\tau})], \tag{30}$$
$$Z_2(t') = \mathrm{e}^{-\gamma_c\tau}[Z_1(t_1) + B_1(t_1)(1 - \mathrm{e}^{-\gamma_m\tau})]. \tag{31}$$

where $\tau = t' - t_1$. Then the atom enters and leaves the $\pi/2$ Ramsey pulse zone at moment t_2 and moment t_3, respectively. Finally, the atomic states are detected at moment t_d, the system will collapse to

$$\begin{aligned}\rho_D^{\pm}(t_d) = & N[Q_2(t_d)|00\rangle\langle 00| + B_2(t_d)|11\rangle\langle 11| \\ & \pm(C_2(t_d)|00\rangle\langle 11| + D_2(t_d)|11\rangle\langle 00|) \\ & +Y_2(t_d)|01\rangle\langle 01| + Z_2(t_d)|10\rangle\langle 10|], \end{aligned} \tag{32}$$

where $N = 1/[Q_2(t_d) + B_2(t_d) + Y_2(t_d) + Z_2(t_d)]$, $Q_2(t_d) = X_2(t_d) + A_2(t_d)$, and the coefficients $A_2(t_d), B_2(t_d), \cdots, Z_2(t_d)$ equal to $A_2(t'), B_2(t'), \cdots,$ $Z_2(t')$ in Eq. (26) with $t' = t_d$. The $+(-)$ sign corresponds to the detected atomic states $|g\rangle(|e\rangle)$, respectively. The concrete forms of $\rho_{ND}^{\pm}(t_d)$ are not given here since they do not affect the collapsed states.

Now we use the concurrence to measure the degree of entanglement of the state $\rho_D^{\pm}(t_d)$, we have

$$C(\rho_D^{\pm}) = 2N\sqrt{|C_1(t_1)D_1(t_1)\mathrm{e}^{-2\Gamma(t_d-t_1)}|}, \tag{33}$$

the two entangled states $\rho_D^{\pm}$ have the same degree of entanglement. Figure 3 shows the evolution of the concurrence versus the scaled time $T_1 = g(t_1 - t_0)$ and $T_d = g(t_d - t_1)$. When the atom is in the COMS, the non-diagonal elements $C_1(t_1)$ and $D_1(t_1)$ will oscillate with the evolution of the system, and the concurrence will show an oscillatory decay with T_1. When the atom leaves the COMS, the product $C_1(t_1)D_1(t_1)$ is a constant, and the effect of Hamiltonian H_2 is only a change of the phase of the system's state, hence the concurrence will exponentially decay with T_d. Due to the decoherence, the non-diagonal elements will gradually decrease with T_1 and T_d, i.e., the coherence of the photon-phonon subsystem will gradually disappear, and the photon-phonon entanglement will gradually disappear.

Then we calculate the standard deviations of the two-mode quadrature component $X_1(X_2)$ of the state $\rho_D^{\pm}(t_d)$. For the state $\rho_D^{-}(t_d)$, the standard deviations of the photon-phonon quadrature components are

$$\Delta X_1^- = \frac{1}{2}\sqrt{1 + N\{K(t_d) - 2\mathrm{Re}[sC_2(t_d)]\}}, \tag{34}$$
$$\Delta X_2^- = \frac{1}{2}\sqrt{1 + N\{K(t_d) + 2\mathrm{Re}[sC_2(t_d)]\}}, \tag{35}$$

where $K(t_d) = 2B_2(t_d) + Y_2(t_d) + Z_2(t_d)$. We also have $\Delta X_1^+ = \Delta X_2^-, \Delta X_2^+ = \Delta X_1^-$. For simplicity here we only discuss the component ΔX_2^- in the following.

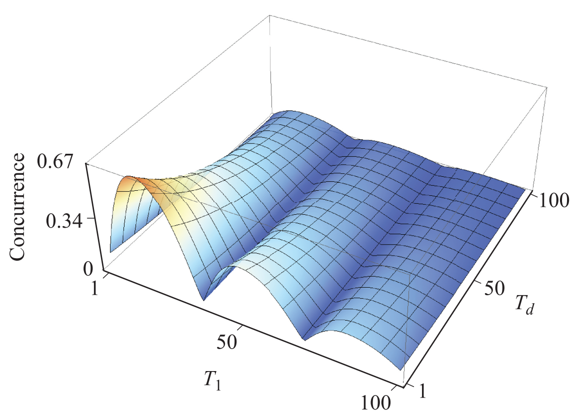

Fig. 3 The concurrence versus the scaled times T_1 and T_d. The parameters are: $\omega_a/\omega_m = 10^2$, $\omega_m/g = 20$, $G/g = 1$, $\gamma_a/g = \gamma_c/g = 2 \times 10^{-2}$, and $\gamma_m/g = 2 \times 10^{-3}$.

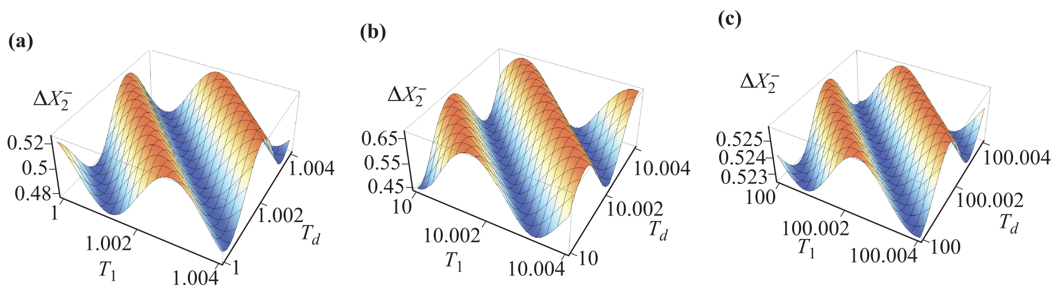

Fig. 4 ΔX_2^- versus the scaled times T_1 and T_d. The parameters are the same as in Fig. 3.

In Fig. 4, we plot ΔX_2^- versus the scaled times T_1 and T_d. Because the non-diagonal elements could generate the squeezing effect, we have also plotted $\mathrm{Re}[sC_2(t_d)]$ versus T_1 and T_d (not shown in this paper) and found that when T_1 and T_d are small, $\mathrm{Re}[sC_2(t_d)]$ is also very small, i.e., the coherence of the photon-phonon subsystem is weak, the amplitude of the photon-phonon squeezing is not obvious. Along with the increasing of T_1 and T_d, $\mathrm{Re}[sC_2(t_d)]$ will gradually increase, the coherence of the photon-phonon subsystem will gradually increase, and the amplitude of the photon-phonon squeezing will gradually increase. However, when T_1 and T_d are large enough, $\mathrm{Re}[sC_2(t_d)]$ will be closer to zero due to the decoherence, so that $K(t_d) \pm 2\mathrm{Re}[sC_2(t_d)] \geq 0$, and the photon-phonon squeezing will disappear.

5 Discussion and conclusion

Next, we make the parameter analysis and discuss the experimental conditions. In our model, the generation time of the photon-phonon entanglement and squeezing is $t = (T_1 + T_d)/g$, and the lifetime of the single-photon (atom) is $1/\gamma_c$ $(1/\gamma_a)$. To generate the entanglement and squeezing, we should guarantee $t < 1/\gamma_c$, $1/\gamma_a$ (γ_m is much smaller than γ_c, hence $1/\gamma_c \ll 1/\gamma_m$). From Fig. 3 and Fig. 4, we find that the photon-phonon entanglement and squeezing can reach the maximum value when $T_1 + T_d \sim 10$. Therefore, we should guarantee $g > \gamma_c$, γ_a, in which $g > \gamma_c$ is the single-photon strong coupling condition in optomechanics. Moreover, we consider the large detuning case, and the condition is $\Delta = \omega_m \gg G, g$. So we should guarantee $\omega_m > \gamma_c$, and this is the resolved sideband condition [36, 37] in optomechanics.

Currently, the resolved sideband regime has been demonstrated in some COMS, so the key challenge is the single photon strong-coupling condition $g > \gamma_c$. In order to achieve this regime, people proposed many schemes to enhance the single-photon optomechanical coupling. These schemes include the use of the cross-Kerr interaction [38], the construction of an array of mechanical resonators [39], mechanical amplification [40], and so on [41].

In summary, we have studied a cavity optomechanical system (COMS), in which a flying atom sequentially passes through and interacts with the COMS and a Ramsey pulse, and then the atomic states are detected. We find that the photon-phonon entanglement and two-mode squeezing can be generated in the COMS only after the atom passes the Ramsey zone and is detected. Using the master equation method, we study the entangling and squeezing properties of the system in the presence of losses. Our study provides the possibility to manipulate cavity optomechanical system with a flying atom and paves the way to achieve the quantum entangled states and squeezed states involving macroscopic objects.

Acknowledgements This work was supported by the National Natural Science Foundation of China (Grant Nos. 11574092, 61775062, 61378012, and 91121023), the National Basic Research Program of China (Grant No. 2013CB921804), and the Innovation Project of Graduate School of South China Normal University (Grant No. 2017LKXM020).

References

1. M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Cavity optomechanics, *Rev. Mod. Phys.* 86(4), 1391 (2014)
2. T. J. Kippenberg and K. J. Vahala, Cavity optomechanics: Back-action at the mesoscale, *Science* 321(5893), 1172 (2008)
3. D. P. DiVincenzo, Quantum computation, *Science* 270(5234), 255 (1995)
4. V. Giovannetti, S. Lloyd, and L. Maccone, Advances in quantum metrology, *Nat. Photonics* 5(4), 222 (2011)
5. J. M. Dobrindt, I. Wilson-Rae, and T. J. Kippenberg, Parametric normal-mode splitting in cavity optomechanics, *Phys. Rev. Lett.* 101(26), 263602 (2008)
6. Z. R. Gong, H. Ian, Y. X. Liu, C. P. Sun, and F. Nori, Effective Hamiltonian approach to the Kerr nonlinearity in an optomechanical system, *Phys. Rev. A* 80(6), 065801 (2009)
7. R. Ghobadi, A. R. Bahrampour, and C. Simon, Quantum optomechanics in the bistable regime, *Phys. Rev. A* 84(3), 033846 (2011)
8. A. H. Safavi-Naeini, T. P. M. Alegre, J. Chan, M. Eichenfield, M. Winger, Q. Lin, J. T. Hill, D. Chang, and O.

Painter, Electromagnetically induced transparency and slow light with optomechanics, *Nature* 472(7341), 69 (2011)

9. Y. C. Liu, B. B. Li, and Y. F. Xiao, Electromagnetically induced transparency in optical microcavities, *Nanophotonics* 6(5), 789 (2017)
10. X. B. Yan, W. Z. Jia, Y. Li, J. H. Wu, X. L. Li, and H. W. Mu, Optomechanically induced amplification and perfect transparency in double-cavity optomechanics, *Front. Phys.* 10(3), 351 (2015)
11. Y. C. Liu, Y. F. Xiao, X. Luan, Q. H. Gong, and C. W. Wong, Coupled cavities for motional ground-state cooling and strong optomechanical coupling, *Phys. Rev. A* 91(3), 033818 (2015)
12. X. Chen, Y. C. Liu, P. Peng, Y. Zhi, and Y. F. Xiao, Cooling of macroscopic mechanical resonators in hybrid atom-optomechanical systems, *Phys. Rev. A* 92(3), 033841 (2015)
13. K. Y. Zhang, L. Zhou, G. J. Dong, and W. P. Zhang, Cavity optomechanics with cold atomic gas, *Front. Phys.* 6(3), 237 (2011)
14. S. Bose, K. Jacobs, and P. L. Knight, Preparation of nonclassical states in cavities with a moving mirror, *Phys. Rev. A* 56(5), 4175 (1997)
15. T. S. Yin, X. Y. Lü, L. L. Zheng, M. Wang, S. Li, and Y. Wu, Nonlinear effects in modulated quantum optomechanics, *Phys. Rev. A* 95(5), 053861 (2017)
16. W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, Towards quantum superpositions of a mirror, *Phys. Rev. Lett.* 91(13), 130401 (2003)
17. D. Vitali, S. Gigan, A. Ferreira, H. R. Böhm, P. Tombesi, A. Guerreiro, V. Vedral, A. Zeilinger, and M. Aspelmeyer, Optomechanical entanglement between a movable mirror and a cavity field, *Phys. Rev. Lett.* 98(3), 030405 (2007)
18. T. P. Purdy, P. L. Yu, R. W. Peterson, N. S. Kampel, and C. A. Regal, Strong optomechanical squeezing of light, *Phys. Rev. X* 3(3), 031012 (2013)
19. R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Quantum entanglement, *Rev. Mod. Phys.* 81(2), 865 (2009)
20. S. Mancini, V. Giovannetti, D. Vitali, and P. Tombesi, Entangling macroscopic oscillators exploiting radiation pressure, *Phys. Rev. Lett.* 88(12), 120401 (2002)
21. X. W. Xu, Y. J. Zhao, and Y. X. Liu, Entangled-state engineering of vibrational modes in a multimembrane optomechanical system, *Phys. Rev. A* 88(2), 022325 (2013)
22. M. Wang, X. Y. Lü, Y. D. Wang, J. Q. You, and Y. Wu, Macroscopic quantum entanglement in modulated optomechanics, *Phys. Rev. A* 94(5), 053807 (2016)
23. X. Y. Lü, G. L. Zhu, L. L. Zheng, and Y. Wu, Entanglement and quantum superposition induced by a single photon, *Phys. Rev. A* 97(3), 033807 (2018)
24. P. D. Drummond and Z. Ficek, Quantum squeezing, Springer Science & Business Media, 2013
25. Y. W. Hu, Y. F. Xiao, Y. C. Liu, and Q. Gong, Optomechanical sensing with on-chip microcavities, *Front. Phys.* 8(5), 475 (2013)
26. A. A. Clerk, F. Marquardt, and K. Jacobs, Back-action evasion and squeezing of a mechanical resonator using a cavity detector, *New J. Phys.* 10(9), 095010 (2008)
27. J. Q. Liao and C. K. Law, Parametric generation of quadrature squeezing of mirrors in cavity optomechanics, *Phys. Rev. A* 83(3), 033820 (2011)
28. X. Y. Lü, J. Q. Liao, L. Tian, and F. Nori, Steady-state mechanical squeezing in an optomechanical system via Duffing nonlinearity, *Phys. Rev. A* 91(1), 013834 (2015)
29. R. Almog, S. Zaitsev, O. Shtempluck, and E. Buks, Noise squeezing in a nanomechanical duffing resonator, *Phys. Rev. Lett.* 98(7), 078103 (2007)
30. W. C. Ge and M. S. Zubairy, Entanglement of two movable mirrors with a single photon superposition state, *Phys. Scr.* 90(7), 074015 (2015)
31. W. C. Ge and M. S. Zubairy, Macroscopic optomechanical superposition via periodic qubit flipping, *Phys. Rev. A* 91(1), 013842 (2015)
32. J. Q. Liao, Q. Q. Wu, and F. Nori, Entangling two macroscopic mechanical mirrors in a two-cavity optomechanical system, *Phys. Rev. A* 89(1), 014302 (2014)
33. J. M. Raimond, M. Brune, and S. Haroche, Manipulating quantum entanglement with atoms and photons in a cavity, *Rev. Mod. Phys.* 73(3), 565 (2001)
34. D. F. V. James and J. Jerke, Effective Hamiltonian theory and its applications in quantum information, *Can. J. Phys.* 85(6), 625 (2007)
35. W. K. Wootters, Entanglement of formation of an arbitrary state of two qubits, *Phys. Rev. Lett.* 80(10), 2245 (1998)
36. A. Schliesser, R. Rivière, G. Anetsberger, O. Arcizet, and T. J. Kippenberg, Resolved-sideband cooling of a micromechanical oscillator, *Nat. Phys.* 4(5), 415 (2008)
37. A. Schliesser, O. Arcizet, R. Rivière, G. Anetsberger, and T. J. Kippenberg, Resolved-sideband cooling and position measurement of a micromechanical oscillator close to the Heisenberg uncertainty limit, *Nat. Phys.* 5(7), 509 (2009)
38. J. Q. Liao, J. F. Huang, L. Tian, L. M. Kuang, and C. P. Sun, Generalized ultrastrong optomechanics, arXiv: 1802.09254
39. A. Xuereb, C. Genes, and A. Dantan, Strong coupling and long-range collective interactions in optomechanical arrays, *Phys. Rev. Lett.* 109(22), 223601 (2012)
40. M. A. Lemonde, N. Didier, and A. A. Clerk, Enhanced nonlinear interactions in quantum optomechanics via mechanical amplification, *Nat. Commun.* 7, 11338 (2016)
41. P. B. Li, H. R. Li, and F. L. Li, Enhanced electromechanical coupling of a nanomechanical resonator to coupled superconducting cavities, *Sci. Rep.* 6(1), 19065 (2016)