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REVIEW ARTICLE

Alpha-clustering effects in heavy nuclei

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The study of cluster structures in light nuclei is extending to the heavy nuclei in these years. As for the stable $N = Z$ nuclei, from the lighter ^{8}Be, ^{12}C nuclei to the heavier ^{20}Ne and even the ^{40}Ca and ^{44}Ti medium nuclei, the α cluster structures have been well studied and confirmed. In heavy nuclei, due to the dominated mean field, the existence of α cluster structure is not clear as light nuclei but some clues were found for indicating the core+α cluster structure in some nuclei, in particular, the ^{208}Pb+α structure in ^{212}Po. We review some recent progress about the theoretical and experimental explorations of the α-clustering effects in heavy nuclei. We also discuss the possible α cluster structure of heavy nuclei from the view of α decay.

Keywords α cluster structure, nuclear cluster model, α correlations, α decay

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Contents

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1 Introduction

Clustering [1–4] is one essential dynamical feature of nuclear many-body systems as well as the mean field. As early as 1937, Wefelmeier proposed [5] that the α particle could be considered as one building block for constituting some $N\alpha$ nuclei. From this perspective, the study of cluster structure in nuclei is even earlier than the nuclear shell-model study. In 1968, the Ikeda diagram [6] was proposed and various cluster structures in light nuclei were predicted to appear around the cluster threshold energies. Subsequently, many light nuclei have been studied by using the developed microscopical cluster models [7] like resonating group method (RGM), generator coordinate method (GCM) or semi-microscopical cluster models like orthogonality condition model (OCM) for half a century. Many important observables like the α-decay widths, electric transition strengths were reproduced well in cluster models while they were not easy to be treated in the traditional shell model in some cases [8]. The cluster structure in light nuclei is highlighted by the famous Hoyle state [9] at 7.65 MeV in ^{12}C, which attract great interests due to its importance for accounting for the stellar abundance of carbon as well as its novel cluster character. Now, it is well described by the so-called THSR wave function [10, 11] and it is considered to be the α

condensate state [12, 13], in which the 3α particles are almost making the $(0S)$ relative motion.

In the past few years, many new nuclear models have been developed for studying the cluster structure in nuclei, such as Antisymmetrized Molecular Dynamics (AMD) [14, 15], Fermionic Molecular Dynamics (FMD) [16, 17], the THSR wave function [10, 13] and so on. These microscopical models provide us with the powerful tools for investigating the cluster structure in light and medium nuclei. The microscopical studies of unstable nuclei, in particular the neutron-rich nuclei have updated our knowledge of clustering. It was found that there are very rich molecular cluster structures in the isotopes of Li, Be, B, and C [18]. Moreover, some excited resonance states above the threshold energy were also found to be the cluster states, e.g., the high-lying excited 0^+ states [19–21] in ^{12}C and ^{16}O. Basically, the clustering structure in light nuclei have been well studied and established.

Clustering, as a basic and fundamental nuclear dynamics, is becoming a key concept for understanding some novel molecular structures in light nuclei. One natural and important question is, whether there exist cluster structure or α clustering correlations in heavy nuclei? As we know, with the increase of the mass number, the shell-model is becoming more and more important. The interesting point is, whether the alpha cluster structure can survive in the mean field and clustering competition from the medium nuclei to the heavy nuclei. Now, both from experimental and theoretical sides, it was found that [22, 23] there exist the inversion doublet bands in the fp-shell ^{40}Ca and ^{44}Ti. Therefore, the ^{40}Ca and ^{44}Ti nuclei could have similar α+core cluster structure analogous to the well-known structure of ^{16}O and ^{20}Ne. These studies of the medium nuclei provide us with certain support for searching for the clustering structure in heavy nuclei.

Considering the great success of the shell model in heavy nuclei [24], traditionally, it is believed that the mean field dominates in the heavy nuclei and it seems that there is no room for clustering. However, just due to the shell-model structure of nuclei, in some cases, analogy to the confirmed alpha cluster structure in medium nuclei ^{40}Ca and ^{44}Ti, the heavy closed-shell core plus the α can also be one possible cluster structure, e.g., the ^{208}Pb+α structure in ^{212}Po [25]. This ^{208}Pb+α structure has been studied and predicted for many years from various nuclear theoretical models. As early as 1978, Tonozuka and Arima [26] studied the enhancement of the α-decay widths of ^{212}Po and found the higher configuration mixing plays a very important role in explaining this width enhancement. Recently, some observed non-natural states [27] provide a direct support for the ^{208}Pb+α clustering in ^{212}Po. Also, it is known that the spontaneous α decay and the α transfer reactions [25] can be viewed as some evidences of clustering in ground states of medium and heavy nuclei. To calculate the alpha decay [25, 28], the α particles are usually assumed to have a chance to exist inside the nucleus in heavy nuclear systems. Thus, the preformation probability of the α cluster in heavy nuclei is much related with the clustering structure. Of course, the calculation of the preformation probability is very difficult to be handled for present nuclear microscopical theories and it is still an open problem.

The early studies for the clustering effect in heavy nuclei are mainly from some phenomenological ways, e.g., the study of the α-clustering correlations from binding energies. In recent years, some attempts [29–31] have been made to study the cluster structure in heavy nuclei by the microscopical cluster models, although it still needs much numerical effort due to the antisymmetric effects from the larger number of nucleons. Some progress has been made recently in the understanding of α-clustering in heavy nuclei.

This article is organized as follows. In Section 2, we start from the inversion doublet bands of ^{20}Ne and then introduce the study of the clustering structure in the medium ^{40}Ca and ^{44}Ti nuclei. Some evidences for the α correlation was discussed in Section 3. Next, as a very typical example, the theoretical and experimental study for the ^{208}Pb+α cluster structure of ^{212}Po was introduced in Section 4. In Section 5, some microscopical study for the α cluster structure was introduced. In Section 6, the clustering structure of heavy nuclei was discussed in the view of the α decay, especially the preformation probability of the α cluster. The summary is given in Section 7.

2 Cluster structures in light and medium nuclei

2.1 The inversion doublet bands in ^{20}Ne

In light nuclei, the cluster structure originating from many-body correlations have been investigated for many years [32–35] and lots of developed cluster states around the threshold energies were found.

The ^{20}Ne can be considered as a transitional nucleus, beyond which (the mass number is greater than 20) the mean field is dominating and the clustering structures do not prevail as much as that of light nuclei. The ^{20}Ne has been studied extensively [14, 36–38] for its rotational bands with transitional character between the α+^{16}O cluster structure and the shell model structure. At the very early state, the cluster structures of nuclei even were not well supported, Horiuchi and Ikeda proposed that [39] the K^π=0$^+$ ground-state band and K^π=0$^-$ band upon 1$^-$ state at 5.78 MeV could be regarded as being the inversion doublet bands with the parity-violating structure of the asymmetric α+^{16}O cluster structure. They analysed further that the existence of inversion doublet bands can be considered as a necessary condition of the asymmetrical cluster structure of nuclei. Arima and Yoshida also found that [40] the alpha widths of 6$^+$ and 8$^+$ states of the ^{20}Ne could not be reproduced in the shell model calculations even the tail of the relative ^{16}O and α cluster wave function was improved. This indicates that certain higher-excited configurations should be included and it is known that this was related with the alpha correlation. After that, the α+^{16}O cluster structure of ^{20}Ne was investigated on the basis of the one-channel RGM [36]. The scattering phase shifts, energy spectra, and the α decay widths of the inversion doublet bands were well reproduced.

Quite recently, the THSR-type wave function [41] was applied to study the inversion doublet bands of ^{20}Ne,

$$\Psi_{\rm Ne}(\beta, \boldsymbol{S}) \propto \mathcal{A}\left[\exp\left(-\frac{8(\boldsymbol{r}-\boldsymbol{S})^2}{5B^2}\right)\phi(\alpha)\phi(^{16}{\rm O})\right]. \quad (1)$$

Here $B^2 = b^2+2\beta^2$, $\boldsymbol{r} = \boldsymbol{X}_1-\boldsymbol{X}_2$. $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ represent the center-of-mass coordinates of the α and ^{16}O clusters, respectively. The calculations here were performed with restriction to axially symmetric deformation, that is, $\boldsymbol{S} \equiv (0, 0, S_z)$. The spin and parity eigenfunctions can be obtained by angular momentum and parity projections techniques [42, 43]. After variational calculations, it was found that the spin and parity projected Hybrid-THSR-Brink wave function reduces to a pure spin and parity projected THSR wave function, i.e. the one with $S_z = 0$, which only serves in an intermediate step to describe negative-parity states.

Figure 1 shows the obtained energy curve of ^{20}Ne with the size parameter β. It can be seen that, a deeper pocket was obtained, which corresponds to the ground state of ^{20}Ne. Usually the cluster states appear around the threshold energy, in which the Coulomb barrier plays an important role for confining the developed α+^{16}O cluster states. For some heavier nuclei, the height of the Coulomb barrier is becoming lower and lower. In this case, the cluster states cannot be formed or be trapped by the Coulomb potential barrier. Therefore, one way to study the possibility of the cluster structure in heavy nuclei is to check the height of the Coulomb barrier, which will be discussed in Fig. 9.

Figure 2 shows that, by using the constructed microscopical cluster model, namely the THSR wave function, the inversion doublet bands of ^{20}Ne were reproduced well without using any adjust parameters. Actually, based on the successful description of the typical rotational bands, the concept of nonlocalzied clustering was proposed [41], which was also applied to many other cluster systems [45–48]. Based on the spirit of the THSR wave function, to study the core+α cluster structure in heavy nuclei, one new proposed new trial wave function [49] can be applied, in which the values of widths of α and the core parts can be different.

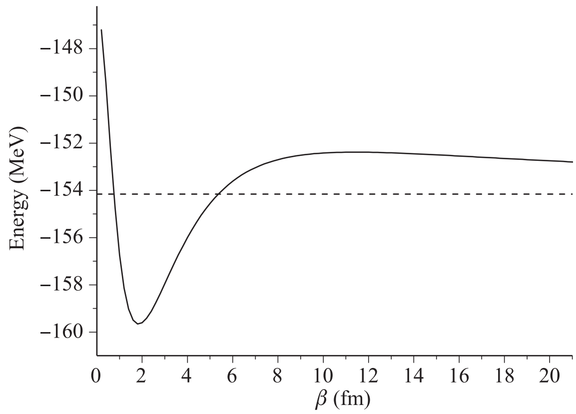

Fig. 1 The energy curve of ^{20}Ne as a function of the size parameter β within the THSR wave function. The dash line represents the corresponding α+^{16}O threshold energy. The same interactions in Ref. [41] are used here.

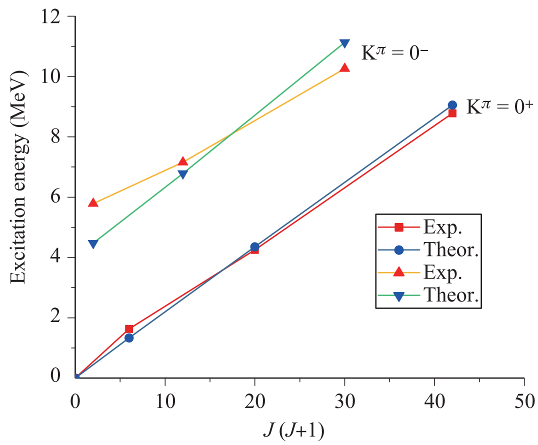

Fig. 2 The energy levels of the inversion doublet bands in ^{20}Ne reproduced by the THSR wave function [44] compared with the experimental levels.

2.2 Cluster structures of ^{40}Ca (^{36}Ar+α) and ^{44}Ti (^{40}Ca+α)

As for the heavier nuclei in the medium mass region, the formation of clustering structure cannot dominate in the competition of the shell model structure due to the strong effect of the spin-orbit coupling interaction. Therefore, the identification of clustering structure both from theoretical and experimental studies is becoming more difficult than the light nuclei like ^{12}C, ^{16}O, and ^{20}Ne. In this section, we will introduce some investigations for the ^{40}Ca(^{36}Ar+α) and ^{44}Ti(^{40}Ca+α) nuclei, which have been predicted to have α+core cluster structure for a long time.

As the heaviest $N = Z$ stable nucleus, the ^{40}Ca is a good starting nucleus to study the structures of medium and heavy nuclei. It is known that [23] the first K^π=0$^+$ band built on $J^\pi = 0_2^+$ state consists of normal-deformed states and they are considered to have dominant 4p-4h configuration. The K^π=2$^+$ band built on $J^\pi = 2_2^+$ state has also been considered to own the normal-deformed states. These normal-deformed states were expected to be some cluster candidate states. By assuming the α+^{36}Ar cluster structure and performing the OCM calculations, Ohkubo *et al.* proposed that [50] the first K^π=0$^+$ band has an α+^{36}Ar structure and they also predicted the corresponding parity-doublet K^π=0$^-$ band. By using the α+^{36}Ar OCM, the K^π=2$^+$ band [23] as well as the K^π=0$^+$ and 0$^-$ band was also obtained. In AMD calculations [51], the density distribution of the intrinsic states of the norm deformed band was found no clear sign of the α+^{36}Ar clustering. However, it was found that the norm-deformed band wave functions have non-negligible component of the overlap with the α+^{36}Ar cluster wave functions.

By studying the α clustering of the parity doublet bands in ^{40}Ca via the (^{6}Li,d) reaction at E=50 MeV, Yamaya *et al.* [22, 52] observed about seventy levels and most importantly, the negative-parity states from the 1$^-$ state at 8.15 MeV up to the 7$^-$ state at 12.65 MeV were identified as possible members of the K^π=0$^-$ band of the parity doublet bands in ^{40}Ca, which can be seen in Fig. 3. This gave a support for α+^{36}Ar clustering structure in ^{40}Ca.

The ^{44}Ti (α+^{40}Ca) is another typical pf shell nucleus expected to have developed cluster structure, which has been studied by many theoretical cluster models. Actually, in the early α+^{40}Ca RGM study [53] of the ^{44}Ti, the lowest 0$^+$ state has been obtained using the reasonable effective nuclear interaction, which can reproduce the elastic scattering cross sections in a wide range. Within a local potential model description, which describes elastic ^{40}Ca(α, α) scattering on broad angular and energy ranges, Michel *et al.* found that [54] a negative parity band starting just above the threshold and composed of narrow states with intermediate cluster character. Now, AMD calculations [55] show that the α+^{40}Ca cluster component in the ground band is considerably dissolved due to the formation of the deformed mean-field and the spin–orbit interaction. Since the ground state still contains non-small amount of the α+^{40}Ca component by about 40%, the nucleus can be excited from the degree of the freedom of the clustering, which can lead to the existence of the K^π=0$^\pm$ bands [55]. Figure 4 shows the density distribution of the intrinsic AMD wave function of the J^π=1$^-$ state of K^π=0$^-$ band [55]. The ^{40}Ca+α clustering structure can be clearly seen. On the other hand, it should be noted that the AMD calculations also show that [55] the mean-field structure and the cluster structure are strongly mixed in many states and the coexistence and interference of α-cluster states and shell model states are important for understanding these features of nuclei.

From the α transfer experiment on ^{40}Ca, the negative-parity rotational band of ^{44}Ti having the ^{40}Ca+α cluster structure was confirmed [22]. The $J^\pi = 1^-$ state around 7 MeV is the band head and these negative-parity states were also found to have larger α decay widths. Therefore, the parity-doublet bands in ^{40}Ca and ^{44}Ti have been successfully established and these findings give a strong support for the existence of the clustering structure in ^{44}Ti and ^{40}Ca. Moreover, the confirmation of cluster structures in ^{44}Ti and ^{40}Ca also gives us the good test to find cluster structures for nuclei heavier than $A = 40$.

In the medium mass region, besides the possible α+core structures, the di-nuclear configurations were another kind of interesting cluster structure, which were

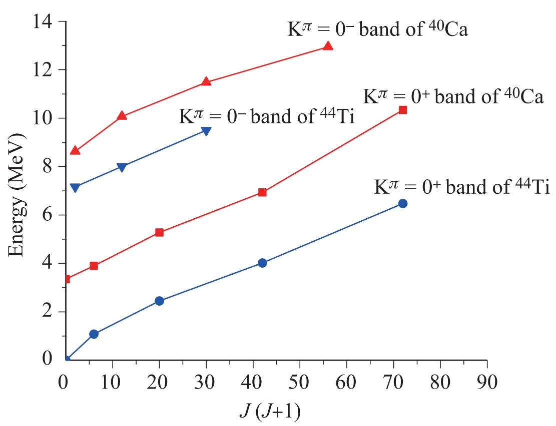

Fig. 3 The experimental observed parity doublet bands in ^{40}Ca and ^{44}Ti [22]. Note that, for the negative-parity states of ^{40}Ca and ^{44}Ti, they are the energy centroids of fragmented levels of each spin.

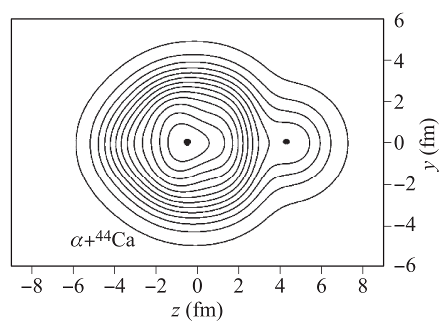

Fig. 4 The density distribution of the intrinsic AMD wave function of the J^π=1$^-$ state of K^π=0$^-$ band in ^{44}Ti. Reproduced from Ref. [55].

supported from the narrow-width resonances observed in heavy-ion scattering experiments [56]. Uegaki [57–59] proposed a molecular model for describing the dinuclear system in the rotating molecular frame. Some stable pole-pole clustering configuration (^{24}Mg+^{24}Mg) and equator-equator configuration (^{28}Si+^{28}Si) were confirmed.

3 The α-clustering correlations in heavy nuclei

3.1 The α-clustering correlations from binding energies

With the development of theoretical models and advancement of experimental methods, the search for clustering structures in medium and heavy nuclei is becoming a very important and interesting topic. However, there are only few theoretical clustering studies for heavy nuclei. Firstly, the cluster structure of heavy nuclei are not well supported from the experiments except for some special cases like the core+α we will discuss later. The knowledge of the clustering effects in heavy nuclei are still very limited. Secondly, the calculations by the traditional microscopical cluster models [60] like RGM, GCM, even the OCM are becoming very complicated due to the antisymmetried effects. In such situations, the phenomenological cluster models or the semiclassical methods are usually acceptance as an alternative.

As we know, the α particle has a very stable and well-saturated character. One way for studying the α-clustering effects in heavy nuclei is from the view of separation energies of proton and neutron pairs. This simple idea is [61], suppose the interaction of correlated proton pairs and correlated neutron pairs can lead to the α clustering, then the quasiparticle energy cannot be simplify additive but can be expressed approximately in the terms of the separation energies of neutron or proton.

Considering the pairing correlations and the corresponding superfluidity are mainly connected with the seniority number s and the energy gap Δ, then $\mathrm{d}E/\mathrm{d}s \approx \Delta$ and the seniority number for the ground state of medium and heavy nuclei is [61]

$$s = \begin{cases} 0, & \text{even-even nuclei,} \\ 1, & \text{even-odd nuclei,} \\ 2, & \text{odd-odd nuclei.} \end{cases} \tag{2}$$

As for the even-even nuclei, the remove of one nucleon requires additional energy due to the α clustering effect. Thus, the binding energy difference can be constructed [61] in terms of the α separation energy $\delta(\alpha)$,

$$\delta B = \begin{cases} \Delta + \delta(\alpha), & \text{even-even nuclei,} \\ 0, & \text{even-odd nuclei,} \\ -\Delta, & \text{odd-odd nuclei.} \end{cases} \tag{3}$$

Table 1 Empirical extracted values of $\delta(a)$ from neutron and proton separation energies for nuclei in the specific region $83 \le Z \le 92$ and $127 \le N \le 142$ [61]. Units are MeV.

N \ Z	83	84	85	86	87	88	89	90	91	92
127	0.36	0.46	0.236	0.299	0.13	0.18	0.074	0.113	–	−0.2
	0.361	0.274	0.215	0.153	0.106	0.082	0.061	0.056	–	–
128	0.3	0.42	0.247	0.307	0.19	0.23	0.143	0.19	−0.03	–
	0.46	0.42	0.321	0.28	0.21	0.17	0.126	0.060	−0.2	–
129	–	0.34	0.229	0.326	0.20	0.25	0.211	0.25	0.02	–
	0.2	0.25	0.234	0.22	0.21	0.20	0.21	0.10	–	–
130	0.1	0.30	0.179	0.295	0.285	0.285	0.262	0.31	0.2	–
	0.3	0.32	0.321	0.306	0.242	0.267	0.25	0.17	–	–
131	0.089	0.212	0.116	0.235	0.329	0.340	0.227	0.297	–	–
	0.116	0.158	0.154	0.275	0.329	0.280	0.324	0.3	–	–
132	0.2	0.208	0.114	0.245	0.37	0.25	0.243	0.30	0.3	0.2
	0.185	0.170	0.197	0.290	0.340	0.287	0.200	0.2	–	–
133	–	0.2	0.127	0.213	0.38	0.13	0.220	0.34	0.25	0.316
	0.2	0.153	0.162	0.33	0.29	0.21	0.34	0.36	0.37	0.4
134	–	–	0.1	0.21	0.40	0.14	0.18	0.25	0.24	0.26
	–	0.2	0.178	0.26	0.23	0.14	0.22	0.23	0.19	0.3
135	–	–	–	0.3	0.461	0.142	0.16	0.21	0.27	0.30
	–	–	0.2	0.35	0.315	0.17	0.21	0.26	0.31	0.2
136	–	–	–	–	0.3	0.168	0.15	0.23	0.21	0.24
	–	–	–	0.4	0.288	0.13	0.16	0.21	0.25	0.2
137	–	–	–	–	–	0.19	0.180	0.256	0.262	0.232
	–	–	–	–	0.2	0.194	0.216	0.227	0.205	0.211
138	–	–	–	–	–	0.22	0.165	0.172	0.273	0.243
	–	–	–	–	–	0.17	0.220	0.291	0.289	0.277
139	–	–	–	–	–	0.23	0.280	0.17	0.26	0.18
	–	–	–	–	–	0.21	0.117	0.154	0.227	0.2
140	–	–	–	–	–	0.3	0.331	0.24	0.20	0.12
	–	–	–	–	–	0.307	0.34	0.28	0.21	0.2
141	–	–	–	–	–	–	0.361	0.19	0.17	0.05
	–	–	–	–	–	0.3	0.23	0.17	0.11	0.04
142	–	–	–	–	–	–	0.4	0.27	0.172	0.119
	–	–	–	–	–	0.4	0.317	0.195	0.113	0.09

It can be easily seen that the $\delta(\alpha) > 0$ indicates the possible existence of α correlations from the extracted energy information. Of course, the extracted $\delta(\alpha)$ quantity should be carefully treated because the binding energy consists of both contributions from the short-range and long-range correlations of nucleons. If the long-range correlations were neglected, $\delta(\alpha)$ can be expressed as [61]

$$\delta(\alpha) \approx (-1)^{Z+N+1}\frac{1}{2} \times [S_n(Z-1,N)-2S_n(Z,N)+S_n(Z+1,N)] \tag{4}$$

$$\approx (-1)^{Z+N+1}\frac{1}{2} \times [S_p(Z,N-1)-2S_p(Z,N)+S_p(Z,N+1)]. \tag{5}$$

Here S_n and S_p are the neutron and proton separation energy, respectively.

Based on the above equations, the $\delta(\alpha)$ can be obtained from the separation of proton and neutron energies, which are shown in Table 1. It can be seen that the extracted $\delta(\alpha)$ in two ways are quite comparable and most $\delta(\alpha)$ have non-negligible positive values. Roughly speaking, these calculated $\delta(\alpha)$ can be considered as a clue for the existence of α correlation in heavy nuclei. In particular, it should be noted that the obtained $\delta(\alpha)$ for the ^{212}Po from the separation of the neutron and proton are exactly have the same value 0.42 and it is also nearly the largest $\delta(\alpha)$ value in Table 1. This result shows the ^{212}Po nucleus has very large α correlations, which is consistent with other theoretical predictions.

On the other hand, to study the mechanism of α-clustering in the interior of heavy nuclei, Koh [30] proposed a simple model for studying the correlated proton pair and neutron pair inside of the heavy nucleus. This model clarifies an interrelation between the paring and the alpha correlation.

The α-like four-nucleon correlations were also investigated from the view of the mean-field method. Hasegawa made a survey [62] of the nuclear binding energies connected with mass formula, which indicates a leading role of the α-like four-nucleon clusters in the single particle mean field. Girod *et al.* [63] studied a number of self-conjugate nuclei for the formation of the α particles with the constrained Hartree–Fock–Bogoliubov approach. They found that beyond a certain low density, those self-conjugate nuclei spontaneously cluster into A/4 cluster particles.

3.2 α-clustering explanations for shell and blocking effects in alpha-transfer reaction

As we mentioned in Section 2, the alpha-transfer reaction is a very important way for studying α clustering in nuclei. It was found that, usually the spectroscopic factors in the alpha-transfer reaction is much smaller for the closed-shell nuclei than that of the mid-shell nuclei, which is called the shell and blocking effects [64, 65] in the alpha-transfer reaction. Similar to the alpha decay, a natural way for explaining the blocking effects is to assume the existence of α clustering in ground states of medium and heavy nuclei. Next, it can be seen that, if it is assumed the α clustering in the ground states of medium and heavy nuclei, by using a simple two-level pairing model, the shell and blocking effects in alpha-transfer reaction can be reasonably explained [66].

The Hamiltonian for the two-level pairing force model is [66]

$$H = H_0(+) + H_0(-) + H_1(+,-), \quad (6)$$

and

$$H_0(\pm) = \pm\epsilon N_F(\pm) - \lambda_0 \sum_{\nu=0,\pm1} B_\nu^\dagger(\pm)B_\nu(\pm), \quad (7)$$

$$H_1(+,-) = -\lambda_1 \sum_{\nu=0,\pm1} B_\nu^\dagger(+)B_\nu(-) + h.c. \quad (8)$$

Here the $\pm\epsilon$ are the single-particle energies of upper and lower levels; The λ_0 and λ_1 represent the intensities of pairing forces with respect to the same level and different levels, respectively,

$$N_F(\pm) = \sum_{m,m_t} a_{m,m_t}^\dagger(\pm)a_{m,m_t}(\pm), \quad (9)$$

$$B_\nu^\dagger(\pm) = \frac{1}{\sqrt{2}}[a^\dagger(\pm)a^\dagger(\pm)]_{M=0,M_T=\nu}^{J=0,T=1}. \quad (10)$$

In the general case, $2\epsilon > \lambda_0, \lambda_1$, $H_1(+,-)$ can be considered as the perturbation. In some sense, this model is too simple, nevertheless the obtained analytical solutions can be very useful for the analysis of the α clustering mechanism in the shell and blocking effects in α-transfer reactions. It should be noted that, in the zero-order approximation, the reduced α-transfer rate cannot be obtained due to the valence neutrons and protons in different shells. The obtained first-order approximation [66] is proportional to the product of reduced transfer rates of neutron and proton pairs.

Figure 5 shows the obtained reduced α-transfer rates $B^{(1)}(Z, N \to Z-2, N-2)$ for $Z = 11$ and 12 with different neutron number N. Figure 6 shows the calculated reduced α-transfer rates $B^{(1)}(Z, N \to Z-2, N-2)$ for N=34 and 35 with different proton number Z. It can be seen the two figures have the quite similar pattern compared with the obtained reduced neutron and proton pair

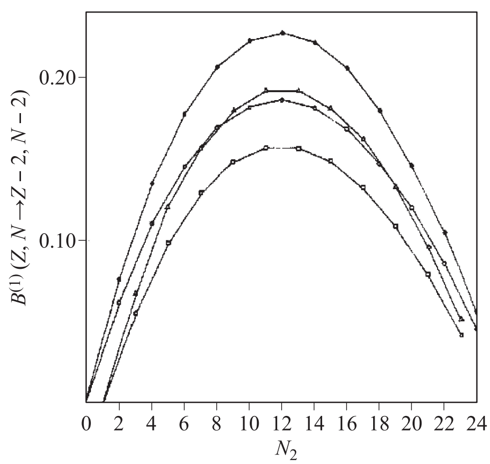

Fig. 5 Calculated reduced α-transfer rates $B^{(1)}(Z, N \to Z-2, N-2)$. The symbols • and △ represent $Z = 12$; The symbols ○ and □ represent $Z = 11$. Reproduced from Ref. [66].

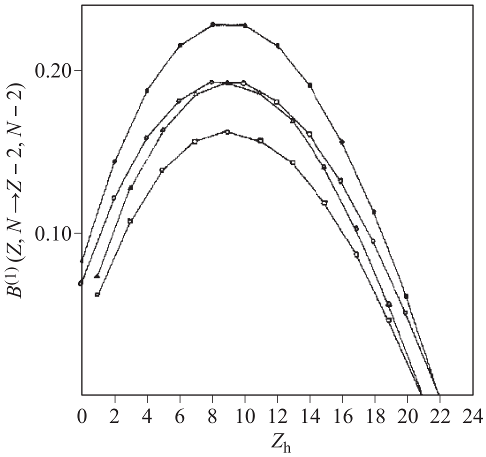

Fig. 6 Calculated reduced α-transfer rates $B^{(1)}(Z, N \to Z-2, N-2)$. The symbols • and △ represent $N = 34$; The symbols ○ and □ represent $N = 35$. Reproduced from Ref. [66].

transfer rates [66]. Moreover, as for some odd-odd nuclei, they have lower $B^{(1)}(Z, N \to Z-2, N-2)$ values shown in the two figures. This indicates the unpaired neutrons and protons have blocking effects to α-transfer reactions. Thus, by using this simple two-level pairing fore model, the shell and blocking effects observed from experiments can be explained. Moreover, this also means the α clustering assumption in heavy nuclei is reasonable.

4 The ^{208}Pb+α cluster structure of ^{212}Po

4.1 Study of the α+^{208}Pb structure of ^{212}Po using a quartetting wave function

The ^{212}Po nucleus has been investigated by numerous nuclear models [25, 26, 67–69] for a long time and many calculations suggest the ^{212}Po could have the cluster structure of α cluster plus the doubly magic core ^{208}Pb. In the shell model calculations, the low-lying spectrum can be reasonably reproduced while the α width of the ground sate of ^{212}Po was much underestimated compared with the experimental value. On the other hand, by using a hybrid-cluster-shell model [70], the α width can be well reproduced and a large amount of α clustering (30%) was also predicted. Therefore, from a theoretical view, ^{212}Po was believed to have the α+^{208}Pb cluster structure.

Recently, Röpke *et al.* [71, 72] obtained an effective alpha particle equation. This equation was derived for cases where an particle is bound to a doubly magic nucleus, which provides us with a microscopic calculation of both cluster preformation probability and decay width.

To solve the in-medium Schrödinger equation from the a four-particle Green function, the wave function $\Psi(\boldsymbol{R}, s_j)$ was assumed to be separated into two parts [71],

$$\Psi(\boldsymbol{R}, s_j) = \varphi^{intr}(\boldsymbol{R}, s_j)\Phi(\boldsymbol{R}). \tag{11}$$

Here $\boldsymbol{R} = \sum_{i=1}^{4} \boldsymbol{r}_i/4$ and s_j $(j = 1, 2, 3)$ is the Jacobian coordinates. Thus, the equation of the wave function for the motion of the center-of-mass can be obtained [71],

$$\begin{aligned}
&-\frac{\hbar^2}{8m}\nabla_R^2\Phi(\boldsymbol{R}) - \frac{\hbar^2}{4m}\int \mathrm{d}s_j\varphi^{intr,*}(\boldsymbol{R}, s_j) \\
&\times[\nabla_R\varphi^{intr}(\boldsymbol{R}, s_j)][\nabla_R\Phi(\boldsymbol{R})] - \frac{\hbar^2}{8m}\int \mathrm{d}s_j\varphi^{intr,*}(\boldsymbol{R}, s_j) \\
&\times[\nabla_R^2\varphi^{intr}(\boldsymbol{R}, s_j)]\Phi(\boldsymbol{R}) + \int \mathrm{d}R' W(\boldsymbol{R}, \boldsymbol{R}')\Phi(\boldsymbol{R}') \\
&= E\Phi(\boldsymbol{R}).
\end{aligned} \tag{12}$$

The adopted potential for the motion of the center-of-mass part is [71]

$$\begin{aligned}
W(\boldsymbol{R}, \boldsymbol{R}') = \int &\mathrm{d}s_j \mathrm{d}s_j' \varphi^{intr,*}(\boldsymbol{R}, s_j)[T[\nabla_{s_j}]\delta(\boldsymbol{R}-\boldsymbol{R}') \\
&\times\delta(s_j - s_j') + BV(\boldsymbol{R}, s_j; \boldsymbol{R}', s_j')]\varphi^{intr}(\boldsymbol{R}', s_j').
\end{aligned} \tag{13}$$

To solve the above equation, the local-density approximation is used for dealing with the Pauli blocking part.

Figure 7 shows the local $W(r)$ potential in the ^{208}Pb+α system. To satisfy the short-range repulsion and long-range attraction, an effective two-term Gaussian interaction was used. After solved the Schrödinger equation for the center-of-mass motion with the $W(r)$ potential, the corresponding wave function can be obtained. By using two sets of potential parameters, the obtained α preformation factor was 0.367 and 0.142 [68]. This result shows that α clustering can occur near the surface of the heavy core. Another saying, the α-like clustering can be found in nuclear system at a low density region.

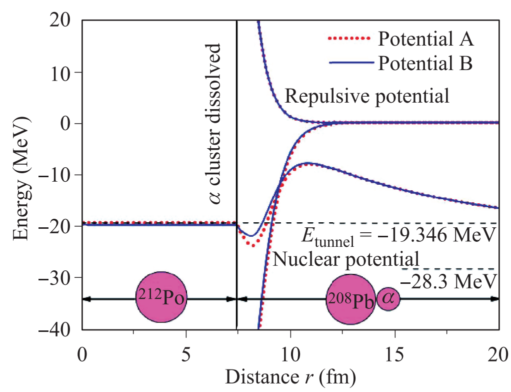

Fig. 7 Different contributions to the potential of the center-of-mass motion of the four-nucleon system on the surface of the ^{208}Pb core. Reproduced from Ref. [68].

4.2 Experimental evidence for the ^{208}Pb+α cluster structure of ^{212}Po

Recently, an important progress for the study of the clustering structure in ^{212}Po was made from the experiment. Astier *et al.* [27] performed the ^{208}Pb(^{18}O,^{14}C) α transfer reaction and observed several enhanced $B(E1)$ transitions related for the non-natural parity states, which can be considered as the experimental evidence of ^{208}Pb+α clustering structure in ^{212}Po.

Figure 8 shows some grouped observed states in their work. Two sets of levels have non-nature parity, namely two groups of even-I positive-parity states around 2 MeV and the two groups of odd-I negative-parity states around 3 MeV (filled circles in Fig. 8). Firstly, these states cannot be described in the low-lying shell model configurations. Moreover, these excited states with non-natural parity have enhanced $E1$ transitions to the yrast states with the same spin. As we know, the strongly enhanced $B(E1)$ transition values are usually found in nuclei with an electric dipole moment, which is closed related with the asymmetry clustering structure [73]. Therefore these observed non-natural states can be considered as the fingerprint of the ^{208}Pb+α structure. It should be noted that the oscillatory motion of the α and the heavy core is very different from the traditional cluster motion in light nuclei.

5 The α cluster structure in heavy nuclei

As we know, the α clustering assumption in heavy nuclei is very important to explain the α decay and α transfer reaction. Indeed, as early as 1937, Wefelmeier [5] ever proposed a cluster model in which the nuclei could be considered as a collection of structureless rigid alpha particles. Brink developed this idea and proposed a microscopic cluster wave function [74] for the cluster structure in nuclei. The Brink wave function has been very successful for the description of light nuclei [75, 76]. In Ref. [29], by using the microscopical Brink cluster model [74], the authors studied systematically the α-chain states and the two-dimmensional α-cluster configuration for some heavy nuclei, which arise from the three-dimmensional α cluster arrangement. Many geometrical cluster configurations were predicted and discussed in their work.

Quite recently, Tohsaki and Itagaki studied the α-clustering geometric configuration [31, 77] with the Brink model. The finite-range three-body interaction force [78] was employed in their microscopical calculations, which can reasonably reproduce the saturation property of the stable $N = Z$ nuclei and nuclear matter and also the phase shift of elastic α-α scattering. Their basic assumption is, some heavy nuclei can have some specific α-clustering geometrical structure and the α cluster is considered as the building block.

The Brink wave function for the $N\alpha$ clusters in nuclei can be written as [31]

$$\Psi(\rho) = \mathcal{A}\{\phi_1(\rho\boldsymbol{R}_1)\dots\phi_N(\rho\boldsymbol{R}_N)\}. \tag{14}$$

The $N\alpha$ clusters are fixed at the generator coordinates $\boldsymbol{R}_1,\dots,\boldsymbol{R}_N$, which are the vertices of five Platonic solids. The kth α cluster wave function is [31]

$$\phi(\rho\boldsymbol{R}_k)=\prod_{i,j}^{2}\left(\frac{1}{\pi b^2}\right)^{3/4}\exp\left[-\frac{1}{2b^2}(\boldsymbol{r}_k^{ij}-\rho\boldsymbol{R}_k)^2\right]\chi_k^{ij}. \tag{15}$$

Figure 9 shows the energy curves per α for different structures, which were obtained by variational calculations. It can be seen that there are stable pockets for each configuration. And the rhombic triacontahedron and the fullerene shape have much larger two-α distance. By checking the barrier height and the energy pockets,

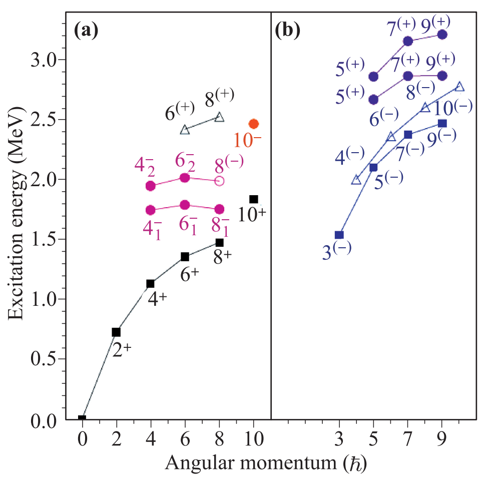

Fig. 8 Excitation energy of the ^{212}Po states as a function of the angular momentum. Reproduced from Ref. [27].

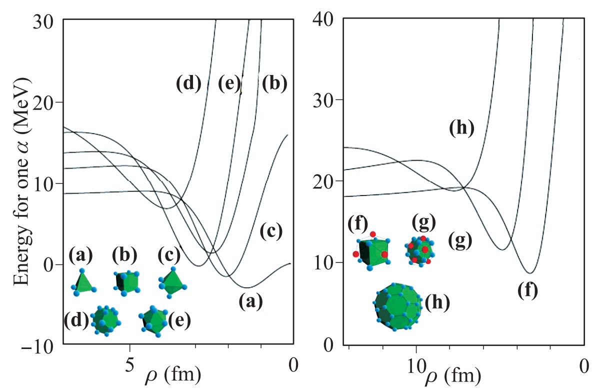

Fig. 9 Energy curves for one α cluster particle as a function of the radius ρ. The shown geometrical structures of α nuclei are **(a)** tetrahedron, **(b)** hexahedron (cube), **(c)** octahedron, **(d)** dodecahedron, **(e)** icosahedron, **(f)** hexahedron-octahedron, **(g)** dodecahedron-icosahedron, and **(h)** fullerene. Reproduced from Ref. [31].

the authors think that the possible existence of α clustering with a geometric shape in heavy nuclei, in particular, the authors pointed out that [31], some ultra-superheavy nuclei (60α) can exist with the hollow structure as one stable configuration, in which the geometric configurations are favored when α clusters form a nucleus and the Coulomb repulsion, most important at large distances, is reduced by forming the hollow structures. This kind of hollow structure in nuclei is a very surprising prediction report from the α cluster model.

The α-clustering effects in heavy nuclei can also be estimated indirectly by investigating the formation of the α clusters in infinite nuclear matter. By arranging the α clusters in each lattice site in the generator coordinate space, then the crystalline α matter can be studied in the microscopical α cluster model. Some calculations [79, 80] indicate that the α-clustering correlations cannot survive in the interior of the heavy nuclei while the α-clustering effects are very important on the surface, which has a very low density and it is about one third of the density of the inside region.

6 The α preformation probability in heavy nuclei

In heavy nuclei, the α decay is a very common type of radioactive decay. Due to the simplicity and high accuracy of alpha-particles observation from alpha-decay experiments, the alpha decay can be a good probe for the nuclear structure. To explain the alpha-decay process, the α preformation factor is introduced and it is defined as the probability of finding the alpha cluster inside the parent nucleus. This preformation factor is very helpful for the analysis of the clustering structure in heavy nuclei, which has been studied by many different nuclear models [25]. In the R-matrix method, the preformation factor is actually the formation amplitude between the wave functions of the daughter nuclei and parent nucleus. As for the typical ^{212}Po nucleus, Varga *et al.* calculated the absolute value of the alpha-decay preformation factors [70] using the hybrid-cluster-shell model, which is about 0.3 and the calculated width on the same footing is consistent with the experimental value. Unfortunately, due to the complexity of both nuclear potential and nuclear many-body problem, for most nuclei, the calculations of the α preformation factor are very difficult in the microscopical models.

For numerous phenomenological models of the alpha decay, one basic assumption is that the existence of the α particle or the α cluster has some possibility to preformed. The alpha decay is considered to be a quantum tunneling process. The decay constant is usually written, $\lambda = S_p \nu P$. The S_p is just the preformation factor of cluster radioactivity. P is the probability of penetration through the Coulomb barrier and the assault frequency of the cluster is ν. By using some cluster models or a formula for the preformation factor [81], the preformation factor and the penetrability can be determined individually and the alpha decay constant can be obtained.

Recently, Qian and Ren [82] studied the α clustering phenomenon of heavy nuclei from the view of the α preformation probability. First, they updated the two-parameter Fermi nucleon density distributions by fitting the available experimental nuclear charge radii and neutron skin thickness data. Within the density-dependent cluster model [28, 83–86], the authors extracted the α preformation factor P_α of even-even nuclei isotopes with $Z = 84$–104, which are ranged from 0.03 to 0.3. It was also found that present α preformation factor varies more smoothly towards the large neutron–proton ratio and the presence of the $Z = 92$ subshell is somewhat suggested by the picture of α formation factor.

Many studies have shown that the α preformation probability have some connections with the pairing gaps [87]. The pairing gaps can be expressed [82] by the experimental binding energies,

$$\Delta_p = \frac{1}{2}[B(Z,N) + B(Z-2,N) - 2B(Z-1,N)], \quad (16)$$

$$\Delta_n = \frac{1}{2}[B(Z,N) + B(Z,N-2) - 2B(Z,N-1)]. \quad (17)$$

Figure 10 shows the comparison between the pairing gaps and the obtained α preformation probability. It can be seen that, as a whole, the general features of the pairing gaps and the corresponding α preformation probability are similar, especially P_α and the $\Delta_p(Z,N)$, which may suggest that proton pairing correlations probably play an more important role in the α-cluster formation in heavy nuclei. On the other hand, since the pairing gaps are directly from the experimental data, this verification of the connection between P_α and $\Delta_{p/n}(Z,N)$ also gives a support for the present obtained α preformation factor. Thus, these non-trivial P_α values indicate that the possible α clustering effect in heavy nuclei.

7 Summary

In summary, we briefly reviewed some related studies for the alpha-clustering effects in heavy nuclei, which includes several aspects. First, we started from the inversion doublet bands of ^{20}Ne, which can be considered as one characteristic originating from the cluster structure. In the medium nuclei, the cluster structure of ^{40}Ca and ^{44}Ti were confirmed just based on their observed inversion doublet bands. The confirmed cluster structures in medium nuclei can be considered as one important step towards studying cluster structure in heavy nuclei. Second, we introduced some possible evidences for the

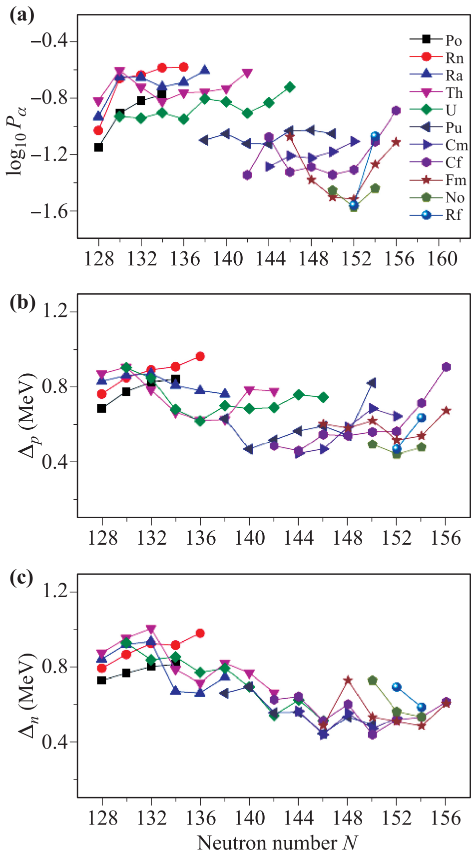

Fig. 10 **(a)** Calculated α preformation factors as a function of the neutron number of parent nuclei for the decays of even–even isotopes with the proton number from 84 to 104. **(b)** The coresponding proton pairing gaps of even–even isotopes. **(c)** For neutron pairing gaps. Reproduced from Ref. [82].

α-clustering correlations in heavy nuclei from the binding energies. Also the shell and blocking effects in alpha-transfer reaction was explained based the α-clustering assumption within a simple two-level model. Third, we introduced the recent theoretical and experimental studies for the ^{208}Pb+α cluster structure in ^{212}Po. In particular, some direct evidence for the ^{208}Pb+α cluster structure in ^{212}Pb has been obtained from experiment. We also introduced the study for the α cluster structure in heavy nuclei based on the microscopical cluster model. Finally, we discussed the cluster structure of heavy nuclei from the view of the α preformation factor. In general, there are a lot of signs indicating the existence of the alpha-clustering effects in heavy nuclei. However, the study of cluster structures in heavy nuclei is a very challenging work and it requires more experimental data and proper theoretical models in the future.

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