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RESEARCH ARTICLE

Heralded amplification of single-photon entanglement with polarization feature

Yu-Yu Jin[1], Sheng-Xian Qin[1], Hao Zu[1], Lan Zhou[1,2,3,†], Wei Zhong[3,4], Yu-Bo Sheng[3,4]

[1] *School of Science, Nanjing University of Posts and Telecommunications, Nanjing 210003, China*
[2] *New Energy Technology Engineering Laboratory of Jiangsu Province, Nanjing University of Posts and Telecommunications, Nanjing 210003, China*
[3] *Key Lab of Broadband Wireless Communication and Sensor Network Technology, Nanjing University of Posts and Telecommunications, Ministry of Education, Nanjing 210003, China*
[4] *Institute of Signal Processing Transmission, Nanjing University of Posts and Telecommunications, Nanjing 210003, China*
Corresponding authors. E-mail: † zhoul@njupt.edu.cn
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Heralded noiseless amplification is beneficial in overcoming transmission photon loss in a noisy quantum channel. We propose a single-photon-assisted heralded noiseless amplification protocol of the single-photon entanglement (SPE), where the single-photon qubit has an arbitrary unknown polarization feature. We focus on both the complete and partial photon loss during the transmission process. After the amplification, the parties can recover the pure less-entangled SPE into a maximally entangled SPE and increase its fidelity. Moreover, the polarization feature of the single-photon qubit will be well preserved and not be leaked. Our protocol can be realized under our current experimental condition. Based on the features above, our protocol may be useful in the quantum secure communication schemes that encode information in the polarization degree of freedom of photons.

Keywords quantum communication, single-photon entanglement, quantum state amplification, polarization feature

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1 Introduction

Currently, quantum entanglement has become an important source of quantum information and quantum computation, such as quantum teleportation [1–5], quantum secure direct communication [6–12], quantum key distribution [13–15], quantum machine learning [16], and other applications [17–24]. In various quantum applications, photons are the best carriers of quantum information owing to their high speed and controllability. The simplest photonic entanglement is the single-photon entanglement (SPE) with the form of $\frac{1}{\sqrt{2}}(|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B)$, which means that the single photon has the same probability to appear in location A and location B. In 2010, Salart *et al.* reported that the purification of the SPE could be applied in remote quantum communication based on quantum repeaters [25]. In 2012, Gottesman *et al.* proposed a protocol for the construction of an interferometric telescope based on the SPE, which has the potential to eliminate the baseline length limit and allows for the interferometers with arbitrarily long baselines [26]. In 2016, Guerreiro *et al.* used the SPE combined with superconducting detectors to demonstrate the quantum nonlocality and the related devices' independent applications [27]. Unfortunately, in practical applications, the environmental noise in a quantum channel may lead to the photon complete loss during the transmission and storage process. The photon complete loss depends exponentially on the transmission length. The correlation between the transmission efficiency (η) and the transmission length (l) can be written as [28]

$$\eta = 10^{-\alpha l/10}, \tag{1}$$

where α is minimal in the two "telecom windows" around 1330 nm ($\alpha \simeq 0.34$ dB/km) and 1550 nm ($\alpha \simeq 0.2$ dB/km). Hence, the transmission efficiency η reduces largely with the growth of the transmission length. Meanwhile, the environmental noise may also make a maximally entangled photon state degrade to a pure less-

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entangled state, which is often called as partial photon loss. Complete and partial photon losses are two primary obstacles in quantum communication. They may reduce the quantum communication efficiency, and worse, they may threaten the security of quantum communication. Therefore, prior to the practical applications of the SPE, it is necessary to solve the complete and partial photon loss problems.

Noiseless linear amplification (NLA) is an efficient method to solve the transmission photon loss problem, which was first presented by Ralph and Lund in 2009 [29]. Henceforth, a large number of NLA protocols for different photon states have been proposed, successively [30–50]. For example, in 2010, Gisin *et al.* proposed a theoretical heralded qubit amplifier for single-photon qubits. They first used the heralded qubit amplifier in a device-independent quantum key distribution (DI-QKD) to lengthen the communication distance. They showed that without an amplifier, no secret key can be established beyond 3.6 km for the trusted detectors. However, an implementation based on a qubit amplifier with heralded single-photon sources can achieve rates of approximately 1 bit/min on distances of 10–20 km and of approximately 1 bit/s on distances of 80–90 km with on-demand single-photon sources [30]. In 2012, Osorio *et al.* experimentally realized the heralded noiseless amplification of a single-photon qubit with single-photon sources and linear optics [34]. Later, the group of Zhang *et al.* extended Gisin's amplifier to protect the two-mode SPE [35]. In 2015, Zhou proposed the first nonlinear recyclable amplification protocol for the SPE using the cross-Kerr nonlinearity [41]. By recycling the protocol, both the success probability and SPE fidelity can be effectively increased. Recently, Monteiro *et al.* proposed a heralded amplification protocol of path-entangled quantum states and adopted this amplification protocol in the DI-QKD. They showed that by exploiting the amplification protocol, the high-fidelity entangled states can be maintained over loss-equivalent distances beyond 50 km [42]. In addition to the single-photon assisted NLA protocols, in 2013, Lemr *et al.* proposed an NLA protocol of a single-photon qubit assisted by an ideal two-photon entanglement, where the success probability does not decrease asymptotically with increasing gain [43]. Later, considering that the ideal auxiliary single photons and auxiliary photon entanglement are difficult to obtain under the current experimental condition, some NLA protocols focused on realizing the amplifications of the SPE under imperfect auxiliary photon states prepared by the current spontaneous parametric down-conversion (SPDC) source [44–46].

Although the NLA protocols of SPE have been widely discussed [35, 41, 42, 44–46], few of previous NLA protocols have discussed the polarization feature of the single-photon qubit. In fact, in the quantum secure communication field, especially in the QKD and DI-QKD applications, information are often encoded in the polarization degree of freedom of the photons. Consequently, when we adopt the NLA in such QKD and DI-QKD protocols, preserving the polarization feature of the photon qubit during the amplification process is meaningful. In 2013, Kocsis *et al.* reported an NLA protocol for a single-photon polarization qubit [50]. Using two polarization beam splitters (PBSs) and some auxiliary single photons, the polarization feature of a single-photon qubit can be preserved during the amplification process. Inspired by this work and previous NLA protocols, we herein propose an NLA protocol of the SPE, which can not only protect the SPE from the transmission complete photon loss and partial photon loss, but can also preserve the polarization feature of the single-photon qubit. Meanwhile, the protocol is based on linear optics so that it may be feasible in the current experimental condition.

The organization of this paper is as follows. In Section 2, we describe the basic principle of our NLA protocol. In Section 3, we present numerical simulations and discussions. In Section 4, we present the conclusion.

2 Amplification protocol of single-photon entanglement

The single-photon source S_0 emits a single-photon qubit and sends it with equal probability to Alice and Bob. We suppose that this photon qubit has an arbitrary polarization feature of $\alpha|H\rangle+\beta|V\rangle$, where $|H\rangle$ ($|V\rangle$) means the horizontal (vertical) polarization, and $|\alpha|^2+|\beta|^2=1$. Hence, the initial maximally entangled photonic state can be written as

$$|\Psi_0\rangle_{AB}=(\alpha|H\rangle+\beta|V\rangle)\otimes\frac{1}{\sqrt{2}}(|1\rangle_{a_0}|0\rangle_{b_0}+|0\rangle_{a_0}|1\rangle_{b_0}). \quad (2)$$

However, considering both the transmission complete and partial photon loss in noisy quantum channel, the single-photon qubit may be lost with the probability of $1-\eta$; meanwhile, the maximally entangled SPE may degrade to a pure less-entangled state. Consequently, the parties finally share a mixed state as

$$\rho_{AB}=\eta|\Psi_1\rangle_{AB}\langle\Psi_1|+(1-\eta)|vac\rangle\langle vac|, \quad (3)$$

where

$$|\Psi_1\rangle_{AB}=(\alpha|H\rangle+\beta|V\rangle)\otimes(m|1\rangle_{a_0}|0\rangle_{b_0}+n|0\rangle_{a_0}|1\rangle_{b_0}). \quad (4)$$

m and n are the entanglement coefficients of the less-entangled SPE and $|m|^2+|n|^2=1$, respectively. Our protocol aims to recover the less-entangled SPE into a maximally entangled SPE, increase the fidelity (η) of the SPE, and preserve the polarization feature of the single-photon qubit. Here, we assume that all the four factors α, β, m, and n are real for simplicity.

The schematic principle of our amplification protocol is shown in Fig. 1. In the whole amplification processing, the parties are not required to know the exact values of α and β; however, they should know the values of m and n. The values of m and n can be obtained by measuring a large number of target states $|\Psi_1\rangle_{AB}$ in the spatial modes of a_0 and b_0 [51–53].

Alice and Bob pass the photons in a_0 and b_0, respectively, through two PBSs, to fully transmit the photon in $|H\rangle$ and reflect the photon in $|V\rangle$. Subsequently, $|\Psi_1\rangle_{AB}$ evolves to

$$\begin{aligned}|\Psi_2\rangle_{AB} = &\alpha(m|H\rangle_{a_1}|0\rangle_{b_1} + n|0\rangle_{a_1}|H\rangle_{b_1})\\ &+\beta(m|V\rangle_{a_2}|0\rangle_{b_2} + n|0\rangle_{a_2}|V\rangle_{b_2}).\end{aligned} \tag{5}$$

If we define $|\Psi_{2H}\rangle_{a_1b_1} = m|H\rangle_{a_1}|0\rangle_{b_1} + n|0\rangle_{a_1}|H\rangle_{b_1}$ and $|\Psi_{2V}\rangle_{a_2b_2} = m|V\rangle_{a_2}|0\rangle_{b_2}+n|0\rangle_{a_2}|V\rangle_{b_2}$, $|\Psi_2\rangle_{AB}$ in Eq. (5) can be simplified as

$$|\Psi_2\rangle_{AB} = \alpha|\Psi_{2H}\rangle_{a_1b_1} + \beta|\Psi_{2V}\rangle_{a_2b_2}. \tag{6}$$

Consequently, by the PBSs, the initial mixed state ρ_{in} is split into two polarization components, which will be amplified simultaneously.

Each of the two parties must prepare two auxiliary photons, the photons in a_3b_3 modes in $|H\rangle$, and the photons in a_6b_6 modes in $|V\rangle$. It is noteworthy that all the photons in $|H\rangle$ ($|V\rangle$) should be indistinguishable. Alice and Bob must also prepare four variable beam splitters (VBSs). A VBS is a common linear optical element, which can generate the spatial entanglement. We define VBS_1 and VBS_2 have the same reflectivity of t_1, while VBS_3 and VBS_4 have the same reflectivity of t_2. For example, as shown in Fig. 1, if a single photon in a_3 mode passes through VBS_1, the single-photon state will evolve to

$$|1\rangle_{a_3} = \sqrt{1-t_1}|10\rangle_{a_4a_5} + \sqrt{t_1}|01\rangle_{a_4a_5}. \tag{7}$$

The two parties make each of the four auxiliary photons enter a VBS, so that they can produce four auxiliary photon states as

$$\begin{aligned}|\psi_1\rangle_{a_4a_5} &= \sqrt{1-t_1}|H\rangle_{a_4}|0\rangle_{a_5} + \sqrt{t_1}|0\rangle_{a_4}|H\rangle_{a_5},\\ |\psi_2\rangle_{b_4b_5} &= \sqrt{1-t_2}|H\rangle_{b_4}|0\rangle_{b_5} + \sqrt{t_2}|0\rangle_{b_4}|H\rangle_{b_5},\\ |\psi_3\rangle_{a_7a_8} &= \sqrt{1-t_1}|V\rangle_{a_7}|0\rangle_{a_8} + \sqrt{t_1}|0\rangle_{a_7}|V\rangle_{a_8},\\ |\psi_4\rangle_{b_7b_8} &= \sqrt{1-t_2}|V\rangle_{b_7}|0\rangle_{b_8} + \sqrt{t_2}|0\rangle_{b_7}|V\rangle_{b_8}.\end{aligned} \tag{8}$$

Considering the initial mixed state, the whole photon state can be described as follows. It is in the state of $|\Psi_2\rangle_{AB}\otimes|\psi_1\rangle_{a_4a_5}\otimes|\psi_2\rangle_{b_4b_5}\otimes|\psi_3\rangle_{a_7a_8}\otimes|\psi_4\rangle_{b_7b_8}$ with the probability of η, and it is in the state of $|vac\rangle\otimes|\psi_1\rangle_{a_4a_5}\otimes|\psi_2\rangle_{b_4b_5}\otimes|\psi_3\rangle_{a_7a_8}\otimes|\psi_4\rangle_{b_7b_8}$ with the probability of $1-\eta$.

We first describe the case where the single photon is not lost. The whole photon state system is distributed spatially between the two parties, which can be described as

$$\begin{aligned}|\Psi_3\rangle =& |\Psi_2\rangle_{AB}\otimes|\psi_1\rangle_{a_4a_5}\otimes|\psi_2\rangle_{b_4b_5}\otimes|\psi_3\rangle_{a_7a_8}\otimes|\psi_4\rangle_{b_7b_8}\\ =&\alpha[m\sqrt{(1-t_1)(1-t_2)}|H\rangle_{a_1}|0\rangle_{b_1}|H\rangle_{a_4}|0\rangle_{a_5}|H\rangle_{b_4}|0\rangle_{b_5}\\ &+m\sqrt{(1-t_1)t_2}|H\rangle_{a_1}|0\rangle_{b_1}|H\rangle_{a_4}|0\rangle_{a_5}|0\rangle_{b_4}|H\rangle_{b_5}\\ &+m\sqrt{t_1(1-t_2)}|H\rangle_{a_1}|0\rangle_{b_1}|0\rangle_{a_4}|H\rangle_{a_5}|H\rangle_{b_4}|0\rangle_{b_5}\\ &+m\sqrt{t_1t_2}|H\rangle_{a_1}|0\rangle_{b_1}|0\rangle_{a_4}|H\rangle_{a_5}|0\rangle_{b_4}|H\rangle_{b_5}\\ &+n\sqrt{(1-t_1)(1-t_2)}|0\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{a_4}|0\rangle_{a_5}|H\rangle_{b_4}|0\rangle_{b_5}\\ &+n\sqrt{(1-t_1)t_2}|0\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{a_4}|0\rangle_{a_5}|0\rangle_{b_4}|H\rangle_{b_5}\\ &+n\sqrt{t_1(1-t_2)}|0\rangle_{a_1}|H\rangle_{b_1}|0\rangle_{a_4}|H\rangle_{a_5}|H\rangle_{b_4}|0\rangle_{b_5}\\ &+n\sqrt{t_1t_2}|0\rangle_{a_1}|H\rangle_{b_1}|0\rangle_{a_4}|H\rangle_{a_5}|0\rangle_{b_4}|H\rangle_{b_5}]\\ &\otimes(\sqrt{1-t_1}|V\rangle_{a_7}|0\rangle_{a_8}+\sqrt{t_1}|0\rangle_{a_7}|V\rangle_{a_8})\\ &\otimes(\sqrt{1-t_2}|V\rangle_{b_7}|0\rangle_{b_8}+\sqrt{t_2}|0\rangle_{b_7}|V\rangle_{b_8})\\ &+\beta[m\sqrt{(1-t_1)(1-t_2)}|V\rangle_{a_2}|0\rangle_{b_2}|V\rangle_{a_7}|0\rangle_{a_8}|V\rangle_{b_7}|0\rangle_{b_8}\\ &+m\sqrt{(1-t_1)t_2}|V\rangle_{a_2}|0\rangle_{b_2}|V\rangle_{a_7}|0\rangle_{a_8}|0\rangle_{b_7}|V\rangle_{b_8}\\ &+m\sqrt{t_1(1-t_2)}|V\rangle_{a_2}|0\rangle_{b_2}|0\rangle_{a_7}|V\rangle_{a_8}|V\rangle_{b_7}|0\rangle_{b_8}\\ &+m\sqrt{t_1t_2}|V\rangle_{a_2}|0\rangle_{b_2}|0\rangle_{a_7}|V\rangle_{a_8}|0\rangle_{b_7}|V\rangle_{b_8}\\ &+n\sqrt{(1-t_1)(1-t_2)}|0\rangle_{a_2}|V\rangle_{b_2}|V\rangle_{a_7}|0\rangle_{a_8}|V\rangle_{b_7}|0\rangle_{b_8}\\ &+n\sqrt{(1-t_1)t_2}|0\rangle_{a_2}|V\rangle_{b_2}|V\rangle_{a_7}|0\rangle_{a_8}|0\rangle_{b_7}|V\rangle_{b_8}\\ &+n\sqrt{t_1(1-t_2)}|0\rangle_{a_2}|V\rangle_{b_2}|0\rangle_{a_7}|V\rangle_{a_8}|V\rangle_{b_7}|0\rangle_{b_8}\\ &+n\sqrt{t_1t_2}|0\rangle_{a_2}|V\rangle_{b_2}|0\rangle_{a_7}|V\rangle_{a_8}|0\rangle_{b_7}|V\rangle_{b_8}]\\ &\otimes(\sqrt{1-t_1}|H\rangle_{a_4}|0\rangle_{a_5}+\sqrt{t_1}|0\rangle_{a_4}|H\rangle_{a_5})\\ &\otimes(\sqrt{1-t_2}|H\rangle_{b_4}|0\rangle_{b_5}+\sqrt{t_2}|0\rangle_{b_4}|H\rangle_{b_5}).\end{aligned} \tag{9}$$

Next, Alice and Bob cause the photons in a_1a_5, b_1b_5, a_2a_8, b_2b_8 to pass through four 50:50 beam splitters

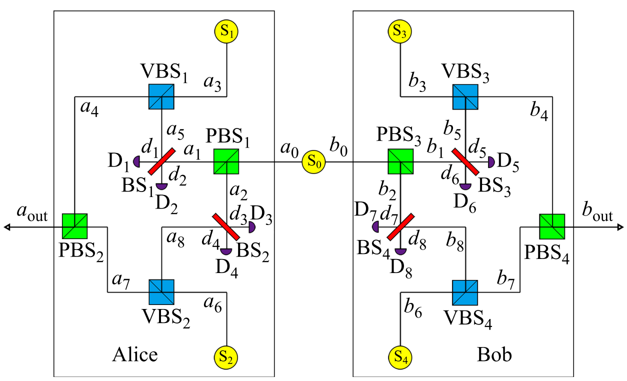

Fig. 1 The schematic principle of our amplification protocol for the SPE. For realizing the amplification and concentration tasks while preserving the polarization feature of single-photon qubit, some auxiliary single photons and linear optical elements are required. BS, VBS, and PBS mean 50:50 beam splitter, variable beam splitter, and polarization beam splitter, respectively. D_i ($i = 1, 2, 3, \cdots, 8$) is the photon-number-resolving detector.

(BSs), respectively, which can yield

$$|1\rangle_{a_5} \longrightarrow \frac{1}{\sqrt{2}}(|1\rangle_{d_1}+|1\rangle_{d_2}), \quad |1\rangle_{a_1} \longrightarrow \frac{1}{\sqrt{2}}(|1\rangle_{d_1}-|1\rangle_{d_2}),$$
$$|1\rangle_{a_2} \longrightarrow \frac{1}{\sqrt{2}}(|1\rangle_{d_3}+|1\rangle_{d_4}), \quad |1\rangle_{a_8} \longrightarrow \frac{1}{\sqrt{2}}(|1\rangle_{d_3}-|1\rangle_{d_4}),$$
$$|1\rangle_{b_5} \longrightarrow \frac{1}{\sqrt{2}}(|1\rangle_{d_5}+|1\rangle_{d_6}), \quad |1\rangle_{b_1} \longrightarrow \frac{1}{\sqrt{2}}(|1\rangle_{d_5}-|1\rangle_{d_6}),$$
$$|1\rangle_{b_2} \longrightarrow \frac{1}{\sqrt{2}}(|1\rangle_{d_7}+|1\rangle_{d_8}), \quad |1\rangle_{b_8} \longrightarrow \frac{1}{\sqrt{2}}(|1\rangle_{d_7}-|1\rangle_{d_8}). \tag{10}$$

Subsequently, they detect the output photons in $d_1 \sim d_8$ spatial modes by the single-photon detector $D_1 \sim D_8$, respectively. The parties select the items that cause only one of the two output modes of each BS to have exactly one photon. Hence, all the single-photon detectors $D_1 \sim D_8$ are required to be photon-number-resolving detectors. Sixteen detection results correspond to the successful cases of the amplification, which are shown in Table 1.

We use the detection result of $D_1D_3D_5D_7$ as an example. From Eq. (9) and Eq. (10), if $D_1D_3D_5D_7$ each registers a single photon, the parties will obtain the state as

$$\begin{aligned}|\Psi_4\rangle = {} & m\sqrt{(1-t_1)t_1t_2^2}(\alpha|H\rangle_{a_1}|H\rangle_{a_4}|0\rangle_{a_5}|0\rangle_{a_7}\\ & |V\rangle_{a_8}|0\rangle_{b_1}|0\rangle_{b_4}|H\rangle_{b_5}|0\rangle_{b_7}|V\rangle_{b_8} + \beta|V\rangle_{a_2}|0\rangle_{a_4}\\ & |H\rangle_{a_5}|V\rangle_{a_7}|0\rangle_{a_8}|0\rangle_{b_2}|0\rangle_{b_4}|H\rangle_{b_5}|0\rangle_{b_7}|V\rangle_{b_8})\\ & +n\sqrt{(1-t_2)t_2t_1^2}(\alpha|0\rangle_{a_1}|0\rangle_{a_4}|H\rangle_{a_5}|0\rangle_{a_7}\\ & |V\rangle_{a_8}|H\rangle_{b_1}|H\rangle_{b_4}|0\rangle_{b_5}|0\rangle_{b_7}|V\rangle_{b_8} + \beta|0\rangle_{a_2}|0\rangle_{a_4}\\ & |H\rangle_{a_5}|0\rangle_{a_7}|V\rangle_{a_8}|V\rangle_{b_2}|0\rangle_{b_4}|H\rangle_{b_5}|V\rangle_{b_7}|0\rangle_{b_8}),\end{aligned} \tag{11}$$

with the probability of

$$\begin{aligned}P_1' = {} & \frac{1}{16}[\alpha^2m^2(1-t_1)t_1t_2^2 + \beta^2m^2(1-t_1)t_1t_2^2\\ & +\alpha^2n^2(1-t_2)t_2t_1^2 + \beta^2n^2(1-t_2)t_2t_1^2]\\ = {} & \frac{1}{16}[t_1t_2(m^2t_2+n^2t_1-t_1t_2)].\end{aligned} \tag{12}$$

After the photon detection, $|\Psi_4\rangle$ can be finally transformed into

$$\begin{aligned}|\Psi_5\rangle = {} & \alpha(m\sqrt{(1-t_1)t_1t_2^2}|H\rangle_{a_4}|0\rangle_{b_4}\\ & +n\sqrt{(1-t_2)t_2t_1^2}|0\rangle_{a_4}|H\rangle_{b_4})\\ & +\beta(m\sqrt{(1-t_1)t_1t_2^2}|V\rangle_{a_7}|0\rangle_{b_7}\\ & +n\sqrt{(1-t_2)t_2t_1^2}|0\rangle_{a_7}|V\rangle_{b_7}).\end{aligned} \tag{13}$$

Finally, Alice and Bob pass the photons in a_4a_7 and b_4b_7 modes through PBS_2 and PBS_4, respectively, causing $|\Psi_5\rangle$ to evolve to

$$\begin{aligned}|\Psi_6\rangle = {} & (\alpha|H\rangle+\beta|V\rangle)\otimes(m\sqrt{(1-t_1)t_1t_2^2}|1\rangle_{a_{out}}|0\rangle_{b_{out}}\\ & +n\sqrt{(1-t_2)t_2t_1^2}|0\rangle_{a_{out}}|1\rangle_{b_{out}}).\end{aligned} \tag{14}$$

If Alice and Bob obtained one of the other 15 detection results in Table 1, they would also finally obtain $|\Psi_6\rangle$ in Eq. (14) by the phase-flipping operations. Therefore, the total success probability is

$$P_1 = 16P_1' = t_1t_2(m^2t_2+n^2t_1-t_1t_2). \tag{15}$$

We found that $|\Psi_6\rangle$ has the similar form as the initial state $|\Psi_1\rangle_{AB}$, while the polarization feature of the single-photon qubit is well preserved.

To obtain the maximally entangled state as $|\Psi_0\rangle_{AB}$ in Eq. (2), the parties need to make the entanglement coefficients of the two spatial modes a_{out} and b_{out} in Eq. (14) to be the same. Hence, they should adjust t_1 and t_2 to meet

$$m\sqrt{(1-t_1)t_1t_2^2} = n\sqrt{(1-t_2)t_2t_1^2}. \tag{16}$$

Eq. (16) can be simplified as

$$\frac{\frac{1}{t_2}-1}{\frac{1}{t_1}-1} = \frac{m^2}{n^2}. \tag{17}$$

Meanwhile, if the initial single photon is lost with the probability of $1-\eta$, Alice and Bob will share a vacuum state. The whole state $|\psi_1\rangle_{a_4a_5}\otimes|\psi_2\rangle_{b_4b_5}\otimes|\psi_3\rangle_{a_7a_8}\otimes|\psi_4\rangle_{b_7b_8}$ can be described as

$$\begin{aligned}|\Psi_7\rangle = {} & |\psi_1\rangle_{a_4a_5}\otimes|\psi_2\rangle_{b_4b_5}\otimes|\psi_3\rangle_{a_7a_8}\otimes|\psi_4\rangle_{b_7b_8}\\ = {} & (\sqrt{1-t_1}|H\rangle_{a_4}|0\rangle_{a_5}+\sqrt{t_1}|0\rangle_{a_4}|H\rangle_{a_5})\\ & \otimes(\sqrt{1-t_2}|H\rangle_{b_4}|0\rangle_{b_5}+\sqrt{t_2}|0\rangle_{b_4}|H\rangle_{b_5})\\ & \otimes(\sqrt{1-t_1}|V\rangle_{a_7}|0\rangle_{a_8}+\sqrt{t_1}|0\rangle_{a_7}|V\rangle_{a_8})\\ & \otimes(\sqrt{1-t_2}|V\rangle_{b_7}|0\rangle_{b_8}+\sqrt{t_2}|0\rangle_{b_7}|V\rangle_{b_8}).\end{aligned} \tag{18}$$

Under the case that only one of the two input modes of each BS has exactly one photon, Alice and Bob will also obtain 16 possible detection results in Table 1. We also use the detection result of $D_1D_3D_5D_7$, each registering a single photon as an example. Under this case, the state

Table 1 The successful detection results of our protocol. "∘" means the protocol is successful under the detection result.

	D_1D_3	D_1D_4	D_2D_3	D_2D_4
D_5D_7	∘	∘	∘	∘
D_5D_8	∘	∘	∘	∘
D_6D_7	∘	∘	∘	∘
D_6D_8	∘	∘	∘	∘

$|\Psi_7\rangle$ will evolve to the vacuum state with the success probability of

$$P_2' = \frac{1}{16}t_1^2t_2^2. \quad (19)$$

If Alice and Bob obtain one of the other 15 detection results in Table 1, they would also obtain a vacuum state. Consequently, the total success probability can be written as

$$P_2 = 16P_2' = t_1^2t_2^2. \quad (20)$$

Therefore, by selecting the items that cause any one of the sixteen detection results in Table 1, and adjusting t_1 and t_2 based on Eq. (17), Alice and Bob can distill a new mixed state as

$$\rho'_{AB} = \eta'|\Psi_0\rangle_{a_{out}b_{out}}\langle\Psi_0| + (1-\eta')|vac\rangle\langle vac|. \quad (21)$$

The total success probability of our protocol is

$$\begin{aligned} P_t &= \eta P_1 + (1-\eta)P_2 \\ &= \eta t_1t_2(m^2t_2 + n^2t_1 - t_1t_2) + (1-\eta)t_1^2t_2^2 \\ &= \eta t_1t_2(m^2t_2 + n^2t_1) + t_1^2t_2^2(1-2\eta t_1t_2). \end{aligned} \quad (22)$$

Compared with our previous amplification work for the SPE that does not consider the polarization feature of the single-photon qubit, the total success probability of our protocol is lower by a factor of t_1t_2. This is because the preservation of the polarization feature of the photon qubit demands more auxiliary photons, so that Alice and Bob are required to perform more parity checks to distill the wanted items, which obviously reduces the success probability.

The fidelity of the new mixed state (η') can be calculated as

$$\eta' = \frac{\eta P_1}{P_t} = \frac{\eta(m^2t_2 + n^2t_1 - t_1t_2)}{\eta(m^2t_2 + n^2t_1) + t_1t_2(1-2\eta t_1t_2)}. \quad (23)$$

Therefore, the amplification factor g of our protocol can be defined as

$$g \equiv \frac{\eta'}{\eta} = \frac{m^2t_2 + n^2t_1 - t_1t_2}{\eta(m^2t_2 + n^2t_1) + t_1t_2(1-2\eta t_1t_2)}. \quad (24)$$

We found that the fidelity of the distilled mixed state and the amplification factor are not related with the polarization feature of the single-photon qubit. To realize the amplification, it is required that $g > 1$. Considering $m^2 + n^2 = 1$ and the relationship between t_1 and t_2 in Eq. (17), we can obtain $g > 1$ under the case that

$$t_2 < \frac{2n^2}{2n^2+1}. \quad (25)$$

Therefore, combined with the transmittance requirement of Eq. (17) and Eq. (25), Alice and Bob can increase the fidelity of the new mixed state, recover the pure less-entangled SPE into the maximally entangled SPE, and preserve the unknown polarization feature of the single-photon qubit, simultaneously.

3 Discussion

Thus far, we have explained our protocol for protecting the SPE from photon loss and decoherence, and preserve the polarization feature of the single-photon qubit. Preserving the polarization feature of the single-photon qubit is the most attractive advantage of our protocol. In practical applications of the SPE, the polarization degree of freedom is often used to carry information. For example, in many QKD protocols [30, 54, 55], the message senders often encode information in the polarization degree of freedom of the photon qubit. Although the NLA is widely used in the QKD field to solve the photon loss problem, few previous NLA protocols consider the polarization feature of photon qubits. We herein consider the case where the SPE encounters complete and partial photon loss simultaneously. Meanwhile, the single-photon qubit is allowed to contain an arbitrary polarization feature. Alice and Bob use PBSs to split the initial SPE into two polarization components, one in $|H\rangle$ and one in $|V\rangle$. The two polarization components are amplified by the two auxiliary single photons, simultaneously. Finally, the two components are combined to recover the polarization feature of the initial single-photon qubit. After operating the amplification protocols, the parties can recover the less-entangled SPE into maximally entangled SPE, increase the SPE fidelity, and preserve the polarization feature of the input single-photon qubit. The preservation of the polarization feature in our protocol does not influence the amplification factor but reduces the total success probability to some extent. Meanwhile, as the two parties do not know the exact polarization information of the single-photon qubit, the information encoded in the polarization degree of freedom will never be leaked in the amplification process. Consequently, our protocol is highly suitable for use in quantum secure communication protocols, which encode information in the photon's polarization degree of freedom. In the protocol, we require ideal single-photon sources to provide auxiliary single photons. In fact, in a practical experiment, the ideal single-photon source can be replaced by the practical imperfect single-photon source. In our previous works, we have investigated the amplifications of single-photon qubits and the SPE under an imperfect single-photon source [44]. We have shown that using an imperfect single-photon source would limit the upper bound of the fidelity of the amplified SPE to the fidelity of the single-photon source. Consequently, for realizing the amplification, we should require the fidelity of the single-photon source to be greater than the fidelity of the initial degraded state. Meanwhile, using an imperfect single-photon source would also reduce the success probability of the protocol. Based on Ref. [44], our protocol can also work under the imperfect single-photon source condition, thus it can be completely realized under current experimental condition.

In the protocol, Alice and Bob realize the amplification and concentration primarily by adjusting the reflectivity of four VBSs. For recovering the less-entangled SPE into the maximally entangled SPE, t_1 and t_2 should satisfy Eq. (17). As the entanglement coefficients m and n of the initial less-entangled state can be obtained, we calculate the value of t_1 as a function of t_2. As shown in Fig. 2. the curves A–F correspond to the spatial entanglement coefficients $m^2 = 0.1, 0.2, 0.4, 0.5, 0.6, 0.8$, respectively. Here, $m^2 = 0.5$ represents that the initial SPE is the maximally entangled state, and $t_1 = t_2$. In other cases, $t_1 \neq t_2$.

To realize the amplification for the SPE, the amplification factor must be $g > 1$. In Fig. 3, we set $m^2 = 0.6$ and show the value of g as a function of t_2. Curves A, B, C, D, E correspond to the transmission lengths $l = 25$ km, 20 km, 15 km, 10 km, 5 km, respectively. We found that g reduces with the growth of t_2. All the curves meet at one point M where $t_2 = 0.444$; when $t_2 < 0.444$, $g > 1$

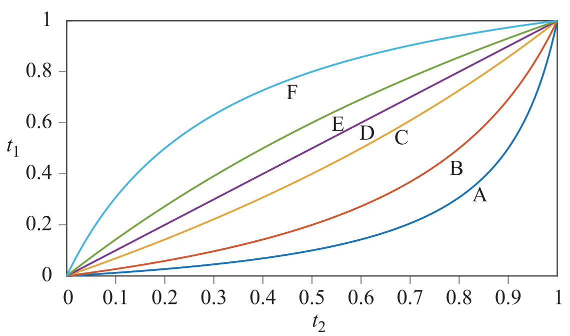

Fig. 2 For recovering the less-entangled SPE into the maximally entangled SPE, t_1 and t_2 should satisfy $m\sqrt{(1-t_1)t_1t_2^2} = n\sqrt{(1-t_2)t_2t_1^2}$. Here, we calculate the value of t_1 as a function of t_2, under the condition that $m^2 = 0.1$ (curve A), $m^2 = 0.2$ (curve B), $m^2 = 0.4$ (curve C), $m^2 = 0.5$ (curve D), $m^2 = 0.6$ (curve E), and $m^2 = 0.8$ (curve F), respectively.

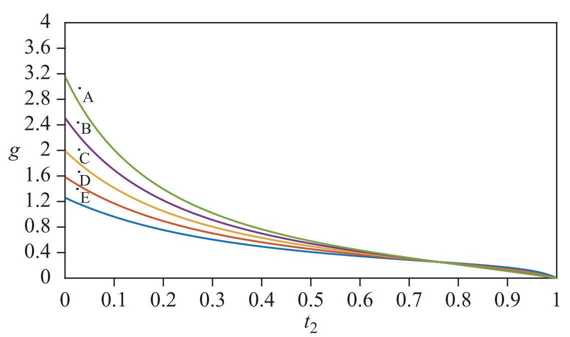

Fig. 3 The relationship between g and t_2. Here, we fix $m^2 = 0.6$. Curve A, B, C, D, E correspond to the transmission length $l = 25$ km, 20 km, 15 km, 10 km, 5 km, respectively.

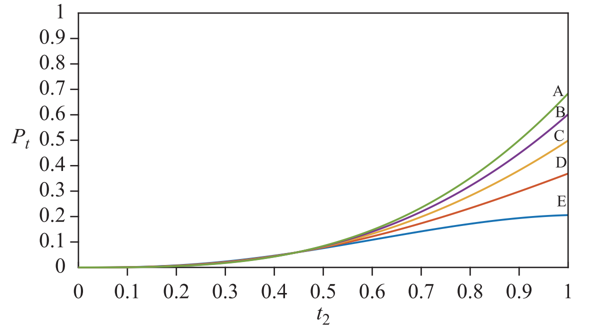

Fig. 4 The total success probability (P_t) alters with t_2 when $m^2 = 0.6$. Curve A, B, C, D, E correspond to the transmission length $l = 25$ km, 20 km, 15 km, 10 km, 5 km, respectively.

is obtained under all the initial fidelity conditions. Consequently, for increasing g, we should select the VBSs with small reflectivity. In particular, when $t_2 \to 0$, we can obtain $g \to \frac{1}{\eta}$, which indicates that the fidelity of the new distilled mixed state is close to one.

Subsequently, we consider the total success probability of our protocol. In Fig. 4, we also set $m^2 = 0.6$ and calculate the relationship between P_t and t_2. Curves A, B, C, D, E represent the transmission lengths $l = 25$ km, 20 km, 15 km, 10 km, 5 km, respectively. We found that P_t increases with the growth of t_2. Interestingly, P_t of our protocol increases with the growth of the transmission length l, which indicates that our protocol has a higher success probability under the high photon loss condition. This feature renders our protocol suitable for the practical long quantum channel condition. Considering the reflectivity requirement for $g > 1$, the high amplification factor must sacrifice the success probability. Therefore, in practical applications, we must consider the two factors and select the VBSs with the suitable reflectivity.

We herein considered a particular noise model, in which only the spatial mode is affected, while the polarization qubit is conserved. Nonetheless, optical fibers are typically responsible for a certain noise rate in the polarization. In fact, the noise mode that affects two or more degrees of freedom of a single photon is called the collective noise mode. The collective noise mode is closer to the practical environmental noise such that it is meaningful to investigate its influence on practical quantum communication protocols. For handling the photon loss and decoherence problem caused by the collective noise mode, the heralded amplification and entanglement concentration for the photonic hyperentanglement must be investigated. Hitherto, some works for hyperentanglement concentration have been proposed [56, 57]; however, the NLA protocols for the photonic hyperentanglement state has been scarcely reported. In our future research work, we plan to investigate the area thereof.

4 Conclusion

We proposed a noiseless linear amplification protocol that can protect the SPE from photon complete loss and partial loss, simultaneously. Further, our protocol can preserve the arbitrary unknown polarization feature of the single-photon qubit. In our protocol, the initial SPE is split into two polarization components, which are amplified by two auxiliary single photons, simultaneously. Finally, the two components are combined to recover the polarization feature of the initial single-photon qubit. The preservation of the polarization feature of the photon qubit does not influence the amplification factor but reduces the success probability of our protocol. Our protocol only relies on linear optical elements; therefore, it can be realized under the current experimental conditions. Moreover, the distilled new mixed state can be retained and used in other applications. These advantages render our amplification protocol suitable for quantum secure communications that encodes messages in the polarization degree of freedom of the photon qubit.

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