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Research article

General hyperentanglement concentration for polarization-spatial-time-bin multi-photon systems with linear optics

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Hyperentanglement has attracted considerable attention recently because of its high-capacity for long-distance quantum communication. In this study, we present a hyperentanglement concentration protocol (hyper-ECP) for nonlocal three-photon systems in the polarization, spatial-mode, and time-bin partially hyperentangled Greenberger–Horne–Zeilinger (GHZ) states using the Schmidt projection method. In our hyper-ECP, the three distant parties must perform the parity-check measurements on the polarization, spatial-mode, and time-bin degrees of freedom, respectively, using linear optical elements and Pockels cells, and only two identical nonlocal photon systems are required. This hyper-ECP can be directly extended to the N-photon hyperentangled GHZ states, and the success probability of this general hyper-ECP for a nonlocal N-photon system is the optimal one, regardless of the photon number N.

Keywords hyperentanglement concentration, linear optics, long-distance quantum communication, high-capacity, polarization-spatial-time-bin hyperentanglement

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1 Introduction

Quantum entanglement is a fundamental resource in quantum information processing (QIP) [1–20] such as quantum teleportation [1–3], quantum dense coding [4, 5], quantum key distribution [6–9], quantum secret sharing [10], and quantum secure direct communication [11–17]. The entangled photon system is one of the most promising candidates because of its manipulability and high-speed transmission. In some conventional protocols of QIP, photon systems are usually entangled in a single degree of freedom (DOF). In reality, more than one DOF in a photon system can be used to carry quantum information. Hyperentanglement [21–28], in which the entanglement is simultaneously present in multiple DOFs of quantum systems, can considerably increase the capacity of QIP and provide other important applications. It is mainly produced by spontaneous parametric downconversion regime. In 2005, Barreiro *et al.* [23] experimentally demonstrated the generation of photonic hyperentanglement simultaneously in polarization, spatial-mode, and time-energy DOFs. Thus far, hyperentanglement has offered some promising applications in hyperparallel photonic quantum computing [29–33], photonic hyperteleportation [34], hyperentanglement swapping [34, 35], and hyperentangled Bell-state analysis [34–39]. It can also be used to assist the efficient quantum repeater [40], complete Bell-state analysis [41–45], deterministic entanglement purification protocols [45–49], and hyperentanglement purification [50, 51].

In an experiment, an entangled quantum system is always prepared locally, and its entanglement is fragile during practical quantum transmission and storage processes. Photon loss and the decoherence of the entanglement in entangled photon systems caused by environment noise may decrease the security and efficiency of long-distance quantum communication. Entanglement concentration is an effective method to distill maximally entangled quantum systems from an ensemble of quantum systems in less-entangled pure states [52]. In 1996, Bennett *et al.* [52] introduced the first entanglement concentration protocol (ECP) for two-qubit systems based on the Schmidt projection method and collective measurements. Since this pioneering work, various effective ECPs have been proposed and discussed [53–70]. Some of them have focused on partially entangled states with unknown parameters [52–56], whereas others have considered those with known parameters [57–59].

In long-distance quantum communication, hyperentanglement can considerably increase channel capacity, and research on hyperentanglement concentration protocols (hyper-ECPs) has attracted considerable attention for improving hyperentanglement in high-capacity long-distance quantum communication [71–80]. In 2013, Ren *et al.* [71] first introduced the parameter-splitting method to concentrate polarization-spatial partially hyperentangled states with known parameters. This novel method is very efficient and simple and can achieve the maximal success probability by using only linear optics. In their study, Ren *et al.* also presented the first linear hyper-ECP for polarization-spatial partially hyperentangled states with unknown parameters. This protocol was later extended to multipartite entanglement by Li and Ghose [74]. Ren and Deng [72] then proposed an efficient hyper-ECP for unknown partially hyperentangled states assisted by diamond nitrogen vacancy (NV) centers in photonic crystal cavities. In 2014, Ren and Long [73] proposed a general hyper-ECP for photon systems assisted by quantum dot spins inside optical microcavities. In 2015, Li and Ghose [77] proposed two hyper-ECPs for time-bin and polarization hyperentangled photon systems with unknown and known parameters. Also in 2015, Ren and Long [76] proposed a two-step hyper-ECP assisted by quantum swap gates, which can significantly improve the success probability of the hyper-ECP. In 2016, Cao *et al.* [78] proposed a hyper-ECP for entangled photon systems by using a photonic module system. Recently, some interesting hyper-ECPs have been proposed for less-hyperentangled multi-photon systems in two DOFs [79, 80].

In this study, we present a hyper-ECP for the three-photon nine-qubit partially hyperentangled Greenberger–Horne–Zeilinger (GHZ) states using the Schmidt projection method. This hyper-ECP can be directly extended to the N-photon hyperentangled GHZ states. Regardless of the number of distant parties that participate in the quantum communication task, only three of the N distant parties must perform the parity-check measurements on the polarization, spatial-mode, and time-bin DOFs of their photon pairs, respectively, with linear optical elements and Pockels cells. In addition, two identical nonlocal photon systems are required in this hyper-ECP, which consumes fewer resources than the extended version of the hyper-ECP for hyperentangled Bell states. This general hyper-ECP for nonlocal N-photon hyperentangled GHZ states theoretically has the maximal success probability, regardless of the photon number N.

2 Hyper-ECP for three-photon nine-qubit partially hyperentangled GHZ states

We define the three-photon polarization-spatial-time-bin maximally hyperentangled GHZ state as

$$|\psi\rangle_{abc} = \frac{1}{2\sqrt{2}}(|HHH\rangle + |VVV\rangle)_{abc} \otimes (|a_1b_1c_1\rangle + |a_2b_2c_2\rangle) \otimes (|SSS\rangle + |LLL\rangle)_{abc}, \tag{1}$$

where the subscripts a, b, and c represent three photons, and $|H\rangle$ and $|V\rangle$ represent the horizontal and vertical polarizations of a photon, respectively. $|i_1\rangle$ and $|i_2\rangle$ are the two spatial modes of photon i $(i = a, b, c)$, and $|S\rangle$ and $|L\rangle$ represent the early and late time bins of a photon, respectively, where the time interval between the two time bins is Δt. In long-distance quantum communication, the maximally hyperentangled GHZ state $|\psi\rangle_{abc}$ may decay to a partially hyperentangled GHZ state $|\psi_0\rangle_{abc}$ by the noisy channels. Here,

$$|\psi_0\rangle_{abc} = (\alpha|HHH\rangle + \beta|VVV\rangle)_{abc} \otimes (\gamma|a_1b_1c_1\rangle + \delta|a_2b_2c_2\rangle) \otimes (\eta|SSS\rangle + \xi|LLL\rangle)_{abc}, \tag{2}$$

where the three photons a, b, and c are held by the three distant parties called Alice, Bob, and Charlie, respectively. The six parameters $\alpha, \beta, \gamma, \delta, \eta$, and ξ are unknown to Alice, Bob, and Charlie, and they satisfy the normalization condition: $|\alpha|^2 + |\beta|^2 = |\gamma|^2 + |\delta|^2 = |\eta|^2 + |\xi|^2 = 1$. To distill the three-photon system in the maximally hyperentangled GHZ state $|\psi\rangle_{abc}$ from those in the partially hyperentangled GHZ state $|\psi_0\rangle_{abc}$, another identical three-photon system $a'b'c'$ is required. Here,

$$|\psi_0\rangle_{a'b'c'} = (\alpha|HHH\rangle + \beta|VVV\rangle)_{a'b'c'} \otimes (\gamma|a'_1b'_1c'_1\rangle + \delta|a'_2b'_2c'_2\rangle) \otimes (\eta|SSS\rangle + \xi|LLL\rangle)_{a'b'c'}. \tag{3}$$

The initial state of six-photon system $abca'b'c'$ can be written as

$$\begin{aligned}|\Psi_0\rangle = {} & (\alpha^2|HHHHHH\rangle + \beta^2|VVVVVV\rangle \\ & +\alpha\beta|HHHVVV\rangle + \alpha\beta|VVVHHH\rangle) \\ & \otimes(\gamma^2|a_1b_1c_1a'_1b'_1c'_1\rangle + \delta^2|a_2b_2c_2a'_2b'_2c'_2\rangle \\ & +\gamma\delta|a_1b_1c_1a'_2b'_2c'_2\rangle + \gamma\delta|a_2b_2c_2a'_1b'_1c'_1\rangle) \\ & \otimes(\eta^2|SSSSSS\rangle + \xi^2|LLLLLL\rangle \\ & +\eta\xi|SSSLLL\rangle + \eta\xi|LLLSSS\rangle).\end{aligned} \tag{4}$$

The photon pairs aa', bb', and cc' belong to Alice, Bob, and Charlie, respectively. The three distant parties perform the bit-flip operations on the polarization, spatial-mode, and time-bin DOFs of photons a', b', and c', respectively. Here, the bit-flip operation on the polarization state can be realized by the half-wave plate (HWP) ($\sigma_x^P = |H\rangle\langle V| + |V\rangle\langle H|$). The bit-flip operation on the spatial-mode state can be completed by exchanging the two spatial modes ($\sigma_x^S = |i_1\rangle\langle i_2| + |i_2\rangle\langle i_1|$) or by combining beam splitters (BSs) with an HWP [81]. In addition, the bit-flip operation on the time-bin state can be completed by the active switch (AS) [82] ($\sigma_x^T = |S\rangle\langle L| + |L\rangle\langle S|$). Then, the state of six-photon system $abca'b'c'$ changes to

$$\begin{aligned}|\Psi_1\rangle = &(\alpha^2|HHHVVV\rangle + \beta^2|VVVHHH\rangle \\ &+\alpha\beta|HHHHHH\rangle + \alpha\beta|VVVVVV\rangle) \\ &\otimes(\gamma^2|a_1b_1c_1a_2'b_2'c_2'\rangle + \delta^2|a_2b_2c_2a_1'b_1'c_1'\rangle \\ &+\gamma\delta|a_1b_1c_1a_1'b_1'c_1'\rangle + \gamma\delta|a_2b_2c_2a_2'b_2'c_2'\rangle) \\ &\otimes(\eta^2|SSSLLL\rangle + \xi^2|LLLSSS\rangle \\ &+\eta\xi|SSSSSS\rangle + \eta\xi|LLLLLL\rangle). \end{aligned} \tag{5}$$

The quantum circuit of our hyper-ECP for the three-photon nine-qubit partially hyperentangled GHZ state with unknown parameters is shown in Fig. 1. In the entire concentration procedure, Alice, Bob, and Charlie perform the polarization parity-check, spatial-mode parity-check, and time-bin parity-check measurements on photon pairs aa', bb', and cc', respectively.

In detail, Alice sends the wave packets from spatial modes a_1 and a_1' to PBS$_1$ and the wave packets from spatial modes a_2 and a_2' to PBS$_2$, as shown in Fig. 1(a). Here, PBS represents a polarizing beam splitter, which is used to perform a polarization parity-check measurement on photons a and a'. If the two photons a and a' have the same polarization state (e.g., HH or VV, defined as even-parity mode), one and only one photon would be detected by the single-photon measurement (SPM), as shown in Fig. 2. Otherwise, both photons a and a' would or would not be detected by the SPM. By post-selecting the even-parity case, the state of six-photon system $abca'b'c'$ is projected to

$$\begin{aligned}|\Psi_2\rangle = &\frac{1}{\sqrt{2}}[|HHHHHH\rangle \\ &\otimes(\gamma^2|a_1'b_1c_1a_2b_2'c_2'\rangle + \delta^2|a_2'b_2c_2a_1b_1'c_1'\rangle \\ &+\gamma\delta|a_1'b_1c_1a_1b_1'c_1'\rangle + \gamma\delta|a_2'b_2c_2a_2b_2'c_2'\rangle) \\ &+|VVVVVV\rangle \\ &\otimes(\gamma^2|a_1b_1c_1a_2'b_2'c_2'\rangle + \delta^2|a_2b_2c_2a_1'b_1'c_1'\rangle \\ &+\gamma\delta|a_1b_1c_1a_1'b_1'c_1'\rangle + \gamma\delta|a_2b_2c_2a_2'b_2'c_2'\rangle)] \\ &\otimes(\eta^2|SSSLLL\rangle + \xi^2|LLLSSS\rangle \\ &+\eta\xi|SSSSSS\rangle + \eta\xi|LLLLLL\rangle). \end{aligned} \tag{6}$$

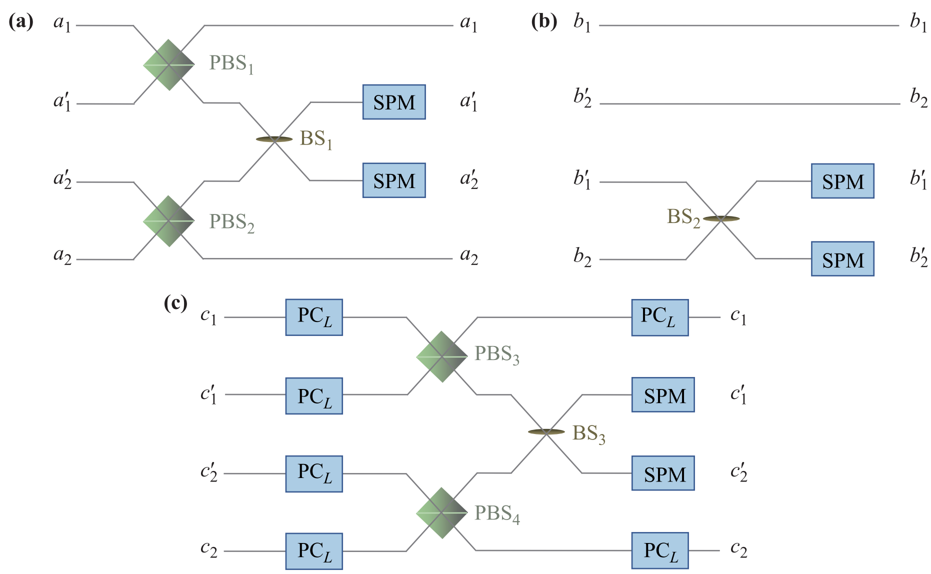

Fig. 1 Schematic of our hyper-ECP for the three-photon nine-qubit partially hyperentangled GHZ state in the polarization, spatial-mode, and time-bin DOFs, where the parameters of the state are unknown to the three distant parties. **(a)** Polarization parity-check measurement performed by Alice. **(b)** Spatial-mode parity-check measurement performed by Bob. **(c)** Time-bin parity-check measurement performed by Charlie. PBS represents a polarizing beam splitter, which transmits the photon in the horizontal polarization state $|H\rangle$ and reflects the photon in the vertical polarization state $|V\rangle$. PC$_L$ (PC$_S$) represents a Pockels cell, which is used to perform a polarization bit-flip operation when the L (S) component is present. BS represents a 50:50 beam splitter, which is used to perform a Hadamard operation on the spatial-mode DOF of a photon [$|i_1\rangle \rightarrow \frac{1}{\sqrt{2}}(|i_1\rangle + |i_2\rangle)$, $|i_2\rangle \rightarrow \frac{1}{\sqrt{2}}(|i_1\rangle - |i_2\rangle)$]. SPM represents a single-photon measurement.

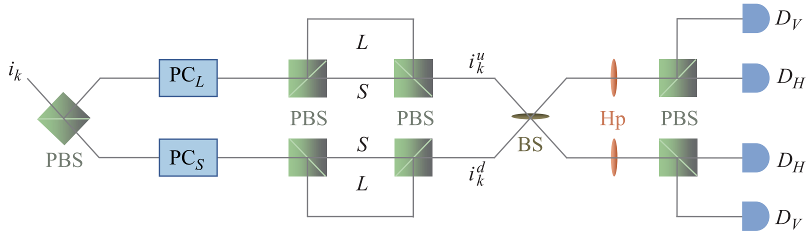

Fig. 2 Schematic of the SPM. H$_P$ represents an HWP, which is used to perform a Hadmard operation on the polarization DOF of a photon $[|H\rangle \to \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle), |V\rangle \to \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)]$. i_k^u and i_k^d represent two spatial modes, where u and d represent the top and bottom spatial modes, respectively. D represents a photon detector.

Bob performs the spatial-mode parity-check measurement on photons b and b', as shown in Fig. 1(b). If the two photons b and b' have the same spatial-mode state (e.g., b_1b_1' or b_2b_2', defined as even-parity mode), one and only one photon would be detected by the SPM. Otherwise, both photons b and b' would or would not be detected by the SPM. By postselecting the even-parity case, the state of six-photon system $abca'b'c'$ is projected to

$$\begin{aligned}|\Psi_3\rangle = \frac{1}{2}[&|HHHHHH\rangle\\ &\otimes(|a_1'b_1c_1a_1b_1'c_1'\rangle + |a_2'b_2'c_2a_2b_2c_2'\rangle)\\ &+|VVVVVV\rangle\\ &\otimes(|a_1b_1c_1a_1'b_1'c_1'\rangle + |a_2b_2'c_2a_2'b_2c_2'\rangle)]\\ &\otimes(\eta^2|SSSLLL\rangle + \xi^2|LLLSSS\rangle\\ &+\eta\xi|SSSSSS\rangle + \eta\xi|LLLLLL\rangle).\end{aligned} \tag{7}$$

Charlie sends the wave packets from spatial modes c_1 and c_1' to PC$_L$ and PBS$_3$, and the wave packets from spatial modes c_2 and c_2' to PC$_L$ and PBS$_4$, as shown in Fig. 1(c), where PC$_L$ and PBS are used to perform a time-bin parity-check measurement on photons c and c'. The PC$_L$ (PC$_S$) represents a Pockels cell [83], which is activated only when the L (S) component is present. If the two photons c and c' have the same time-bin state (e.g., LL or SS, defined as even-parity mode), one and only one photon would be detected by the SPM. Otherwise, both photons c and c' would or would not be detected by the SPM. By postselecting the even-parity case and considering the indistinguishability of photons aa', bb', and cc', the state of six-photon system $abca'b'c'$ is projected to

$$\begin{aligned}|\Psi_4\rangle = \frac{1}{2\sqrt{2}}[&(|HHHHHH\rangle + |VVVVVV\rangle)|SSSSSS\rangle\\ &+(|HHVHHV\rangle + |VVHVVH\rangle)|LLLLLL\rangle]\\ &\otimes(|a_1b_1c_1a_1'b_1'c_1'\rangle + |a_2b_2c_2a_2'b_2'c_2'\rangle).\end{aligned} \tag{8}$$

With the effect of another PC$_L$ on spatial modes c_1 and c_2, the state of six-photon system $abca'b'c'$ changes to

$$\begin{aligned}|\Psi_5\rangle = \frac{1}{2\sqrt{2}}[&(|HHHHHH\rangle + |VVVVVV\rangle)|SSSSSS\rangle\\ &+(|HHHHHV\rangle + |VVVVVH\rangle)|LLLLLL\rangle]\\ &\otimes(|a_1b_1c_1a_1'b_1'c_1'\rangle + |a_2b_2c_2a_2'b_2'c_2'\rangle).\end{aligned} \tag{9}$$

Finally, Alice, Bob, and Charlie measure photons a', b', and c', respectively, with BS and SPM. If the three photons $a'b'c'$ emit from the spatial modes $a_1'b_1'c_2'$, $a_2'b_1'c_1'$, $a_1'b_2'c_1'$ or $a_2'b_2'c_2'$, a spatial-mode phase-flip operation σ_z^S ($\sigma_z^S = |b_1\rangle\langle b_1| - |b_2\rangle\langle b_2|$) must be performed on photon b. Otherwise, no spatial-mode feed-forward operation is required. The feed-forward operations on polarization and time-bin DOFs of photon b are dependent on the results of the SPM. For example, a photon in state $|H^S\rangle$ is injected into the SPM from the spatial mode i_k. After it passes through PBS, PC$_S$, PBS, PBS, BS, and H$_P$ shown in Fig. 2, the state of the photon evolves as follows:

$$\begin{aligned}|H^S\rangle \otimes |i_k\rangle &\xrightarrow{PBS,PC_S} |V^S\rangle \otimes |i_k^d\rangle\\ &\xrightarrow{PBS,PBS} |V^{SL}\rangle \otimes |i_k^d\rangle\\ &\xrightarrow{BS} \frac{1}{\sqrt{2}}|V^{SL}\rangle \otimes (|i_k^u\rangle - |i_k^d\rangle)\\ &\xrightarrow{H_p} \frac{1}{2}(|H^{SL}\rangle - |V^{SL}\rangle) \otimes (|i_k^u\rangle - |i_k^d\rangle).\end{aligned} \tag{10}$$

Here, $|H^S\rangle$ (or $|V^S\rangle$) indicates that the polarization of the photon is $|H\rangle$ (or $|V\rangle$) and the time-bin of the photon is $|S\rangle$. $|H^{SL}\rangle$ (or $|V^{SL}\rangle$) means that the state $|H^S\rangle$ (or $|V^S\rangle$) passes through the path L. After the three photons a', b', and c' are detected by SPM, the feed-forward operations on polarization, spatial-mode, and time-bin DOFs are performed on photon b. The relations of the results of the SPM on $a'b'c'$, the corresponding output states of abc, and the feed-forward operations on polarization and time-bin DOFs of photon b are shown in Table 1. The four states in Table 1 are as follows:

$$|\phi^{++}\rangle_{PT}^{abc} = \frac{1}{2}(|HHH\rangle + |VVV\rangle) \otimes (|SSS\rangle + |LLL\rangle)$$

$$|\phi^{+-}\rangle_{PT}^{abc} = \frac{1}{2}(|HHH\rangle + |VVV\rangle) \otimes (|SSS\rangle - |LLL\rangle)$$

Table 1 Results of the SPM on $a'b'c'$, the corresponding output states of abc, and the feed-forward operations on photon b.

Detection ($a'b'c'$)	State of abc	Operation (b)
$H^uH^uH^u$, $V^uV^uH^u$, $H^uV^uV^u$, $V^uH^uV^u$, $H^uH^dH^d$, $V^uV^dH^d$, $H^uV^dV^d$, $V^uH^dV^d$, $H^dH^dV^u$, $V^dV^dV^u$, $H^dV^dH^u$, $V^dH^dH^u$, $H^dH^uV^d$, $V^dV^uV^d$, $H^dV^uH^d$, $V^dH^uH^d$	$\|\phi^{++}\rangle_{PT}^{abc}$	I
$H^uH^uV^u$, $V^uV^uV^u$, $H^uV^uH^u$, $V^uH^uH^u$, $H^uH^dV^d$, $V^uV^dV^d$, $H^uV^dH^d$, $V^uH^dH^d$, $H^dH^dH^u$, $V^dV^dH^u$, $H^dV^dV^u$, $V^dH^dV^u$, $H^dH^uH^d$, $V^dV^uH^d$, $H^dV^uV^d$, $V^dH^uV^d$	$\|\phi^{+-}\rangle_{PT}^{abc}$	σ_z^T
$H^uH^uH^d$, $V^uV^uH^d$, $H^uV^uV^d$, $V^uH^uV^d$, $H^uH^dH^u$, $V^uV^dH^u$, $H^uV^dV^u$, $V^uH^dV^u$, $H^dH^dV^d$, $V^dV^dV^d$, $H^dV^dH^d$, $V^dH^dH^d$, $H^dH^uV^u$, $V^dV^uV^u$, $H^dV^uH^u$, $V^dH^uH^u$	$\|\phi^{-+}\rangle_{PT}^{abc}$	σ_z^P
$H^uH^uV^d$, $V^uV^uV^d$, $H^uV^uH^d$, $V^uH^uH^d$, $H^uH^dV^u$, $V^uV^dV^u$, $H^uV^dH^u$, $V^uH^dH^u$, $H^dH^dH^d$, $V^dV^dH^d$, $H^dV^dV^d$, $V^dH^dV^d$, $H^dH^uH^u$, $V^dV^uH^u$, $H^dV^uV^u$, $V^dH^uV^u$	$\|\phi^{--}\rangle_{PT}^{abc}$	σ_z^T, σ_z^P

$$|\phi^{-+}\rangle_{PT}^{abc} = \frac{1}{2}(|HHH\rangle - |VVV\rangle) \otimes (|SSS\rangle + |LLL\rangle)$$
$$|\phi^{--}\rangle_{PT}^{abc} = \frac{1}{2}(|HHH\rangle - |VVV\rangle) \otimes (|SSS\rangle - |LLL\rangle). \quad (11)$$

After the conditional feed-forward operations are performed on photon b, Alice, Bob, and Charlie can obtain the three-photon nine-qubit maximally hyperentangled GHZ state $|\psi\rangle_{abc} = \frac{1}{2\sqrt{2}}(|HHH\rangle + |VVV\rangle)_{abc} \otimes (|a_1b_1c_1\rangle + |a_2b_2c_2\rangle) \otimes (|SSS\rangle + |LLL\rangle)_{abc}$ with a total success probability of $P = 8|\alpha\beta\gamma\delta\eta\xi|^2$.

3 Discussion and summary

Hyperentanglement concentration is critical in high-capacity long-distance quantum communication for improving the entanglement of nonlocal hyperentangled photon systems, which can reduce the number of resources consumed in quantum communication with entanglement in multiple DOFs. In this study, we presented a hyper-ECP for the three-photon nine-qubit partially hyperentangled GHZ state using the Schmidt projection method. In the extended version of the hyper-EPC for the hyperentangled Bell state [84], all three distant parties must perform all three parity-check measurements on the three DOFs of their photon pairs. Thus, six identical nonlocal three-photon systems are required for these operations by using linear optics. In our hyper-ECP, three distant parties perform only the polarization parity-check, spatial-mode parity-check, and time-bin parity-check measurements, respectively, on their photon pairs with linear optical elements and Pockels cells. Therefore, only two identical nonlocal three-photon systems are required, thus consuming fewer resources than the extended version of the hyper-ECP for hyperentangled Bell states. The success probability of our hyper-ECP is $P = 8|\alpha\beta\gamma\delta\eta\xi|^2$ (e.g., the relationship of the success probability of our hyper-ECP to the parameters of the partially hyperentangled GHZ state is shown in Fig. 3 for the case $|\alpha| = |\delta| = |\xi|$, and the relationship is similar for other cases.).

Obviously, the parity-check measurements on the three DOFs are the main techniques used by our hyper-ECP, and distinguishing the 1-photon case from the 0- and 2-photon cases is essential. One means of solving this problem is to use the photon-number-resolving detector. Another is to use the single-photon detector and postselection. If the efficiency of the photon detector is less than 100%, the postselection is necessary in this hyper-ECP.

Our hyper-ECP is also suitable for concentrating the nonlocal partially hyperentangled N-photon GHZ state $|\psi_N\rangle$ with unknown parameters. In detail, only three of the N distant parties (e.g., Alice, Bob, and Charlie) are required to perform the parity-check measurements on the three DOFs of their photon pairs, respectively, as shown in Fig. 1. In addition, the other distant parties only need to perform the local spatial-mode Hadmard operations and SPMs on the corresponding photons of their photon pairs in each step, which is simpler than the extended version of the hyper-ECP for hyperentangled Bell states. Here,

$$\begin{aligned}|\psi_N\rangle = &(\alpha|HH\cdots H\rangle + \beta|VV\cdots V\rangle)_{ab\cdots z}\\ &\otimes(\gamma|a_1b_1\cdots z_1\rangle + \delta|a_2b_2\cdots z_2\rangle)\\ &\otimes(\eta|SS\cdots S\rangle + \xi|LL\cdots L\rangle)_{ab\cdots z},\end{aligned} \quad (12)$$

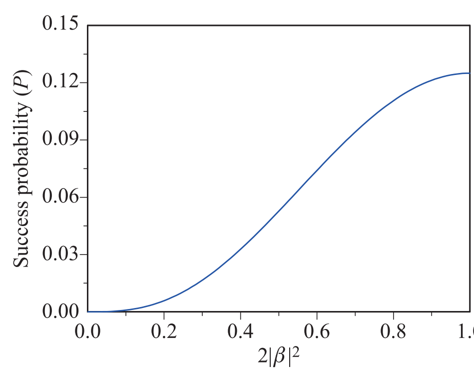

Fig. 3 Success probability of our hyper-ECP for three-photon systems in a partially hyperentangled GHZ state with unknown parameters. The parameters of the partially hyperentangled GHZ state are chosen as $|\alpha| = |\delta| = |\xi|$.

where the subscripts a, b, $\cdots$, and z represent the photons that are held by the N distant parties, named Alice, Bob, $\cdots$, and Zach, respectively. The success probability of the hyper-ECP for partially hyperentangled N-photon GHZ states remains $P = 8|\alpha\beta\gamma\delta\eta\xi|^2$ regardless of the photon number N.

In summary, we proposed a general hyper-ECP for nonlocal partially hyperentangled GHZ states in the polarization, spatial-mode, and time-bin DOFs using the Schmidt projection method. This method can also be extended to improve the entanglement of the nonlocal N-photon systems entangled in m DOFs ($N \geq m$), where m of the N distant parties are required to perform the parity-check measurements on the m DOFs of their photon pairs, and two identical nonlocal N-photon systems are required. Our hyper-ECP considers linear optical elements and Pockels cells and does not need to know the parameters of the state beforehand, thus making it more practical for high-capacity long-distance quantum communication.

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