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LETTER

A feasible approach to field concentrators of arbitrary shapes

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We propose a simple method to design field concentrators of arbitrary shapes based on Fabry–Pérot resonances. The material parameters are feasible in terms of metallic layered structures and gradient index dielectrics. The functionalities are well confirmed by numerical simulations.

Keywords field concentrator, Fabry–Pérot resonance, transformation optics

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Transformation optics [1, 2] offers a great tool for designing novel devices [3], such as invisibility cloaks [4], field rotators [5, 6], field concentrators [7, 8], etc. The material parameters are typically anisotropic and inhomogeneous, making implementation considerably challenging. Recently, Fabry–Pérot (FP) resonances are introduced in transformation optical device designs [9]. Such devices are usually easy to implement, with flexibility to work at multiple frequencies. For example, a circular field concentrator for multiple frequencies is realized in microwaves [9]. In this letter, we will further propose a method to design such field concentrators of arbitrary shapes, which are not feasible for conversional transformation optics in general [10]. We will start with a circular case, and show how to achieve field concentrators of arbitrary shapes, if the inner and outer boundaries are carefully chosen. The theory is simple, and we will verify the functionalities using finite element methods from COMSOL Multiphysics. Finally, we will provide a detailed design with feasible materials.

We start from a mapping of a field concentrator in two dimensions. This mapping compresses the region of $c < r' < b$ [in Fig. 1(a)] into $a < r < b$ [in Fig. 1(b)], which is called a field concentrator [7]. It can collect all of the energy that originally locates in $r' < c$ [in Fig. 1a)] into the region of $r < a$ [in Fig. 1(b)]. The inner core has a refractive index of c/a ($c > a$) if the inner mapping is a linear function of $r' = \frac{c}{a}r$ ($\theta' = \theta, z' = z$). The electromagnetic (EM) parameters in the region of $a < r < b$ are mostly anisotropic and depends on r. Let us consider the transverse magnetic (TM) polarization for example. With the limit of $c \to b$ [see Fig. 1(c)], the set of EM parameters becomes $\varepsilon_r \to \infty$, $\varepsilon_\theta \to 0$, and $\mu_z \to 0$, which is extremely difficult to implement. Nevertheless, one can use $\varepsilon_r \to \infty$, $\varepsilon_\theta = \varepsilon(r) = n^2(r)$, and $\mu_z = 1$ as a replacement [9]. Such a version could be implemented by an alternative layered structure of perfect electric conductors (PECs) and gradient index dielectrics along r direction, see in Fig. 1(d). The refractive index profile of the gradient index dielectrics is simply $n(r)$ if the filling ratio of PECs is tiny. This new type of field concentrators is based on FP resonances and can work at multiple wavelengths of $\int_a^b n(r)\mathrm{d}r = \frac{m\lambda}{2}$ (where m is an integer) [8]. If the inner core is chosen as $\varepsilon = \frac{b^2}{a^2}$, $\mu_z = 1$, and $n(a) = \frac{b}{a}$, $n(b) = 1$, the field concentrator becomes nearly invisible. For instance, we can choose $n(r) = \frac{b}{r}$ to obtain a perfect field concentrator. Let us then consider a more general case. Suppose we have another mapping that can compress the region of $c(\theta) < r' < b(\theta)$ [in Fig. 1(c)] into $a(\theta) < r < b(\theta)$ along r direction, where $a(\theta) = \frac{1}{\alpha}b(\theta)$ and $c(\theta) = \frac{1}{\beta}b(\theta)$. The field concentrator based on this mapping can collect all of the energy that is originally localized at $r' < c(\theta)$ [in Fig. 1(c)] into the region of $r < a(\theta)$. Hence it is a field concentrator of an arbitrary shape [10]. The inner core has a refractive index of $\frac{c(\theta)}{a(\theta)} = \frac{\alpha}{\beta}$ ($\alpha > \beta$) if the inner mapping is a linear function of $r' = \frac{\alpha}{\beta}r$. By taking the limit of $c(\theta) \to b(\theta)$ (or $\beta \to 1$), the EM parameters in $a(\theta) < r < b(\theta)$ become $\varepsilon_r \to \infty$, $\varepsilon_\theta \to 0$, and $\mu_z \to 0$ as well. Likewise, we can use $\varepsilon_r \to \infty$, $\varepsilon_\theta = \varepsilon(r) = n^2(r)$, and $\mu_z = 1$ as a replacement for the region of $a(\theta) < r < b(\theta)$. It is amazing that if we take $n(r) = \frac{n_0}{r}$, this type of field concentrators can satisfy the FP resonance conditions:

$$\int_{a(\theta)}^{b(\theta)} n(r)\mathrm{d}r = \int_{a(\theta)}^{b(\theta)} \frac{n_0}{r}\mathrm{d}r = n_0 \log \frac{b(\theta)}{a(\theta)} = n_0 \log \alpha = \frac{m\lambda}{2}, \quad (1)$$

where m is an integer. Therefore, this version can also be implemented by an alternative layered structure of PECs and gradient index dielectrics with the profile of $n(r) = \frac{n_0}{r}$, see the schematic plot in Fig. 1(d). However, it is not a perfect version similar to the circular one, as the impedance cannot be matched at every position along $r = a(\theta)$ and $r = b(\theta)$. In the following sections, we will set $\varepsilon = \alpha^2 = 4$ ($\alpha = 2$), $\mu_z = 1$ for the inner core, and $n_0 = 1$, as an example.

To show the functionalities of the above field concentrators, we will perform numerical simulations for different shapes using the EM parameters of $\varepsilon_r \to \infty$, $\varepsilon_\theta = \varepsilon(r) = \frac{1}{r^2}$, and $\mu_z = 1$ at $\lambda = \frac{2\log 2}{m}$ (in simulations, we set $\varepsilon_r = 100\,000$ and $m = 2$). First, we plot the magnetic field pattern for a circular field concentrator with $b(\theta) = b = 1$ in Fig. 2(a), which demonstrates almost perfect invisibility. We then change $b(\theta)$ so that it becomes a square, a hexagon, and a pentagram, and plot the related magnetic field patterns in Figs. 2(b), (c), and (d), respectively. All of the above field patterns with nearly perfect invisibility show that the field concentrators based on FP resonances could be achieved for any

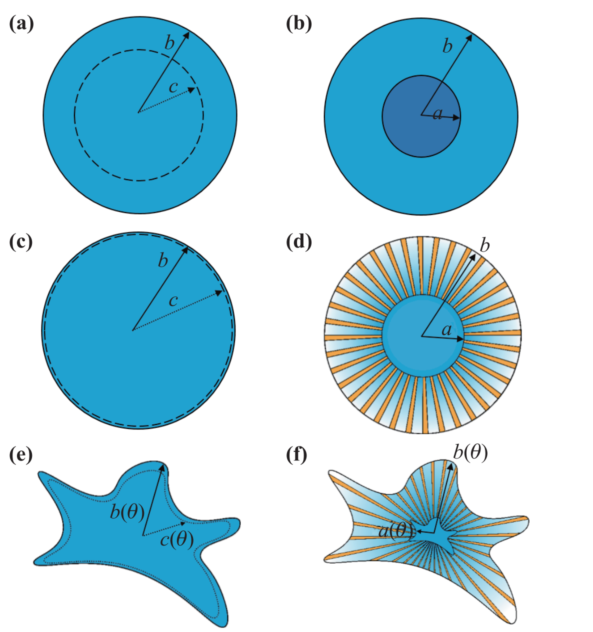

Fig. 1 **(a)** Virtual space (r' space, a flat space) of a circular field concentrator. **(b)** Physical space (r space) of a circular field concentrator. **(c)** Virtual space (r' space) of a circular field concentrator with $c \to b$. **(d)** Implementation of a circular field concentrator based on FP resonances. **(e)** Virtual space (r' space, a flat space) of a field concentrator of an arbitrary shape with $c(\theta) \to b(\theta)$. **(f)** Implementation of a field concentrator of an arbitrary shape based on FP resonances.

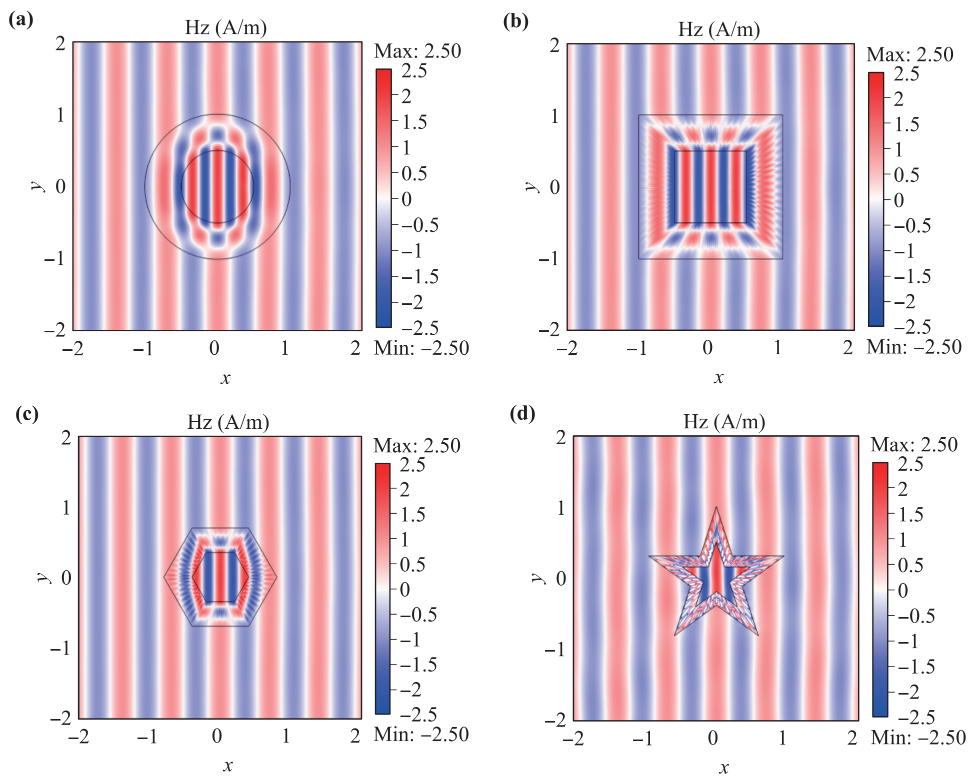

Fig. 2 Magnetic field patterns for field concentrators of **(a)** a circle, **(b)** a square, **(c)** a hexagon, and **(d)** a pentagram. The wavelength is chosen at $m = 2$.

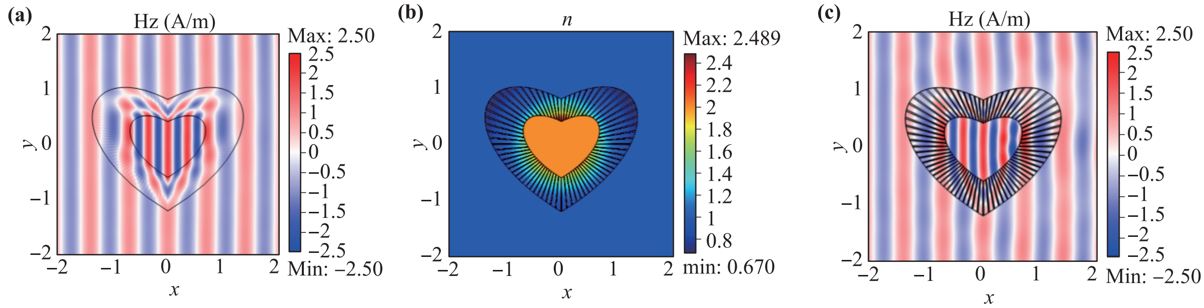

Fig. 3 **(a)** Magnetic field pattern for an ideal field concentrator of a heart shape. **(b)** Refractive index profile of the field concentrator with PECs. **(c)** Magnetic field pattern for the alternative layered structure of PECs and dielectric profiles. The filling ratio of PECs is set as 0.2.

arbitrary shapes.

How to realize such a field concentrator? We follow the proposal of Ref. [9] in Figs. 1(d) and (f). Starting from an ideal version for a heart shape, we plot the field pattern in Fig. 3(a) (similar to those in Fig. 2). By replacing the region of $a(\theta) < r < b(\theta)$ with an alternative layered structure of PECs and a refractive index profile of $\frac{1}{r}$ [see Fig. 3(b)], we plot the field pattern in Fig. 3(c) to prove the validity of this structure. The functionality is very good with small scattering. In Fig. 3(b), a region with refractive index of less than unity remains, which could be overcome by choosing a larger value of n_0. Therefore, all regions involving field concentrators could be realized with normal dielectrics.

In conclusion, we have proposed a simple method for designing field concentrators of arbitrary shapes with pure dielectric profiles and PECs. As they are based on FP resonances, in principle they can function for multiple wavelengths. As the fields inside the core medium could be enhanced strongly, such field concentrators will find applications not only in light concentration and manipulation in plasmonics, but also in the design of devices such as sensitive detections and solar cells [11]. In addition, it could be extended to other dynamic waves, including acoustic waves [12], surface water waves, and ocean waves [13].

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References

1. U. Leonhardt, Optical conformal mapping, *Science* 312(5781), 1777 (2006)
2. J. B. Pendry, D. Schurig, and D. R. Smith, Controlling electromagnetic fields, *Science* 312(5781), 1780 (2006)
3. H. Y. Chen, C. T. Chan, and P. Sheng, Transformation optics and metamaterials, *Nat. Mater.* 9(5), 387 (2010)
4. D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, Metamaterial electromagnetic cloak at microwave frequencies, *Science* 314(5801), 977 (2006)
5. H. Y. Chen and C. T. Chan, Transformation media that rotate electromagnetic fields, *Appl. Phys. Lett.* 90(24), 241105 (2007)
6. H. Y. Chen, B. Hou, S. Chen, X. Ao, W. Wen, and C. T. Chan, Design and experimental realization of a broadband transformation media field rotator at microwave frequencies, *Phys. Rev. Lett.* 102(18), 183903 (2009)
7. M. Rahm, D. Schurig, D. A. Roberts, S. A. Cummer, D. R. Smith, and J. B. Pendry, Design of electromagnetic cloaks and concentrators using form-invariant coordinate transformations of Maxwell's equations, *Photon. Nanostru.- Fund. Appl.* 6, 87 (2008)
8. M. M. Sadeghi, H. Nadgaran, and H. Chen, Perfect field concentrator using zero index metamaterials and perfect electric conductors, *Front. Phys.* 9(1), 90 (2014)
9. M. M. Sadeghi, S. Li, L. Xu, B. Hou, and H. Chen, Transformation optics with Fabry-Pérot resonances, *Sci. Rep.* 5(1), 8680 (2015)
10. W. X. Jiang, T. J. Cui, Q. Cheng, J. Y. Chin, X. M. Yang, R. Liu, and D. R. Smith, Design of arbitrarily shaped concentrators based on conformally optical transformation of nonuniform rational B-spline surfaces, *Appl. Phys. Lett.* 92(26), 264101 (2008)
11. J. A. Schuller, E. S. Barnard, W. Cai, Y. C. Jun, J. S. White, and M. L. Brongersma, Plasmonics for extreme light concentration and manipulation, *Nat. Mater.* 9(3), 193 (2010)
12. H. Y. Chen and C. T. Chan, Acoustic cloaking and transformation acoustics, *J. Phys. D Appl. Phys.* 43(11), 113001 (2010)
13. M. Farhat, S. Enoch, S. Guenneau, and A. B. Movchan, Broadband cylindrical acoustic cloak for linear surface waves in a fluid, *Phys. Rev. Lett.* 101(13), 134501 (2008)