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Research article

Multi-hop teleportation in a quantum network based on mesh topology

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In this paper, we propose a mesh-topology-based multi-hop teleportation scheme for a quantum network. By using the proposed scheme, quantum communication can be realized between two arbitrary nodes, even when they do not share a direct quantum channel. Einstein–Podolsky–Rosen pairs are used as quantum channels. The source node (initial sender) and all intermediate nodes make Bell measurements independently. They send the results to the destination node (final receiver) by classical channels. The quantum state can be determined from the Bell measurement result, and only the destination node is required for simple unitary transformation. This method of simultaneous measurement contributes significantly to quantum network by reducing the hop-by-hop transmission delay.

Keywords multi-hop teleportation, quantum network, mesh topology

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1 Introduction

Mesh topology is a network setup where nodes are connected to one another; it allows most transmissions to be distributed even when one of the connections fails [1, 2]. Specifically, mesh topology is dynamically self-organized and self-configured; mesh connectivity in the network can be automatically established and maintained by the nodes. Mesh networks have many advantages, they have low up-front costs, are easy to maintain, robust, and provide reliable service coverage. However, there are few existing studies [3, 4] on multi-hop teleportation in quantum networks based on mesh topology.

Quantum teleportation is critical for the realization of quantum communication in quantum networks [5–7]. There have been studies [8–13] into the transmission of a pair of unknown entangled particles between two nodes. However, for two nodes which do not share a quantum channel, the study of quantum channels may waste quantum resources and have a large footprint. For instance, the quantum channel is made up of a W state and an Einstein–Podolsky–Rosen (EPR) pair [8]. A Greenberger–Horne–Zeilinger (GHZ) state and a Bell pair can also be used as the quantum channel [9]. An EPR pair is the best choice for the quantum channel because it uses the minimum amount of resourced and has maximal entanglement characteristics [14, 15]. When it comes to the transmission of two unknown entangled particles, a quantum channel consisting of two EPR pairs provides a better solution than other entanglement resources.

Bell measurements should be made in the quantum teleportation process. Partner particles will automatically collapse the remaining particles into an entangled state [16–19]. For one Bell measurement, four types of measured results will be obtained and the remaining particles will automatically collapse into a new entangled quantum state. So far, few studies have explored the relationship between the Bell measurement result (BMR) and the quantum state.

In this paper, we propose a scheme of multi-hop teleportation in a quantum network based on mesh topology, which enables quantum teleportation to be realized between two nodes without a direct quantum channel. The required EPR pairs are distributed between two nodes by intermediate nodes using the entanglement swapping method. In this study, we use simultaneous Bell measurements in the source node (initial sender) and all of the intermediate nodes where the measurement results are sent directly to the destination node (final receiver). Only the destination node is required for unitary transformation; hence, this method reduces the transmission time. We also make a specific calculation of the recov-

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ering operations which correspond to all combinations of the measurement results and find the relationship between the BMR and the quantum state.

2 A quantum network system based on mesh topology

Figure 1 shows multi-hop teleportation in a quantum network based on mesh topology (QNM). The QNM is composed of spatially separated quantum nodes and classical nodes. There is a one-to-one relationship between the classical and quantum nodes. Quantum nodes are linear optical components with quantum information encoded in a number of photons which fly from node to node. They serve as quantum memories, which store information, and perform gate operations based on quantum interference effects among indistinguishable photons. In this network, most of the packets of information transmitted are quantum bits (qubits) which are represented as a linear combination of the computational basis states $|0\rangle$ and $|1\rangle$. It is supposed that the teleported quantum state is $|\psi\rangle = (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)$, where a, b, c, and d are complex probability amplitudes which satisfy $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$.

In the QNM, quantum teleportation is realized among nodes via quantum and classical channels. As shown in Fig. 1, quantum and classical channels are established for transmitting quantum and classical information, respectively. The dashed and solid lines represent classical channels and quantum channels, respectively. In our scheme, two arbitrary nodes share classical channels, and there are insufficient quantum channels shared by these nodes.

Two nodes which share two EPR pairs are adjacent quantum nodes named "Alice" and "Bob". Alice needs to do two Bell measurements. The four types of Bell pairs used in the quantum networks are defined as

$$
\begin{aligned}
|\phi^+\rangle &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \\
|\phi^-\rangle &= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle), \\
|\varphi^+\rangle &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle), \\
|\varphi^-\rangle &= \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle). \qquad (1)
\end{aligned}
$$

In Fig. 1, all of the outcomes from the Bell measurements are transmitted directly by classical channels. Quantum and classical channels can be different. Adjacent quantum nodes can transfer quantum information due to the existence of direct quantum and classical channels; otherwise, direct communication between the nodes would be forbidden. For instance, as shown in Fig. 1, a

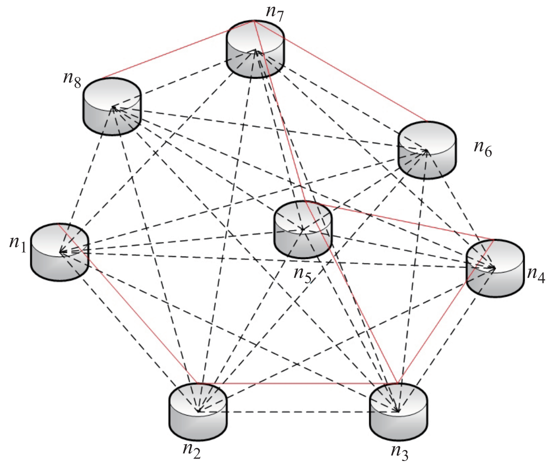

Fig. 1 Multi-hop teleportation in a quantum network based on mesh topology. The dashed lines and solid lines represent classical channels and quantum channels, respectively.

direct classical channel can be identified between n_1 and n_3 regardless of the fact that there is no direct quantum channel between them. Fortunately, with the help of intermediate nodes, quantum channels between the nodes using entanglement swapping technology can be established. In Fig. 1, n_1 can communicate with n_3 through $n_1 \to n_2 \to n_3$. In fact, n_1 and n_2 transmit Bell measurement outcomes to n_3 through classical channels in a direct manner. For example, n_4 can communicate with n_8 through $n_4 \to n_5 \to n_7 \to n_8$.

3 Teleportation in a quantum network based on mesh topology

3.1 One-hop teleportation based on different BMRs

Quantum teleportation has been widely studied in order to transmit quantum states from one node to another. The EPR pairs are entanglement resources containing a minimal number of particles. In the QNM, adjacent quantum nodes share two EPR pairs. The sender performs two Bell measurements and sends the results to the receiver via a classical channel. The quantum circuit for one-hop quantum teleportation is presented in Fig. 2. In this figure, Alice wants to send $|\psi\rangle$ to Bob using two EPR pairs. The whole quantum system can be written as

$$
\begin{aligned}
|\psi\rangle_{t1} &= |\psi\rangle_{12} \otimes |\psi\rangle_{34} \otimes |\psi\rangle_{56} \\
&= (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)_{12} \\
&\quad \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{34} \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{56}. \qquad (2)
\end{aligned}
$$

First, Alice needs to make two Bell measurements and send the measurement results to n_2. Sixteen kinds

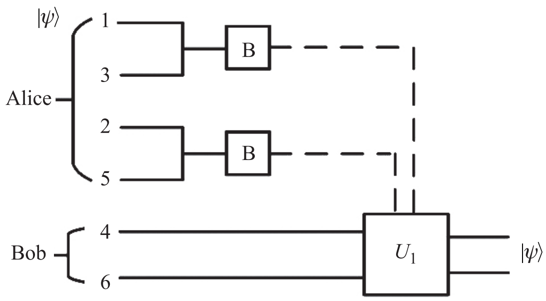

Fig. 2 Quantum circuit for one-hop quantum teleportation. Dashed lines: Classical channels; solid lines: Quantum channels.

of results can be obtained, $|\phi^{\pm}\rangle_{13}|\phi^{\pm}\rangle_{25}$, $|\phi^{\pm}\rangle_{13}|\varphi^{\pm}\rangle_{25}$, $|\varphi^{\pm}\rangle_{13}|\phi^{\pm}\rangle_{25}$, $|\varphi^{\pm}\rangle_{13}|\varphi^{\pm}\rangle_{25}$. Then, as shown in Table 1, Bob performs the relevant unitary transformation on its particles to recover the initial quantum state in accordance with the BMR. The unitary transformations U_1 are expressed as

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{3}$$

$$(\sigma_x)_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \tag{4}$$

$$(\sigma_x)_6 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{5}$$

$$(\sigma_x)_4(\sigma_x)_6 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \tag{6}$$

Table 1 shows the unitary transformations which Bob needs to perform to recover the initial quantum state according to the BMR sent by Alice. For example, when the BMR received by Bob is $|\phi^{+}\rangle_{13}|\varphi^{+}\rangle_{25}$, Bob must perform $(\sigma_x)_6$. It is obvious that the minus signs before the states $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$ are global phases for the final states which make no difference to the teleportation results. Therefore, they are ignored in the following treatment. Table 1 lists the unitary transformations corresponding to the 16 possible system states in Bob for different BMRs. The relation between the unitary transformation and the possible system state varies according to the result of the Bell measurements. Consequently, different unitary transformations should be carried out if the BMRs measured by Alice are inconsistent. For example, if the BMR is $|\phi^{+}\rangle_{13}|\phi^{+}\rangle_{25}$, there is no need for Bob to do anything, whereas if the BMR is $|\varphi^{+}\rangle_{13}|\varphi^{+}\rangle_{25}$, then Bob needs to do X unitary transformations on both particle 4 and particle 6.

In practice, all BMRs are not the same. Hence, the BMR in the teleportation process is necessary and significant for determining the corresponding unitary transformations required to recover the initial quantum state.

3.2 Two-hop teleportation based on different BMRs

In the QNM, no direct EPR pairs are shared by two arbitrary nodes when classical channels exist. In this case, the multi-hop method can be regarded as a good choice for realizing teleportation through the use of entanglement swapping technology. The two-hop model is the most basic. Alice wants to communicate with Bob, but there is no direct EPR pair. We use entanglement swapping technology to establish contact between them. Alice and Charlie do Bell measurements and send the results to Bob by direct classical channels.

Figure 3 shows the quantum circuit for two-hop quantum teleportation, Alice → Charlie → Bob. The whole

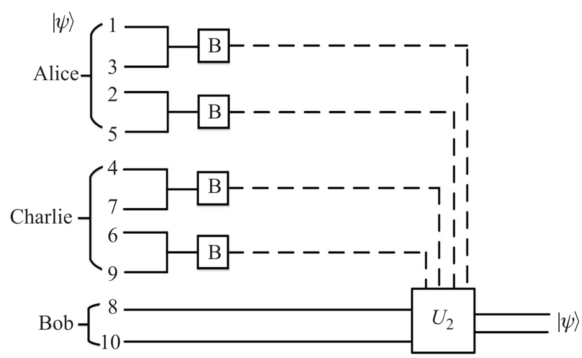

Fig. 3 Quantum circuit for two-hop quantum teleportation.

Table 1 The relations among BMR, quantum state and unitary matrix.

Alice	BMR	Bob	U_1
$a\|00\rangle + b\|01\rangle + c\|10\rangle + d\|11\rangle$	$\|\phi^{\pm}\rangle_{13}\|\phi^{\pm}\rangle_{25}$	$(a\|00\rangle \pm b\|01\rangle \pm c\|10\rangle \pm d\|11\rangle)_{46}$	I
	$\|\phi^{\pm}\rangle_{13}\|\varphi^{\pm}\rangle_{25}$	$(a\|01\rangle \pm b\|00\rangle \pm c\|11\rangle \pm d\|10\rangle)_{46}$	$(\sigma_x)_6$
	$\|\varphi^{\pm}\rangle_{13}\|\phi^{\pm}\rangle_{25}$	$(a\|10\rangle \pm b\|11\rangle \pm c\|00\rangle \pm d\|01\rangle)_{46}$	$(\sigma_x)_4$
	$\|\varphi^{\pm}\rangle_{13}\|\varphi^{\pm}\rangle_{25}$	$(a\|11\rangle \pm b\|10\rangle \pm c\|01\rangle \pm d\|00\rangle)_{46}$	$(\sigma_x)_4(\sigma_x)_6$

system can be expressed as

$$\begin{aligned}|\psi\rangle_{t2} &= |\psi\rangle_{12} \otimes |\psi\rangle_{34} \otimes |\psi\rangle_{56} \otimes |\psi\rangle_{78} \otimes |\psi\rangle_{9,10} \\ &= (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)_{12} \\ &\quad \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{34} \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{56} \\ &\quad \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{78} \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{9,10}. \end{aligned} \tag{7}$$

Similarly, Alice needs to do two Bell measurements and sends the results to Bob. Meanwhile, Charlie gets the teleported quantum state and does two Bell measurements without any unitary transformations. Charlie then sends the results to Bob. Finally, Bob performs the related unitary transformation on its particles to recover the initial quantum state according to the BMRs, as presented in Table 2. The unitary transformations U_2 are expressed as

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{8}$$

$$(\sigma_x)_8 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \tag{9}$$

$$(\sigma_x)_{10} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{10}$$

$$(\sigma_x)_8(\sigma_x)_{10} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \tag{11}$$

As shown in Table 2, the quantum state will be obtained as one of four types irrespective of the BMR. We define S as a quantum state, which is one of the four possible states S_i $(i = 1, 2, 3, 4)$ given by

$$S_1 = a|00\rangle \pm b|01\rangle \pm c|10\rangle \pm d|11\rangle, \tag{12}$$

$$S_2 = a|01\rangle \pm b|00\rangle \pm c|11\rangle \pm d|10\rangle, \tag{13}$$

$$S_3 = a|10\rangle \pm b|11\rangle \pm c|00\rangle \pm d|01\rangle, \tag{14}$$

$$S_4 = a|11\rangle \pm b|10\rangle \pm c|01\rangle \pm d|00\rangle. \tag{15}$$

Therefore, the final unitary transformation is one of four types. We also define A as a unitary transformation, which is one of the four possible unitary transformations A_j $(j = 1, 2, 3, 4)$ given by

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{16}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \tag{17}$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{18}$$

Table 2 The relations among BMR, quantum state and unitary matrix.

BMR/Charlie	BMR/Bob	U_2
$\|\phi^\pm\rangle_{13}\|\phi^\pm\rangle_{25}/(a\|00\rangle \pm b\|01\rangle \pm c\|10\rangle \pm d\|11\rangle)_{46}$	$\|\phi^\pm\rangle_{47}\|\phi^\pm\rangle_{69}/(a\|00\rangle \pm b\|01\rangle \pm c\|10\rangle \pm d\|11\rangle)_{8,10}$	I
	$\|\phi^\pm\rangle_{47}\|\varphi^\pm\rangle_{69}/(a\|01\rangle \pm b\|00\rangle \pm c\|11\rangle \pm d\|10\rangle)_{8,10}$	$(\sigma_x)_{10}$
	$\|\varphi^\pm\rangle_{47}\|\phi^\pm\rangle_{69}/(a\|10\rangle \pm b\|11\rangle \pm c\|00\rangle \pm d\|01\rangle)_{8,10}$	$(\sigma_x)_8$
	$\|\varphi^\pm\rangle_{47}\|\varphi^\pm\rangle_{69}/(a\|11\rangle \pm b\|10\rangle \pm c\|01\rangle \pm d\|00\rangle)_{8,10}$	$(\sigma_x)_8(\sigma_x)_{10}$
$\|\phi^\pm\rangle_{13}\|\varphi^\pm\rangle_{25}/(a\|01\rangle \pm b\|00\rangle \pm c\|11\rangle \pm d\|10\rangle)_{46}$	$\|\phi^\pm\rangle_{47}\|\phi^\pm\rangle_{69}/(a\|01\rangle \pm b\|00\rangle \pm c\|11\rangle \pm d\|10\rangle)_{8,10}$	$(\sigma_x)_{10}$
	$\|\phi^\pm\rangle_{47}\|\varphi^\pm\rangle_{69}/(a\|00\rangle \pm b\|01\rangle \pm c\|10\rangle \pm d\|11\rangle)_{8,10}$	I
	$\|\varphi^\pm\rangle_{47}\|\phi^\pm\rangle_{69}/(a\|11\rangle \pm b\|10\rangle \pm c\|01\rangle \pm d\|00\rangle)_{8,10}$	$(\sigma_x)_8(\sigma_x)_{10}$
	$\|\varphi^\pm\rangle_{47}\|\varphi^\pm\rangle_{69}/(a\|10\rangle \pm b\|11\rangle \pm c\|00\rangle \pm d\|01\rangle)_{8,10}$	$(\sigma_x)_8$
$\|\varphi^\pm\rangle_{13}\|\phi^\pm\rangle_{25}/(a\|10\rangle \pm b\|11\rangle \pm c\|00\rangle \pm d\|01\rangle)_{46}$	$\|\phi^\pm\rangle_{47}\|\phi^\pm\rangle_{69}/(a\|10\rangle \pm b\|11\rangle \pm c\|00\rangle \pm d\|01\rangle)_{8,10}$	$(\sigma_x)_8$
	$\|\phi^\pm\rangle_{47}\|\varphi^\pm\rangle_{69}/(a\|11\rangle \pm b\|10\rangle \pm c\|01\rangle \pm d\|00\rangle)_{8,10}$	$(\sigma_x)_8(\sigma_x)_{10}$
	$\|\varphi^\pm\rangle_{47}\|\phi^\pm\rangle_{69}/(a\|00\rangle \pm b\|01\rangle \pm c\|10\rangle \pm d\|11\rangle)_{8,10}$	I
	$\|\varphi^\pm\rangle_{47}\|\varphi^\pm\rangle_{69}/(a\|01\rangle \pm b\|00\rangle \pm c\|11\rangle \pm d\|10\rangle)_{8,10}$	$(\sigma_x)_{10}$
$\|\varphi^\pm\rangle_{13}\|\varphi^\pm\rangle_{25}/(a\|11\rangle \pm b\|10\rangle \pm c\|01\rangle \pm d\|00\rangle)_{46}$	$\|\phi^\pm\rangle_{47}\|\phi^\pm\rangle_{69}/(a\|11\rangle \pm b\|10\rangle \pm c\|01\rangle \pm d\|00\rangle)_{8,10}$	$(\sigma_x)_8(\sigma_x)_{10}$
	$\|\phi^\pm\rangle_{47}\|\varphi^\pm\rangle_{69}/(a\|10\rangle \pm b\|11\rangle \pm c\|00\rangle \pm d\|01\rangle)_{8,10}$	$(\sigma_x)_8$
	$\|\varphi^\pm\rangle_{47}\|\phi^\pm\rangle_{69}/(a\|01\rangle \pm b\|00\rangle \pm c\|11\rangle \pm d\|10\rangle)_{8,10}$	$(\sigma_x)_{10}$
	$\|\varphi^\pm\rangle_{47}\|\varphi^\pm\rangle_{69}/(a\|00\rangle \pm b\|01\rangle \pm c\|10\rangle \pm d\|11\rangle)_{8,10}$	I

$$A_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \tag{19}$$

The relation between S_i and A_j is shown in Fig. 4. We can easily recover the initial quantum state from the teleported quantum state according to Fig. 4.

3.3 Multi-hop teleportation based on different BMRs

Analogous to two-hop teleportation, for multi-hop teleportation with an arbitrary node, we get the teleported quantum state, which is one of the four types. We define B as the BMR, which is one of the four possible values B_k $(k = 1, 2, 3, 4)$ given by

$$B_1 = \{|\phi^+\rangle|\phi^+\rangle, |\phi^+\rangle|\phi^-\rangle, |\phi^-\rangle|\phi^+\rangle, |\phi^-\rangle|\phi^-\rangle\}, \tag{20}$$

$$B_2 = \{|\phi^+\rangle|\varphi^+\rangle, |\phi^+\rangle|\varphi^-\rangle, |\phi^-\rangle|\varphi^+\rangle, |\phi^-\rangle|\varphi^-\rangle\}, \tag{21}$$

$$B_3 = \{|\varphi^+\rangle|\phi^+\rangle, |\varphi^+\rangle|\phi^-\rangle, |\varphi^-\rangle|\phi^+\rangle, |\varphi^-\rangle|\phi^-\rangle\}, \tag{22}$$

$$B_4 = \{|\varphi^+\rangle|\varphi^+\rangle, |\varphi^+\rangle|\varphi^-\rangle, |\varphi^-\rangle|\varphi^+\rangle, |\varphi^-\rangle|\varphi^-\rangle\}. \tag{23}$$

The transformations of the quantum state can then form a finite quantum state machine (QSM) based on the BMR as shown in Fig. 5. As marked in section 3.2, the four circles denote the four quantum states, and the arrow lines represent transformations of the quantum states under the effect of the BMR marked on the arrows. Thanks to the QSM, the quantum state can be determined using the BMR measured by the source node and intermediate nodes along the path. Moreover, the quantum state is transformed back into its original state after a number of BMRs.

The source node and all intermediate nodes make Bell measurements and send the BMRs directly to the destination node. Finally, the destination node performs a related unitary transformation, one of four types. Figure 6 shows the multi-hop quantum teleportation with k intermediate nodes.

The combination of Fig. 4 and Fig. 5 shows that, if for example, S_1 is the initial state, then the BMR group (B_1, B_2, B_3, B_2, B_4, and B_3) is required to illustrate the function of the QSM. This is a six-hop communication. The quantum state starts at S_1. Under the effect of B_1, it remains unchanged. The quantum state then changes to S_2 because of the BMR B_2; it will then change into S_4, S_3, S_2, and finally S_4 under the effects of other BMRs B_3, B_2, B_4, and B_3. As a result, we gain the final quantum state $a|11\rangle \pm b|10\rangle \pm c|01\rangle \pm d|00\rangle$. Using the quantum state along with Table 2 allows the corresponding unitary transformations for the recovery of the initial quantum state to be obtained.

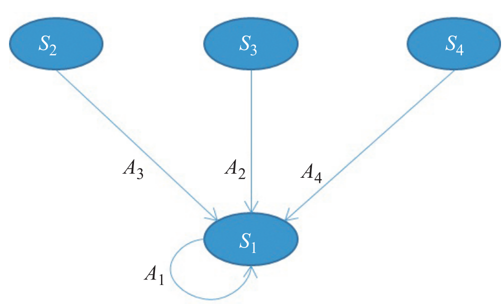

Fig. 4 The relation among quantum states S_i and unitary transformations A_j.

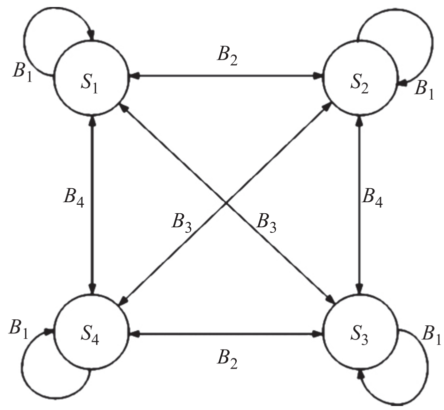

Fig. 5 Finite quantum state machine (QSM) based on BMRs.

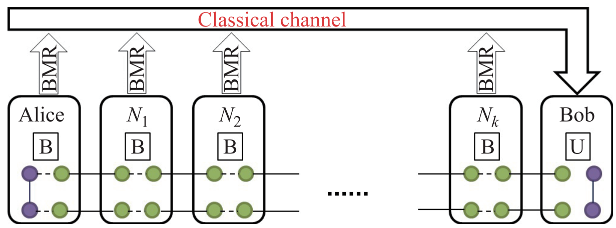

Fig. 6 Multi-hop quantum teleportation with k intermediate nodes.

4 Example of teleportation in the QNM

We present an example to illustrate the whole process of teleportation in the QNM. For example, as shown in Fig. 1, we assume that the source node n_1 wants to send a two-particle quantum state $|\psi\rangle = (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)$ to the destination node n_8. However, there are no available EPR pairs shared between n_1 and n_8. According to this representation, a multi-hop method would be a good choice to realize successful transmission. We choose the quantum path $n_1 \to n_2 \to n_3 \to n_5 \to n_7 \to n_8$ according to the shortest path algorithm. There are direct classical channels shared among the source node, intermediate nodes, and destination node. The transmission mode is shown in Fig. 7.

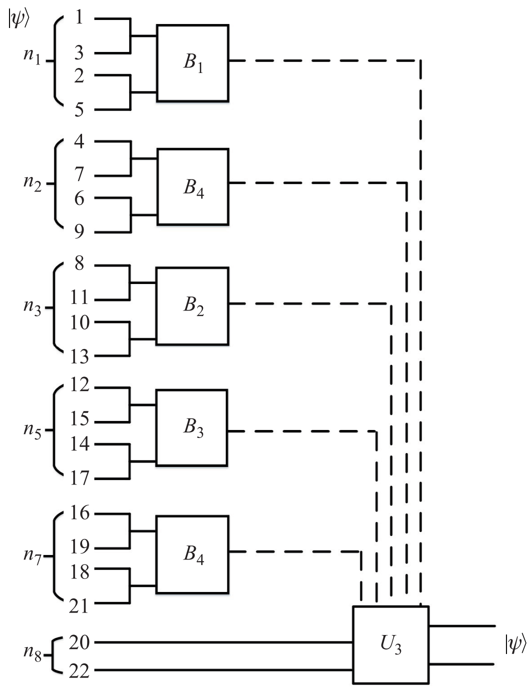

Fig. 7 Example of multi-hop teleportation in the QNM.

During transmission, two EPR pairs are shared among adjacent nodes. The source node (n_1) and intermediate nodes (n_2, n_3, n_5, and n_7) do Bell measurements, and the results are independently sent to the destination node (n_8) by classical channels. After successfully collecting all of the classical information from these five nodes, the destination node (n_8) calculates the logical relation between the measured results. If we assume that the measured results are B_1, B_4, B_2, B_3, and B_4, then the destination node (n_8) will obtain the teleported quantum state S_4 according to the QSM (see Fig. 5). Finally, the destination node (n_8) performs unitary transformations A_4 to recover the initial quantum state according to Fig. 4.

5 Discussion

In this paper, we propose multi-hop teleportation in a quantum network based on mesh topology. All nodes can transmit classical information directly to any node, whereas most cannot transmit quantum information directly between each other. Communication can be realized with the help of intermediate nodes using the entanglement swapping method. The source and intermediate nodes perform Bell measurements and independently send the results to the destination node; the destination node then does the corresponding unitary transformations to recover the initial quantum state.

Mesh topology is a network setup where all of the nodes are interconnected with one another, and it has some advantages over other networks. In particular, mesh topology can handle a large amount of traffic, enabling most transmissions to be distributed even when one of the connections goes down. Furthermore, adding additional devices does not disrupt data transmission among other devices. When deploying this topology, the source and intermediate nodes can send the BMRs to the destination node independently (see Fig. 6). These simultaneous measurements can reduce hop-by-hop communication delay [20].

In our scheme, we use EPR pairs as the quantum channels. EPR pairs are the simplest entanglement resources which reduces quantum resources and storage space. Furthermore, there is no need for intermediate nodes to do any unitary transformations as different types of BMRs, and the destination node only needs to do one of four types of simple unitary transformation.

6 Conclusion

In conclusion, we studied multi-hop teleportation in a quantum network based on mesh topology and proposed a scheme for quantum teleportation among nodes in a network that have no direct Bell pairs shared. We used simultaneous measurements and the entanglement swapping method to calculate the one- and two-hop cases in detail and generalize them to the multi-hop case. The source node and all of the intermediate nodes make Bell measurements independently and send measurement results to the destination node by classical channels. Once all of the classical information has been obtained, the destination node performs unitary transformations to recover the initial quantum state. With the help of the QSM, the quantum state can be determined by the BMRs measured by the source and intermediate nodes. Throughout the process, only the destination node needs to do simple unitary transformations. Our scheme reduces the overall transmission time and it is very promising to be realized in practice.

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