

# Quantum connectivity optimization algorithms for entanglement source deployment in a quantum multi-hop network

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At first, the entanglement source deployment problem is studied in a quantum multi-hop network, which has a significant influence on quantum connectivity. Two optimization algorithms are introduced with limited entanglement sources in this paper. A deployment algorithm based on node position (DNP) improves connectivity by guaranteeing that all overlapping areas of the distribution ranges of the entanglement sources contain nodes. In addition, a deployment algorithm based on an improved genetic algorithm (DIGA) is implemented by dividing the region into grids. From the simulation results, DNP and DIGA improve quantum connectivity by 213.73% and 248.83% compared to random deployment, respectively, and the latter performs better in terms of connectivity. However, DNP is more flexible and adaptive to change, as it stops running when all nodes are covered.

**Keywords** entanglement source deployment, quantum connectivity, deployment algorithm

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## 1 Introduction

Quantum entanglement is a fundamental resource to most advanced fields of quantum information processing, such as quantum teleportation, quantum key distribution, and quantum secure direct communications. The realization of quantum internet [1] also requires scientific capabilities for generating and characterizing quantum coherence and entanglement.

For quantum teleportation, Bennett *et al.* [2] first proposed teleportation of an arbitrary single qubit in 1993, which was based on an Einstein–Podolsky–Rosen (EPR) pair shared between two nodes. Then it was experimentally demonstrated [3]. Afterwards, schemes of teleporting a single-qubit state were proposed through Greenberger–Horne–Zeilinger (GHZ) state [4–7], W state [8], mixed state [9, 10], etc. To teleport an arbitrary two-qubit state, many protocols were proposed using tensor products of two Bell states [11], genuine five-qubit entangled state [12], multi-qubit cluster state [13], etc. In addition, schemes to teleport an arbitrary  $N$ -qubit state were proposed through non-maximally entangled Bell state

[14], composite GHZ–Bell channel [15], genuine multipartite entanglement channel [16], and  $n$  pairs of EPR channel [17], respectively. Regarding quantum key distribution, Ekert [18] reported a key distribution process in cryptography in 1991, which was based on Bell’s theorem, and Ursin *et al.* [19] experimentally demonstrated entanglement-based quantum key distribution over 144 km. In addition, a protocol known as quantum secure direct communication is proposed by Deng *et al.* [20], using blocks of EPR pairs. Hu *et al.* [21] reported the first experimental demonstration of quantum secure direct communication based on the DL04 protocol and equipped with single photon. These studies underlie the proposals of quantum communication networks.

Entanglement distribution is performed through optical fibers or free-space links, which are known as quantum channels. For optical fibers, the experiments in [22] realize entanglement distribution over distances ranging from several hundred meters up to 10 km. However, because of absorptive losses of the present fiber and detector technology, the distance over which entanglement can be shared is limited [23]. In addition, some researchers [24–29] adopt free-space channels to implement entangle-

ment distribution with less dispersion and zero birefringence, which would not alter the polarization state of a photon. Fedrizzi *et al.* [30] demonstrate free-space quantum teleportation over 16 km with a single pair of entangled photons. Furthermore, Pan *et al.* [31] report quantum teleportation of independent qubits over a 97-km one-link free-space channel with multi-photon entanglement, and demonstrate entanglement distribution over a two-link channel, in which the entangled photons are separated by 101.8 km. These experiments verify the maturity and applicability of such technologies in real-world scenarios.

For large-scale quantum communication networks, one of the essential challenges is to transfer an unknown quantum state over arbitrary distances with the help of quantum teleportation and entanglement distribution. However, according to the present technology, quantum teleportation over arbitrary distances cannot be realized, and is restricted to the distribution range of entanglement. In order to teleport an unknown quantum state between two distant nodes without directly sharing entanglement pairs, some researchers [32–37] introduced intermediate nodes for multi-hop teleportation in a quantum network. In the multi-hop quantum path, there are entangled resources among adjacent nodes; therefore, qubit states can be transmitted from source node to destination. Shi *et al.* [33] proposed a quantum wireless multi-hop network in which an arbitrary single qubit can be teleported hop by hop based on shared W states. A protocol for quantum communication through entanglement swapping based on arbitrary types of Bell pairs is demonstrated by Wang *et al.* [35]. Then, Zou *et al.* [37] proposed a composite GHZ-Bell channel to realize multi-hop quantum teleportation of arbitrary two-qubit states. These protocols improve transmission flexibility and efficiency of the quantum network by transmitting measurement outcomes of all nodes in parallel and reducing hop-by-hop teleportation delay.

In quantum communication networks, a device used to generate and distribute entangled photon pairs is called an entanglement source (ES). In order to build a path for multi-hop teleportation, entanglement pairs are shared between adjacent nodes in a multi-hop path by ES. For a large-scale quantum network, more than one ES may be needed to provide entanglement. The entanglement source deployment has a great effect on the quantum connectivity of the network, which is not only an important measure of robustness, but also a critical property pertaining to the overall network performance. However, the problem of how to deploy the entanglement sources is not considered in previous studies of multi-hop teleportation in a wide quantum communication network.

In this paper, for a given network, we first investigate a quantum connectivity algorithm to optimize the

quantum connectivity of a quantum multi-hop network (QMN) with limited entanglement sources. We consider two nodes to be connected when they share entanglement directly or when a multi-hop quantum path exists between them. Therefore, quantum connectivity is defined as the ratio of the connected node pairs to all node pairs. Then, two adaptive algorithms are proposed to provide simple solutions with good connectivity, called deployment algorithm based on node position (DNP) and deployment algorithm based on improved genetic algorithm (DIGA). For DNP, according to the distribution range of ES, we reconstruct the network graph and select optimal quantum paths to deploy ES at mid-points between the adjacent nodes. Simulation results demonstrate that the average quantum connectivity is improved by 213.73% compared to deploying ESs randomly. In addition, DIGA is proposed to solve entanglement source deployment (ESD) problems by dividing the network region into grids, which increased quantum connectivity by 248.83%. Therefore, DIGA performs with better connectivity. But in some cases, DNP is more flexible, as it would stop running while all nodes are covered by ESs.

The rest of the paper is organized as follows. In Section 2, we describe the architectures of QMN, and in Section 3, we introduce quantum distribution links for transmission of entangled photon pairs. In Section 4, we define the ESD problem. Then, the mathematical model of the ESD problem is established in Section 5. In Section 6, DNP and DIGA are proposed to optimize quantum connectivity in QMN. Finally, we evaluate the algorithms in Section 7 in comparison with deploying ESs randomly, followed by conclusions in Section 8.

## 2 Quantum multi-hop network

In classical communication networks, the architectures are typically composed of two fundamental elements: nodes and links. Nodes represent the communicating entities. Links represent one-hop communication paths between adjacent nodes. The data are transmitted between nodes by links.

However, in QMN, as shown in Fig. 1, nodes interact with quantum states, and they can store quantum particles and process information. The data transmitted in QMN are mostly quantum bits (qubits), instead of classical bits (c-bits). Qubits are the units of quantum information, which is represented as a linear combination of computational basis states  $|0\rangle$  and  $|1\rangle$ . Generally, we use  $|\chi\rangle = \alpha|0\rangle + \beta|1\rangle$  to model the qubit, where  $\alpha$  and  $\beta$  are complex probability amplitudes satisfying  $|\alpha|^2 + |\beta|^2 = 1$ . In addition, there are two kinds of links in QMN: quantum links, which are built by entanglement pairs, are

used for teleporting qubits, and classical links, which are employed to transfer classical information, like classical Bell-state measurement results.

In the quantum multi-hop network, one of the important elements is the entanglement source (ES), which generates and distribute entangled photon pairs to nodes to construct quantum channels. For entanglement distribution in Ref. [26], the ES first sends the entangled photon pairs to specified nodes via telescopes mounted on a platform rotatable in azimuth and elevation, and then entangled photons are collected by telescopes on each node.

As Fig. 1 shows, we assume that ESs can distribute entangled photon pairs in all directions; their distributed areas are encircled by dashed lines. Therefore, two nodes in one circle can establish quantum links and teleport quantum state directly, like Nodes 1, 2, 3, 5, and 8. For example, Fig. 2 shows quantum links in a QMN between some nodes who share entanglement pairs.

Supposing Node 1 wants to send an unknown qubit state  $|\chi\rangle$  to Node 2, who shares EPR pairs to a build one-hop path. The quantum state of the whole system is

$$\begin{aligned}
 |\Psi_1\rangle &= |\chi\rangle \otimes |\varphi\rangle \\
 &= \frac{1}{\sqrt{2}}(\alpha|0\rangle + \beta|1\rangle) \otimes (|00\rangle + |11\rangle) \\
 &= \frac{1}{\sqrt{2}}(\alpha|000\rangle + \beta|100\rangle + \alpha|011\rangle + \beta|111\rangle). \quad (1)
 \end{aligned}$$

Then, Node 1 performs a Bell state measurement and transmits the measurement results to Node 2 through a classical channel. According to the measurement results, Node 2 applies suitable unitary operations to recover the initial two-qubit state. The Pauli matrices  $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ ,  $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ ,  $\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ , and identity matrix  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  are the basic quantum operators.

However, if Node 1 wants to transfer a one-qubit state to Node 9 in Fig. 2, Node 1 must select a path in the quantum multi-hop network, such as Node 1  $\rightarrow$  Node 8  $\rightarrow$  Node 7  $\rightarrow$  Node 9. In the multi-hop quantum path, there are entangled photon pairs among adjacent

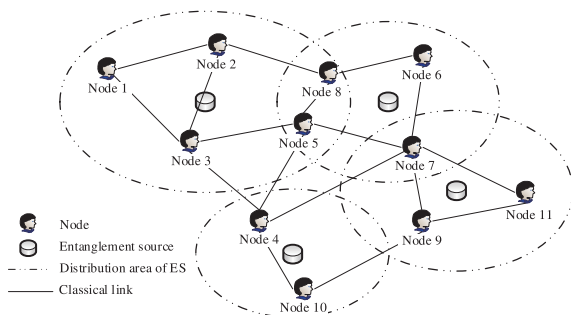


Fig. 1 Quantum multi-hop network architecture diagrams.

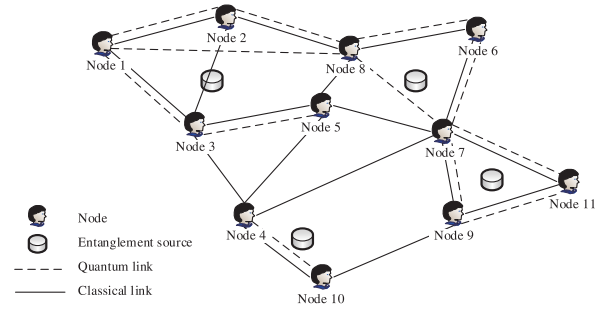


Fig. 2 Quantum multi-hop network with quantum links.

nodes, so the entire system state is written as

$$\begin{aligned}
 |\Psi_2\rangle &= |\chi\rangle \otimes |\varphi\rangle^3 = \frac{1}{2\sqrt{2}}(\alpha|0\rangle + \beta|1\rangle) \\
 &\otimes (|00\rangle + |11\rangle) \otimes (|00\rangle + |11\rangle) \otimes (|00\rangle + |11\rangle). \quad (2)
 \end{aligned}$$

Then, Node 1, Node 8, and Node 7 shall perform Bell state measurements. As for processing measurement results, multi-hop quantum teleportation would be more complex than a one-hop path, which has been introduced in Refs. [27–32].

In addition, from Fig. 1 and Fig. 2, we can see that Node 4 cannot teleport quantum state to any nodes except Node 10, and there is no intermediate node that could be chosen to establish a multi-hop path. Therefore, if two nodes in different distribution areas want to make quantum communications, the overlapping area should have nodes to help them establish a multi-hop path, like Node 5, Node 8, and Node 7.

### 3 Quantum distribution links

The entanglement sources distribute entangled photon pairs to nodes through optical fibers or free space. During entanglement transmission, they must be protected from environmental noise to reduce error transmission.

#### 3.1 Single-mode fibers

In Ref. [38], single-mode fibers, which guide a single spatial mode with a small fiber core, are very well suited to carry single quanta. In past decades, researchers have put a great deal of effort into reducing transmission losses, so the attenuation is as low as 2 dB/km at a 890-nm wavelength, 0.35 dB/km at 1310 nm, and 0.2 dB/km at 1550 nm. A single-mode fiber with perfect cylindrical symmetry shall provide an ideal quantum distribution link. However, there are four polarization effects in all real fibers as follow: geometric phase, birefringence, polarization mode dispersion, and polarization-dependent

losses.

### 3.2 Free-space links

Another link known as free space is also introduced in Ref. [38] for application in the cases where optical fibers are not available. Transmission over free space is weakly dispersive, so the polarization state of a photon would not be altered. However, there are higher and varying transmission losses, because the energy transmitted via a free-space link would spread out. Furthermore, loss of energy, ambient daylight, or even moonlight at night would lead to a higher error rate at the receiver.

However, because of unavoidable noise in the quantum distribution links, the entanglement between two particles becomes more and more degraded the further they propagate. The entanglement purification and concentration are two powerful approaches to distill the maximum entanglement from the degraded entanglement. Bennett *et al.* [39] applied local operations to prepare a smaller number of entangled pairs of arbitrarily high purity. A scheme for the entanglement purification of general mixed entangled states is presented by Pan *et al.* [40], which requires only simple linear optical elements. Deng *et al.* [41] proposed two economical one-step error-correction protocols for multipartite polarization-entangled systems in a GHZ state. In addition, Sheng *et al.* [42] introduced a universal way to concentrate an arbitrary  $N$ -particle less-entangled W state into a maximally entangled W state with different parity check gates. Zhou *et al.* [43] investigated the first entanglement purification protocol for logic-qubit entanglement.

In this paper, there are no restrictions on which quantum distribution links to transmit entangled photon pairs in a QMN; our scheme for entanglement source deployment can apply in any form of quantum network. However, the distribution range of the ES is limited by entanglement distribution technology.

## 4 Problem of entanglement source deployment

In this section, we define the problem of entanglement source deployment (ESD) in a quantum multi-hop network. We assume the network area is an  $A \times A$  km<sup>2</sup> square region. The initial network topology is represented by an undirected geometric graph  $G = (V_{\text{Node}}, E)$ , where  $V_{\text{Node}} = \{N_1, N_2, \dots, N_n\}$  is the set of nodes (vertices) and  $E = \{(N_i, N_j) : d(N_i, N_j) \leq R_c, N_i, N_j \in V_{\text{Node}}\}$  is the set of classical links (edges).  $R_c$  is the maximum transmission range of a classical link and  $d(N_i, N_j)$  is the Euclidean distance between  $N_i$  and  $N_j$ . Let a set  $P_{\text{Node}} = \{(x_N^1, y_N^1), (x_N^2, y_N^2), \dots, (x_N^n, y_N^n)\}$  denote

the coordinates of the nodes. Note that the network  $G = (V_{\text{Node}}, E)$  is without quantum links now, because there are no ESs to generate and distribute entangled photon pairs. The problem is how to deploy the entanglement sources for network  $G = (V_{\text{Node}}, E)$  in this square region, and build as many quantum connections over all nodes as possible.

As mentioned in Section 2, if two nodes obtain entangled photon pairs from the same ES, they will establish a quantum link and teleport the quantum state directly. Otherwise, intermediate nodes should be introduced to help establish multi-hop quantum paths. The intermediate nodes must exist in an overlapping area of the entanglement source distribution range. If there is at least one quantum path between two nodes, regardless of whether they share entanglement directly or with the help of intermediate nodes, those nodes are considered connected.

In comparison with the base station deployment in classical wireless communication, the communication quality for cell edge users is disturbed by the overlaying signal from different base stations nearby. Most researchers focus on studying effective approaches to interference coordination and cancellation. Some schemes mitigate interference by ensuring orthogonality between transmitted signals in time and frequency, as well as space, or by actively removing and canceling interfering signals from the desired signal. In addition, LTE provides a flexible dynamic inter-base station approach to interference coordination [44]. However, for the problem of ESD in QMNs, if nodes in two distribution ranges of neighbor ESs need to communicate, intermediate nodes in overlapping areas are necessary to help establish a quantum path.

Therefore, our objective is to determine the position of entanglement sources in the network region, and optimize the quantum connectivity of the QMN with limited entanglement sources. That means the amount of deployed ESs should be less than the upper limit numbers of ESs.

## 5 The mathematical programming model

Before establishing a mathematical programming model of the ESD problem, there are some assumptions, definitions, and notations about QMN that should be mentioned.

### 5.1 Nodes

Assuming that the nodes in a QMN can prepare the quantum state that they want to teleport, store quantum particles, and process information based on Bell-state measurement results, then for the initial network

topology  $G = (V_{\text{Node}}, E)$ ,  $V_{\text{Node}} = \{N_1, N_2, \dots, N_n\}$  is the set of nodes, and the number of nodes is  $n$ .  $P_{\text{Node}} = \{(x_N^1, y_N^1), (x_N^2, y_N^2), \dots, (x_N^n, y_N^n)\}$  is used to denote the coordinates of the nodes.

### 5.2 Entanglement sources (ESs)

Assume that an ES can distribute entangled photon pairs all around, and the power of distribution in all directions is same. Furthermore, the distributed area of all ESs is circle with radius  $R_q$ , which is dependent on entanglement distribution technology. That means nodes in one circle can establish quantum links and teleport quantum state directly. According to the experiment of Pan *et al.* [26], each distributed distance between an ES and nodes via a two-link channel is about 50 km.

Let the upper limit number of the ESs be  $m$ , and the positions of the ESs is expressed by a set  $P_{ES} = \{(x_{ES}^1, y_{ES}^1), (x_{ES}^2, y_{ES}^2), \dots, (x_{ES}^m, y_{ES}^m)\}$ . In addition, the set  $ES(i), (i = 1, 2, \dots, m)$  contains nodes which could distribute entangled photon pairs by the  $i$ th ES. For node  $j (N_j, j = 1, 2, \dots, n)$ ,  $N_j \in ES(i)$  when  $\sqrt{(x_{ES}^i - x_N^j)^2 + (y_{ES}^i - y_N^j)^2} \leq R_q$ .

### 5.3 Intermediate nodes

As described in Section 2, intermediate nodes help to build multi-hop paths, whose positions are in overlapping areas of the ES's distribution range. When  $ES(i) \cap ES(j) \neq \emptyset$ , it means that the overlapping area between the  $i$ th and  $j$ th ES's distribution range contains intermediate nodes, so any node in those two distribution areas can make quantum communications. Therefore, the problem of ESD with limited ES is different from the vertices covering problem, not only should all nodes be covered using as few ESs as possible, but also the overlapping distribution area should contain a number of intermediate nodes to improve quantum connectivity.

### 5.4 Shortest path

Assuming all the network graphs discussed in this paper are unweighted graphs, if a pair of nodes are connected by edges, the weight of their edge is set to 1, otherwise, 0. The shortest path between node  $i (N_i)$  and node  $j (N_j)$  is the path with minimum hop, and we regard the total hops as the distance of their path.

### 5.5 Quantum connectivity (QC)

In a QMN with  $n$  nodes, the numbers of all pairs of nodes is  $n \cdot (n - 1)/2$ . Let  $A_{ij}$  denote the connectivity of node  $i (N_i)$  and node  $j (N_j)$ , if there is at least one quantum path between  $N_i$  and  $N_j$ , which share entanglement di-

rectly or contain a multi-hop quantum path,  $A_{ij}$  is set to 1. If not,  $A_{ij}$  is set to 0. We define the connectivity of QMNs as the ratio of connected node pairs to all node pairs. Therefore, the quantum connectivity (QC) of QMNs is formulated as

$$QC = \frac{\sum_{i,j \in V_{\text{Node}}, i > j} A_{ij}}{n \cdot (n - 1)/2}. \tag{3}$$

Based on above illustration, the optimization model for the ESD problem is as follows:

**maximize**

$$\frac{\sum_{i,j \in V_{\text{Node}}, i > j} A_{ij}}{n \cdot (n - 1)/2}, \tag{4}$$

**subject to**

$$A_{ij} = \begin{cases} 1 & \text{Exist path from } N_i \text{ to } N_j; \\ 0 & \text{None path from } N_i \text{ to } N_j. \end{cases} \tag{5}$$

$i, j \in V_{\text{Node}}, i \neq j.$

## 6 Proposed optimization algorithm

In this section, we first propose two different algorithms to optimize quantum connectivity with limited entanglement sources in a given QMN. One algorithm improves quantum connectivity by guaranteeing all overlapping areas of ESs' distribution ranges contain nodes, and is called the deployment algorithm based on node position. In addition, a deployment algorithm based on an improved genetic algorithm is proposed to solve ESD problem through dividing region into grids.

### 6.1 Deployment algorithm based on node position (DNP)

#### Step 1: Transform network graph

We transform the given network  $G = (V_{\text{Node}}, E)$  to  $G' = (V_{\text{Node}}, E')$ , where  $E' = \{(N_i, N_j) : d(N_i, N_j) \leq 2 \cdot R_q, N_i, N_j \in V_{\text{Node}}\}$ , which represents the possible quantum links of network. When  $d(N_i, N_j) \leq 2 \cdot R_q$ , we deploy ES at the midpoint of the connecting line between  $N_i$  and  $N_j$ , so the entanglement distribution area can cover  $N_i$  and  $N_j$ , and they can establish a direct quantum link between them.

#### Step 2: Deploy entanglement source

Before entanglement source deployment, we perform program initialization. Let an empty set  $P_{ES} = \emptyset$  record the ES's coordinates which have been deployed, and an empty set  $M = \emptyset$  loads nodes which have been covered by ES's distribution area. The remainder is recorded in

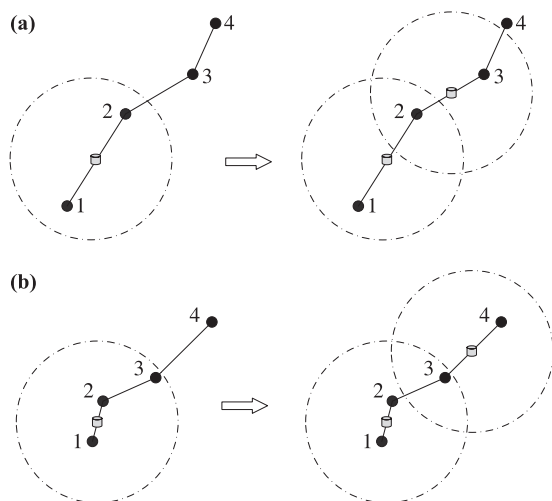
$\bar{M} = V_{\text{Node}} \setminus M$ . First, the pair of nodes with the longest Euclidean distance in a set  $\bar{M}$  is determined by

$$(u, v) = \max_{N_i, N_j \in V_{\text{Node}}} d(N_i, N_j), \quad (6)$$

where  $d(N_i, N_j)$  is the Euclidean distance between  $N_i$  and  $N_j$ . Note that if there is no connective path between  $N_i$  and  $N_j$ , we should reselect the nodes with the second-longest distance. Then, a classical algorithm, Yen's algorithm [44], which is used for ranking the  $K$  shortest loopless paths between a pair of nodes, is introduced to search all shortest paths between Node  $u$  and Node  $v$  in network graph  $G' = (V_{\text{Node}}, E')$ . We add the judgment of shortest path's distance to control  $K$  in Yen's algorithm, so that the output set of top  $K$  shortest paths is equal to all paths with a minimum distance.

Let  $SP$  be a set of all shortest paths between  $u$  and  $v$ , and the  $i$ th shortest path is described as  $SP(i) i = 1, 2, \dots, K$ , where  $K$  denotes the length of the shortest path. For the  $i$ th shortest path  $SP(i)$ , we plan to deploy ES at the midpoint of the connecting line between adjacent nodes in turn. Note that there are two cases for deployment shown in Fig. 3. As for Fig. 3(a), the distribution area of ESs which have been deployed do not cover the next node, node 3, of the path, so the next ES should be sited at the midpoint of node 2 and node 3. However, if the next node has been covered, we should choose the edge between node 3 and node 4 to arrange ES, as shown in Fig. 3(b). Therefore, before deploying ES, we shall check whether the next node is covered or not.

According to the result of deployment in all shortest paths, we compare the number of nodes covered by en-



**Fig. 3** Two deployment cases at the midpoint between adjacent nodes. (a) The next node 3 is not covered. (b) The next node 3 has been covered.

tanglement source distribution area. As for  $SP(i)$ , the amount of covered nodes ( $CNA_i$ ) is formulated as

$$CNA_i = \sum_{j=1}^{S_i} \text{Card}(ES(j)), \quad (7)$$

where  $S_i$  denotes the number of ES deployed in the  $i$ th shortest path  $SP(i)$ , and  $ES(j)$ , ( $j = 1, 2, \dots, S_i$ ) is a set of nodes covered by the  $j$ th ES's distribution area. In addition,  $\text{Card}(\cdot)$  is used to calculate the number of elements in a set.

Then a shortest path  $SP(k)$  with maximum covered nodes is selected by

$$CNA_k = \max_{i=1}^K \sum_{j=1}^{S_i} \text{Card}(ES(j)), \quad (8)$$

where  $K$  denotes the length of the shortest path. This deployment scheme would optimize quantum connectivity of the network, because it is guaranteed to establish quantum links over all nodes which have been covered by deployed ESs. Finally, the positions of the ESs deployed in  $SP(k)$  will be added to the set  $P_{ES}$ , the nodes which are covered by newly added ESs in  $SP(k)$  will be recorded in the set  $M$ , and we eliminate nodes which have been covered from the complement set  $\bar{M} = V_{\text{Node}} \setminus M$ .

For an updated set  $\bar{M}$ , we repeat the above operations:

- (i) Determine  $u$  and  $v$  with longest Euclidean distance in a set  $\bar{M}$ ;
- (ii) Calculate all shortest paths between  $u$  and  $v$  to record in set  $SP$ ;
- (iii) Deploy ES at the midpoint of the connecting line between adjacent nodes;
- (iv) Select a shortest path  $SP(k)$  with maximum covered nodes;
- (v) Add ES's coordinates and covered nodes in  $SP(k)$  to  $P_{ES}$  and  $M$ , and update  $\bar{M} = V_{\text{Node}} \setminus M$ .

When  $\bar{M}$  is empty, the deployment of ESs is completed. However,  $\bar{M}$  may only contains one element sometimes; our approach for this case is to select a closest node. If the distance of those two nodes is less than  $2 \cdot R_q$ , we deploy a new ES at midpoint of them, or the last node will not be attended to. In addition, we shall note that the upper limit number of ES is set to  $m$  in Section 5. It is necessary to check whether the number of deployed ES is over a set value  $m$  before deploying another ES. When it reaches the upper value, entanglement source deployment shall stop.

The deployment algorithm based on node position is composed by following steps, which are shown in the appendix.

### 6.2 Quantum connectivity calculation

Before presenting a deployment algorithm based on improved genetic algorithm, we introduce the method of calculating quantum connectivity in this paper. In an  $n$ -node QMN in which ESs have been deployed, the coordinates of the nodes  $P_{\text{Node}}$  and entanglement sources  $P_{ES}$  have been determined. The collections  $ES(i), (i = 1, 2, \dots, m')$  could also be known, and contain nodes that distributed entangled photon pairs by the  $i$ th ES. These sets are expressed as follows:

$$\begin{aligned}
 P_{\text{Node}} &= \{(x_N^1, y_N^1), (x_N^2, y_N^2), \dots, (x_N^n, y_N^n)\}; \\
 P_{ES} &= \{(x_{ES}^1, y_{ES}^1), (x_{ES}^2, y_{ES}^2), \dots, (x_{ES}^{m'}, y_{ES}^{m'})\}, \quad m' \leq m; \\
 ES(i), (i = 1, 2, \dots, m'); \\
 &if \sqrt{(x_{ES}^i - x_N^j)^2 + (y_{ES}^i - y_N^j)^2} \leq R_q, \quad N_j \in ES(i),
 \end{aligned}
 \tag{9}$$

where  $m'$  denotes the number of already-deployed ESs, and  $V_{\text{Node}}$  contains all nodes in this QMN.

In order to calculate QC, the Floyd-Warshall algorithm is introduced to find the distance of the shortest paths between all pairs of vertices in the graph. First, we shall construct a network graph  $G''(V'', E'')$ , where  $V'' = \{v_1, \dots, v_n, v_{n+1}, \dots, v_{n+m'}\}$  is a set of all nodes and ESs with determined positions, nodes are marked from 1 to  $n$ , and ES from  $n + 1$  to  $n + m'$ . As for a set  $E'' = \{(u, v), u, v \in V''\}$ , it contains edges which connected between  $i$ th ESs and nodes in  $ES(i), (i = 1, 2, \dots, m')$ . The weights of all edges are set to 1. Second, let a  $(n + m') * (n + m')$  adjacent matrix  $W$  of graph  $G''$  be initialized to  $\infty$ , and  $W[i, j]$  denotes the weight of the edge between node  $i$  and  $j$ . For  $(u, v) \in E''$ ,  $W[u, v] = W[v, u] = 1$ . Taking a 5-node network with two ESs as example, the collections  $ES(i), (i = 1, 2)$  is given by  $ES(1) = \{1, 2, 4, 5\}, ES(2) = \{2, 3, 5\}$ . Figure 4 shows the corresponding network graph  $G''_1(V''_1, E''_1)$ , nodes are marked from 1 to 5, and ES from 6 to 7. Hence, the sets and adjacent matrix  $W$  are expressed as follows:

$$\begin{aligned}
 V''_1 &= \{1, 2, 3, 4, 5, 6, 7\}; \\
 E''_1 &= \{(1, 6), (2, 6), (4, 6), (5, 6), (2, 7), (3, 7), (5, 7)\}.
 \end{aligned}
 \tag{10}$$

$$W = \begin{bmatrix}
 \infty & \infty & \infty & \infty & \infty & 1 & \infty \\
 \infty & \infty & \infty & \infty & \infty & 1 & 1 \\
 \infty & \infty & \infty & \infty & \infty & \infty & 1 \\
 \infty & \infty & \infty & \infty & \infty & 1 & \infty \\
 \infty & \infty & \infty & \infty & \infty & 1 & 1 \\
 1 & 1 & \infty & 1 & 1 & \infty & \infty \\
 \infty & 1 & 1 & \infty & 1 & \infty & \infty
 \end{bmatrix}.
 \tag{11}$$

Therefore, based on adjacent matrix  $W$ , we determine the distance of the shortest paths between all pairs of vertices in graph  $G''$  by the Floyd-Warshall algorithm. Finally, we count the connected pairs over all nodes, if the distance of the shortest paths between node  $i$  and

node  $j$  is not  $\infty$ ,  $A_{ij}$  is set to 1. If not,  $A_{ij}$  is set to 0. Then the QC is calculated by formula (3).

### 6.3 Deployment algorithm based on improved genetic algorithm (DIGA)

#### Step 1: Mesh region

In section 4, the whole region of QMN is assumed as an  $A \times A$  km<sup>2</sup> square region. Now, we divide network region into some same-sized areas and define ESs should be deployed at the center of these grid. Each grid has its own grid ID, and grid ID can be used to calculate the center of the corresponding grid by

$$\begin{cases}
 px = (\text{mod}((x - 1), b) + 0.5) \times (A/b); \\
 py = (\text{floor}((x - 1)/b) + 0.5) \times (A/a),
 \end{cases}
 \tag{12}$$

where  $x$  is grid ID,  $a$  and  $b$  are the number of rows and columns of the region after meshing, respectively,  $\text{floor}(\cdot)$  is the operator to make decimals round down integers, and  $\text{mod}(\cdot)$  is the modulo operator.

For example,  $10 \times 10$  km<sup>2</sup> region is divided into 20 ( $a = 5, b = 4$ ) grids in Fig. 5. For grid ID 10 at the third line and the second column, the central position is (3.75, 5) which is calculated by formulas (12).

#### Step 2: Deploy entanglement source

We define a grid ID set  $X = \{x_1, x_2, \dots, x_m\}$ , where  $x_i$  is an integer between 1 and the maximum grid ID, which denotes  $m$  ESs deployed at the center of these

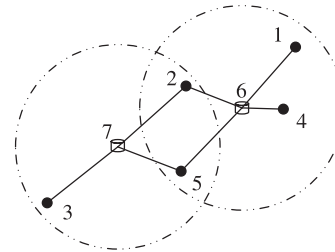


Fig. 4 Network graph: ESs make links with its covered nodes.

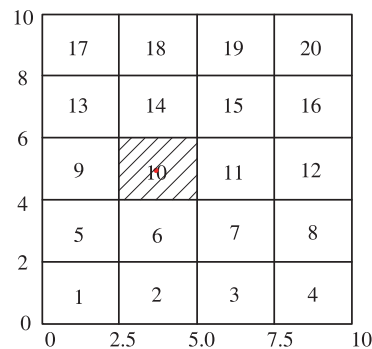
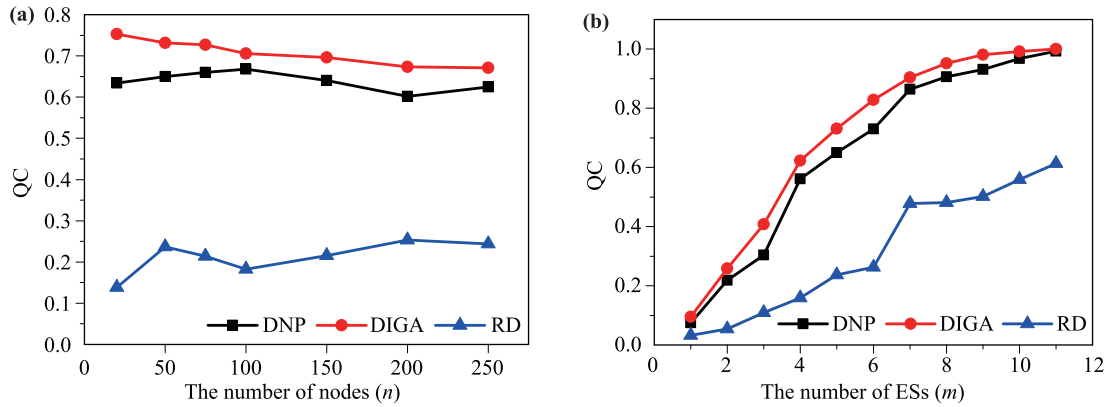


Fig. 5 Divide  $10 \times 10$  km<sup>2</sup> region into 20 grids.



**Fig. 6** (a) The average quantum connectivity with different number nodes. (b) The average quantum connectivity with different number ESs.

grids.  $X$  can be used to calculate the position of ESs  $P_{ES} = \{(x_{ES}^1, y_{ES}^1), (x_{ES}^2, y_{ES}^2), \dots, (x_{ES}^m, y_{ES}^m)\}$  by formula (12). According to Section 6.2, quantum connectivity can be calculated.

Therefore, the ESD problem can be formulated as:  
**maximize**

$$\frac{\sum_{i,j \in V_{Node}, i > j} A_{ij}}{n \cdot (n-1)/2} = f(X), \quad X = \{x_1, x_1, \dots, x_m\}, \quad (13)$$

**subject to**

$$1 \leq x_i \leq \max ID, \quad i = 1, 2, \dots, m, \quad (14)$$

where  $\max ID$  is maximal grid ID in region, and all elements in  $X$  should be integer. Then based on genetic algorithm, we can solve the above ESD problem.

## 7 Performance evaluation

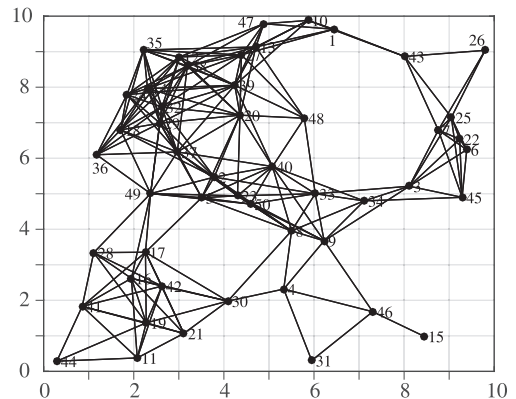
In this section, we evaluate the performance of DNP and DIGA. In the simulation study, all nodes are uniformly distributed in an  $A \times A$  km<sup>2</sup> square region, and ESs are deployed by different algorithms. Each result point is the average value of 500 simulation runs.

In the first set of simulations, we assume that the size of the region is fixed at  $10 \times 10$  km<sup>2</sup>, the distributed range of all ESs is 2.5 km, and the upper limit  $m$  is 5. In DIGA, the network region is divided into 100 grids, so the maximum grid ID is 100. We vary the number of nodes in the region from 20 to 250. Based on DNP, DIGA, and deploying ESs randomly (RD), Figure 6(a) shows the average quantum connectivity of QMNs with different numbers of nodes. From the simulation results, the performance of DNP and DIGA are notably better compared to random deployment. In detail, quantum connectivity is increased by 213.73% for DNP and by

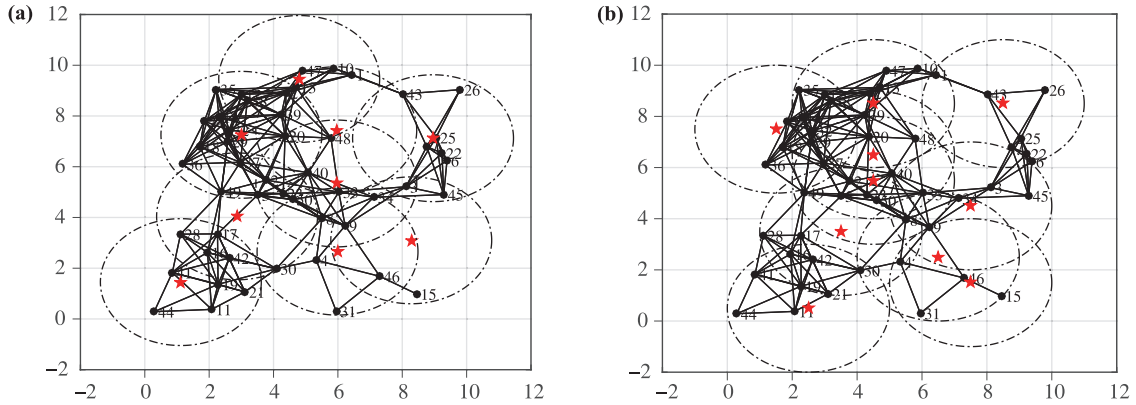
248.83% for DIGA. In addition, DIGA performs with better connectivity, as DIGA improves quantum connectivity by an average of 10.76% compared to DNP.

In the second set of simulations, the region size and distributed range of the ESs are the same as above. However, the number of nodes is fixed at 50. We vary the ES's upper limit  $m$  from 1 to 11. Figure 6(b) shows the average quantum connectivity of QMNs with different numbers of ESs. The results indicate that while the number of ESs increase, the quantum connectivity would be increased significantly. DIGA improves quantum connectivity by an average of 12.24% compared to DNP.

We observe a 50-node network deployed on a  $10 \times 10$  km<sup>2</sup> square region. The network topology as well as the node physical positions are shown in Fig. 7. We define the distributed range of ES as 2.5 km, and the upper limit  $m$  is 10. We first run the DNP to find a set of ESs' positions, which are shown in Table 1. The results indicate only 9 ESs are needed to make all nodes connect. Figure 8(a) is a corresponding network with 9 ESs deployed by DNP. Therefore, DNP would stop running when all nodes are



**Fig. 7** A 50-node network deployed on  $10 \times 10$  km<sup>2</sup> square region.



**Fig. 8** (a) Network with 9 ESs deployed by DNP. (b) Network with 10 ESs deployed by DIGA.

**Table 1** The result of ESs' position calculated by DNP and DIGA.

ES ID	DNP-ESPos	DIGA	
		Grid ID	ESPos
ES1	(1.119, 1.453)	85	(4.5, 8.5)
ES2	(2.867, 4.043)	48	(7.5, 4.5)
ES3	(5.951, 5.340)	89	(8.5, 8.5)
ES4	(8.957, 7.124)	34	(3.5, 3.5)
ES5	(3.002, 7.258)	18	(7.5, 1.5)
ES6	(8.279, 3.099)	55	(4.5, 5.5)
ES7	(5.989, 2.653)	27	(6.5, 2.5)
ES8	(5.949, 7.440)	65	(4.5, 6.5)
ES9	(4.798, 9.460)	3	(2.5, 0.5)
ES10	/	72	(1.5, 7.5)

covered by ESs, and it is more flexible and adaptive to cover all nodes with less ESs. In DIGA, the network region is divided into 100 grids. After running DIGA, we get a set of grid IDs and quantum connectivity is 1. The results of DIGA are shown in Table 1 and Fig. 8(b), respectively.

Therefore, DNP and DIGA improve quantum connectivity significantly, and DIGA performs better than DNP on average. However, DNP is more flexible for the ESD problem, since it would stop running when all nodes are

covered by ESs.

## 8 Conclusions

In this paper, we introduce the differences between the quantum network and classical wireless communication network in the deployment of resources, and study how to deploy a set of entanglement sources to optimize quantum connectivity in QMN. For the ESD problem, we develop two adaptive algorithms to provide simple solutions as well as good performance, and describe a method to calculate quantum connectivity. DNP is a heuristic algorithm to guarantee that all nodes that have been covered by the deployed ESs are connected. DIGA is based on a genetic algorithm and solves the problem by dividing the region into grids. From the simulation results, DNP and DIGA improve the quantum connectivity of QMN by 213.73% and 248.83%, respectively. Moreover, the results indicate that DIGA can perform better than DNP for the ESD problem on an average, but DNP is more flexible and adaptive to cover all nodes with fewer ESs.

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### Appendix Deployment algorithm based on node position

**Input:** A network graph  $G(V_{\text{Node}}, E)$ ; Upper limit of ES  $m$ ; Distribution range of ES  $R_q$ .

**Output:** Coordinates of ES  $P_{ES} = \{(x_{ES}^1, y_{ES}^1), (x_{ES}^2, y_{ES}^2), \dots, (x_{ES}^{m'}, y_{ES}^{m'})\}, m' \leq m$ .

**Initialization:** Coordinates of already deployed ES  $P_{ES} = \emptyset$ ; Covered nodes  $M = \emptyset$ ;

Complement set  $\bar{M} = V_{\text{Node}} \setminus M$ .

#### (1) Transform network graph

for each pair of nodes in  $V_{\text{Node}}$

if  $d(N_i, N_j) \leq 2 \cdot R_q$

```

    ( $N_i, N_j$ ) belongs to  $E'$ 
  end
end
Construct a new network graph:  $G' = (V_{\text{Node}}, E')$ 
(2) Deploy entanglement source
while  $\text{Card}(M) > 1$ 
   $d(u, v) = \max_{N_i, N_j \in M} d(N_i, N_j)$ 
  If connective paths do not exist, reselect  $u$  and  $v$ ;
  Calculate all shortest paths between  $u$  and  $v$ ;
  for each shortest path  $SP(i)$ 
    for each hop in  $SP(i)$ 
      if the amount of deployed ES has not reached  $m$ 
        Check whether the next node is covered or not;
        Deploy ES at midpoint between adjacent nodes;
      else
        break;
      end
    end
  end
end
Select  $SP(k)$  with maximum covered nodes;
Update  $P_{ES}$ ,  $M$  and  $\bar{M} = V_{\text{Node}} \setminus M$ .
end
if  $\text{Card}(\bar{M}) = 1$ 
  Select a closest node;
  if the distance is less than  $2 \cdot R_q$  and the deployed ES does not reach  $m$ 
    Deploy a new ES at the midpoint of them;
    Update  $P_{ES}$  and  $M$ 
  end
end
end
end

```

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