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REVIEW ARTICLE

Recent progress in econophysics: Chaos, leverage, and business cycles as revealed by agent-based modeling and human experiments

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Agent-based modeling and controlled human experiments serve as two fundamental research methods in the field of econophysics. Agent-based modeling has been in development for over 20 years, but how to design virtual agents with high levels of human-like "intelligence" remains a challenge. On the other hand, experimental econophysics is an emerging field; however, there is a lack of experience and paradigms related to the field. Here, we review some of the most recent research results obtained through the use of these two methods concerning financial problems such as chaos, leverage, and business cycles. We also review the principles behind assessments of agents' intelligence levels, and some relevant designs for human experiments. The main theme of this review is to show that by combining theory, agent-based modeling, and controlled human experiments, one can garner more reliable and credible results on account of a better verification of theory; accordingly, this way, a wider range of economic and financial problems and phenomena can be studied.

Keywords agent-based modeling, controlled human experiment, minority game, econophysics, chaos, leverage, business cycle

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Contents

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1 Introduction

Since the emergence of modern economics, basic science has never been far removed from its development. Numerous data have impeded the traditional ways of classical economics, and this has left economists appealing to mathematical and physical models and methods. Now that we are in the age of "big data", more and more physicists are starting to work in the field of economics. The fusion of different disciplines could be beneficial to all involved.

Generally, in our opinion, there are basically three methods of analyses in econophysics — namely, empirical analyses (mainly data analyses of historical information), computer simulations (such as agent-based modeling), and controlled human experiments (real people participating in designed econophysical experiments). The following briefly introduces econophysics and each of these approaches.

1.1 The birth of econophysics and original research methods

The timing of the birth of physics and of economics is difficult to confirm, since even ancient humans were able to handle forces and fire, and use shells as a transaction medium. It is obvious that advances in physics have brought significant changes to human lives (e.g., steam power, electricity, computers, and communication satellites). It is therefore reasonable that the methods of physics could be used in service of nature and human society.

Economics has become an increasingly important discipline, as people in modern society conduct transactions with others more frequently than was the case in ancient times. For this reason, mathematics has been introduced into the field of economics to more effectively handle data and numbers that pertain to transactions. However, given that the subjects of transactions are human beings — which are extremely difficult to abstract — real and basic principles and processes regarding transactions remain far from satisfactory. This challenge appeals to physicists (especially statistical physicists), and they have sought to produce further advances in economics by using physical models.

The concept of "econophysics" first appeared in the literature in an article by Stanley [1]. The thrust of this discipline is the study of financial and economic problems through the use of physical ideas and models [2]. The main advantage inherent in this approach is that traditional physics can provide a different perspective by which to scrutinize some complex financial or economics problems that require quantification and modeling.

The original methods used in econophysics mainly comprise mathematical and statistical analyses of existing data (in what is sometimes called "empirical analysis") [3–5], as well as primary theoretical analysis (mostly with agent-based modeling [e.g., the distinguished "minority game"] [6–10]).

In the early days of econophysics, most research focused on empirical analysis, since scientists should first derive some stylized facts based on existing data from real financial markets. Some theoretical analyses have also been undertaken to supplement empirical studies. In terms of research that embraces these methods, some monographs [11–14] and reviews [2, 15–17] are worth reading.

1.2 Brief introduction to agent-based models

The thinking behind agent-based modeling is borrowed from molecular dynamics simulations [18] and Monte Carlo simulations [19, 20]. This means of modeling is a way of deriving macroscopic results, and it is done by simulating the microscopic units of a system under certain evolutionary mechanisms. Econophysicists would do well in undertaking considerable and effective research, given its simplicity and underlying intuition [12, 13, 21–23].

Although the concept of agent-based modeling has a long history, it did not proliferate widely until the 1990s, when the performance of modern computers reached a certain threshold of sophistication [24]. Textbooks [25] and specialized journals (e.g., the *Journal of Artificial Societies and Social Simulation* [*JASSS*] and *Complex Adaptive Systems Modeling* [*CASM*]) subscribe to this particular research method. In its early days, agent-based modeling was mainly used to study certain issues, such as migration, disease propagation, communications, and social networks [26–28]. More recently, agent-based simulations have been applied to a wider range of areas, including human cognition [29] and decision making [30].

Since we are talking about econophysics here, a simple overview of the application of agent-based modeling to the field of economics would be of benefit. The agents here are individual units within complex adaptive systems, and they interact with each other under certain evolutionary rules based on initial system configurations. System behaviors and stylized facts are obtained at the end of the evolutionary process. Numerical simulations could generate new findings that are unavailable through conventional methods. Learning processes and external interference are sometimes involved during the simulation; as such, agent-based modeling is actually a bottom-up approach to studying economics [31]. It acts as a supplement to theoretical results and a verification of real-world data. Agent-based modeling has been applied to a variety of research fields, such as competition and collaboration [32], asset pricing [33], market structure [34], hedging [35], financial crises [36, 37], and even macroeconomics [38].

1.3 The advantages of experimental econophysics

Controlled human experiments in the field of econophysics were first introduced by Platkowski and Ramsza in 2003 [39], much later than the birth of this research area. Some people believe that agent-based simulations should also be incorporated into the scope of experimental econophysics, since such simulations constitute one kind of "experiment". However, in this review, we limit the denomination of "experimental econophysics" to controlled human experiments done in the field of econophysics. In this context, the word "controlled" means that these laboratory experiments that involve real people are controlled by the experiment organizer by tuning one or more variables or conditions among experiment sessions. There are mainly two advantages inherent in experimental econophysics: (i) in both experiments and real markets, the subjects are humans, and this allows for direct comparisons of results, and (ii) causality can be confirmed or disproven by controlling the experimental conditions.

Since a combination of experimentation and theory is absolutely valid in traditional physics, it is natural to conduct human experiments in econophysics [2]. However, as the subjects are human, some points should be emphasized. First, besides human experiments, empirical analyses should also be undertaken, and their results compared to those obtained in the laboratory. Second, the experiments should be conducted for specific purposes (e.g., verifying specific properties found in empirical analyses); this means that the mechanisms of the human experiments need to be carefully designed. Third, the provision of theoretical analyses (if possible) can better verify the causality studied if they resonate with reality. Nevertheless, it is sometimes difficult to undertake a full complement of theoretical, empirical, and experimental analyses, given the complexity of human society and psychology.

1.4 Chaos, leverage, and business cycles

In this review, we look to present and explain new results regarding chaos, leverage, and business cycles, all of which are obtained by undertaking agent-based modeling and human experiments. It is worthwhile first to present some basic knowledge concerning these research areas.

The concept of "chaos" describes a group of system behaviors that are extremely sensitive to initial conditions (where even the systems are deterministic and nonrandom) and which therefore make it impossible to make long-term predictions [40, 41] (i.e., the predictability of chaotic systems is approximate). Chaotic behaviors are common in weather and climate, as well as in geological, mathematical, social, physical, economic [42, 43], financial [44, 45], engineering, biological, ecological, and psychological [46] systems.

Leverage has been widely used in financial markets worldwide to multiply gains and losses [47], chiefly by borrowing "funds". Obviously, risk is enhanced given the possibility of shrinkage of the borrowed money. Actually, leverage, mortgage debts, options, and future contracts can also be treated as products. Given the widespread use of leverage, banks and supervisors within the market have imposed various kinds of limitations and regulations [48]. Even so, many attribute the 2007–2009 financial crisis to leverage.

Typically, the term "business cycles" refers to the upward and downward movement of gross domestic product against its trend [49]. More extensively, a single business cycle can refer to a sequential boom and contraction. The causes of business cycles were originally thought to be overproduction and underconsumption, both of which are caused by wealth inequality. Later, multiple explanations for business cycles were proposed; for example, Keynesians believe that business cycles are caused by employment levels that are higher or lower than the equilibrium, and that they can be resolved by implementing monetary and fiscal policies.

2 Relevant knowledge

Here, we present some relevant knowledge in preparation for reading this review. Having a thorough understanding of these concepts will be useful in considering the information in the following sections.

2.1 The methodology for agent-based modeling

In deriving agent-based simulations, it is essential that one start from general evolutionary laws and the principles of each agent, and then gather from each agent the feedback that forms the behaviors of the overall system [50–52]. This is a bottom-up approach to studying complex systems.

The basic components of an agent-based model are: (i) a large number of agents, (ii) learning rules or adaptive processes for these agents, and (iii) a complex system comprises specific topologies. Typically, there are two ways of designing learning rules or adaptive processes—namely, abstracting real-world systems and borrowing physical models [2]. Although the rules may be simple, agent-based modeling could obtain system behaviors different from those in equilibria. The system topologies almost approximate network or lattice structures. In other words, agent-based modeling is more like an inductive process by which researchers look to determine phenomena after agents interact with each other under certain rules.

The goal of agent-based modeling is to understand the microscopic dynamics of a particular behavior; this represents the single greatest potential benefit of agent-based modeling. Besides equilibria, the robustness of the system can also be tested through agent-based modeling; this is relevant to the diversity and connectedness of the agents. The relationship between agent-based models and complex network-based models has been strengthened [53] by rapid developments in network science. The agent environment could also be of importance [54]. One final point is that agent-based models still need to be verified and validated, to ensure that they are working correctly [55].

2.2 The methodology for controlled human experiments

For hundreds of years, physicists have conducted experiments to verify theories. While studying the potential cause of a behavior, controlled experiments are undertaken to better understand exact causes. Since this study examines human actions and market interactions within the field of econophysics, we should surely conduct controlled human experiments to better explain related theories. In reality, many psychologists have already undertaken many human-behavior-based experiments, ranging from simple to complex. During such experiments, the organizer may give the subjects certain orders and instructions, and the subjects behave as they wish, according to certain rules. Given the large number of agents required for most experiments, many human experiments today are processed online, in a manner seen in much our research.

Figure 1 is a schematic flow chart that explains the process of preparing computer programs for human experiments. The left panel shows the roles of the server; these include distributing necessary information to clients, and storing and processing data. Users need to upload their choices according to different environments. Most of the time, for clarity, the client is designed as a web page. In most of our research concerning controlled human experiments, we use the PHP language to compile web pages, and MySQL to store and process data. Figure 2 is a photograph taken during our human experiment that was conducted on September 21, 2014.

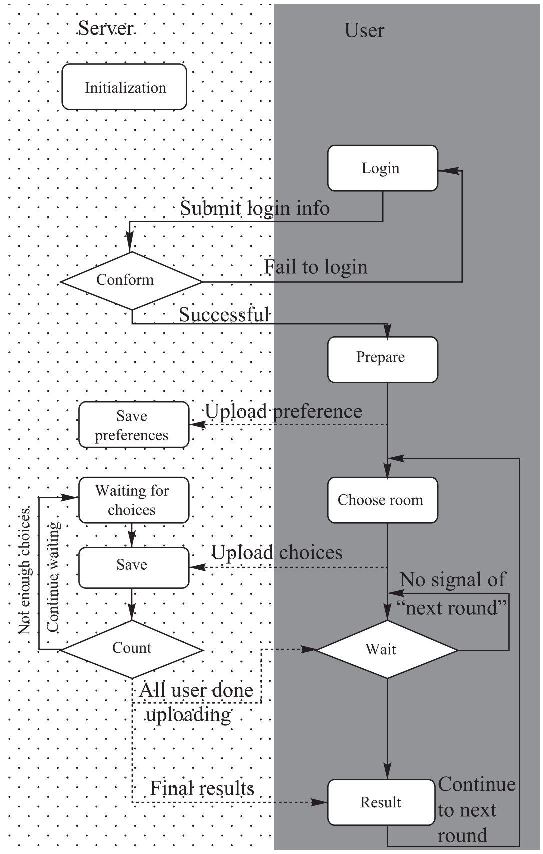

Fig. 1 Schematic flow chart for designing computer programs for controlled human experiments. Revisions can be made according to specific circumstances of the researched system. Reproduced from Ref. [56].

2.3 El Farol Bar problem

The El Farol Bar problem was first addressed by Arthur, in 1994 [57]; it has become one of the most significant problems (i.e., dilemmas) in the field of game theory. A detailed description of this problem is as follows. In the city of Santa Fe, New Mexico, there is a small bar called El Farol Bar. A certain population of people like to go there every Thursday night. However, the bar is small, and so it cannot accommodate all the people who want to have fun there. The following are the assumptions made: (i) if less than 60% of the population goes to the bar, those at the bar will have a better time than if they stay at home, and (ii) if more than 60% of the population goes to the bar, those at the bar will have a worse time than if they were to stay at home. Unfortunately, it is compulsive for everyone to decide whether he or she will go to the bar at the same time, without consulting the others (i.e., no one knows how many people will go to the bar before he or she makes his or her own decision).

What makes this problem interesting is that there does not exist a pure-strategy Nash equilibrium — that is, if

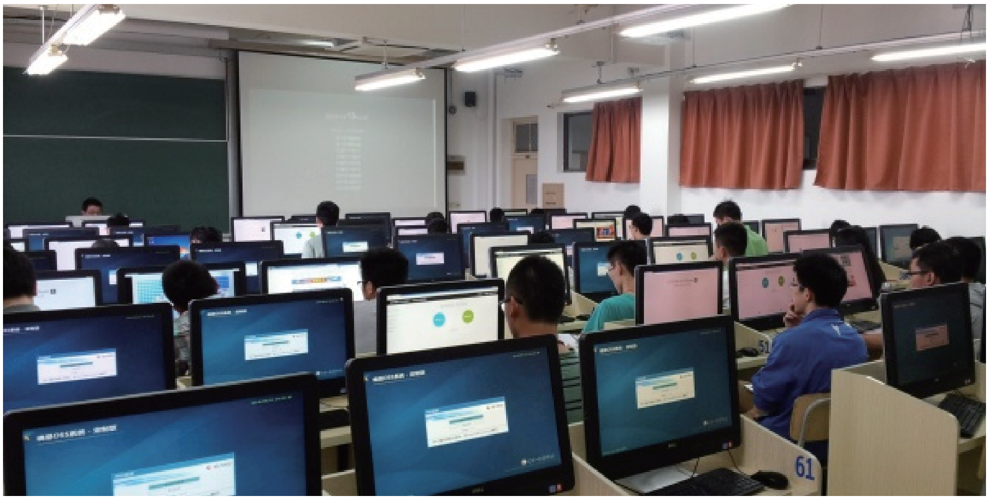

Fig. 2 A photo taken at our human experiment conducted on September 21, 2014.

everyone uses the same pure strategy, no one will win the "game". There only exists a mixed-strategy Nash equilibrium, which allows each person to make a choice with a particular probability. (Here, in this case, it is likely that 60% will go.) This is the only symmetric Nash equilibrium for this problem. Some asymmetric Nash equilibria are discussed in Ref. [58].

The El Farol Bar problem became famous because it reveals an important focal point in modern society — namely, resource allocation. The fact is that people are always competing, quite spontaneously, for limited resources. The El Farol Bar problem merely oversimplifies this competitive behavior. Resource allocation is especially and obviously more important in the field of economics, and many assumptions made in former economics theories do not easily hold in real markets. Problems such as the El Farol Bar problem usually appear when there are no dominated pure strategies for each person. Even when dominated strategies do exist, it is still difficult for everyone to discover and implement these strategies, on account of their limited knowledge, rationality, and deductive-reasoning abilities [2].

Game theory serves as an abstraction of real-world economics-based situations, in which people compete with others to benefit themselves. One concept vital to theory is "complete knowledge", in which everyone knows how others will act under all circumstances. The solution (mixed strategy) for the El Farol Bar problem is obtained under such circumstances. However, it is impractical to assume that human beings will act fully rationally. Each person has his or her own strategies for predicting bar attendance, while considering others' strategies. Ultimately, some kind of equilibrium will be reached. This problem gave birth to the distinguished "minority game" in econophysics, which will be introduced in the following subsection.

2.4 Minority game

The "minority game" was proposed by Challet and Zhang [6, 13]. The setup of this game is simple: there are an odd number of players, and each player should independently make one of two choices in each game round. After all players have made a choice, those who end up on the minority side win. This process continues for many rounds. Its similarity to the El Farol Bar problem is that no dominated pure strategies exist. In the mixed-strategy symmetric Nash equilibrium, everyone chooses a side with a 50% probability. The authors also analyzed the collective behavior of the players in this complex adaptive system. A considerable body of research has been undertaken regarding the minority game and its applications [12, 13].

Historical information plays an important role in a minority game. The agents in a simulation can learn from history in order to improve their performance in the forthcoming round. Nevertheless, there should not be a universal dominated pure strategy. For a memory length of m, there are 2^{2^m} possible strategies. These strategies are randomly allocated to each player at the beginning of the game.

One of the applications of a minority game is to simulate stock markets, since the two sides in the game are similar to the two choices of stock market participants (i.e., buy and sell) [59, 60]. Some research results support the viewpoint that stock prices vibrate around their equilibria. Furthermore, competition and cooperation problems can also be studied through the use of a minority game [61, 62].

2.5 Complex networks and network of networks

A large number of interacting individuals form a network, and if the network has nontrivial topological features (i.e., they do not appear in simple-structure networks), it is then called a complex network. Most real-world systems are complex networks, because there are rarely simple topologies or interactions. Initially, in the fields of computer science, brain science, and social science, scientists began to study complex networks. With the growing sophistication of computer performance, the study of complex networks has become an appealing area.

The word "complex" means that interactions among the elements of the network cannot be simply described. Such features are invisible in regular lattice networks or random networks [63]. Two types of well-known complex networks are scale-free networks [64] and small-world networks [65, 66]. The node degrees of a scale-free network are power-law distributed; a small-world network, on the other hand, has a short edge length and a high clustering coefficient. Therefore, both of these network types have nontrivial features. Other main research areas include node failures and recoveries [67], the control of disease propagation, and the modeling of city traffic [68], among others.

Recently, research about complex networks has been expanded to the concept of a "network of networks" [69]. The interdependence of the networks makes them more fragile to external attacks and internal node failures, generating cascading failures and exhibiting first-order percolation transitions [70]. Chen, Zhang, and Huang proposed in 2007 the origins of a "network of networks" [71]: they create a normal Watts–Strogatz network of vertices and replace each vertex with another Watts–Strogatz network with a specific degree (Fig. 3). The subvertices in a specific vertex have the same number of connections, while subvertices in different vertices have different numbers of connections. The number of subvertices in each vertex is distributed according to a particular probability distribution. This network setup is analogous to a hierarchy structure. As one can see, the subvertices here are similar to a small-world network, and these networks are connected by edges; this is identical to the concept of "network of networks."

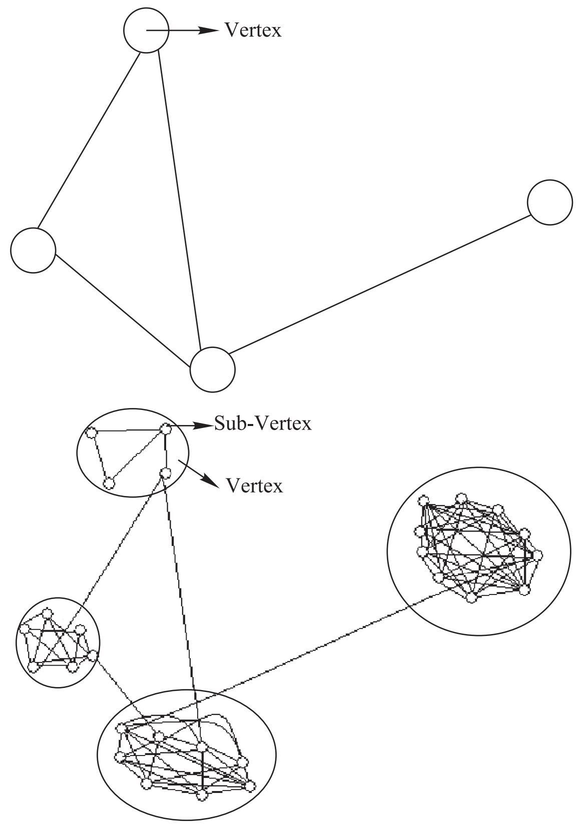

Fig. 3 Upper panel: Sketch graph of the Watts–Strogatz network, with each vertex connected to its two nearest neighbors. Lower panel: Sketch graph of the network built by the authors (i.e., each vertex actually comprises many subvertices). Reproduced from Ref. [71].

3 Principles for the design of agent-based models

Agent-based models typically imitate the microscopic models in statistical physics. The only difference is that units in agent-based models represent human beings or social organizations, while those in microscopic models are interacting entities with zero intelligence.

The term "intelligence" is used here to describe one's capacity for logic, understanding, learning, creativity, and problem-solving, and so it is reasonable that we should define "intelligence" for agents in agent-based models. A better agent-based model should adopt more intelligent agents. However, there still lacks a universal principle by which to define agent intelligence.

Hence, in Ref. [72], our group proposes three principles for designing highly intelligent agents (i.e., *iAgents*). These principles are set according to the needs in modeling financial markets. We use the processing of multiple information, inductive learning, and a dynamic genetic algorithm to explain improvements in *iAgent* intelligence. We acquire the previously defined intelligence level of *iAgents* (i.e., their accumulated profits in trading) by letting them do virtual index trading in stock markets — namely, the S&P 500 and NKY 225 indices. We also compare the intelligence level of our *iAgents* to those of agents from other agent-based models.

3.1 Principles

We propose three principles, and the design of highly intelligent agents should follow the following rules.

Principle 1: *iAgents* should have a high level of information-processing ability. Since considerable amounts of information flow into and out of the financial markets, *iAgents* should be able to handle such information.

Principle 2: *iAgents* should have a high level of learning ability. Real traders in financial markets are always learning from others, new and incoming information, and the whole market, in order to improve their own strategies. This means that there should exist a mechanism by which the *iAgents* learn.

Principle 3: *iAgents* should have a high level of ability to adapt to environmental changes. Given the ever-changing internal and external environment of financial markets, traders should be adaptive to all kinds of situations in order to see steady gains over the long term. That is, *iAgents* should update their strategies in line with different circumstances.

3.2 Model

We design an agent-based model that satisfies the three aforementioned principles. The details of the model are as follows.

Information in financial markets can be classified in many respects, such as fundamental information, technical information, and many other specific types of information. Our *iAgents* should be able to handle three kinds of technical information — price change series $\{R_t\}$, trading volume change series $\{V_t\}$, and market volatility change series $\{A_t\}$ ($A_t = |R_t| - |R_{t-1}|$). By monitoring the signs of these three series (which result in three sign series $\{M_t\}$, $\{D_t\}$, and $\{Y_t\}$), the *iAgents* decide their stock positions. The values in the three sign series could be -1 (negative values in the original series), 0 (0 in the original series), and $+1$ (positive values in the original series). The influence of these values will decay with time in an exponential form [73]. At time step t, the influence of information on a single *iAgent* can be normalized into:

$$I_M(t) = \sum_{\tau=1}^{T_M} M_{t-\tau} \exp\left(-\frac{\tau-1}{T_M}\right) \Big/ \sum_{\tau=1}^{T_M} \exp\left(-\frac{\tau-1}{T_M}\right),$$
$$I_D(t) = \sum_{\tau=1}^{T_D} D_{t-\tau} \exp\left(-\frac{\tau-1}{T_D}\right) \Big/ \sum_{\tau=1}^{T_D} \exp\left(-\frac{\tau-1}{T_D}\right),$$
$$I_Y(t) = \sum_{\tau=1}^{T_Y} Y_{t-\tau} \exp\left(-\frac{\tau-1}{T_Y}\right) \Big/ \sum_{\tau=1}^{T_Y} \exp\left(-\frac{\tau-1}{T_Y}\right), \tag{1}$$

where T_M denotes the half-life time, and $I_M(t)$, $I_D(t)$, $I_Y(t) \in [-1, +1]$, $\forall t$, are impact factors of the technical information.

iAgents should also learn from information. A single *iAgent* could choose from a variety of weights $\alpha, \beta, \gamma \in [0, 1]$ ($\alpha + \beta + \gamma = 1$) and directional parameters e_M, e_D, e_Y (-1 or $+1$) for the three impact factors. The position-changing level $L(t)$ is defined as

$$L(t) = \alpha e_M I_M(t) + \beta e_D I_D(t) + \gamma e_Y I_Y(t) \in [-1, +1], \forall t, \tag{2}$$

and the position for a particular agent at time step t is defined as

$$P(t) = [L(t) P_{\max}], \tag{3}$$

where $P_{\max}$ is the position limit for this *iAgent*. It is obvious that e_M, e_D, and e_Y determine the influence of the impact factors on the agents' stock positions.

Furthermore, the *iAgents* should be able to optimize the model, which tunes the parameters of α, β, γ, e_M, e_D, e_Y, T_M, T_D, and T_Y. Different parameter choices can lead to different simulation results, and so it is reasonable that the *iAgents* be able to tune the parameters automatically through a learning process to reach an optimized state. Therefore, we define the strategy forms as $\{\alpha,\ \beta,\ \gamma,\ e_M,\ e_D,\ e_Y,\ T_M,\ T_D,\text{ and } T_Y\}$. Each *iAgent* has S strategies, and all of them will be used to undertake fake stock-trading at each time step, while his or her accumulated profits will be calculated and recorded. The *iAgents* will always use the strategy that derives the largest profit. Since the *iAgents* can tune the parameters themselves, problems concerning the manual calibration of too many parameters do not exist.

We also introduce the dynamic genetic algorithm (DGA) method to improve the *iAgents*' strategies. In every g_a time steps (the DGA period) there would occur crossovers, mutations, and communications. Figure 4 is a schematic diagram of DGA, and some details of DGA can be found in Ref. [72].

3.3 Results

We let the *iAgents* trade virtually on the S&P 500 and NKY 225 indices, so the intelligence level of each *iAgent* is assessed. We then compare the performance of our *iAgents* with those of random traders, wealth game (WG) agents [74], and WG-DGA agents (i.e., upgraded WG agents whose evolution adheres to the DGA method). The differences among these four kinds of agents are listed in Table 1.

We evaluate agent performance as follows. Initially, they have no cash or stock. They can borrow any amount of money or short-sell any stock units, only if they satisfy $|P(t)| \le P_{\max} = 10$. The accumulated profit of an

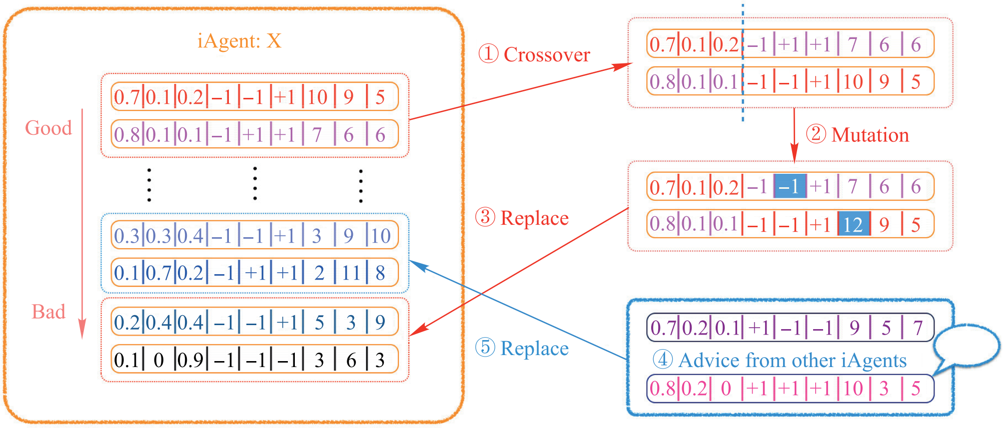

Fig. 4 A schematic diagram of DGA used in the evolution of the *iAgents*' strategies. Reproduced from Ref. [72].

Table 1 Intelligence level of four types of agents, based on the three principles. Reproduced from Ref. [72].

	Random traders	WG agents	WG-DGA agents	*iAgents*
Information processing	none	low	low	high
Learning	none	low	low	high
Adaptation	none	low	high	high

agent is his or her total value of cash and stocks. Market impact and trading costs are not considered.

We divide the two traded indices into two parts — namely, the optimization period and the test period — as shown in Fig. 5. For the optimization period, we simulate our *iAgent* model and WG-DGA model for 1000 cycles to find their optimal strategies. For the random agents and WG agents, we need to simulate the former period only once, since they cannot optimize their strategies themselves. The accumulated profits of all agents are set back to zero at the end of the optimization cycle (the beginning of the test period), and their strategies remain. Details of the other simulation settings can be found in [72].

We simulate 10 000 agents for each of the four types; their distributions of accumulated profits in the two periods and with the two indices are shown in Figs. 6 and 7 and summarized in Tables 2 and 3. In analyzing the distribution patterns of the agents, we decide to use the mean accumulated profits to describe the intelligence level of each agent type.

We can see from the graphs and tables that the mean wealth of random agents is close to zero; this is reasonable. The WG and WG-DGA agents perform at a level worse than the average market performance level. However, in the test period, the mean wealth of the *iAgents*

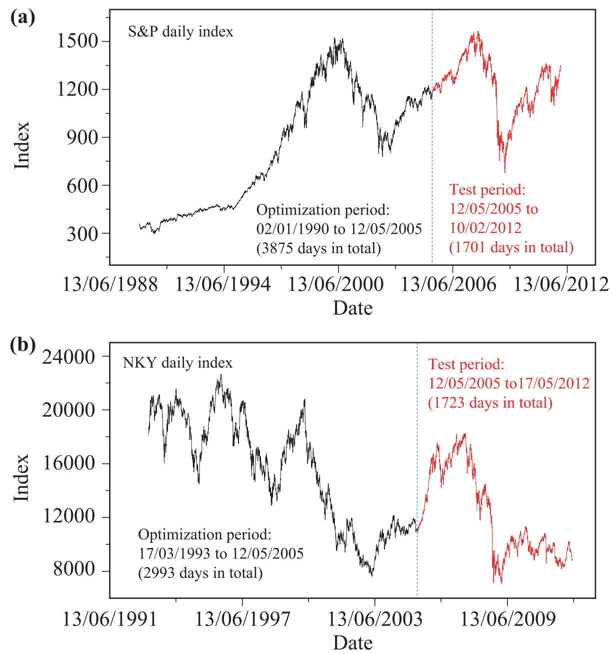

Fig. 5 The S&P 500 and NKY 225 indices used to assess agent intelligence level. Each index is divided into two periods, as shown by the black and red lines. Reproduced from Ref. [72].

Table 2 Agents' mean wealth from index trading on the S&P 500 index, at the end of the optimization and the test periods. Reproduced from Ref. [72].

	Optimization period		Test period	
Market performance	7996.7		1832.8	
	Mean wealth	*SD*	*Mean wealth*	*SD*
Random traders	35.66	3865.67	−13.22	4036.10
WG agents	5657.24	2075.80	1132.34	2450.09
WG-DGA agents	10147.21	654.09	1619.46	1066.86
iAgents	1243.27	2409.27	15679.84	2362.30

Table 3 Agents' mean wealth from index trading on the NKY 225 index, at the end of the optimization and the test periods. Reproduced from Ref. [72].

	Optimization period		Test period	
Market performance	70954.3		22013.5	
	Mean wealth	*SD*	*Mean wealth*	*SD*
Random traders	624.71	72227.74	−211.28	46446.92
WG agents	−43498.95	58847.92	−2908.69	36037.43
WG-DGA agents	73843.27	43030.82	−61501.22	30347.29
iAgents	45438.43	56210.40	16120.84	31853.90

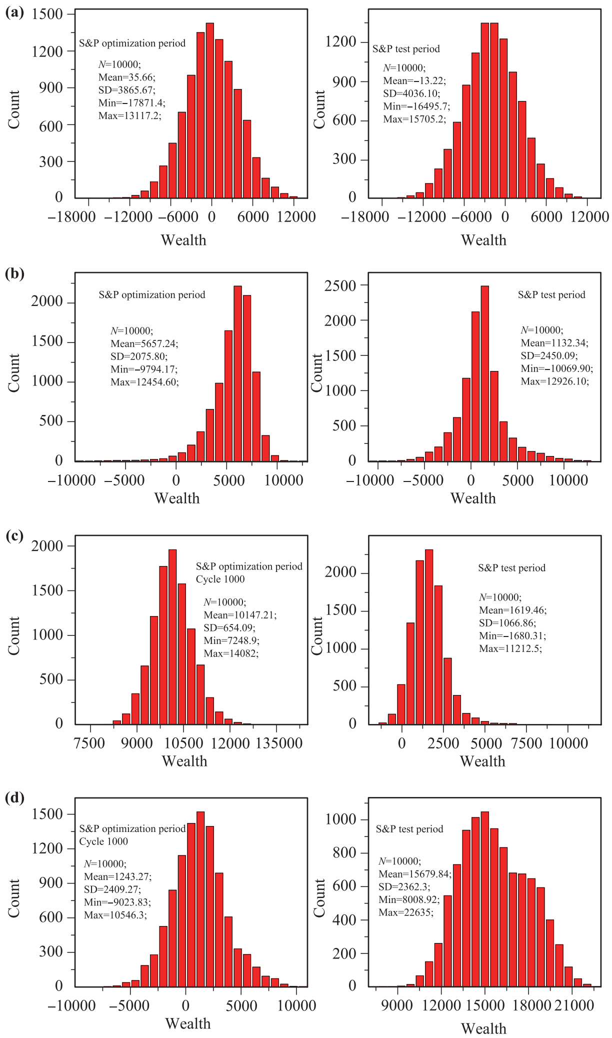

Fig. 6 Accumulated profit distributions of the 10 000 agents from each of the four types — namely, **(a)** random traders, **(b)** WG agents, **(c)** WG-DGA agents, and **(d)** *iAgents* — by trading on the S&P 500 index. The graphs in the left-hand column show the agents' wealth at the end of the optimization period; those in the right-hand column show the agents' wealth at the end of the test period. Reproduced from Ref. [72].

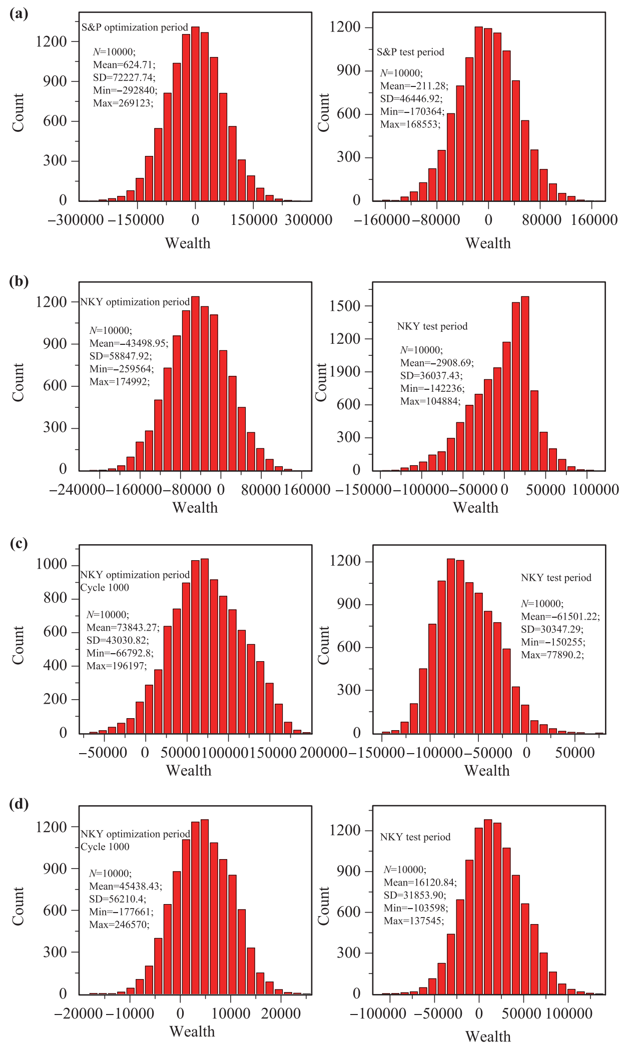

Fig. 7 Accumulated profit distributions of the 10 000 agents from each of the four types — namely, **(a)** random traders, **(b)** WG agents, **(c)** WG-DGA agents, and **(d)** *iAgents* — by trading on the NKY 225 index. The graphs in the left-hand column show the agents' wealth at the end of the optimization period; those in the right-hand column show the agents' wealth at the end of the test period. Reproduced from Ref. [72].

could be 8.56 times larger than the market performance of the S&P 500 index. Similar results are seen in the summary of simulations on the NKY 225 index. Among the four types of virtual agents, our *iAgents* perform best.

3.4 Concluding remarks

There is a further note regarding the algorithmic trading of the *iAgents*. We set the number of strategies S for each *iAgent* to be a small value, in order to reduce the simulation time. It is certain that increasing S can further improve *iAgent* performance. In addition, the evolutionary strength of their strategies is determined by g_a, g_m, and g_p (the exact meanings of these parameters are found in [72]), and the number of optimization cycles. High evolutionary strength may result in agent overlearning; therefore, a moderate parameter setting is preferred for our *iAgents*.

This section presented our principles and the design of highly intelligent agents in simulating financial markets. We defined three principles for determining the intelligence level of financial market traders — which is to say, their ability to process information, learn, and adapt. We designed a kind of highly intelligent agent, an *iAgent*, and these agents are compared to three other kinds of virtual agents in terms of virtual trading on the S&P 500 and NKY 225 indices. Ultimately, we found that the *iAgents* perform best. It is worth noting that the three principles proposed here can also be useful in modeling other human-based complex adaptive systems.

4 Chaos and periodicity

Minority games [6] have been applied to the study of various kinds of human collective behaviors and self-organized systems, such as financial markets and social problems [75–80]. In 2009, our group proposed a market-directed resource allocation game (MDRAG) to model the equilibrium of a system with biased resource allocation [81]. Most of the extensions of the minority game dynamically evolve in a single direction (i.e., from an initial state to an equilibrium). Here, we propose a two-sided minority game [82] in which resources are movable and can be transferred and controlled.

Generally, in two-sided minority games [83, 84], there are two groups of participants who play two distinct roles (i.e., buyers and sellers, or consumers and suppliers), and they gain payoffs by trading with others under specific rules. In our game, the two parties are set to correlate, and this promises an end state in the game where there is an equilibrium.

Phase transitions have been observed in many social systems [85–87]; within our game, we too discover phase transitions, by spectrum analysis. The critical point of this phase transition is determined by the complexity of the system, which is described by an entropy-like quantity and measures the relationships among participants in different groups. This can be better explained by the counterpart of quantum mechanics. However, if the critical point is exceeded, the overall behaviors of the participants become convergent and periodic.

4.1 Theory

We can describe complex adaptive systems that comprise a large number of interacting units by the many-body Schrödinger equation in quantum mechanics:

$$H\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2,\cdots,\boldsymbol{r}_N)=\left[\sum_i^N -\frac{\hbar^2}{2m}\nabla_i^2+\sum_i^N V(\boldsymbol{r}_i)+\sum_{i<j}U(\boldsymbol{r}_i,\boldsymbol{r}_j)\right]\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2,\cdots,\boldsymbol{r}_N), \tag{4}$$

where $\Psi(\boldsymbol{r})$ is the wave function of N electrons with potential V. If we regard the individuals in games as electrons in particles, we can write the "Schrödinger equation" for complex adaptive systems as

$$H\Psi(1,2,\cdots,N)=\left[\sum_i^N(T_i+V_i)+\sum_{i<j}U(i,j)\right]\times\Psi(1,2,\cdots,N). \tag{5}$$

Here, the wave function $\Psi=\Psi(1,2,\cdots,N)$ represents the ensemble of choices in a particular game, T_i is a stochastic function, and V_i denotes the external resource ratio. Since the choices available to the individuals in our game are limited to two options, the Hamilton operator can be expressed by spins in the Ising model with up or down orientations.

For example, in a MDRAG, the wave function can be predicted as a normalized form of

$$\Psi(1,2,\cdots,N)=\sqrt{\frac{N_1}{N}}\left|1\right\rangle+\sqrt{\frac{N_2}{N}}\left|2\right\rangle, \tag{6}$$

where N_1 and N_2 are the number of agents entering Room 1 and Room 2, respectively, and $|i\rangle$ $(i=1,2)$ represents the orthogonal basis of the winning agents who choose Room i. Thus, the reduced-density matrix for the wave function can be expressed as

$$\rho=\left|\Psi\right\rangle\left\langle\Psi\right|=\frac{N_1}{N}\left|1\right\rangle\left\langle 1\right|+\frac{N_2}{N}\left|2\right\rangle\left\langle 2\right|. \tag{7}$$

Eq. (7) indicates that the probability of the individuals making two choices is equal to the resource ratio, which determines the equilibrium.

4.2 Model and results

Our two-sided minority game is designed on the basis of a MDRAG, by adding suppliers to the game; this is done to control the resource supply. Similar to the agents in the MDRAG, the suppliers could also choose a room in which to supply their resources. Now that the game result is completely and internally determined, it is a better analogy to real markets. In short, the market is isolated, and there are two kinds of participants (suppliers and consumers) trading two commodities (C_1 and C_2). The memory length P and number of strategies for each agent S are introduced, as in all kinds of minority games.

Figure 8 is a sketch graph of our model. The introduction of suppliers differentiates our model from a traditional minority game, since the resource ratios are also determined by agents and could be ever-changing.

In one round of the game, the numbers of suppliers (consumers) entering Rooms 1 and 2 are denoted as M_1 and M_2 (N_1 and N_2), respectively. Since every supplier (or consumer) is equal, the winning judgement of our game is as follows. After the end of a particular round (i.e., all the participants have made their choices simultaneously), if $M_1/N_1 > M_2/N_2$, the consumers who purchased C_1 win; this is because there is an excess supply of C_1, and so its price is relatively low (i.e., these consumers are on the relatively minority side). Similarly, suppliers who choose to supply C_2 also win. The winning sides will be reversed if $M_1/N_1 < M_2/N_2$. The winners of the round get 1 point; the losers get nothing.

Given the fact that the two types of agents interact with each other, we can regard them as two subsystems — namely, N and M. Thus, the whole system can be expressed as a superposition of multinominal product states. We define the Hilbert space of our system as $H_{NM} = H_N \times H_M$, and $|1\rangle$ and $|2\rangle$ as the orthogonal bases. Thus, the states of the two subsystems can be written as $|\psi\rangle_M = \sqrt{\frac{M_1}{M}}\,|1\rangle_M + \sqrt{\frac{M_2}{M}}\,|2\rangle_M$ and $|\psi\rangle_N = \sqrt{\frac{N_1}{N}}\,|1\rangle_N + \sqrt{\frac{N_2}{N}}\,|2\rangle_N$, and the state of the whole system

$$
\begin{aligned}
|\psi\rangle_{NM} &= \sum_{i,j} c_{ij}\,|i\rangle_N \times |j\rangle_M \\
&= \hat{c}_{12}\,|1\rangle_N \times |2\rangle_M + \hat{c}_{21}\,|2\rangle_N \times |1\rangle_M\,,
\end{aligned} \tag{8}
$$

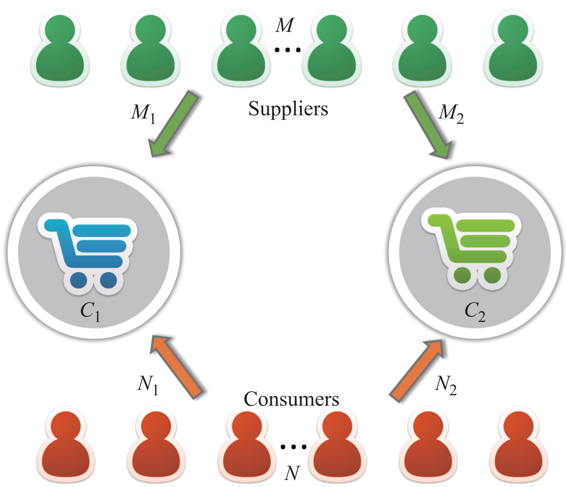

Fig. 8 A sketch of our two-sided minority game. Two commodities C_1 and C_2 are freely traded in the two rooms. M suppliers and N consumers choose from the two commodities. Reproduced from Ref. [82].

where c_{ij} is the composite coefficient, $|i\rangle_N$ is the basis state of the winning room for the consumers' subsystem, and normalized factors $\hat{c}_{12}$ and $\hat{c}_{21}$

$$
\begin{aligned}
\hat{c}_{12} &= \sqrt{\frac{N_1 M_2}{N_1 M_2 + N_2 M_1}}, \\
\hat{c}_{21} &= \sqrt{\frac{N_2 M_1}{N_1 M_2 + N_2 M_1}}.
\end{aligned} \tag{9}
$$

Since the reduced-density matrix for one of the subsystems is the partial trace over the other, we are able to define an entropy-like quantity to describe the system's complexity:

$$
Q_N = Q_M = -\mathrm{Tr}(\boldsymbol{\rho} \log_2 \boldsymbol{\rho}) = -\sum_{i=1}^{2} \rho_i \log_2 \rho_i, \tag{10}
$$

where $\rho_i = \frac{N_i/M_i}{N_1/M_1 + N_2/M_2}$. When the value of Q is large, this indicates a random system state where agents have plenty of "possible states". When the value of Q is small, this indicates reduced system complexity, since arbitrage opportunities emerge. Thus, it can be seen that the value of Q also indicates the market's efficiency (i.e., an efficient market does not exist with arbitrage opportunities).

In our game settings, only suppliers are able to know the system complexity $[Q(t) = -\sum_{i=1}^{2} \rho_i(t) \log_2 \rho_i(t)]$ before they make their choices. At equilibrium, $Q(t)$ will be highest, and arbitrage opportunities will be driven away from the market. We also define a psychological threshold value Q_{th} for the suppliers, at which point they are able to change their choices in order to find new arbitrage opportunities.

Suppose the strategy space for suppliers is $\{(\alpha, 1-\alpha)\}$, where α is the proportion of agents providing commodity C_1 (i.e., M_1/M). The strategy space for consumers is $\{(\beta, 1-\beta)\}$, and α or $\beta \in [0, 1]$. We also suppose that $p(t)$ is the historic information at time step t, then the suppliers' and consumers' strategies can be given as

$$
\alpha(t) = \frac{1}{N} \sum_{i}^{N} c_i(s_i^*(t), p(t)), \tag{11}
$$

$$
\beta(t) = \frac{1}{M} \sum_{j}^{M} c_j(s_j^*(t), p(t)), \tag{12}
$$

respectively, where $s_i^*(t)$ is the best strategy of agent i and c_i is his or her choice (i.e., the value of 1 for the choice of Room 1, and 0 for the choice of Room 2). If $\alpha < \beta$, the suppliers in Room 1 and consumers in Room 2 win, and the total payoff $r_{\alpha\beta} = M\alpha + N(1-\beta)$. If $\alpha > \beta$, $r_{\alpha\beta} = M(1-\alpha) + N\beta$. The expected value of the system payoff

$$
r = \sum_{\alpha,\beta} r_{\alpha\beta} P_{\alpha\beta}, \tag{13}
$$

where

$$P_{\alpha\beta} = 1.175\mathrm{e}^{-[(\alpha-0.5)^2+(\beta-0.5)^2]}\mathrm{d}\alpha\mathrm{d}\beta. \tag{14}$$

The detailed calculation process can be found in subsection 3.3 of Ref. [82].

Figure 9 shows the payoffs of different combinations of α and β. Large values of total payoff are shown in dark grey, and small values in light grey. The case of $\alpha = \beta$ corresponds to Pareto optimality and the Nash equilibrium. The system will always fluctuate around the equilibrium. By introducing the threshold Q_{th}, the system finds a way to move away from the equilibrium.

To express the influence of the threshold value Q_{th} on the α and β strategies, we illustrate in Figs. 10(a) and (c) the change in strategy α over time, where $Q_{th} = 0.88$ and 0.72, respectively. With a low threshold, suppliers behave so as to stay in one market for a while (and the system approaches equilibrium simultaneously), and to switch markets whenever the real-time $Q(t)$ is below Q_{th}. This is not the case when Q_{th} is high, as it is difficult for suppliers to jump out of the threshold "trap".

We know that a fractional Gaussian/Brownian noise/motion can be expressed by a pseudo-random series, and it has a power spectrum density of $1/f^\gamma$, where f denotes the frequency of the noise and γ determines the pattern of the signal [88, 89]. If $\gamma > 1$, the noise behaves as a fractional Brownian motion; if $\gamma < 1$, it behaves as a fractional Gaussian noise. By calculating the value of γ under different values Q_{th} in our simulations, we find a phase transition around $\gamma = 1$ (Fig. 11). For large Q_{th} values, $\gamma > 1$, and the system is in a periodic phase with nonstationary Brownian motion; for small Q_{th} values, $\gamma < 1$, and the system is in a chaotic phase with stationary Gaussian noise.

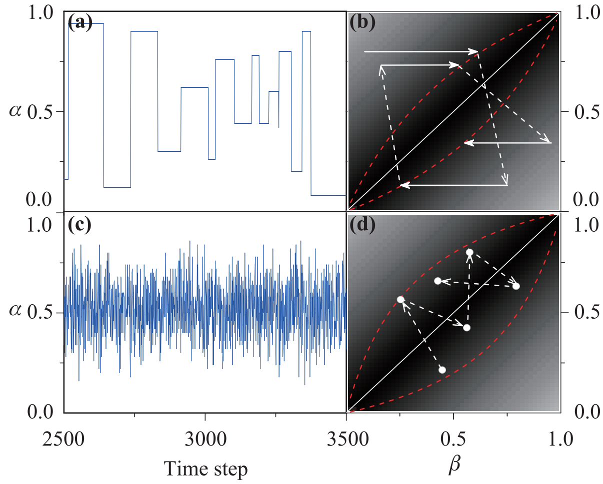

Fig. 10 **(a, b)** The time series of the α strategy and its possible trace in the payoff map under a relatively high entanglement threshold ($Q_{th} = 0.88$). **(c, d)** The same conditions as for (a) and (b), but with a relatively low threshold ($Q_{th} = 0.72$). The time series shown comprise time steps 2500–3500. In (b) and (d), variations in suppliers' strategies are plotted with white solid arrows. Whenever the strategy falls into the "trap", it will try to jump out of it (white dashed arrows). For the periodic phases in (a), the traces of strategy α are almost horizontal; for the phases in (c), however, their traces are more chaotic. Reproduced from Ref. [82].

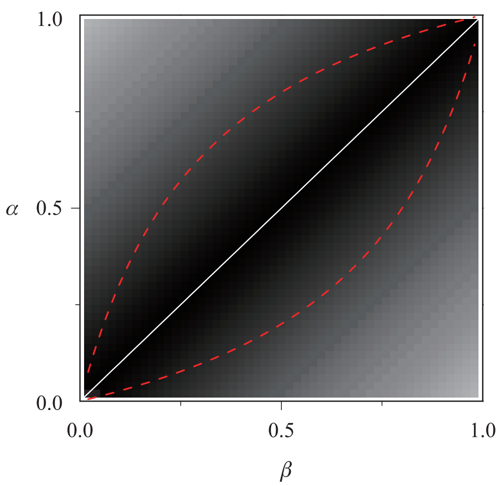

Fig. 9 The total payoff map of the combinations of strategy α and β. The gray level in the map indicates the value of the total payoff $r_{\alpha\beta}$. The strategy combinations under the white solid line lead to a result where the suppliers of commodity C_1 win; those above the white solid line lead to results where they lose. The two red dashed lines are drawn with $Q_{th} = 0.72$. If the suppliers' strategies fall into the "trap" between the red lines, they are likely to find a new strategy that is far from the equilibrium, in order to find arbitrage opportunities. Reproduced from Ref. [82].

4.3 Experiment

We designed a human experiment that made use of web pages; we then recruited 38 students from the Department of Physics at Fudan University to participate in our game. As incentives, they received rewards based on their overall game performance. We conducted five sessions of experiments with $Q_{th} = 0.5$, 0.9, 0.92, 0.98, and 0.99. The participants were randomly assigned as suppliers or consumers. Whenever the real-time market complexity $Q(t)$ exceeded the threshold Q_{th}, the suppliers were able to make new choices. Each game session lasted more than 30 rounds.

We calculate strategies α for each session; these are plotted in Fig. 4(b). The results echo those of former simulations, revealing two phases with a critical point around $Q_{th} = 0.9$.

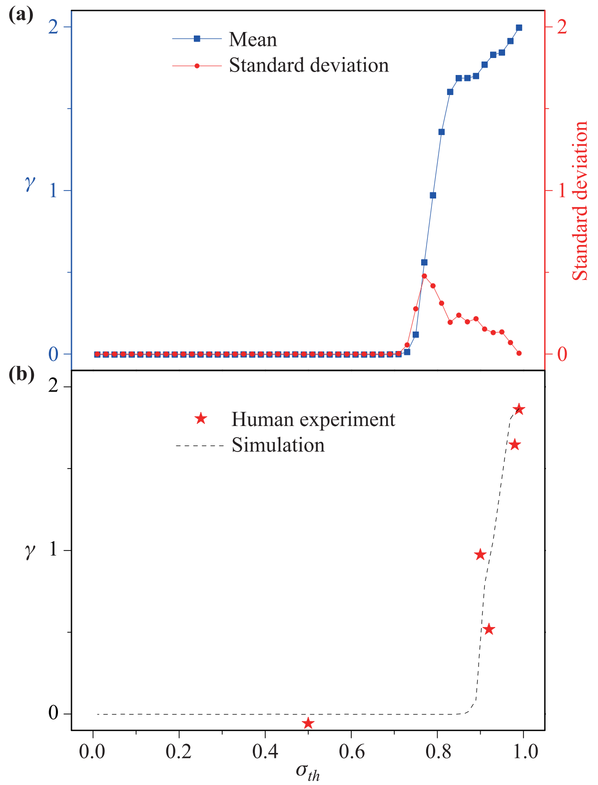

Fig. 11 **(a)** The relationship between the value of γ and Q_{th}. When Q_{th} grows beyond a critical value, γ will approximate 2, and the series is more like Brownian motion (periodic phase). If the threshold is smaller than the critical value, the value of γ will be small, and the series will behave chaotically as Gaussian noise (chaotic phase). **(b)** The red stars show the values of γ obtained in five of our human experiments. The dotted line shows simulation results with parameters $S = 4$ and $P = 32$. Reproduced from Ref. [82].

4.4 Concluding remarks

We designed a two-sided minority game, where two parties (suppliers and consumers) traded in two commodities. By introducing the suppliers as agents, the resource ratio in our game was completely controlled by the agents themselves. We used a quantum mechanics approach to describe the system, and defined an entropy-like quantity to characterize the system complexity. We calculated the power spectrum exponent and found a system transition from a chaotic phase to a periodic phase; this transition was found to be relevant to the system complexity threshold. Human experiments were further conducted to verify our findings.

The results of this work suggest that most real-world markets (since they are bipartite) could exist in a transition between chaotic and periodic phases; as such, they provide insights into a better understanding of market states, especially from the perspective of quantum mechanics.

4.5 Relevant research

Recently, some agent-based models have been designed and developed to study many aspects of chaotic and periodic behaviors. The following provides brief introductions of, and excerpts from, these works.

Gharehchopogh and Dizaji developed an agent-based model to predict road accidents [90]. Their model can provide solutions with regards to accident factors, using plenty of information and advanced data-mining techniques. They use two algorithms — namely, the particle swarm optimization algorithm and chaos optimization algorithms. Their results indicate that their model can predict such road accidents with 98% accuracy.

Das *et al.* propose a novel swarm dynamics in better explorations of agent-based systems [91]. The particle motions in the swarm are determined by an attractant–repellent profile that indicates the distance between particles. They use the Lyapunov function to undertake stability and chaos analyses of the swarm. They explore the parameters of chaotic behaviors, and this enables them to control the system. They also propose a real-world application for this swarm dynamics.

Yang *et al.* studied the periodic behaviors of discrete-time second-order multiagent systems [92]. They consider two system types where the agent dynamic is a double integrator. The periodic behaviors are derived by establishing conditions on the feedback gains of the linear consensus law.

5 Leverage

Leverage is an important concept in modern financial markets [93–95]. Generally, the term "leverage" refers to borrowing money to buy more of something. The leverage ratio is defined as an investor's total assets (his or her own assets, plus borrowed assets) divided by his or her own assets. Leverage is considered an effective way of amplifying investor profits; meanwhile, higher risk is also derived, since the investors must return the loans, and those loans may have shrunk during their investment. A considerable amount of research has examined the exact influences of leverage on investors [96], firms [97–99], and financial markets [100–103]. The excessive use of leverage is believed to be harmful to firms and to be one of the main reasons for the 2008 global financial crisis. Therefore, the study of leverage is helpful to individual investors and policy-makers alike [104, 105].

In Ref. [106], we designed an artificial stock market to study the influences of leverage. Generally, there are two kinds of agents in the market: intelligent agents (i.e., large-scale investors who can use leverage and heterogeneous strategies while making decisions) and quasi-random agents (i.e., small-scale investors who cannot use leverage). Furthermore, we do human experiments to validate our theory and simulation results. We change only the value of the leverage ratio; all other parameters remain fixed during the simulations and experiments, to filter out the direct effect of leverage.

5.1 Model

There is only one stock in the market, and at any discrete time step t, the participants are able to buy or sell the stock. We simplify the trading process so that they need to decide only whether to buy or sell and the order sizes. After all participants have made their decision, the total demand $O_d(t)$ (the sum of all buy orders) and total supply $O_s(t)$ (the sum of all sell orders) are known. The change in stock price is determined by the excess demand [107, 108]:

$$\ln P(t+1) - \ln P(t) = \lambda[\ln O_d(t) - \ln O_s(t)], \tag{15}$$

where λ is the market depth, which describes the range of influence of excess demand. We also assume that the transaction price of time t is the weighted average price of time t and $t+1$:

$$P_{trans}(t) = (1-\beta)P(t) + \beta P(t+1), \tag{16}$$

where $\beta \in [0, 1]$.

For simplicity, we set leverage rules in three respects: leverage qualification, leverage ratio, and margin call. (i) At the beginning of the trading, we give each participant 10 000 cash and 1000 shares of stock at a price of $P(0) = 10$ (i.e., the initial wealth for everyone is 20 000). At any time t, the wealth of a participant can be expressed as

$$W(t) = D(t) + E(t)P(t), \tag{17}$$

where $D(t)$ is the cash he or she holds and $E(t)$ is the shares of stock he or she holds at time t. We stipulate that those whose wealth exceed $W_L = 25\ 000$ are able to trade with leverage. (ii) Let L_r denote the leverage ratio, and M_I the least margin proportion required. Thus, at any time t', the total wealth of a participant $W(t') = L_r W_L$. (iii) Another value concerning leverage is the maintenance margin requirement (M_M). It is generally lower than M_I, and investors should always keep the margin larger than M_M, lest their stocks be automatically sold to return the loan (in what is called a margin call). We further introduce the critical percentage loss l_c of the margin from W_L, and $(1-l_c)W_L$ is the critical margin level. We have

$$M_M = \frac{1-l_c}{L_r - l_c}. \tag{18}$$

Here, we set the value of l_c to be 40% — that is, if a participant's margin falls below 15 000 after trading with leverage, he or she will be forced to close out. If his or her wealth once again exceeds 25 000, he or she will become requalified to use leverage.

5.2 Simulations

In the simulations, there exists a mechanism by which intelligent agents make decisions on their own, under specific market circumstances. Similar to the minority game, to fulfill this purpose, we use strategy tables comprising history series and corresponding decisions. Table 4 is a schematic strategy table, and detailed explanations can be found in Refs. [74] and [106]. Each intelligent agent is assigned S random strategies at the beginning of the simulation, and the agents will always use the most successful strategy (i.e., that which would have derived the most virtual points).

We define an investment ratio $r_i(t)$

$$r_i(t) = 1 - \exp\{-cW_i(t)\} \tag{19}$$

to determine the order size, where c is a positive constant. The order size of agent i at time t can be calculated by

$$\text{shares to buy} = \lfloor r_i(t)D_i(t)/P(t)\rfloor, \tag{20}$$

$$\text{shares to sell} = \lfloor r_i(t)E_i(t)\rfloor. \tag{21}$$

Since the quasi-random agents are small-scale investors, they usually trade randomly and contribute a small proportion of the trading volume. We therefore consider them as a whole. We suppose the total buy orders and sell orders of intelligent agents at time t to be $O_d^{int}(t)$ and $O_s^{int}(t)$, respectively; then, the number of buy orders from quasi-random agents is $R_d\{O_d^{int}(t) + O_s^{int}(t)\}$, and the number of sell orders is $R_s\{O_d^{int}(t) + O_s^{int}(t)\}$. R_d and R_s are two random numbers whose values are on the scale of [0.1, 0.3].

Some detailed parameter settings can be found in Ref. [106]. Generally, we recruit 1000 agents in our sim-

Table 4 A schematic strategy table with memory length of 3. Reproduced from Ref. [106].

History	Choice
000	1
001	−1
010	0
011	0
100	1
101	−1
110	1
111	−1

ulations, and the value of L_r varies from 1 to 9 at a 0.2 interval. Under each simulation session with a specific value of L_r, we run 100 rounds with 10 000 time steps in each round.

5.3 Experiment

We recruited students from Fudan University to play the role of intelligent agents in human experiments. All the other settings of the market are the same as those in the simulations. We undertook two series of experiments; one was conducted on July 8, 2013 with 22 participants, and the other on September 27, 2013 with 46 participants. The value of L_r was set to be 1 and 5 in the first series of experiments, while in the second series it was set to 2, 3, and 4. For a single value of L_r, we conducted 60 rounds of trading. Some parameter settings of the human experiment are depicted in subsection 2.3 of Ref. [106].

5.4 Results

Stock price fluctuations under different circumstances are shown in Fig. 12. That figure shows that a large L_r leads to large stock price fluctuations — which are, in turn, a negative outcome of leverage. There are mainly two reasons for this phenomenon: (i) when L_r goes high, more money will swarm into the market, and (ii) when L_r goes high, margin calls occur more frequently and small stock market drops may trigger greater crashes (Fig. 13). Simulations and experiments show the same trend. Numerical differences are caused by the parameter settings and experimental limitations.

We further study the kurtosis of the market, using simulation data. The tails of a probability density function (PDF) represent the likelihood of extreme cases. There are always fatter tails in financial market return distributions than in Gaussian distributions [109]. Kurtosis is used to measure the tail of a distribution, and it is

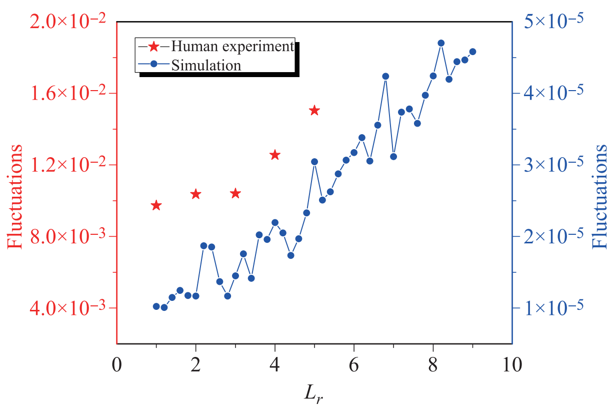

Fig. 12 Market fluctuations under different leverage ratios for simulations (blue dots and vertical scale) and human experiments (red stars and vertical scale). For simulations, the fluctuations are calculated as the ensemble average of 100 simulation rounds. Reproduced from Ref. [106].

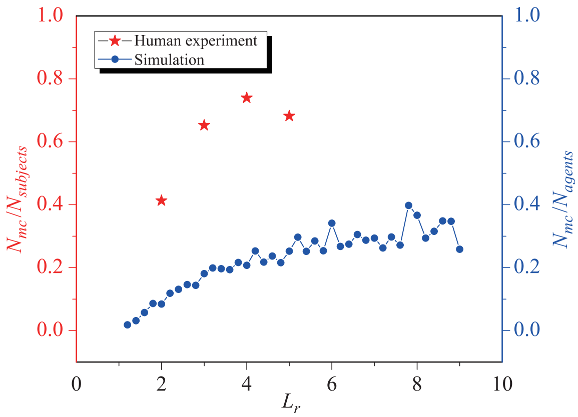

Fig. 13 The number of margin calls occurred N_{mc} over the number of intelligent agents N_{agents} (human subjects $N_{subjects}$ in experiments) under different leverage ratio values. Reproduced from Ref. [106].

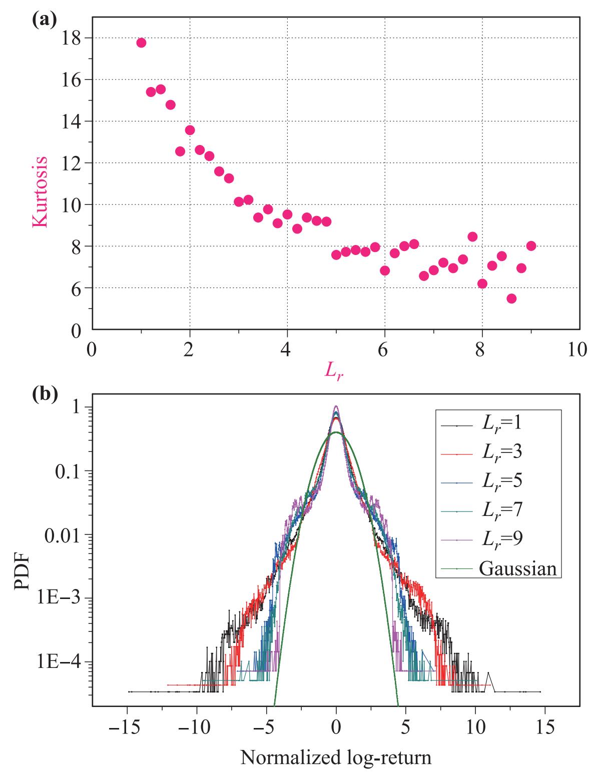

Fig. 14 The effect of leverage on fat tails of the PDF of the price return series: **(a)** Kurtosis as a function of leverage ratio. **(b)** The PDF curves of the normalized log-return series of the market under different leverage ratio values. Reproduced from Ref. [106].

defined as

$$\text{kurtosis} = \frac{\mu_4}{\sigma^4} - 3, \tag{22}$$

where μ_4 is the fourth central moment and σ is the standard deviation of the series. For a Gaussian distribution, kurtosis equals 0. Figure 14 shows the kurtosis and PDF under different L_r values. It is especially interesting to find that a higher L_r leads to thinner tails.

Figure 15 offers a possible explanation for this phenomenon. $N_{mc}/N_{leverage}$ is the percentage of agents trading with leverage who also end up with margin calls. We see from Fig. 15 that with tremendous growth in L_r, $N_{mc}/N_{leverage}$ does not grow much, and its highest value is only 0.557. This means that many of the market participants trading with leverage were never confronted with margin calls. It is therefore reasonable to think that leverage qualification and margin call are two investor filters that "screen out" the best-performing ones. These work to protect the market from irrational crashes.

5.5 Concluding remarks

In this study, we designed intelligent agents that are much more reasonable than those found in previous research; furthermore, our model successfully produced the fat-tail distribution of price returns under various leverage ratio values.

In summary, we designed an artificial stock market, and studied the influence of leverage with the help of controlled simulation and experimentation. We found that leverage has two opposite effects on the market. On one hand, it generates large fluctuations; on the other hand, a larger leverage ratio leads to thinner tails in the return distribution, which in turn reduces the probability of market crashes and crises. Our work could help inform financial market regulators as they work to create healthy financial markets.

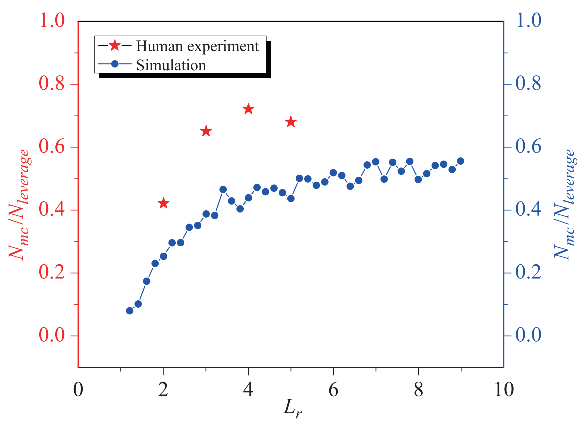

Fig. 15 The number of margin calls N_{mc} divided by the number of times that participants gained the right to trade with leverage $N_{leverage}$, under different leverage ratio values. Reproduced from Ref. [106].

5.6 Relevant research

In Ref. [110], our group studied the effects of leverage on wealth distribution, through the use of controlled human experiments. We found that higher leverage leads to a higher Gini coefficient, a measure that represents unequal wealth distribution due to the higher risk introduced by high leverage and the diversified trading abilities of participants. The results of this study could be helpful to regulators as they adjust wealth distribution by means of leverage.

Aymanns and Farmer propose an agent-based model of leveraged investors in financial systems [111]. The agents can manage their risk via analyses of the value-at-risk constraint and historical price series. Their results indicate the presence of endogenous irregular oscillations with leverage cycles. They further studied this phenomenon by modeling the dynamics and nature of feedback loops; a flexible leverage regulation policy was introduced to tune the system states and reduce the amplitude of leverage cycles.

Breuer *et al.* developed an agent-based model to exchange leveraged assets under double auction [112]. They verified some results concerning endogenous leverage and asset pricing in general equilibrium theory.

Fischer and Riedler propose an agent-based model with boundedly rational agents trading a risky asset [113]. The price of the asset is endogenously determined and the balance sheet of the asset is given to the agents. Fischer and Riedler found that a log-normal distribution of balance sheet size emerges, and higher leverage leads to greater inequality among agents and more bankruptcies and systemic crashes. These phenomena can be attributed to the credit frictions inherent in various bank types.

6 Business cycles

Competition and cooperation exist everywhere in human society [114–120]. It is important to study these phenomena, given the importance of understanding and predicting human behaviors [121].

In Ref. [122], we propose an agent-based model free of competition, with two products available. The participants are divided into suppliers and consumers, and they should decide which product they want to produce/buy. We call this the "Janus game". We show that the game may be helpful to understanding human behavioral regularity. Business cycles can be observed as periodic phases in our game, whose origin is still far from clarified [123–

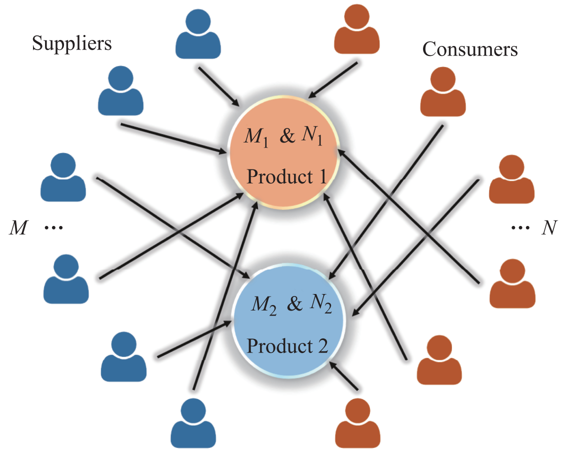

Fig. 16 Sketch of the Janus game with two M suppliers and N consumers competing for two products (Product 1 and Product 2). Reproduced from Ref. [123].

125].

Business cycles comprise prosperities and depressions. However, most research concerning business cycles focus on their properties. The current work provides a bottom-up microscopic understanding of business cycles.

6.1 Model

Figure 16 is a sketch of our Janus game. M suppliers and N consumers are randomly assigned and placed in an isolated market in which they trade in two products. Each supplier could produce one unit of the product he or she chooses, and each consumer could buy one unit of the product he or she chooses. The game is repeated, but the suppliers are not allowed to change their choices unless some conditions are met.

This game is actually a two-sided minority game, like that introduced in Section 4. The minority of suppliers and consumers win. The market reaches the equilibrium when $M_1/N_1 = M_2/N_2$. If $M_1/N_1 > M_2/N_2$, then suppliers producing P_2 and consumers purchasing P_1 win; if $M_1/N_1 < M_2/N_2$, then suppliers producing P_1 and consumers purchasing P_2 win.

6.2 Simulations

The decision-making process inherent in our Janus game is the same as that of an MDRAG [81]. We assume that the number of situations an agent would encounter is P, and that $1 < P < 2^m$, where m is the memory length of the agent. Each strategy corresponds to a choice of 1 (P_1) or 0 (P_2). The size of the strategy pool is 2^P. At the beginning of the simulation, S strategies are randomly assigned to each agent. Hereafter, all agents should make choices based on their best strategy. Each strategy involving the correct choice in a particular game round would be given one point.

In our Janus game, we give suppliers the ability to decide the time (the time interval ΔT) to supply goods, since they will all be looking for new arbitrage opportunities once the system is in equilibrium with maximized entropy. We further define an arbitrage index a_I according to Shannon entropy, to show how far the system state has deviated from the equilibrium: $a_I = 1 + \sum_{i=1}^{2} p_i \log_2 p_i$, where p_i is the nonbalanced degree of the system and is defined as $p_i = \frac{M_i/\langle N_i\rangle}{M_1/\langle N_1\rangle + M_2/\langle N_2\rangle}$. A lower a_I value indicates that the system is closer to equilibrium. Once a_I falls below the warning value A_I, the suppliers will make new choices in producing products.

We conduct the simulation under $M = N = 100$, and each combination of parameters is simulated for 5000 time steps in 20 iterations. A segment of N_1/N and M_1/M values in one of the sessions is displayed in Fig. 17(a). Generally, a time series $y(t)$ can be separated into two parts — namely, $y(t) = f(t) + \varepsilon(t)$, where $f(t)$ is the underlying smooth trend and $\varepsilon(t)$ is the noise around the trend [126]. We use local linear kernel regression to estimate the trend [127]. Figures 17(b) and (d) show the noise and "kernels" of N_1/N and M_1/M, respectively, with clear periodicity indicated. Furthermore, we use

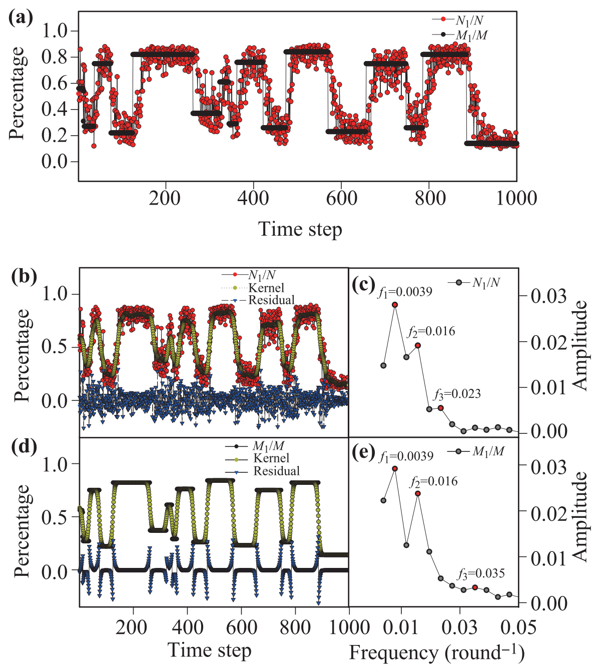

Fig. 17 **(a)** A segment of the simulation result with $S = 32$, $P = 32$, and $A_I = 0.05$. **(b)** The local linear kernel regression results for N_1/N. **(c)** The frequency spectrum for N_1/N. **(d)** The local linear kernel regression results for M_1/M. **(e)** The frequency spectrum for M_1/M. Reproduced from Ref. [123].

the discrete Fourier transform to analyze the periodic properties of the series. The three frequency peaks in Figs. 17(c) and (e) indicate this cyclical property. These frequencies are mainly determined by the adaptability levels of the agents and the arbitrage warning threshold.

The power spectral density distribution for the self-affine time series can exhibit in the form of $D(f) \sim 1/f^{\beta}$ [128–132], where β represents the strength of the time series persistence. We therefore also use the periodogram method [133] to analyze the "kernel" of the time series. Here, $\beta > 1$ corresponds to nonstationarity with strong persistence, while $\beta < 1$ corresponds to stationarity with weak persistence.

We calculate the values of β, and the results are shown in Figs. 18(a) and (b). Basically, two kinds of series are observed. We neglect the tails in the linear fitting, since they are meaningless in the context of our research. We think that Brownian motion is the driving process for our time series, and different parameter settings lead to different subdiffusive coefficients (which are relevant to β). Time series with $\beta > 1$ have a large subdiffusive coefficient with nonstationarity (periodicity), while those with $\beta < 1$ behave randomly.

Figure 18(c) shows how A_I affects β. When A_I is low, β is close to 2, and the series is cyclical; when A_I is high, β is close to 0. There is also an abrupt drop in β when A_I gradually increases; this indicates a phase transition from a periodic phase to a nonperiodic phase. In addition, the number of strategies for each agent S (the ability of agents' decision-making) can also affect β. Small S values and large P values indicate agents that are not overly "clever"; meanwhile, large S values and small P values indicate high adaptability levels. A three-dimensional map is displayed in Fig. 18(d), and it shows the relationships among S, P, and β. A clear phase transition can also be observed.

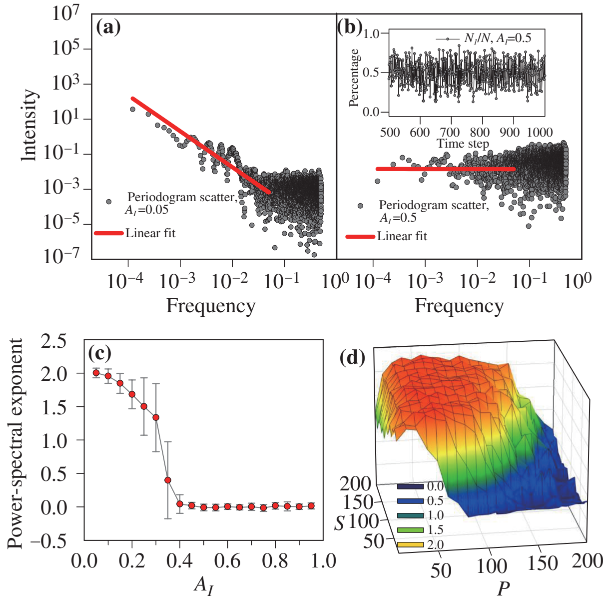

Fig. 18 **(a)** The results derived from the periodogram method for one simulation result of N_1/N with $S = 32$, $P = 32$, and $A_I = 0.05$. **(b)** The results for N_1/N with $S = 32$, $P = 32$, and $A_I = 0.5$. The inset shows the corresponding time series (the 500th to 1000th time steps). **(c)** The relation between averaged β and the warning threshold A_I. **(d)** The surface map of β as a function of S and P. Reproduced from Ref. [123].

6.3 Experiments

To verify the aforementioned simulation results, we conducted controlled human experiments. The rules and settings for our experiment are as follows. We recruited 50 students from Fudan University to participate in the Janus game, and they were given some essential instructions before the experiment began. All participants were randomly assigned as suppliers and consumers; the assumption was that they would make their choices exactly as the agents in simulations had done. In total, we conducted three sessions of experiments, with 40–50 time steps in each session. ΔT was set to be 10, 6, and 3 in the three sessions. Those who won in one time step were each awarded one point. As incentives for the participants, the total points they received could be exchanged into cash at a ratio of 1 : 1.

We present the time series of N_1/N and M_1/M in Fig. 19(a). It can be seen that N_1/N always fluctuates around M_1/M. To make the trend clearer, we adopt local linear kernel regression in the time series. The series following the regression (i.e., the kernel part and the residual part) are shown in Figs. 19(b) and (d). We further employ frequency spectral analysis, the results of which are shown in Figs. 19(c) and (e). It is interesting that there exists a nontrivial relationship between the peak frequencies f_2, f_3, f_4 and the time interval ΔT [i.e., $f_i \approx \frac{1}{2\Delta T}(i = 2, 3, 4)$], while f_1 refers to the intrinsic periodicity of the kernel series.

In summary, we found that there was indeed periodicity in human behavior. The results of this experiment indicate that in markets under free competition, business cycles may occur for endogenous reasons.

6.4 Concluding remarks

We designed a Janus game to study human behavior, which can be considered a fractional Brownian motion with phase transition between periodic and nonperiodic phases. Business cycles can be explained in terms of this periodic phase.

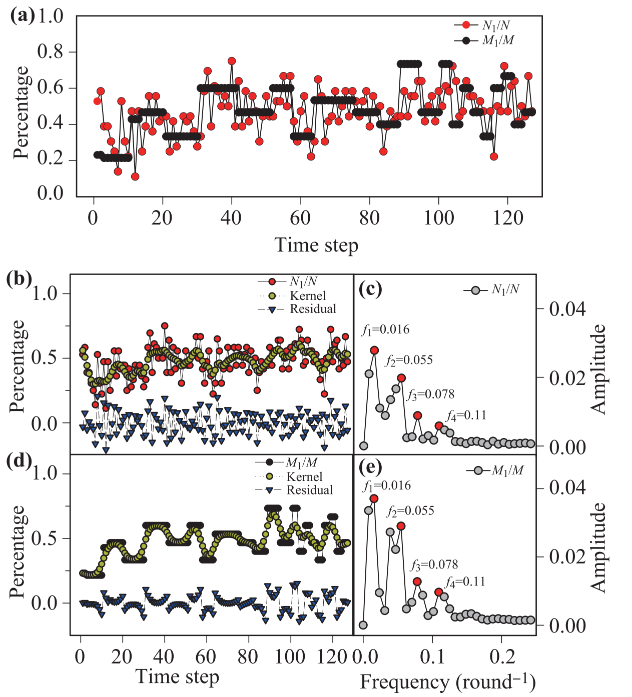

Fig. 19 **(a)** The controlled human experiment result of N_1/N and M_1/M. **(b)** The local linear kernel regression results for time series N_1/N. **(c)** The frequency spectrum analysis results for N_1/N. **(d)** The local linear kernel regression results for M_1/M. **(e)** The frequency spectrum analysis results for M_1/M. Reproduced from Ref. [123].

6.5 Relevant research

The authors of [134] studied housing and mortgage markets and a credit agent-based network. They conducted a series of simulations to explore the influence of households' creditworthiness conditions when granted a mortgage. Easier access to credit leads to the inflation of housing prices and short-run output expansion.

The authors of Ref. [135] studied the effect of credit on macroeconomics, as well as the reasons behind financial and credit instability. They developed an agent-based model to analyze the evolution of an economy; this model could help reveal the source of recent financial crises. Specifically, the repercussions of interbank connectivity on bank performance were simulated, and bankruptcy waves and business cycles were observed.

7 Summary and perspectives

In the current review, we presented some newly obtained results regarding chaos, leverage, and business cycles in financial markets, by making use of agent-based modeling and human experiments. We also reviewed a standard or principle for agent-based modeling. As such, this review covers some basic concepts and principles, as well as the latest research on popular topics in the field of econophysics.

We have sought to combine theory, agent-based modeling, and human experiments in each of our research projects [136]. Since we are facing real-world problems, theory is not fully applicable, given the complexity of human behavior. Thus, a controlled human experiment can serve as a bridge between theory and reality, because while the participants are human beings, their behavior in the experiment is limited (i.e., they can do only one of a few actions that the organizers want them to do). However, in view of the limited time and number of participants in experiments, it is difficult to simulate a macroscopic financial problem solely in a laboratory environment. The use of agent-based modeling allows us to simulate such problems on computers and verify possible factors and reasons for the studied problem.

The development of agent-based modeling has been undertaken on a full scale, and more and more high-quality scholarly works have featured numerous agent-based models. However, as we pointed out in Section 3, the "intelligence" of agents should be further enhanced. They should possess the abilities needed to process information, learn, and adapt to environmental changes. Future agent-based models should be developed with an eye to adhering to certain principles [72] that would make them more human-like.

For the controlled human experiment, in consideration of its shortcomings, a considerable amount of effort should continue to be made, as it is a relatively new method in the field of econophysics. It seems essential to design online experiments to recruit more participants and conduct within the laboratory a greater number of experimental sessions. Besides validating theories and simulations, on account of their effectiveness, such controlled human experiments should be used to predict future real-world events (e.g., crises).

In any case, agent-based modeling and controlled human experiments require further development; they should better simulate real-world phenomena. Both methods have their own shortcomings; nonetheless, when used in concert, relatively comprehensive results could be obtained. In other words, developments in both methods complement each other. Many research fields other than econophysics use agent-based modeling and human experiments; hence, there is a vast depth of experience to be referenced from a variety of scientific disciplines. This richness of experience will be key to maintaining the prosperity of the field of econophysics.

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