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RESEARCH ARTICLE

Entanglement of coherent superposition of photon-subtraction squeezed vacuum

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A new kind of non-Gaussian quantum state is introduced by applying nonlocal coherent superposition $(\tau a + sb)^m$ of photon subtraction to two single-mode squeezed vacuum states, and the properties of entanglement are investigated according to the degree of entanglement and the average fidelity of quantum teleportation. The state can be seen as a single-variable Hermitian polynomial excited squeezed vacuum state, and its normalization factor is related to the Legendre polynomial. It is shown that, for $\tau = s$, the maximum fidelity can be achieved, even over the classical limit (1/2), only for even-order operation m and equivalent squeezing parameters in a certain region. However, the maximum entanglement can be achieved for squeezing parameters with a π phase difference. These indicate that the optimal realizations of fidelity and entanglement could be different from one another. In addition, the parameter τ/s has an obvious effect on entanglement and fidelity.

Keywords non-Gaussian operation, quantum entanglement, squeezed state

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1 Introduction

As a wonderful property of quantum mechanics, quantum entanglement plays an important role in quantum computation and quantum communication [1], such as quantum teleportation, quantum cloning, and quantum dense encoding. Thus, it is of great importance in preparing nonclassical quantum states and enhancing the entanglement degree of quantum states. It is well known that although local unitary operations such as squeezing can be used to enhance the nonclassical properties of quantum states, they cannot be employed to improve the entanglement, owing to the limit of the no-go theorem [2]. Therefore, the preparation of quantum states with higher entanglement is still a very challenging task, which is critical for quantum teleportation.

To more effectively realize quantum information processing and computation tasks, nonclassical and high-entanglement quantum states are usually required. Many researchers have paid much attention to achieving this aim theoretically and experimentally [3–12]. For example, a photon-added coherent state was proposed by Agarwal and Tara [13], which can be prepared experimentally using parametric down-conversion and homodyne detection. Then, photon-added coherent state and photon-subtracted squeezed states are used in quantum key distribution [14, 15], which showed better performance than others. Recently, a kind of Laguerre polynomial excited coherent state was proposed via the unbalanced beam splitter and quantum catalysis [8, 16]. As a natural extension, these non-Gaussian operations are also used to enhance the entanglement of quantum states with continuous variables [4, 7, 9, 12, 17–21]. For instance, the photon-subtraction operation was demonstrated experimentally to realize an obvious entanglement distillation of the two-mode squeezed vacuum in a low-squeezing region [22]. In addition, quantum catalysis was extended to enhance the entanglement of the two-mode squeezed vacuum and the teleportation fidelity of quantum states [23]. These results show that local non-Gaussian operations can not only prepare higher nonclassical states (including the entanglement), but also improve the performance in the field of quantum information.

On the other hand, nonlocal coherent superposition operations are also often used to prepare non-Gaussian states, such as $a^\dagger + b^\dagger$, $a + b$, $a^2 + b^2$ and $a^{\dagger 2} + b^{\dagger 2}$ [9, 24–26]. For instance, Bandyopadhyay et al. considered a nonlocal superposition operator $(xa^\dagger + \mathrm{i}yb^\dagger)^m$ on a pair of single-mode squeezed vacuum states (SSVs), which, in fact, presents a quantum optical elliptic vortex. The Wigner function distribution was then discussed and the entanglement was calculated by using the logarithmic negativity [27, 28]. Starting from generalized wave-packet states with a vortex structure, we also proposed a kind of Hermite polynomial excited squeezed vacuum, which includes the case of $(xa^\dagger + iyb^\dagger)^m$ on the SSVs as a special case [29]. It is shown that these parameters in the generalized state can modulate all nonclassical properties including the vortex structure. In addition, a two-mode N-photon entangled state can be prepared by high-order a nonlocal operation $(a^\dagger - ib^\dagger)^m$ [24] and the inseparability of photon-added Gaussian states is investigated, which are generated from two-mode Gaussian states by applying $a^\dagger + \mu b^\dagger$. It is found that the entanglement of the states is involved with high-order moment correlations and the fidelity of teleporting coherent states cannot be raised by employing the photon-added Gaussian states as a quantum channel of teleportation [30]. It is interesting to note that there is a commutative relation $[a, a^\dagger] = 1$, which implies that these two non-Gaussian operations generally present different roles when operated on quantum states. Then, a question naturally arises: can the nonlocal operation such as $\tau a + sb$ can be used to raise both the entanglement and fidelity of teleportation, which may be useful for both quantum information and quantum computation.

In this paper, we shall introduce a new kind of non-Gaussian entangled state by applying nonlocal coherent superposition operation of photon-subtraction $(\tau a+sb)^m$ to two separable SSVs with r_1 and r_2 being real squeezing parameters. Then, we examine the degree of entanglement by using the linear entropy and the average fidelity of quantum teleportation to explore how these properties can be affected by the coherent superposition operation. It is found that there exists entanglement after the nonlocal photon-subtraction operation only when both r_1 and r_2 are not equal to zero, but the maximum entanglement can be achieved in the region characteristic of $r_1 = -r_2$ rather than $r_1 = r_2$ for $\tau/s = 1$. This is not the case for the fidelity of teleportation. It is interesting to note that fidelity over the classical limit (1/2) can be obtained in a small squeezing parameter region characteristics of $r_1 = r_2$. This indicates that a high fidelity is not always beneficial for high entanglement. As far as we know, there is no report about this issue in the literature to date.

This paper is arranged as follows. In Section 2, we introduce the high-order coherent photon-subtraction superposition single-mode squeezed vacuum states (CPS-SSVs), and subsequently derive the normalization factor by using the completeness of coherent state, normal ordering form of squeezed vacuum, and generating function of the new Legendre polynomials. It is found that the factor is related to the Legendre polynomials. In Section 3, we will introduce the coherent state representation of the characteristic function (CF) and derive the CF of the CPS-SSV, which shows a strong correlation with single-variable Hermitian polynomial. This result will be used to investigate the fidelity and entanglement in Sections 4 and 5, respectively. It is shown that an effective fidelity of teleportation for coherent state can be obtained for $r_1 = r_2$ and even number m, but the maximum entanglement can be achieved at $r_1 = -r_2$ rather than $r_1 = r_2$ for a certain squeezing parameter. The last section is devoted to drawing a conclusion.

2 High-order CPS-SSV and its normalization

We firstly introduce the high-order CPS-SSV. Here, we should point out that the nonlocal photon-subtraction superpositions are successfully prepared experimentally, which are used to create long-range entanglement based upon two single-mode cat states as inputs [31, 32]. Following the method in Refs. [31, 32], if two single-mode squeezed states are used as the inputs, then the photon detections with single-photon and vacuum at the output ports of a symmetrical beam splitter will lead to the superposition of photon subtraction. In addition, the coherent photon-subtraction superposition state may be produced by using the technique of state reduction and a Λ three-level atom system with initial coherence between two ground states [33]. In theory, a single-mode squeezed state could be acquired by operating the single-mode squeezing operator on a vacuum state:

$$S_1(r_1)|0\rangle_a = \exp\left\{\frac{r_1}{2}(a^2 - a^{\dagger 2})\right\}|0\rangle_a,$$
$$S_2(r_2)|0\rangle_b = \exp\left\{\frac{r_2}{2}(b^2 - b^{\dagger 2})\right\}|0\rangle_b, \tag{1}$$

where $S_j(r_j) = \exp\{r_j(k^2 - k^{\dagger 2})/2\}$, $(j = 1, 2; k = a, b)$ is single-mode squeezing operator, r_j is squeezing parameter, and $a^\dagger$, $b^\dagger$ (a, b) are the Bose creation operators and annihilation operators for the two modes. $a^\dagger$, $b^\dagger$ (a, b) have the commutation relation $[a, a^\dagger] = [b, b^\dagger] = 1$. For simplicity, we consider the squeezing parameter to be real. If we operate the photon-subtraction coherent superposition operation $(\tau a + sb)^m$ on two single-mode squeezed states, we then obtain a non-Gaussian quantum state:

$$|\Psi\rangle = N_m(\tau a + sb)^m S_1 S_2|00\rangle, \tag{2}$$

where N_m is the normalized coefficient, which could be obtained by solving the equation $\langle\Psi|\Psi\rangle = 1$, to yield (see Appendix):

$$N_m^{-2} = m!(B^2 - 4A^2)^{\frac{m}{2}} P_m\left(\frac{B}{\sqrt{B^2 - 4A^2}}\right). \tag{3}$$

Equation (3) shows that the normalization coefficient N_m^{-2} is related to the Legendre polynomial $P_m(x)$ and it is the analytical expression of an arbitrary order two-mode CPS-SSV, demonstrating great importance in the investigations of entanglement and other nonclassical properties of quantum states.

In addition, Eq. (2) can be rewritten as (see Appendix):

$$|\Psi\rangle = N_m \left(\mathrm{i}\sqrt{B_0}\right)^m H_m\left(\frac{\hat{A}_0}{2\mathrm{i}\sqrt{B_0}}\right) S_1 S_2 |00\rangle, \tag{4}$$

where $\hat{A}_0 = -(\tau a^\dagger \tanh r_1 + s b^\dagger \tanh r_2)$, $B_0 = -\frac{1}{2}(\tau^2 \tanh r_1 + s^2 \tanh r_2)$. Equation (4) indicates that photon-subtraction superposition SSVs could be seen as a production of single-variable Hermitian polynomial excited superposition squeezed vacuum states. Hermitian polynomial excited states have also been investigated recently [35, 36].

3 The coherent state representation of the CF and the CF of $|\Psi\rangle$

In this part, we will introduce the coherent state representation of the CF and derive the CF of $|\Psi\rangle$. According to the definition of the CF, the CF of an arbitrary two-mode quantum state $\rho_{1,2}$ is

$$\chi(\alpha, \beta) = \mathrm{tr}[D_1(\alpha) D_2(\beta) \rho_{1,2}], \tag{5}$$

in which $D_1(\alpha) = \exp\{\alpha a^\dagger - \alpha^* a\}$, $D_2(\beta) = \exp\{\beta b^\dagger - \beta^* b\}$ are the displacement operators for modes a and b.

Consider that the displacement operator has the anti-normal production form: $D_1(\alpha) = \mathrm{e}^{\frac{|\alpha|^2}{2}} \mathrm{e}^{-\alpha^* a} \mathrm{e}^{\alpha a^\dagger}$ and the completeness relation of the coherent states [24], the displacement operator in the coherent state representation would be

$$D_1(\alpha) = \int \frac{\mathrm{d}^2 z_1}{\pi} \mathrm{e}^{\frac{|\alpha|^2}{2} - \alpha^* z_1 + \alpha z_1^*} |z_1\rangle\langle z_1|; \tag{6}$$

thus, in the coherent states representation, Eq. (5) could be expressed as

$$\chi(\alpha, \beta) = \mathrm{e}^{\frac{|\alpha|^2 + |\beta|^2}{2}} \int \frac{\mathrm{d}^2 z_1 \mathrm{d}^2 z_2}{\pi^2} Q(z_1, z_2) \times \mathrm{e}^{\alpha z_1^* - \alpha^* z_1 + \beta z_2^* - \beta^* z_2}, \tag{7}$$

where $Q(z_1, z_2) \equiv \langle z_1, z_2 | \rho_{1,2} | z_1, z_2\rangle$ is the Q function of entangled state $\rho_{1,2}$. Therefore, if the Q function or normal ordering form of $\rho_{1,2}$ is given, we could work out the characteristic function of $\rho_{1,2}$ based on Eq. (7), which is a new method of calculating the CF [37].

If $\rho_{1,2} = |\Psi\rangle\langle\Psi|$, by making use of equation (A8), we have

$$\begin{aligned} &Q(z_1, z_2) \\ &= \overline{N}_m^2 \mathrm{e}^{-|z_1|^2 - |z_2|^2 - \frac{z_1^2 + z_1^{*2}}{2} \tanh r_1 - \frac{z_2^2 + z_2^{*2}}{2} \tanh r_2} \\ &\times \frac{\partial^{2m}}{\partial t^m \partial t'^m} \mathrm{e}^{-\frac{t^2 + t'^2}{2}(\tau^2 \tanh r_1 + s^2 \tanh r_2)} \\ &\times \mathrm{e}^{-t(\tau z_1^* \tanh r_1 + s z_2^* \tanh r_2) - t'(\tau z_1 \tanh r_1 + s z_2 \tanh r_2)}|_{t,t'=0}. \end{aligned} \tag{8}$$

Substituting Eq. (8) into Eq. (7) and utilizing integration of Eq. (A11), we have

$$\chi(\alpha, \beta) = \overline{N}_m^2 G \frac{\partial^{2m}}{\partial t^m \partial t'^m} \mathrm{e}^{C(t^2 + t'^2) + Dt - D^* t' + Ett'}\Big|_{t=t'=0}, \tag{9}$$

where we have set

$$\begin{aligned} G &= \cosh r_1 \cosh r_2 \mathrm{e}^{-\frac{1}{2}(|\alpha|^2 \cosh 2r_1 + |\beta|^2 \cosh 2r_2)} \\ &\quad \times \mathrm{e}^{-\frac{1}{4}\left[(\alpha^{*2} + \alpha^2) \sinh 2r_1 + (\beta^{*2} + \beta^2) \sinh 2r_2\right]}, \\ C &= -\frac{1}{4}(\tau^2 \sinh 2r_1 + s^2 \sinh 2r_2), \\ D &= \tau(\alpha^* \sinh r_1 \cosh r_1 + \alpha \sinh^2 r_1) \\ &\quad + s(\beta^* \sinh r_2 \cosh r_2 + \beta \sinh^2 r_2), \\ E &= \tau^2 \sinh^2 r_1 + s^2 \sinh^2 r_2. \end{aligned} \tag{10}$$

In order to obtain a simpler expression of Eq. (9), we exponentially expand tt' and make use of the generation function of the single-variable Hermitian polynomial:

$$\begin{aligned} &\frac{\partial^m}{\partial t^m} \exp\{Ct^2 + Dt\}|_{t=0} \\ &= \left(\mathrm{i}\sqrt{C}\right)^m H_m\left(\frac{D}{2\mathrm{i}\sqrt{C}}\right) \\ &= \left(-\mathrm{i}\sqrt{C}\right)^m H_m\left(\frac{D}{-2\mathrm{i}\sqrt{C}}\right), \end{aligned} \tag{11}$$

we have

$$\begin{aligned} \chi(\alpha, \beta) = \overline{N}_m^2 G \sum_{l=0}^{m} &\frac{(m!)^2 E^l C^{m-l}}{l! \left[(m-l)!\right]^2} \\ &\times H_{m-l}\left(\frac{D}{2\mathrm{i}\sqrt{C}}\right) H_{m-l}\left(\frac{D^*}{2\mathrm{i}\sqrt{C}}\right), \end{aligned} \tag{12}$$

where finally we use the recursion relation of the single-variable Hermitian polynomial

$$\frac{\mathrm{d}^l}{\mathrm{d}x^l} H_m(x) = \frac{2^l m!}{(m-l)!} H_{m-l}(x). \tag{13}$$

Equation (12) reveals that the CF of the coherent photon-subtraction superposition single-mode squeezed state shows strong correlation with the single-variable

Hermitian polynomial. Therefore, we have derived the coherent state representation of the CF and the CF of the photon-subtraction coherent superposition squeezed states. Owing to the introduction of the photon subtraction and the coherent superposition operation, the CF becomes related to the single-variable Hermitian polynomial, and therefore shows non-Gaussian properties.

4 Fidelity of the quantum teleportation

We investigate the fidelity of quantum teleportation by using the photon-subtraction coherent superposition squeezed states as the entangled resources. Here we make use of the Kimble–Brauntein scheme [38, 39]. In this scheme, we first use the symmetric beam-splitter and the results of unit gain measurement to realize the unitary transformation, and then generate the final quantum state that we require. We let the input and output states of the quantum teleportation be ρ_{in} and ρ_{out}. If we use $\mathrm{tr}(\rho_{in}\rho_{out})$ to describe the quantum teleportation fidelity with input state ρ_{in} and output state ρ_{out}, the fidelity could be written as

$$F = \int \frac{\mathrm{d}^2\lambda}{\pi} \chi_{in}(\lambda)\chi_{out}(-\lambda), \tag{14}$$

where $\chi_{in}(\lambda)$ and $\chi_{out}(-\lambda)$ are the CFs of ρ_{in} and ρ_{out}, respectively. Through the Weyl order of the density operator, it shows that $\chi_{in}(\lambda)$, $\chi_{out}(-\lambda)$ and $\chi_{1,2}$ have the following relation [40, 41]: $\chi_{out}(\lambda) = \chi_{in}(\lambda)\chi_{1,2}(\lambda^*, \lambda)$, where $\chi_{1,2}$ is the CF of the entangled resource. Therefore, the fidelity of the quantum teleportation is given by

$$F = \int \frac{\mathrm{d}^2\lambda}{\pi} \chi_{in}(\lambda)\chi_{in}(-\lambda)\chi_{1,2}(-\lambda^*, -\lambda). \tag{15}$$

From Eq. (15), one can see that when the CFs of both the input state and entangled resource are obtained, one can analytically calculate the fidelity of teleportation by performing the above integration.

In order to clearly discuss the performance for quantum teleportation of the current entangled resource, we need to specify the input state to be teleported. For quantum system with continuous variables, both coherent states and squeezed states are usually used as the target states. Generally, the teleportation fidelity for certain states depends on the parameter related to the states. This is true for teleporting coherent qubits using the same scheme. One can refer to Refs. [31, 42, 43] for discussions on teleportation of coherent qubits. Next, for simplicity, we investigate the fidelity by considering the coherent state $|\gamma\rangle$ input to be teleported. An ideal Kimble–Brauntein scheme (including the symmetric beam-splitter) demonstrates that the fidelity is not associated with the amplitude of the coherent state when the input is a coherent state. Thus, we can use the coherent vacuum state as the input and its CF is

$$\chi_{in}(\lambda) = \mathrm{Tr}\left[|0\rangle\langle 0|\mathrm{e}^{\lambda a^\dagger - \lambda^* a}\right] = \exp\left(-\frac{1}{2}|\lambda|^2\right). \tag{16}$$

Substituting Eq. (16) into Eq. (15), we then have the fidelity of the teleportation

$$F = \int \frac{\mathrm{d}^2\lambda}{\pi} \exp(-|\lambda|^2)\chi_{1,2}(-\lambda^*, -\lambda). \tag{17}$$

Equation (17) indicates that we can determine the fidelity as long as the two-mode entangled source $\rho_{1,2}$ is given.

For the entangled resource $|\Psi\rangle$, the CF $\chi_{1,2}(-\lambda^*, -\lambda)$ can be obtained from $\chi(\alpha, \beta)$ in Eq. (12) by replacing α, β with $-\lambda^*, -\lambda$, respectively. After the replacements, we see

$$\begin{aligned} &G \to \cosh r_1 \cosh r_2 \mathrm{e}^{-(\omega_0 - 1)|\lambda|^2 - \frac{1}{2}(\lambda^{*2} + \lambda^2)\omega_3}, \\ &D \to -\lambda^*\omega_1 - \lambda\omega_2, \end{aligned} \tag{18}$$

where, for simplicity, we let

$$\begin{aligned} \omega_0 &= \cosh^2 r_1 + \cosh^2 r_2, \\ \omega_1 &= \tau \sinh^2 r_1 + s \cosh r_2 \sinh r_2, \\ \omega_2 &= s \sinh^2 r_2 + \tau \cosh r_1 \sinh r_1, \\ \omega_3 &= \frac{1}{2}(\sinh 2r_1 + \sinh 2r_2). \end{aligned} \tag{19}$$

Substituting Eq. (18) into Eq. (17) and using Eq. (3), we have

$$\begin{aligned} F_m &= \frac{\overline{N}_m^2}{\sqrt{A_1 B_1 R_0}} \frac{\partial^{2m}}{\partial t^m \partial t'^m} \mathrm{e}^{\overline{C}(t^2 + t'^2) + \overline{E}tt'}|_{t=t'=0} \\ &= \frac{\overline{N}_m^2}{\sqrt{A_1 B_1 R_0}} m! D^m P_m(\overline{E}/D), \end{aligned} \tag{20}$$

in which $R_0 = \omega_0^2 - \omega_3^2$, $A_1 = 1 - \tanh^2 r_1$, $B_1 = 1 - \tanh^2 r_2$, $\overline{C} = C + R_1$, $\overline{E} = E + R_2$, $D = (\overline{E}^2 - 4\overline{C}^2)^{1/2}$, and

$$\begin{aligned} R_1 &= \frac{1}{2R_0}[2\omega_0\omega_1\omega_2 - (\omega_1^2 + \omega_2^2)\omega_3], \\ R_2 &= \frac{1}{R_0}[2\omega_1\omega_2\omega_3 - (\omega_1^2 + \omega_2^2)\omega_0]. \end{aligned} \tag{21}$$

Equation (20) is the analytical expression of the quantum teleportation fidelity for a coherent state realized with the CPS-SSV. We can see that the fidelity is related to the Legendre polynomial $P_m(x)$. In particular, when $m = 0$, which means no coherent superposition operation (no entanglement), the fidelity is

$$F_0 = [\cosh 2r_1 + \cosh 2r_2 + \cosh^2(r_1 - r_2) + 1]^{-\frac{1}{2}}. \tag{22}$$

This is the fidelity obtained without entanglement. Equation (22) reveals that the fidelity is always smaller than the maximum value of the fidelity without entanglement (1/2). When $r_1 = r_2 = r$, $F_0 = 1/(2\cosh r)$, which is the maximum value of the fidelity and will decrease with the increasing of the squeezing parameter.

We then study how the non-Gaussian operation — photon-subtraction superposition would affect the quantum teleportation fidelity; namely, we investigate how τ and s, squeezing parameters r_1, r_2, and the coherent superposition order would affect the fidelity. In Fig. 1, we show the picture of fidelity as a function of squeezing parameters r_1, r_2 for a given value of $\tau/s = 1$, and several different values of m. It is clearly seen that only for even numbers m, there are three regions near the center of phase space spanned by r_1 and r_2, in which the fidelity could be higher than the classic limit value (1/2), whereas the region with $r_1 = r_2$ is wider than others. However, for odd numbers m, there is no region over the limit. These results show that we can get an effective fidelity over 1/2 along the axis of $r_1 = r_2$ rather than $r_1 = -r_2$ for the case $\tau/s = 1$. We should note here that for the case of $\tau/s = -1$, we can get the same pictures as Fig. 1 by replacing (r_1, r_2) with $(-r_1, -r_2)$. In fact, the different values of τ/s could obviously affect the distribution form of fidelity and its symmetry (as an example, see Fig. 2). In addition, the case $\tau/s = 1$ is optimal for teleporting a coherent state when $r_1 = r_2$.

In order to clearly compare the effects of different values of m on the fidelity of teleportation, now we further take $r_1 = r_2 = -r$ and $\tau/s = 1$, and plot the fidelity as a function of r in Fig. 3. Figure 3 shows that: (i) When the order of the photon-subtraction superposition is odd, the fidelity is worse than the that without the superposition operation, but the fidelity would increase as the value of m increases, and it is interesting to note that although there is entanglement for odd number m, the teleportation fidelity cannot be enhanced; (ii) For the even-order superposition case, the fidelity could be better than that without superposition operation when the squeezing parameters are in a small region. For example, when $m = 2, 4$ and the squeezing parameters are smaller than 0.68 and 0.45, respectively, the fidelity could be enhanced. This threshold will decrease as the value of m increases. In addition, when $m = 2, 4$ and the squeezing parameters are smaller than 0.39 and 0.34, respectively, the fidelity could be higher than the classic fidelity (1/2). From (i) and (ii), one can see that higher entanglement does not always improve the fidelity, and sometimes it can make the fidelity even worse. (iii) As the value of m increases, we could obtain a better fidelity when the squeezing parameter is small (≤ 0.25). On the contrary, when the order is odd, the fidelity would increase as the value of m increases, regardless of the values of the squeezing parameters.

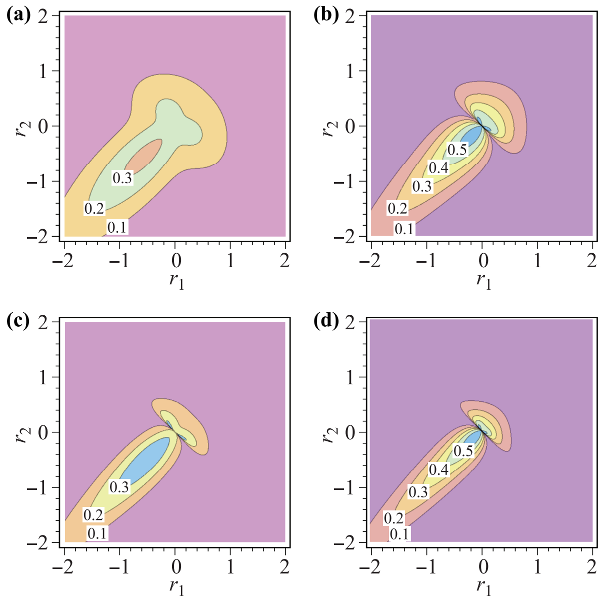

Fig. 1 Fidelity as a function of two squeezing parameters r_1, r_2. **(a)**, **(b)**, **(c)**, and **(d)** correspond to $m = 1, 2, 3, 4$, respectively, where $\tau/s = 1$.

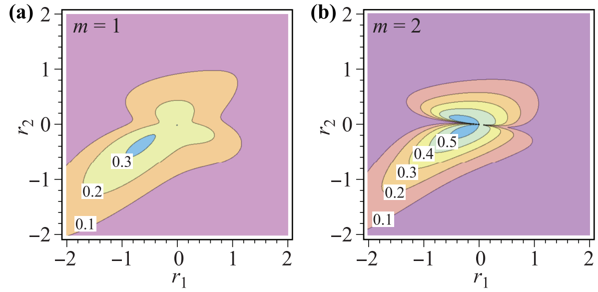

Fig. 2 Fidelity as a function of two squeezing parameters r_1, r_2 and $\tau/s = 0.3$.

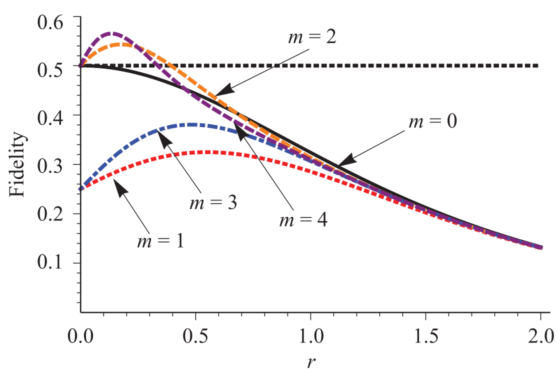

Fig. 3 Fidelity dependence on the squeezing parameters $r_1 = r_2 = -r$ for different values of $m = 0$, 1, 2, 3, 4 with $\tau/s = 1$.

5 Quantification of entanglement

Next, we consider the quantification of entanglement for the CPS-SSV. In order to realize our purpose, we appeal to the linear entropy, which is actually the upper bound of the von Neumann entropy and monotonous for pure bipartite states. For the two-mode quantum system $\rho_{1,2}$ shown in Eq. (2), the linear entropy is defined as

$$E = 1 - \mathrm{Tr}(\rho_1^2), \tag{23}$$

where $\rho_1 = \mathrm{Tr}_2(\rho_{1,2})$ is the reduced density operator.

To obtain the analytical expression of the linear entropy for the states in Eq. (2), using Eq. (A5), we have

$$\begin{aligned}\rho_{1,2} =& |\Psi\rangle\langle\Psi| \\ =& \frac{(\overline{N}_m)^2 \partial^{2m}}{\partial t^m \partial t'^m} \mathrm{e}^{-\frac{t^2+t'^2}{2}(\tau^2 \tanh r_1 + s^2 \tanh r_2)} \overline{\rho_1 \rho_2}\Big|_{t=t\prime=0},\end{aligned} \tag{24}$$

where we have set

$$\begin{aligned}\overline{\rho_1} &= \mathrm{e}^{-(t\tau a^\dagger + \frac{a^{\dagger 2}}{2})\tanh r_1}|0\rangle\langle 0|\mathrm{e}^{-(t'\tau a + \frac{a^2}{2})\tanh r_1}, \\ \overline{\rho_2} &= \mathrm{e}^{-(tsb^\dagger + \frac{b^{\dagger 2}}{2})\tanh r_2}|0\rangle\langle 0|\mathrm{e}^{-(t'sb + \frac{b^2}{2})\tanh r_2}.\end{aligned} \tag{25}$$

Further using the coherent state completeness relation, it is easy to derive

$$\begin{aligned}\mathrm{Tr}_2(\overline{\rho_2}) &= \int \frac{\mathrm{d}^2\alpha}{\pi} \mathrm{e}^{-|\alpha|^2 - (t's\alpha + ts\alpha^* + \frac{\alpha^2}{2} + \frac{\alpha^{*2}}{2})\tanh r_2} \\ &= \cosh r_2 \mathrm{e}^{(tt' - \frac{t'^2+t^2}{2}\tanh r_2)s^2 \sinh^2 r_2},\end{aligned} \tag{26}$$

where we have used the integration formula (A11). Thus, the corresponding reduced density operator for $\rho_{1,2}$ is given by

$$\begin{aligned}\rho_1 &= \mathrm{Tr}_2(\rho_{1,2}) \\ &= \cosh r_2 \frac{\partial^{2m}}{\partial t^m \partial t'^m} \mathrm{e}^{-\frac{t^2+t'^2}{2}(\tau^2 \tanh r_1 + s^2 \tanh r_2)} \\ &\quad \times (\overline{N}_m)^2 \mathrm{e}^{(tt' - \frac{t'^2+t^2}{2}\tanh r_2)s^2 \sinh^2 r_2} \overline{\rho_1}\Big|_{t=t\prime=0}.\end{aligned} \tag{27}$$

In order to calculate E, we employ the CF; from Eq. (14), one can see that

$$\mathrm{Tr}(\rho_1^2) = \int \frac{\mathrm{d}^2\alpha}{\pi} \chi_1(\alpha)\chi_1(-\alpha), \tag{28}$$

where $\chi_1(\alpha)$ is the CF of the reduced density operator ρ_1. Thus, we need to derive $\mathrm{Tr}[\overline{\rho_1} D_1(\alpha)]$. Using the coherent state representation of the displacement operator Eq. (6), we can get

$$\begin{aligned}\mathrm{Tr}[\overline{\rho_1} D_1(\alpha)] &= \mathrm{Tr}\left[\overline{\rho_1} \int \frac{\mathrm{d}^2 z}{\pi} \mathrm{e}^{\frac{|\alpha|^2}{2} - \alpha^* z + \alpha z^*} |z\rangle\langle z|\right] \\ &= \mathrm{e}^{\frac{|\alpha|^2}{2}} \int \frac{\mathrm{d}^2 z}{\pi} \mathrm{e}^{-|z|^2 - \alpha^* z + \alpha z^*} \mathrm{e}^{-(zt'\tau + z^* t\tau + \frac{z^2 + z^{*2}}{2})\tanh r_1} \\ &= \cosh r_1 \mathrm{e}^{\frac{|\alpha|^2}{2} - (\alpha^* + t'\tau \tanh r_1)(\alpha - t\tau \tanh r_1)\cosh^2 r_1} \times \mathrm{e}^{-\frac{\sinh 2r_1}{4}[(\alpha^* + t'\tau \tanh r_1)^2 + (\alpha - t\tau \tanh r_1)^2]}.\end{aligned} \tag{29}$$

Thus, by combining Eqs. (29) and (27), we have

$$\begin{aligned}\chi_1(\alpha) &= (\overline{N}_m)^2 \frac{\partial^{2m}}{\partial t^m \partial t'^m} \mathrm{e}^{-\frac{t^2+t'^2}{2}(\tau^2 \tanh r_1 + s^2 \tanh r_2)} \times \cosh r_2 \mathrm{e}^{(tt' - \frac{t'^2+t^2}{2}\tanh r_2)s^2 \sinh^2 r_2} \mathrm{Tr}[\overline{\rho_1} D_1(\alpha)]\Big|_{t=t\prime=0} \\ &= (\overline{N}_m)^2 \frac{\partial^{2m}}{\partial t^m \partial t'^m} \mathrm{e}^{-\frac{t^2+t'^2}{2}(\tau^2 \tanh r_1 + s^2 \tanh r_2)} \times \cosh r_1 \cosh r_2 \mathrm{e}^{(tt' - \frac{t'^2+t^2}{2}\tanh r_2)s^2 \sinh^2 r_2} \\ &\quad \times \mathrm{e}^{\frac{|\alpha|^2}{2} - (\alpha^* + t'\tau \tanh r_1)(\alpha - t\tau \tanh r_1)\cosh^2 r_1} \times \mathrm{e}^{-\frac{\sinh 2r_1}{4}[(\alpha^* + t'\tau \tanh r_1)^2 + (\alpha - t\tau \tanh r_1)^2]}\Big|_{t=t\prime=0},\end{aligned} \tag{30}$$

and $\chi_1(-\alpha)$ is not shown, which can be obtained from Eq. (30) by replacing α^* and α by $-\alpha^*$ and $-\alpha$, respectively. Here, we should note the difference between the differential variables.

Substituting Eq. (31) and $\chi_1(-\alpha)$ into Eq. (28) and using the integration formula (A11), we finally obtain

$$\mathrm{Tr}(\rho_1^2) = \epsilon^2 R, \tag{31}$$

where we have set

$$\begin{aligned}R =& \frac{\partial^{4m}}{\partial t^m \partial t'^m \partial \bar{t}^m \partial \overline{t'}^m} \exp\left\{A(t^2 + t'^2 + \bar{t}^2 + \overline{t'}^2)\right\} \\ &\times \exp\left\{\bar{B}(tt' + \overline{t'}\bar{t}) + \varpi(t\overline{t'} + \bar{t}t')\right\}\Big|_{t=t'=\bar{t}=\overline{t'}=0},\end{aligned} \tag{32}$$

which leads to the linear entropy E_m for Eq. (4), given by

$$E_m = 1 - \epsilon^2 R, \tag{33}$$

where $A = -\frac{1}{4}(s^2 \sinh 2r_2 + \tau^2 \sinh 2r_1)$, $\bar{B} = s^2 \sinh^2 r_2$, $\varpi = \tau^2 \sinh^2 r_1$ and

$$\epsilon = \left[m! \left(\bar{B}^2 - 4A^2\right)^{\frac{m}{2}} P_m \left(\frac{\bar{B}}{\sqrt{\bar{B}^2 - 4A^2}} \right) \right]^{-1}. \tag{34}$$

In particular, when $m = 0$, by using Eqs. (32) and (33) as well as (34), we see that $E_0 = 0$ due to $\epsilon^2 R = 1$, as expected, since the case corresponds to two separable SSVs. While for the case of $m = 1$ and $s = \tau$, by using Eqs. (33) and (34), we are able to calculate the linear entropy E_1, i.e.,

$$E_1 = \frac{2\sinh^2 r_1 \sinh^2 r_2}{\left(\sinh^2 r_1 + \sinh^2 r_2\right)^2} \leq \frac{1}{2}. \tag{35}$$

Equation (35) indicates that there is entanglement between the two-mode states after the coherent superposition only when the two squeezing parameters, both r_1 and r_2, are not equal to zero, but the maximum is always less than 0.5. This is to say, the non-Gaussian operation can effectively entangle two separable quantum states. In addition, when $s = \tau$, the degree of entanglement of $|\Psi\rangle$ is independent of s or τ for any m. Equation (35) is just a special example. Here, we should note that when r_1 or r_2 is zero, there is no entanglement.

Using Eq. (33), we can further discuss the entanglement property for different values of r_1, r_2, m, and τ/s. For simplicity, and without loss of generality, we take $s = 1$ in the following discussions. In Fig. 4, we plot the linear entropy as a function of r_1 and r_2 for a given $\tau/s = 1$ and several different values of $m = 1, 2, 3$. From Fig. 3, it is clear that the entanglement distribution in the space of r_1 and r_2 is symmetrical about the two axes, both $r_1 = r_2$ and $r_1 = -r_2$. This point can be made clearer by recalling the definitions of A, B, and ϖ. In addition, it is interesting to note that there are two maximal values for E_m when $r_1 = r_2$ or $r_1 = -r_2$. However, the maximal value E_- for the latter $(r_1 = -r_2)$ is always larger than that E_+ for the former. For instance, at points with $r_1 = \pm r_2 = 1.0$, these maximal values (E_+, E_-) are $(0.5, 0.5)$ $(0.43, 0.63)$ $(0.52, 0.69)$, respectively, for different values of $m = 1, 2, 3$. Furthermore, it is interesting to note that E_- increases with increasing m, while the changing of E_+ depends on the even and odd state of m. These results indicate that under the symmetrical coherent photon-subtraction case with $s = \tau$, one can obtain a higher degree of entanglement for the case of $r_1 = -r_2$. In other words, the phase difference of squeezing parameters for the two SSVs may be an important factor for generating optimal entanglement.

In order to see clearly the effects of parameters τ and s on the entanglement E_m, we plot the linear entropy as a function of r_1 and r_2 for several different values of τ/s and $m = 1, 2$ in Fig. 5. Here, for simplicity, we only consider the case with $\tau/s < 1$, because $\tau/s > 1$ provides a symmetrical description. From Fig. 4, it can be seen that, owing to the effect of asymmetrical superposition $(\tau/s < 1)$, (i) the linear entropy does not have the symmetrical axis $r_1 = r_2$ or $r_1 = -r_2$, which implies how the parameters τ/s affect the distribution of linear entropy in the space of r_1 and r_2; (ii) in a certain center region of the squeezing parameter space, the maximum values of linear entropy become smaller than those with symmetrical superposition $(\tau/s = 1)$. These show that for a small squeezing parameter, the symmetrical superposition of photon-subtraction shows a better performance than the asymmetrical one for generating higher entanglement.

Finally, we make a comparison between the fidelity and entanglement. From Figs. 1 and 4 with $\tau/s = 1$, it is interesting to note that effective fidelity over 1/2 can be obtained in only one region characteristics of $r_1 = r_2 (< 0)$, while entanglement can be generated in four regions characteristics of $r_1 = r_2$ and $r_1 = -r_2$. In addition, in the common region characteristics of $r_1 = r_2 (< 0)$, al-

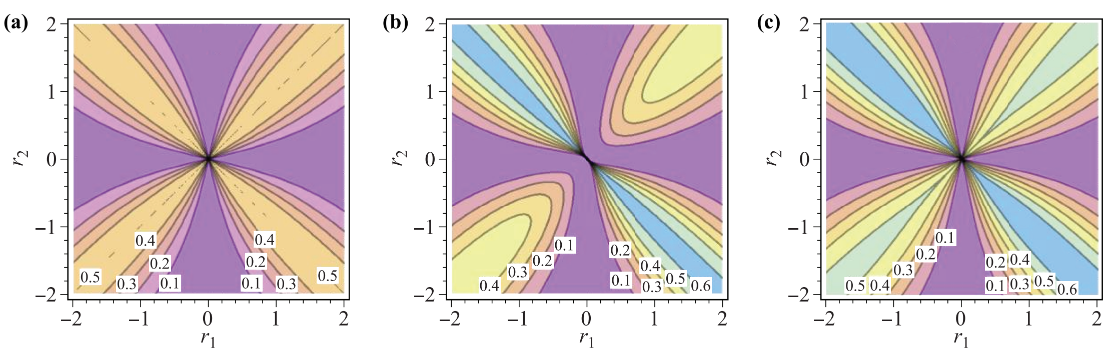

Fig. 4 Linear entropy of the CPS-SSV as a function of r_1 and r_2 for **(a)** $m = 1$, **(b)** $m = 2$, **(c)** $m = 3$, as well as $\tau = s$.

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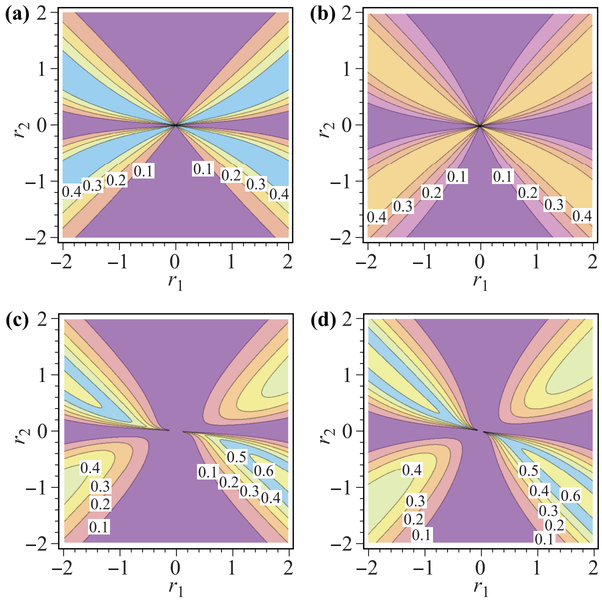

Fig. 5 Linear entropy of the CPS-SSV as a function of r_1 and r_2 for **(a)** $m = 1$, $\tau/s = 0.3$; **(b)** $m = 1$, $\tau/s = 0.5$; **(c)** $m = 2$, $\tau/s = 0.3$; **(d)** $m = 2$, $\tau/s = 0.5$.

though the fidelity can exceed the classical limit of 1/2, the maximum value of entanglement cannot be obtained. As shown above, the maximum entanglement is present in the region characteristics of $r_1 = -r_2$. This indicates that a higher degree of entanglement cannot always lead to higher fidelity of teleportation. This has been pointed out in other literature [44, 45]. Thus, realizing either maximum fidelity or entanglement requires different choices of r_1 and r_2.

6 Conclusion

In this paper, we introduced a new non-Gaussian quantum state with continuous variable — photon-subtraction superposition squeezed state. This state is realized theoretically by operating the photon superposition operator $(\tau a+sb)^m$ on two independent SSVs. We derived the normalization coefficient by using the normal ordering of the squeezed states and the completeness relation of the coherent state, which is very important to investigate the nonclassical properties of quantum states. It is shown that the coefficient could be expanded by the Legendre polynomial function $P_m(x)$. In addition, it is interesting to note that the new state can be considered as a Hermite superposition-excited squeezed vacuum state.

By exploiting the antinormal ordering form of the displacement operator, we introduced the coherent state representation of the CF, and then we obtained the CF of the CPS-SSV, which could be expressed with two single-variable Hermitian polynomials. As an application, based on the Kimble–Brauntein quantum teleportation scheme, we discussed the teleportation of the coherent states by taking the CPS-SSV as the quantum entangled source. We also investigated the relationship between the fidelity and the squeezing parameters based on the analytical expression of the fidelity. It is found that: (i) the fidelity reaches its maximum when the squeezing parameters are equal to one another; (ii) the odd-order operation makes the fidelity lower than the fidelity without the superposition operation, while for the even order operation, the fidelity could be higher than the fidelity without entanglement when the squeezing parameters are in a certain region, and it could be even higher than the maximum classical fidelity (1/2); (iii) the fidelity can be improved as the value of m increases. In short, the efficient fidelity of teleportation can be realized only for the case associated with $r_1 = r_2$ and even-order operation.

Finally, we investigated the degree of entanglement for the new state by using the linear entropy. These results show that the entanglement can be formed by the photon-subtraction superposition only when $r_1 \neq 0$ and $r_2 \neq 0$. Under the symmetrical case, i.e., $\tau/s = 1$, one can find the maximal degree of entanglement in the region characteristics of $r_1 = -r_2$ rather than $r_1 = r_2$, which increases with increasing m. This provides an effective way to prepare an optimal entanglement under symmetrical photon-subtraction superposition. A comparison between the fidelity and entanglement shows that a higher degree of entanglement cannot always lead to a higher fidelity of teleportation, and the methods of improving fidelity and entanglement may be different from one another. In addition, the parameters τ and s present clearly a kind of modulating effect on the entanglement and fidelity of teleportation. It would be interesting to further examine some applications of the proposed non-Gaussian state in the fields of quantum information and quantum computation.

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Appendix A Derivation of Eqs. (4) and (3)

In order to get the value of N_m, we firstly work out the normal ordering form of $\rho_{12} \equiv |\Psi\rangle\langle\Psi|$, which is the density operator of the quantum state $|\Psi\rangle$. By using the normal ordering form of the single-mode squeezing operator $S_j(r_j)$,

$$S_1(r_1) = \mathrm{e}^{-\frac{a^{\dagger 2}}{2}\tanh r_1}\mathrm{e}^{(a^\dagger a+\frac{1}{2})\ln \mathrm{sech} r_1}\mathrm{e}^{\frac{a^2}{2}\tanh r_1},$$
$$S_2(r_2) = \mathrm{e}^{-\frac{b^{\dagger 2}}{2}\tanh r_2}\mathrm{e}^{(b^\dagger b+\frac{1}{2})\ln \mathrm{sech} r_2}\mathrm{e}^{\frac{b^2}{2}\tanh r_2}, \quad \text{(A1)}$$

the single-mode squeezed vacuum state (SSV) could be written as

$$S_1S_2|00\rangle = (\mathrm{sech} r_1 \mathrm{sech} r_2)^{\frac{1}{2}}\mathrm{e}^{-\frac{1}{2}(a^{\dagger 2}\tanh r_1+b^{\dagger 2}\tanh r_2)}|00\rangle. \quad \text{(A2)}$$

On the other side, we could write the high-order coherent photon-subtraction superposition operation in a differential form:

$$(\tau a+sb)^m = \frac{\partial^m}{\partial t^m}\mathrm{e}^{(\tau a+sb)t}\bigg|_{t=0}. \quad \text{(A3)}$$

According to Eqs. (A2) and (A3), Eq. (2) could be written as

$$|\Psi\rangle = \overline{N}_m\frac{\partial^m}{\partial t^m}\mathrm{e}^{(\tau a+sb)t}\mathrm{e}^{-\frac{a^{\dagger 2}}{2}\tanh r_1-\frac{b^{\dagger 2}}{2}\tanh r_2}|00\rangle\bigg|_{t=0}, \quad \text{(A4)}$$

where, for simplicity, we let $\overline{N}_m = N_m$ (sech $r_1)^{\frac{1}{2}}$ $\cdot$(sech $r_2)^{\frac{1}{2}}$. Based on the identical equations $\mathrm{e}^{\lambda a}a^\dagger\mathrm{e}^{-\lambda a} = a^\dagger+\lambda$ and $\mathrm{e}^{A+B} = \mathrm{e}^A\mathrm{e}^B\mathrm{e}^{-\frac{[A,B]}{2}}$, in which $[A,[A,B]] = [B,[A,B]] = 0$ [34], we can then write Eq. (A4) as

$$|\Psi\rangle = \overline{N}_m\frac{\partial^m}{\partial t^m}\exp\left\{-\frac{t^2}{2}(\tau^2\tanh r_1+s^2\tanh r_2)\right\}$$
$$\times\exp\left\{-t(\tau a^\dagger\tanh r_1+sb^\dagger\tanh r_2)\right\}$$
$$\times\exp\left\{-\frac{a^{\dagger 2}}{2}\tanh r_1-\frac{b^{\dagger 2}}{2}\tanh r_2\right\}|00\rangle\bigg|_{t=0}. \quad \text{(A5)}$$

On one hand, by combining the SSV in Eq. (A2), Eq. (A5) could be written as

$$|\Psi\rangle = N_m\frac{\partial^m}{\partial t^m}\exp\left\{-\frac{t^2}{2}(\tau^2\tanh r_1+s^2\tanh r_2)\right\}$$
$$\times\exp\left\{-t(\tau a^\dagger\tanh r_1+sb^\dagger\tanh r_2)\right\}S_1S_2|00\rangle\bigg|_{t=0}. \quad \text{(A6)}$$

Notice the generation function of single-variable Hermitian polynomial:

$$\frac{\partial^m}{\partial t^m}\exp\{At+Bt^2\}\bigg|_{t=0} = \left(\mathrm{i}\sqrt{B}\right)^m H_m\left(\frac{A}{2\mathrm{i}\sqrt{B}}\right); \quad \text{(A7)}$$

therefore, Eq. (A6) could be written as Eq. (4).

On the other hand, in the coherent state $\langle\alpha,\beta|$ representation, the wavefunction of the state vector $|\Psi\rangle$ in Eq. (A5) could be written as

$$\langle\alpha,\beta|\Psi\rangle = \overline{N}_m\mathrm{e}^{-\frac{1}{2}(|\alpha|^2+|\beta|^2+\alpha^{*2}\tanh r_1+\beta^{*2}\tanh r_2)}$$
$$\times\frac{\partial^m}{\partial t^m}\exp\left\{-\frac{t^2}{2}(\tau^2\tanh r_1+s^2\tanh r_2)\right\}$$
$$\times\exp\left\{-t(\tau\alpha^*\tanh r_1+s\beta^*\tanh r_2)\right\}\bigg|_{t=0}. \quad \text{(A8)}$$

Considering the completeness relationship of the coherent states:

$$\int\frac{\mathrm{d}^2\alpha\mathrm{d}^2\beta}{\pi^2}|\alpha,\beta\rangle\langle\alpha,\beta| = 1, \quad \text{(A9)}$$

the normalization coefficient of the non-Gaussian quantum $|\Psi\rangle$ state could be obtained by

$$N_m^{-2} = N_m^{-2}\int\frac{\mathrm{d}^2\alpha\mathrm{d}^2\beta}{\pi^2}|\langle\alpha,\beta|\Psi\rangle|^2. \quad \text{(A10)}$$

Using the integration formula:

$$\int\frac{\mathrm{d}^2z}{\pi}\exp\left(\zeta|z|^2+\xi z+\eta z^*+fz^2+gz^{*2}\right)$$
$$= \frac{1}{\sqrt{\zeta^2-4fg}}\exp\left(\frac{-\zeta\xi\eta+\xi^2g+\eta^2f}{\zeta^2-4fg}\right), \quad \text{(A11)}$$

N_m^{-2} could finally be written as

$$N_m^{-2} = \frac{\partial^{2m}}{\partial^m t\partial^m t'}\exp\{-A\left(t^2+t'^2\right)+Btt'\}\bigg|_{t=t'=0}, \quad \text{(A12)}$$

where

$$A = -\frac{1}{4}\left(s^2\sinh 2r_2+\tau^2\sinh 2r_1\right),$$
$$B = \tau^2\sinh^2 r_1+s^2\sinh^2 r_2. \quad \text{(A13)}$$

By utilizing the new Legendre polynomial generation function proposed in Ref. [3], Eq. (A12) could be simplified to Eq. (3).

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