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RESEARCH ARTICLE

Spin in the extended electron model

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It has been found that a model of extended electrons is more suited to describe theoretical simulations and experimental results obtained via scanning tunnelling microscopes, but while the dynamic properties are easily incorporated, magnetic properties, and in particular electron spin properties pose a problem due to their conceived isotropy in the absence of measurement. The spin of an electron reacts with a magnetic field and thus has the properties of a vector. However, electron spin is also isotropic, suggesting that it does not have the properties of a vector. This central conflict in the description of an electron's spin, we believe, is the root of many of the paradoxical properties measured and postulated for quantum spin particles. Exploiting a model in which the electron spin is described consistently in real three-dimensional space – an extended electron model – we demonstrate that spin may be described by a vector and still maintain its isotropy. In this framework, we re-evaluate the Stern–Gerlach experiments, the Einstein–Podolsky–Rosen experiments, and the effect of consecutive measurements and find in all cases a fairly intuitive explanation.

Keywords spin, extended electron model, geometric algebra, Stern–Gerlach experiment, Einstein–Podolsky–Rosen, magnetism

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1 Introduction

Magnetic fields are the manifestation of charge in rotation around a centre [1]. In single atoms, the orbit of electrons around a nucleus is accounted for by the so-called orbital magnetic dipole moment [2, 3]. This describes a magnetic field with a well know magnitude and orientation. However, it has been observed that atoms with no orbital magnetic moments, like silver, experience a force upon application of an external magnetic field [4], which has been attributed to the spin of the atom's outer electron(s). Here, the classical picture of magnetic moments breaks down and, in the standard model, we conclude that electron spin is not an object in real space because it is isotropic and therefore does not have the properties of a vector.

In the standard model, electrons are modelled as point particles [5], which is, we believe, the fundamental problem with conventional interpretations. With this restriction, the only way to reconcile that electron spin is isotropic in one case and vector-like in another is to rely on abstract mathematics. In addition, recent experimental evidence cast further doubt on this assumption [6], since current STM measurements appear able to resolve a density distribution on noble metal surfaces that cannot be explained as a consequence of a probability distribution of detection events without violating Heisenberg's uncertainty relations [7]. If, instead, we relax this condition and employ an extended electron model [8, 9], it is possible to render these two properties of electron spin in real space. This extended electron model is based on four postulates. Firstly, the wave properties of electron are a *real* property of electrons in motion. This accounts for the high resolution in STM experiments. Secondly, electrons in motion possess intrinsic electromagnetic potentials and, thirdly, these give rise to the intrinsic magnetic moment, or spin, of electrons. Finally, in equilibrium, the energy density is constant throughout the space occupied by an electron. Within this framework, formulated using geometric algebra [10, 11], it is possible to characterise an electron spin vector in real three-dimensional space while reproducing the results of experiment and maintaining isotropy.

In the following, we present the standard approach along with the extended electron approach and discuss their implications with regard to experimental results.

2 Pauli algebra

Within the standard model, spin is accounted for by the Pauli matrices, which, along with the identity matrix, form a complete basis for all 2×2 Hermitian matrices and, as observables correspond to Hermitian operators, they span the space of observables of the 2-dimensional Hilbert space [12],

$$\begin{aligned} \sigma_x &= |0\rangle\langle 1| + |1\rangle\langle 0|, \\ \mathrm{i}\sigma_y &= |0\rangle\langle 1| - |1\rangle\langle 0|, \\ \sigma_z &= |0\rangle\langle 0| - |1\rangle\langle 1|. \end{aligned} \tag{1}$$

Each matrix has eigenvalues of ± 1 representing spin-up and spin-down. In the case of spin-1/2 particles, we define spin operators, $\boldsymbol{S}_a = \hbar\sigma_a/2$, where a is axis (x, y, or z) and the corresponding eigenvectors in Hilbert space are given by

$$\begin{aligned} |x^{\pm}\rangle &= \frac{1}{\sqrt{2}}\left(|0\rangle \pm |1\rangle\right), \quad |z^{+}\rangle = |0\rangle, \\ |y^{\pm}\rangle &= \frac{1}{\sqrt{2}}\left(|0\rangle \pm \mathrm{i}|1\rangle\right), \quad |z^{-}\rangle = |1\rangle. \end{aligned} \tag{2}$$

Generally, the quantum state of a particle, with respect to spin, is represented by a two component spinor, $\Psi = \Psi_0|0\rangle + \Psi_1|1\rangle$, which contains a superposition of both states. When the spin of this particle is measured with respect to a given axis, $a = x, y, z$, the probability that a spin of $\pm\hbar/2$ will be measured is $|\langle a^{\pm}|\Psi\rangle|^2$. Following the measurement, the spin state of the particle is said to collapse into the corresponding eigenstate and all equivalent measurements will yield the same eigenvalue, but when a measurement is performed on another axis, $b \neq a$, the probability of finding a spin of $\pm\hbar/2$ is then $|\langle b^{\pm}|a^{\pm}\rangle|^2 = 1/2$. Going on to remeasure along the original axis, we find we are equally likely measure either spin-up or spin-down, so there is no memory of the original measurement. Mathematically, this is due to the non-commutativity of the Pauli matrices, $[\sigma_b, \sigma_a] \neq 0$. Physically, the explanation is not clear, which is a consequence of our failure to define the physical process responsible for the wavefunction collapse.

Describing this process has been problematic. Objective collapse theories like the Ghirardi–Rimini–Weber theory [13] or the Penrose interpretation [14] adopt a more rigorous version than the Copenhagen interpretation [15], but these have been challenged experimentally [16]. Adopting de Broglie's ontological approach [17], as opposed to Schrödinger's more epistemological approach [18], one may argue that electron spin can be described in real space. In the de Broglie–Bohm model [19, 20], or in other hidden-variable approaches, we find non-local potentials due to Bell's inequalities [21]. Suggested solutions to this problem include the many-worlds interpretation [22], superdeterminism [23, 24], and retrocausality [25, 26], but these depend on a somewhat profound metaphysical shift in our description of the universe. We note that many loopholes exist in the Bell's inequalities experiments [27]. Indeed, strictly speaking, no Bell experiment can exclude all conceivable local hidden-variable theories [28] and, as there is no physical reality ascribed to the imaginary component of the phase of the two measured objects, the description is limited from certain viewpoints [29]. This will be explored in more detail in the following sections.

3 Extended electrons

In the extended electron model, we exploit the framework of geometric algebra to parametrise electron spin in real space. Firstly, we define three perpendicular unit vectors, $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, and $\boldsymbol{e}_3$. Correspondingly, we may define three perpendicular bivector terms as the plane cast by each combination of two unit vectors, $\boldsymbol{e}_1\boldsymbol{e}_2$, $\boldsymbol{e}_2\boldsymbol{e}_3$, and $\boldsymbol{e}_3\boldsymbol{e}_1$. No plane is cast by two parallel vectors, so $\boldsymbol{e}_i\boldsymbol{e}_i = 1$. We then define the trivector, which corresponds to the unit volume defined by the three unit vectors. This we call the pseudoscalar, $\boldsymbol{i} = \boldsymbol{e}_1\boldsymbol{e}_2\boldsymbol{e}_3$. Multiplying the pseudoscalar with a vector gives the bivector perpendicular to the vector, $\boldsymbol{i}\boldsymbol{e}_1 = \boldsymbol{e}_2\boldsymbol{e}_3$. We note that the behaviour of the Pauli matrices is implicitly reproduced by the elements of geometric algebra [30]. Indeed, the Pauli matrices are a matrix description of rotations in three dimensional space, described in geometric algebra by the bivectors.

We now define a vector of motion, $\boldsymbol{e}_v$, and the bivector term perpendicular to the vector of motion, $\boldsymbol{i}\boldsymbol{e}_v$. This bivector term may be visualised as the product of two vectors, which are perpendicular to one another and to the vector of motion, $\boldsymbol{e}_E$ and $\boldsymbol{e}_H$, such that the bivector is given by $\boldsymbol{e}_E\boldsymbol{e}_H = \boldsymbol{i}\boldsymbol{e}_v$ (see Fig. 1). These vectors correspond respectively to the direction of the electric, $\boldsymbol{E}$, and magnetic, $\boldsymbol{H}$, field, which we introduce in accordance with the second postulate of the extended electron model. An additional phase is also added to account for the energy conservation of electrons at the local level [8], but for simplicity we set this term to zero in our notation.

Since the geometric product is anti-commutative, we may also define an antiparallel bivector, $\boldsymbol{e}_H\boldsymbol{e}_E = -\boldsymbol{i}\boldsymbol{e}_v$. Thus, this bivector term gives rise to a spin vector, with a direction corresponding to the spin unit vector, $\boldsymbol{e}_S$, that can be either parallel or antiparallel to the vector of motion. The electron spin is defined by the helictiy and relative direction of the electromagnetic field terms, which satisfies the third postulate of the extended electron model. Indeed, we may define the a Poynting-like vector, which we call the spin density, $S = |\boldsymbol{E}|\,|\boldsymbol{H}|$, that gives the energy density of the field components of the

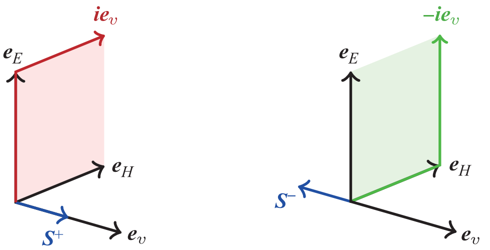

Fig. 1 Schematic of electron spin and field vectors ($\boldsymbol{S}$, $\boldsymbol{e}_E$, and $\boldsymbol{e}_H$, respectively) and the vector of motion, $\boldsymbol{e}_v$, for an electron, with both the parallel (left) and antiparallel (right) behaviour. The direction of the electron spin vector, $\boldsymbol{S}^{\pm}$, is shown by the short arrows on the $\boldsymbol{e}_v$ axis in both cases.

electron. In this framework, the wavefunction may be written in terms of the mass density, ρ, the spin density, and the direction of the spin vector,

$$\Psi\left(\boldsymbol{r}\right) = \rho^{1/2}\left(\boldsymbol{r}\right) + \boldsymbol{ie}_S\left(\boldsymbol{r}\right) S^{1/2}\left(\boldsymbol{r}\right). \tag{3}$$

Here, all terms depend on position, $\boldsymbol{r}$, which we omit from further equations, but note that rather than describing a probabilistic distribution of point-like electron states, this wavefunction describes the physical properties within the volume of an extended electron.

The duality operation, $\Psi^{\dagger}$, is represented by a change in the helicity of the bivector term and, hence, a change in sign of the spin vector. The product of Ψ and $\Psi^{\dagger}$ complies with the Born rule and corresponds to the inertial number density of the electron,

$$\Psi\Psi^{\dagger} = \rho + S = \rho_0, \tag{4}$$

which corresponds to the requirement of energy conservation and the fourth postulate of the extended electron model; that the energy density at every point of the extended electron is a constant. Here, the wave properties are related to oscillations in the mass density of the electron, which are supplemented by equal and opposite oscillations in the spin density, $\dot{S} = -\dot{\rho}$. So the first postulate of the extended electron model is satisfied.

Figure 1 shows a schematic representation of these bivector notation, where the spin vector of an electron is given by [8]

$$\boldsymbol{S}^{\pm} = \frac{\hbar}{2}\Psi\boldsymbol{e}_S^{\pm}\Psi^{\dagger} = \pm\frac{\hbar}{2}\rho_0\boldsymbol{e}_v, \tag{5}$$

Electron spin, defined in this way, is a constant vector associated with the direction perpendicular to the plane of the electromagnetic field terms, which are defined by the velocity of the electron. Electrons with vanishing velocity, therefore, contain no field components to their energy density and thus do not possess spin. The spin of an electron in motion is only isotropic in relation to rotations in the bivector plane perpendicular to the vector of motion, $\boldsymbol{ie}_v$, but since this direction is due to the motion of the electron, a statistical manifold of equal number spin-up, $\boldsymbol{S}^+$, and spin-down, $\boldsymbol{S}^-$, electrons is fully isotropic.

If the electrons are free, a magnetic field, $\boldsymbol{B}$, will alter their trajectory according to the classical Lorentz forces, but if, on the other hand, they are not free – instead moving along a constrained trajectory – their spin will be affected. This effect is modelled by a modified Landau–Lifshitz equation [8],

$$\dot{\boldsymbol{e}}_S = \text{const} \cdot \boldsymbol{e}_S \times \left(\boldsymbol{u} \times \dot{\boldsymbol{B}}\right), \tag{6}$$

where $\boldsymbol{u}$ is the electron's velocity. For a finite static field, the induced spin vector, $\boldsymbol{S}'$, may be described by the first order term,

$$\boldsymbol{S}' = \text{const} \cdot \boldsymbol{S} \times \left(\boldsymbol{u} \times \boldsymbol{B}\right). \tag{7}$$

So, in response to an external magnetic field, the spin vector rotates in either a parallel or antiparallel direction depending on electron spin, which gives rise to two induced spin densities. The induced spin densities will lead to a precession around the magnetic field in two directions, which will give rise to induced magnetic moments parallel, or anti parallel to the field. In an inhomogeneous field the force of deflection is then directed either parallel or antiparallel to the field gradient, leading to the alternate trajectories seen in Stern–Gerlach–type experiments [4]. For example, in the case of the hydrogen atom, the wavefunction is an exponentially decaying wavefunction, similar to that in the standard picture [5], but with the electron spin direction parallel to the radial vector and pointing outward (spin-up) or inward (spin-down). These spin components are acted upon by the magnetic field and split the trajectory of the atoms accordingly. Here, there is no wavefunction collapse, we simply reveal the direction of the spin vector with respect to the vector of motion. The conventional framework omits the possibility that measurements directly affect the electron spin properties of a system. We see in the extended electron model that the measurement has an explicit effect. Thus, we can explain why measurements on different axes are non-commutative: the measurement is felt by the electron and the spin vector is realigned with each new measurement.

4 Spooky action at a distance

We now consider the famous Einstein–Podolsky–Rosen (EPR) thought experiment [31, 32], which concludes that communication between two measurements seems to violate the principle of causality. We imagine a source that emits an electron pair. The spin of the two particles are measured separately, but due to their common source, the measurements implicitly depend on one another. If the z-axis of the first electron is measured to be spin-up, then it is known that the z-axis measurement of the second electron will be spin-down. In the standard model, this is because the initial measurement has collapsed the wavefunction [33]. On the other hand, if the measurement on the second electron is performed along the y-axis, there is an equal chance of measure spin-up or spin-down. The implication is that the second electron somehow *knows* on which axis the first measurement was performed, a phenomenon that Einstein dubbed spooky action at a distance. Experimental evidence has thus far shown a correlation between these two measurements [34], but that on its own is not enough to prove a causal link. In the framework of extended electrons, we find that this communication is an epiphenomenon of an underlying correlation [29], which is contained mathematically in local variables.

The spin vector can either be parallel or antiparallel to the vector of motion so if measurements are taken parallel to this axis for both electrons, the correlation between the two measurement is explained trivially. Indeed, this argument can be extended to all measurement angles except perfectly perpendicular, at which point the probability of measuring spin-up or spin-down are equal and correlations between the measurements are harder to explain. Here, we assume the measurement contains rotations in the plane perpendicular to the direction of motion and the spin vector, which in geometric algebra, are described by the multiplication of two vectors. The term itself is given the name rotor. We describe a rotation on the plane $\boldsymbol{e}_E\boldsymbol{e}_H$ through an angle of φ, by

$$R(\varphi) = \mathrm{e}^{(\boldsymbol{e}_E\boldsymbol{e}_H)\boldsymbol{e}_S\varphi} = \cos\varphi + \boldsymbol{i}\sin\varphi. \quad (8)$$

Then the probability of detecting an angle of rotation, φ, is given by the square of the scaler part of the rotor,

$$p\left(\varphi\right) = \cos^2\varphi. \quad (9)$$

This is true regardless of whether electron spin is parallel or antiparallel, since that effect is only apparent in the pseudoscalar term, $\boldsymbol{i}\sin\varphi$, which changes sign from positive to negative respectively. It is here that the model diverges from Bell's original derivation of his inequalities, in which he assumes the correlation probability is the product of the two measurement probabilities. Instead, to account for the two rotations, we take the product of the rotors for each electron, assuming that the latter spin is antiparallel,

$$R(\varphi_1)\cdot R(\varphi_2) = \mathrm{e}^{\boldsymbol{i}(\varphi_1-\varphi_2)}. \quad (10)$$

The square of the scaler term then gives the correlation probability in a form similar to that derived in the Clauser–Horne–Shimony–Holt formalism [35],

$$p\left(\varphi_1,\varphi_2\right) = \cos^2\left(\varphi_1-\varphi_2\right). \quad (11)$$

The difference between this approach and the assumptions made in Bell's inequalities is that the pseudoscalar terms in the rotors have an effect on the correlation probabilities. Thus, we find that measurements conducted on the same axis are expected to be fully correlated, whereas perpendicular measurements will be uncorrelated. This correlation is explicitly contained in local variables, unlike the phase correlations proposed by de Broglie [36], and later Bohm [19], which are manifestly non-local. In the current model, the superluminal communication is simply an artefact of the phase correlation and, since this correlation does not violate local causality, then there is no paradox.

5 Conclusion

In conclusion, within the standard approach, it is assumed that electron spin cannot have the properties of a vector and still maintain its isotropy. However, in order to interact with a magnetic field, electron spin *must* have the properties of a vector. We assert this conflict is the source of many of the paradoxes related to electron spin and that exploiting a model in which the spin is described consistently in real three-dimensional space allows us to resolve these paradoxes while maintaining the isotropy.

The essential difference between this model and conventional interpretations is that the electrons are modelled as spatially extended entities as opposed to point-particles. In this way, the wave properties are encoded into oscillating mass and spin densities, which comply with the Born rule to give the inertial number density. Spin-up and spin-down are represented by spin vectors that are respectively parallel and antiparallel to the vector of motion of the electron, which is itself an extended vector field. The isotropy of the electron spin is reproduced in a statistical manifold with an equal number of spin-up and spin-down electrons. Moreover, this behaviour is a manifestation of the helicity of electromagnetic field components, the orientation of which may be affected by an external magnetic field, giving rise to the results of Stern–Gerlach–type experiments. In principle, this process is deterministic, since the spin density determines the result. In practice, the spin density is unknown and the experimental results must still be analysed statistically.

We have also shown that the non-commutativity of electron spin measurements on different axes is well explained by the interaction between the spin vectors and the measurement field. Finally, EPR-type experiments were interpreted through the lens of extended electrons and the spectre of spooky action at a distance was found to be nothing more than an underlying correlation.

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