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RESEARCH ARTICLE

Broadband illusion optical devices based on conformal mappings

Zhan Xiong, Lin Xu, Ya-Dong Xu, Huan-Yang Chen†

College of Physics, Optoelectronics and Energy & Collaborative Innovation Center of Suzhou Nano Science and Technology, Soochow University, Suzhou 215006, China
Corresponding author. E-mail: † chy@suda.edu.cn
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In this paper, we propose a simple method of illusion optics based on conformal mappings. By carefully developing designs with specific conformal mappings, one can make an object look like another with a significantly different shape. In addition, the illusion optical devices can work in a broadband of frequencies.

Keywords optical devices, llusion optics, onformal transformation

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1 Introduction

Transformation optics (TO) [1–3], as a powerful tool, has been used to design various devices with intriguing properties, such as cloaks [1, 4], concentrators [5, 6], and rotators [7–9]. More recently, conformal transformation optics (CTO) [10] has made fast progress owing to the success of carpet cloaks [11–14]. It has been proved that only normal dielectrics are required for designing such devices, easing the design and fabrication of the devices. In fact, CTO has been proved to be able to be applied to construct several devices that were originally thought to be achievable only by using general TO [10]. In this letter, we consider designing illusion optical devices based on CTO. Illusion optics [15] was originally proposed based on negative refractive index metamaterials (NIMs) [16], and in principle, can be used to make an object appear like another. However, owing to the slow progress in experiments of NIMs, only one example has been presented in transmission line systems [17]. It has been found that compressing mappings of concentrators can also be used to construct illusion optics devices [18–20], and the implementations are anisotropic and complicated, but not require NIMs. Hence, we have to determine whether it is possible to design illusion optical devices using conformal mappings and whether the devices are workable in a broadband of frequencies.

2 Design and simulations

Let us start with CTO. It has been proved that, for transverse electric (TE) polarizations, the electric field E_z will follow the Helmholtz equation [2, 10],

$$(\partial x^2 + \partial y^2 + n_z^2 k^2)E_z = (4\partial z \partial z^* + n_z^2 k^2)E_z = 0, \quad (1)$$

where $n_z(x, y)$ is the refractive index profile and $k = 2\pi/\lambda$ is the wave vector. By performing a conformal mapping $w(z)$, the electric field in a new space (w-space) will also follow the Helmholtz equation, but with a new refractive index profile $n_w(u, v)$. The conformal mapping will ensure the optical path is unchanged so that the refractive index profiles in z-space (physical space) and w-space (virtual space) will have the following relationship:

$$n_z(x, y) = n_w(u, v) \left| \frac{\mathrm{d}w}{\mathrm{d}z} \right|. \quad (2)$$

It is then reasonable to consider illusion optics from the conformal mappings. It should be straightforward using TO to “compress” a big object, i.e., cloaks, or to “enlarge” a small object, i.e., concentrators/superscatterers. However, this is not the sole concept of illusion optics. We mean that not only the size, but also the shape should be changed in a substantial way. The question we are faced with is how to design illusion optical devices

Frontiers of Physics

using conformal mappings? Let us consider a Zhukowski mapping $w(z) = z + \frac{1}{z}$ as an example. Figure 1(a) shows the plot for conformal illusion optics in physical space, where an elliptical perfect electric conductor (PEC) is enclosed with a dielectric profile of $n_z(x, y) = |1 - \frac{1}{z^2}|$. Such a structure will appear like a sunglass-shaped PEC (or two PECs connected with each other), as shown in Fig. 1(b). Hence, such an idea can be used to construct a simple illusion optical device to separate an object into two (therefore completely changing the object's appearance). It is very neat to choose an elliptical PEC like this. There are at least three advantages of this design. The first one is that this device can work for a broadband of frequencies, which is not possible with illusion optics with NIMs. The second is that the inner unit circle can be excluded to avoid considering the branch cut effect, i.e., to eliminate the singularity at the origin. Finally, the two points, $(0, 1)$ and $(0, -1)$ are mapped to $(0, 0)$ in virtual space, which makes the illusion effect dramatic such that one object appears as two objects touching each other.

For details, we choose an elliptical PEC with a prolate axis with a length of 6 and a minor axis with a length of 1. Figure 2(a) shows the refractive index profile of the illusion device, where the minimum value is $\sim$0.88 and the maximum is 2. We then assume a plane wave is incident from the $-y$ direction (with a wavelength of 2.5 for instance), and plot the electric field pattern in Fig. 2(b). The pattern is exactly the same as that for a bare sunglass-shaped PEC in air, as plotted in Fig. 2(c). It is noted that the phenomenon will be the same if the plane wave is incident from other directions.

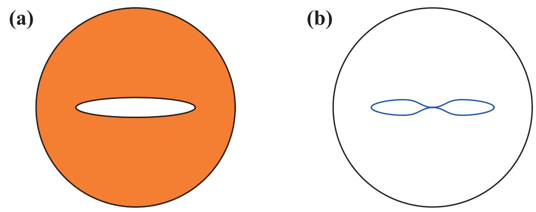

Fig. 1 Schematic plot of conformal illusion optics. **(a)** Physical space, an elliptical PEC covered with a dielectric profile (in orange color). **(b)** Virtual space, a sunglass-shaped PEC in air.

As the minimum value approaches 1, we further modify the illusion device, i.e., by replacing all the materials with a refractive index less than one with air. Therefore, we will obtain a new refractive index profile, as shown in Fig. 2(d). Such a device can be realized by using normal dielectrics, and can work for a broadband of frequencies, even for visible frequencies [13]. For example, we also plot the field pattern for the modified illusion device in Fig. 2(e). It also appears to be two PECs touching each other in air when compared with Fig. 2(c).

To visualize the effect, we plot the field pattern for a bare elliptical PEC in air in Fig. 2(f). As shown, the device indeed makes an object appear as another one (or two connected ones). To demonstrate the broadband functionality of our design, we plot the field patterns of the modified illusion device, a bare sunglass-shaped PEC, and a bare elliptical PEC in Figs. 3(a)–(c) with a wavelength of 1 and Figs. 3(d)–(f) with a wavelength of 4. It is clear that the field patterns are almost the same

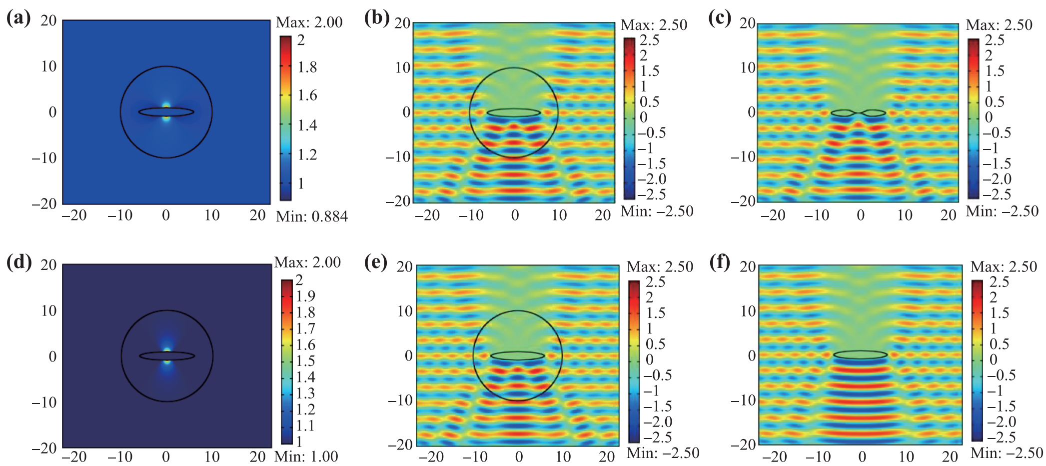

Fig. 2 **(a)** Refractive index profile of the conformal illusion optical device. **(b)** The electric field pattern for an incident TE plane wave interacting with the device. **(c)** The electric field pattern for the plane wave impinging at a bare sunglass-shaped PEC. **(d)** Refractive index profile of the modified version of the illusion optical device. **(e)** The electric field pattern for an incident TE plane wave interacting with the modified device. **(f)** The electric field pattern for the plane wave impinging at a bare elliptical PEC. The working wavelength is 2.5.

for the modified illusion device and the bare sunglass-shaped PEC for both wavelengths, while they are quite different from those of the bare elliptical PEC.

We then plot the angular scattering cross section for the above cases at a wavelength of 2.5. Figure 4(a) shows the cross section for a perfect illusion device, while Fig. 4(b) shows that for the modified device. Both devices have almost the same scattering angular distribution as that of a bare sunglass-shaped PEC in air, as shown in Fig. 4(c). For comparison, we also plot the angular scattering cross section for a bare elliptical PEC, as shown in Fig. 4(d).

Such an illusion effect can be extended further. Several years ago, we proved that NIMs can be used to make two PECs look like one [21]. Later, we demonstrated that a conformal lens can manipulate the numbers of light sources (active objects) [22]. However, it should not be very practical to make one passive object (or PEC) into two. The reason for this is simple. There is a multiple scattering effect between objects, which cannot occur for only one object. However, if we reduce the multiple scattering effect, at least approximately, it is possible to make an illusion device to transform one object into two. Now, we consider the sunglass-shaped PEC [plotted in Fig. 5(a)] and re-plot the field pattern of Fig. 2(c) as Fig. 5(b) for further comparison. The angular scattering cross section in Fig. 4(c) is also re-plotted as Fig. 5(c). From the special shape at the meeting point we can make two approximations, one with a distance of 0.8 m [Fig. 5(d)] and another with a larger distance of 2.6 m [Fig. 5(g)]. We plot the field pattern of the first distance in Fig. 5(e) and the related angular scattering cross section in Fig. 5(f), which shows scattering similar to that for the connected ones. However, when the separate distance increases, as shown by the field pattern in Fig. 5(h) and the related angular scattering cross section in Fig. 5(i), the scattering differs from that of the connected ones. Hence, the multiple scattering effect between two separate objects should not be ignored as the distance increases. Therefore, the current device can partly change one object into two, if the distance between the two objects is small enough.

3 Conclusions

In conclusion, we have proposed a practical illusion device to make one object seem like two connected ones (or

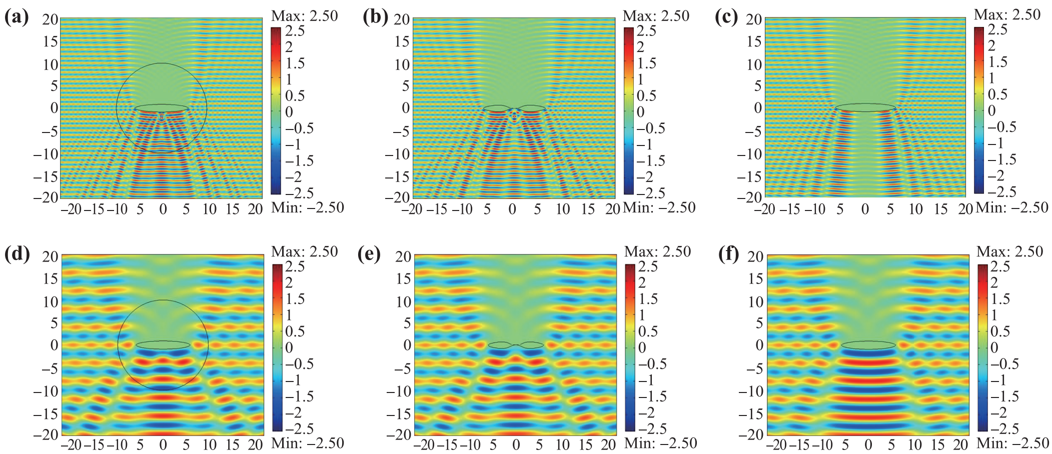

Fig. 3 The field patterns of **(a)** the modified illusion device, **(b)** a bare sunglass-shaped PEC, and **(c)** a bare elliptical PEC at a wavelength of 1; and the field patterns of **(d)** the modified illusion device, **(e)** a bare sunglass-shaped PEC and **(f)** a bare elliptical PEC at a wavelength of 4.

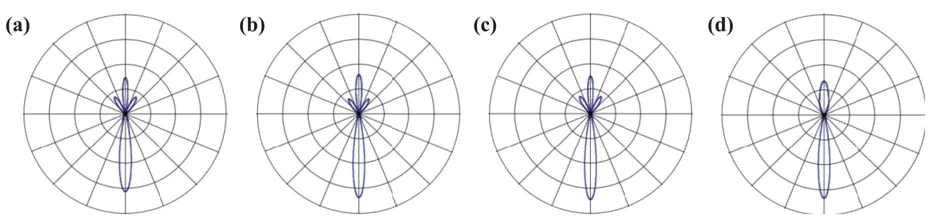

Fig. 4 The angular scattering cross sections of **(a)** the perfect illusion device, **(b)** the modified device, **(c)** a bare sunglass-shaped PEC, and **(d)** a bare elliptical PEC.

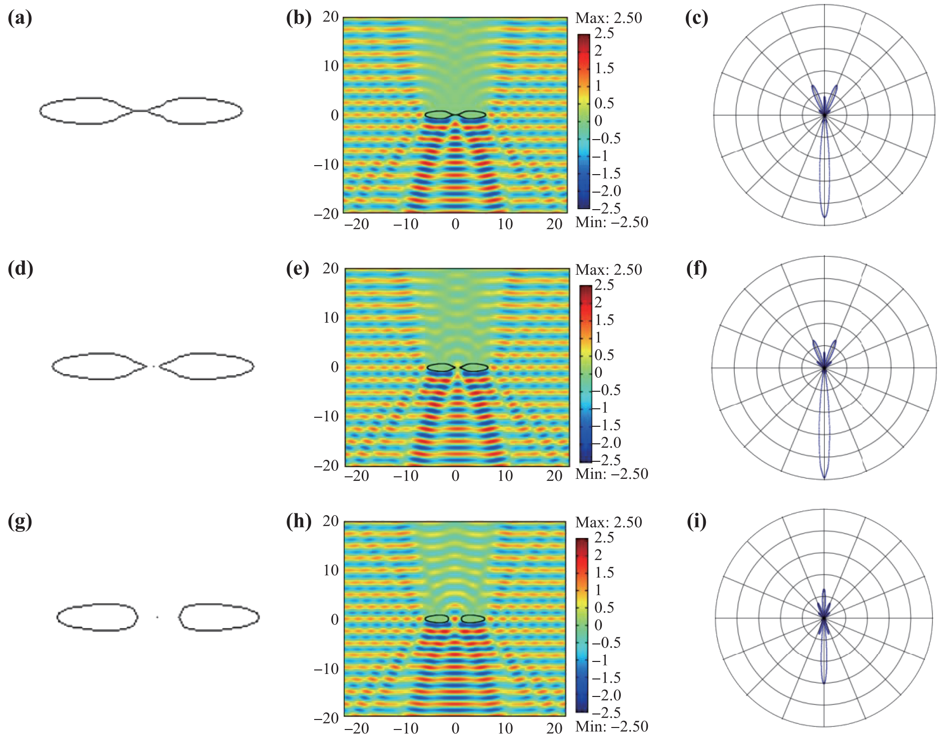

Fig. 5 **(a)** Schematic plot of the bare sunglass-shaped PEC. **(b)** The electric field pattern for the bare sunglass-shaped PEC. **(c)** The angular scattering cross section for the bare sunglass-shaped PEC. **(d)** Schematic plot of two separate PECs at a small distance. **(e)** The electric field pattern for two separate PECs with a small distance. **(f)** The angular scattering cross section for two separate PECs at a small distance. **(g)** Schematic plot of two separate PECs at a big distance. **(h)** The electric field pattern for two separate PECs at a big distance. **(i)** The angular scattering cross section for two separate PECs at a big distance.

two separate ones) using conformal mappings. The device can be implemented by using normal dielectrics and can work in a broadband of frequencies. More interesting illusion devices can be fabricated by using other designs of conformal mappings. We hope that the current design can be put into practical application soon.

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