

RESEARCH ARTICLE

Some new symmetric relations of quark masses and prediction of fourth-generation quark masses

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In this study, we discover a mass space defined by generalized Koide relations, named here as k -relations, and achieve some new symmetric relations. These relations can be further used to predict the fourth-generation quark masses in terms of dilation magnitude and angular rotation ratios in the general mass space. Thus far, no theory has been proposed that can constrain the number of generations of quarks; this theory naturally limits the number of generations of quarks.

Keywords fourth generation of quarks

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1 Introduction

The standard model (SM) consists of bosons and fermions, and fermions are further divided into leptons and quarks. The present form of SM contains three generations of both leptons and quarks, and thus far, no experimental evidence for the presence of a fourth generation has been reported for leptons or quarks. However, problems such as the hierarchy problem could be solved if a fourth generation exists. A fourth generation might also help resolve the problem of baryogenesis [1]. Heavy particles may also be candidates for the elusive dark matter.

The SM does not give any prediction about the number of generations and only requires the existence of at least three generations, which is concluded using the width of the Z boson. Fourth-generation particles, through large Yukawa couplings, could themselves be the initiators of the electroweak symmetry breaking [2, 3]. In short, the present form of SM does not answer various questions that can be addressed by considering models beyond SM.

Koide's formula, which gives the precise value of masses of the three generations of leptons [4, 5], is expressed as follows:

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2. \quad (1)$$

Eq. (1) should also be applicable to the masses of three generations of quarks. The masses of quarks are listed

in Table 1. Like leptons, quarks are divided into three generations, although our universe is mostly made up of first-generation particles.

The remainder of this paper is organized as follows. In Section 2, we declare some k -relations considering quarks generation-wise. In Section 3, we define some new symmetric relations amongst the masses of quarks, which are then used to predict the mass values of the fourth generation of quarks. In Section 4, we show how no fifth generation of quarks can exist. Section 5 presents the summary and conclusion.

2 Generation-wise k -relations

If we consider three generations of quarks and separately calculate the k -relation for each generation, interesting results can be obtained. For the first generation,

Table 1 Masses of quarks [6].

Quarks	Mass (eV)	Charge (e)
<i>up(u)</i>	$2.3 \pm 0.0011 \times 10^6$	2/3
<i>down(d)</i>	$4.8 \pm 0.0000035 \times 10^6$	-1/3
<i>strange(s)</i>	$95 \pm 5 \times 10^6$	-1/3
<i>charm(c)</i>	$1.275 \pm 0.025 \times 10^9$	2/3
<i>bottom(b)</i>	$4.18 \pm 0.03 \times 10^9$	-1/3
<i>top(t)</i>	$160 \pm 4.0 \times 10^9$	2/3

$$k_{First}^2 = \frac{m_u + m_d}{(\sqrt{m_u} + \sqrt{m_d})^2} = 0.51654$$

$$\approx \frac{1}{2}.$$

For the second generation,

$$k_{Second}^2 = \frac{m_s + m_c}{(\sqrt{m_s} + \sqrt{m_c})^2} = 0.663098$$

$$\approx \frac{2}{3},$$

while for the third generation,

$$k_{Third}^2 = \frac{m_b + m_t}{(\sqrt{m_b} + \sqrt{m_t})^2} = 0.760436$$

$$\approx \frac{3}{4},$$

which employs the fact that the hierarchy tends to increase with the number of generations. Further, the fractional results given above cannot be a mere coincidence.

3 Symmetric relations

We express relations for up quarks and down quarks as follows:

$$k_i^2 = \frac{\sum_i m_i}{(\sum_i \sqrt{m_i})^2}, \quad \text{for } i = 1 \text{ to } 3. \quad (5)$$

The above relations are called the k -relations, where i denotes the generation number of quarks. By using the above equation and the masses of three generations of quarks given in Table 1, we can solve the following relations. In the case of up quarks, we obtain the following. For $i = 1$,

$$k_{1Qu}^2 = \frac{m_u}{(\sqrt{m_u})^2} = 1; \quad (6)$$

For $i = 2$,

$$k_{2Qu}^2 = \frac{m_u + m_c}{(\sqrt{m_u} + \sqrt{m_c})^2} = 0.921836; \quad (7)$$

For $i = 3$,

$$k_{3Qu}^2 = \frac{m_u + m_c + m_t}{(\sqrt{m_u} + \sqrt{m_c} + \sqrt{m_t})^2} = 0.843657. \quad (8)$$

In the case of down quarks, for $i = 1$,

$$k_{1Qd}^2 = \frac{m_d}{(\sqrt{m_d})^2} = 1; \quad (9)$$

For $i = 2$,

$$k_{2Qd}^2 = \frac{m_d + m_s}{(\sqrt{m_d} + \sqrt{m_s})^2} = 0.70031; \quad (10)$$

For $i = 3$,

$$k_{3Qd}^2 = \frac{m_d + m_s + m_b}{(\sqrt{m_d} + \sqrt{m_s} + \sqrt{m_b})^2} = 0.729579. \quad (11)$$

Now, we can calculate the difference between two consecutive generations of k -relations, i.e., for up quarks, we have

$$\Delta_1 = k_{1Qu}^2 - k_{2Qu}^2 = 0.0781645,$$

$$\Delta_2 = k_{2Qu}^2 - k_{3Qu}^2 = 0.0781787. \quad (12)$$

For down quarks, we obtain

$$\delta_1 = k_{1Qd}^2 - k_{2Qd}^2 = 0.29969,$$

$$\delta_2 = k_{2Qd}^2 - k_{3Qd}^2 = -0.0292692. \quad (13)$$

It is worth noting that, in the case of up quarks, the difference between the k -relations of the 1st generation and 2nd generation is equal to the difference between those of the 2nd generation and 3rd generation.

By applying the idea given by Ref. [7], one can consider the k -relations as vectors. We consider a mass space defined by these k -relations and then calculate the magnitude dilation and angle variation between each successive generation of k -relations, which are used to predict the quark masses of the next generation.

3.1 Magnitude

By using the above relations, we can calculate γ , which is defined as the sum of up quarks and down quarks k -relations for each generation:

$$\gamma_1 = k_{1Qu}^2 + k_{1Qd}^2 = 2,$$

$$\gamma_2 = k_{2Qu}^2 + k_{2Qd}^2 = 1.62215,$$

$$\gamma_3 = k_{3Qu}^2 + k_{3Qd}^2 = 1.57324. \quad (14)$$

From the above, we can calculate ζ , which is defined as the difference between two γ :

$$\zeta_1 = \gamma_2 - \gamma_1 = -0.377854,$$

$$\zeta_2 = \gamma_3 - \gamma_2 = -0.0489094. \quad (15)$$

Further, we define α as the ratio between two ζ values, such that

$$\alpha = \frac{\zeta_{i+1}}{\zeta_i}, \quad \forall \quad i = 1, 2, 3, \quad (16)$$

which is called the magnitude dilation ratio in the mass space and gives

$$\frac{\zeta_2}{\zeta_1} = 0.12944. \quad (17)$$

With α known, we can calculate ζ_3 :

$$\zeta_3 = \alpha * \zeta_2 = -0.00633083. \quad (18)$$

From the above, we can calculate

$$\begin{aligned} \zeta_3 &= \gamma_4 - \gamma_3, \\ \gamma_4 &= \zeta_3 + \gamma_3 = 1.56691. \end{aligned} \tag{19}$$

Now, by using γ_4 , which is the sum of the 3rd and 4th generation of k -relations, we obtain

$$\begin{aligned} \gamma_4 &= k_{4Qu}^2 + k_{4Qd}^2 \\ &= \frac{m_u + m_c + m_t + m_{t'}}{(\sqrt{m_u} + \sqrt{m_c} + \sqrt{m_t} + \sqrt{m_{t'}})^2} \\ &\quad + \frac{m_d + m_s + m_b + m_{b'}}{(\sqrt{m_d} + \sqrt{m_s} + \sqrt{m_b} + \sqrt{m_{b'}})^2}. \end{aligned}$$

In the above equation, $m_{t'}$ and $m_{b'}$ represent the masses of the fourth-generation quarks. Upon simplification, it becomes

$$\gamma_4 = \frac{1.61277 \times 10^{11} + m_{t'}}{(437224 + \sqrt{m_{t'}})^2} + \frac{4.2798 \times 10^9 + m_{b'}}{(76590.6 + \sqrt{m_{b'}})^2}. \tag{20}$$

To find the masses of the fourth-generation quarks, we need to solve the above equation having two unknowns.

3.2 Angle

One can also find the angle θ between the k -relations, which are

$$\begin{aligned} \theta_1 &= \arctan \frac{k_{1Qu}^2}{k_{1Qd}^2} = 45^\circ, \\ \theta_2 &= \arctan \frac{k_{2Qu}^2}{k_{2Qd}^2} = 52.7764^\circ, \\ \theta_3 &= \arctan \frac{k_{3Qu}^2}{k_{3Qd}^2} = 49.1473^\circ. \end{aligned}$$

The ratios between two consecutive angles are

$$\begin{aligned} \beta_1 &= \frac{\theta_2}{\theta_1} = 1.17281, \\ \beta_2 &= \frac{\theta_3}{\theta_2} = 0.931237. \end{aligned} \tag{21}$$

To find β_3 , we can similarly define a relation

$$\eta = \frac{\beta_{(i+1)}}{\beta_i}, \quad \forall \quad i = 1, 2, 3, \tag{22}$$

where η is called the angular rotation ratio in the mass space. The value of η is fixed as 0.794023, which in turn yields the value of β_3 as 0.739424. We can use the obtained value of β_3 to find θ_4 , i.e.,

$$\begin{aligned} \beta_3 &= \frac{\theta_4}{\theta_3}, \\ \theta_4 &= \beta_3 \theta_3 = 36.3407^\circ, \end{aligned} \tag{23}$$

where θ_4 by definition is

$$\theta_4 = \arctan \frac{k_{4Qu}^2}{k_{4Qd}^2}. \tag{24}$$

On using the value of θ_4 in Eq. (23), the above equation becomes

$$\frac{k_{4Qu}^2}{k_{4Qd}^2} = 0.735667, \tag{25}$$

meaning

$$\begin{aligned} \frac{k_{4Qu}^2}{k_{4Qd}^2} &= \frac{\frac{m_u + m_c + m_t + m_{t'}}{(\sqrt{m_u} + \sqrt{m_c} + \sqrt{m_t} + \sqrt{m_{t'}})^2}}{\frac{m_d + m_s + m_b + m_{b'}}{(\sqrt{m_d} + \sqrt{m_s} + \sqrt{m_b} + \sqrt{m_{b'}})^2}} \\ &= \frac{(1.61277 \times 10^{11} + m_{t'}) \times (76590.6 + \sqrt{m_{b'}})^2}{(4.2798 \times 10^9 + m_{b'}) \times (437224 + \sqrt{m_{t'}})^2}. \end{aligned} \tag{26}$$

Further, through numerical calculations to solve Eq. (20) and Eq. (26) having two unknown variables, we can find the values of b' and t' . Through numerical calculation, one can obtain two sets of solutions, one of which can be discarded by considering the constraint that the mass value of the fourth-generation quarks should be greater than the mass of the third-generation quarks. Thus, we finally deduce

- (i) the mass of b' is predicted to be equal to 2.04361 TeV,
- (ii) the mass of t' is predicted to be equal to 2.78181 TeV.

The predicted masses are of the TeV scale and can provide the matter-field substrate in models beyond SM in which the Higgs field is not considered elementary [8, 9].

4 Fifth generation

Extending the work by following similar steps will lead to the masses of the fifth generation of quarks. The k -relation for the fourth-generation up quarks becomes

$$k_{4Qu}^2 = \frac{m_u + m_c + m_t + m_{t'}}{(\sqrt{m_u} + \sqrt{m_c} + \sqrt{m_t} + \sqrt{m_{t'}})^2} = 0.664137, \tag{27}$$

and that for the fourth-generation down quarks becomes

$$k_{4Qd}^2 = \frac{m_d + m_s + m_b + m_{b'}}{(\sqrt{m_d} + \sqrt{m_s} + \sqrt{m_b} + \sqrt{m_{b'}})^2} = 0.902768. \tag{28}$$

We can calculate the difference between two successive generations of k -relations; i.e., for up quarks, we have

$$\Delta_3 = k_{3Qu}^2 - k_{4Qu}^2 = 0.17952, \tag{29}$$

while for down quarks, we have

$$\delta_3 = k_{3Qd}^2 - k_{4Qd}^2 = -0.173189. \quad (30)$$

By using the magnitude dilation ratio α defined in previous section, we can obtain ζ_4 :

$$\zeta_4 = \alpha * \zeta_3 = -0.000819462. \quad (31)$$

By using ζ_4 , which is the difference between γ_5 and γ_4 , where we know the value of γ_4 from Eq.(19), we can express and calculate

$$\begin{aligned} \zeta_4 &= \gamma_5 - \gamma_4, \\ \gamma_5 &= \zeta_4 + \gamma_4, \\ \gamma_5 &= 1.56609. \end{aligned} \quad (32)$$

By definition, γ can be expressed as

$$\begin{aligned} \gamma_5 &= k_{5Qu}^2 + k_{5Qd}^2 \\ &= \frac{m_u + m_c + m_t + m_{t'} + m_{t''}}{(\sqrt{m_u} + \sqrt{m_c} + \sqrt{m_t} + \sqrt{m_{t'}} + \sqrt{m_{t''}})^2} \\ &\quad + \frac{m_d + m_s + m_b + m_{b'} + m_{b''}}{(\sqrt{m_d} + \sqrt{m_s} + \sqrt{m_b} + \sqrt{m_{b'}} + \sqrt{m_{b''}})^2}. \end{aligned}$$

In the above equation, $m_{t''}$ represents the mass of the fifth-generation up quark, while $m_{b''}$ represents the mass of the fifth-generation down quark. Upon simplification, it becomes

$$\begin{aligned} \gamma_5 &= \frac{2.94308 \times 10^{12} + m_{t''}}{(2.1051 \times 10^6 + \sqrt{m_{t''}})^2} \\ &\quad + \frac{2.04789 \times 10^{12} + m_{b''}}{(1.50614 \times 10^6 + \sqrt{m_{b''}})^2}. \end{aligned} \quad (33)$$

To solve this equation having two unknown variables, we need one more equation, which can be obtained by using the angle dilation ratio between the k -relations.

We already know the value of θ_4 from Eq. (23), but the value of β_4 is unknown. To find the value of β_4 , we need the angle dilation ratio η , defined by Eq. (22), and β_3 , which gives

$$\beta_4 = \eta\beta_3 = 0.587119.$$

By using the value of β_4 , we can find the value of θ_5 , since

$$\begin{aligned} \beta_4 &= \frac{\theta_5}{\theta_4}, \\ \theta_5 &= \beta_4\theta_4 = 21.3363^\circ. \end{aligned} \quad (34)$$

By using the definition of θ_5 , we can write

$$\theta_5 = \arctan \frac{k_{5Qu}^2}{k_{5Qd}^2},$$

which on simplification becomes

$$\frac{k_{5Qu}^2}{k_{5Qd}^2} = 0.390614, \quad (35)$$

meaning

$$\begin{aligned} \frac{k_{5Qu}^2}{k_{5Qd}^2} &= \frac{\frac{m_u + m_c + m_t + m_{t'} + m_{t''}}{(\sqrt{m_u} + \sqrt{m_c} + \sqrt{m_t} + \sqrt{m_{t'}} + \sqrt{m_{t''}})^2}}{\frac{m_d + m_s + m_b + m_{b'} + m_{b''}}{(\sqrt{m_d} + \sqrt{m_s} + \sqrt{m_b} + \sqrt{m_{b'}} + \sqrt{m_{b''}})^2}} \\ &= \frac{(2.94308 \times 10^{12} + m_{t''}) \times (1.50614 \times 10^6 + \sqrt{m_{b''}})^2}{(2.1051 \times 10^6 + \sqrt{m_{t''}})^2 \times (2.04789 \times 10^{12} + m_{b''})}. \end{aligned} \quad (36)$$

Simultaneously solving Eq. (33) and Eq. (36) gives imaginary solutions, showing that no real solution exists for the fifth-generation particles; hence, only four generations of quarks exist in nature.

5 Summary and conclusion

In this paper, we achieved some new symmetric relations, named k -relations, and used them to calculate the magnitude dilation and angular rotation ratio in the general mass space to predict the masses of the fourth-generation quarks. The obtained mass values for fourth-generation quarks are not ruled out by the present experimental studies [6]. This paper is based on models beyond SM because the simplest four-generation extension of SM is inconsistent with the most-recent LHC data [10]. The predicted TeV-scale masses of the fourth-generation quarks are considered in various models beyond SM [11, 12]. The masses of the fourth-generation quarks predicted here cannot be accurately searched for with the current experimental capabilities, but the required experimental capabilities will hopefully be developed in the near future. We also extended the above model to predict the masses of the fifth-generation particles and discovered that this model rules out the presence of fifth-generation particles as there exist no real solution for fifth-generation particles; hence, according to this model, only four generations of particles exist in nature. Thus, to our knowledge, this paper presents the first theoretical model that naturally constrains the number of generations of quarks.

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