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Research article

Solving the Dyson–Schwinger equation around its first singularities in the Borel plane

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The Dyson–Schwinger equation of the massless Wess–Zumino model is written as an equation over the anomalous dimension of the theory. Its asymptotic behavior is derived and the procedure to compute the perturbations of this asymptotic behavior is detailed. This procedure uses ill-defined objects. To solve this, the Dyson–Schwinger equation is rewritten for the Borel plane. It is shown that the ill-defined procedure in the physical plane can be applied in the Borel plane. Other results obtained in the Borel plane are stated and the proof for one result is described.

Keywords Dyson–Schwinger equation, Wess–Zumino model, Borel transform

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1 Introduction

In this paper, we will present our procedure to deal with the Dyson–Schwinger equation of the massless Wess–Zumino model. This model was first presented in Ref. [2]. It is an exactly supersymmetric model due to which we can consider only one equation instead of the system. However, this equation (hereafter, referred to as the Dyson–Schwinger equation) is nonlinear in that the integrand is linked to the product of the Green function of the theory. We have two goals when working with this equation:

- to obtain precise non-perturbative information about the model (i.e., beyond the asymptotic behavior of its anomalous dimension),
- to propose a well-defined procedure.

The massless Wess–Zumino model is clearly a toy model since no supersymmetric partners have ever been observed; thus, if supersymmetry exists, it has to be broken. Therefore, the second point is crucial, i.e., we want to be sure that every point of the computation is solid before tackling more physically relevant theories.

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We will start by introducing the Dyson–Schwinger equation that we will consider. Next, our procedure for computing the perturbations of the asymptotic behavior of the anomalous dimension of the model in the physical plane (where the parameter of expansion is the fine structure constant of the theory) will be presented. After a brief introduction of the Borel transform, we will explain how this procedure is reinterpreted in the Borel plane (where the parameter of expansion is the Borel transform of the fine structure constant of the theory) and how the Borel plane approach can be used to tackle very technical questions to be addressed in the physical plane.

2 The physical plane

2.1 Dyson–Schwinger equation of the Wess–Zumino model

Let us start by expanding the two-point function as a series of the logarithm of the external impulsion:

$$G(L) = 1 + \sum_{k=1}^{+\infty} \gamma_k \frac{L^k}{k!}, \tag{1}$$

where $L = \ln(p^2/\mu^2)$. Moreover, γ_ns are functions of a, which is the fine structure of the theory, $\gamma_n = \gamma_n(a)$. By

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using the Hopf algebraic method, it has been shown in [6] and [1] that this two-point function obeys the renormalization group equation (RGE)

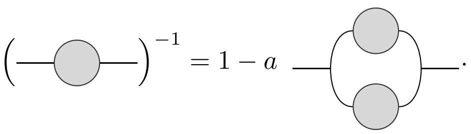
$$\partial_L G = \gamma(1 + 3a\partial_a)G. \tag{2}$$

Thus, γ_n obeys an induction relation

$$\gamma_{k+1} = \gamma(1 + 3a\partial_a)\gamma_k \tag{3}$$

with $\gamma := \gamma_1$ being the anomalous dimension of the theory. Hence, if γ is known, the dressed two-point function is completely known (at least in principle). Therefore, we will write the Dyson–Schwinger equation as an equation over γ rather than an equation over $G(L)$ in the physical plane as well as the Borel plane.

Now, the Dyson–Schwinger equation of the Wess–Zumino model can be simply written as



(4)

We are working with an exactly supersymmetric theory. Hence, using superspace techniques, it can be shown that there is no vertex renormalization because of the difference of signs between bosonic and fermionic loops. Therefore, we will use the singular throughout this paper when referring to the Dyson–Schwinger equation, i.e., there is no infinite tower of equations in this theory. Due to this, it very suitable for developing techniques for Dyson–Schwinger equations in general. Now, the loop integral on the right hand side of Eq. (4) is

$$\int \mathrm{d}^4 q \frac{G(q^2)}{q^2}\frac{G((p-q)^2)}{(p-q)^2}.$$

It can be evaluated by performing a Mellin transform, which can be performed by

$$\left(\ln\frac{p^2}{\mu^2}\right)^k = \left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^k \left(\frac{p^2}{\mu^2}\right)^x \Bigg|_{x=0}.$$

Then, after permuting sums, derivatives, and integral, and taking a derivative with respect to L and evaluating it at $L = 0$, we have the following equation:

$$\gamma = a\left(1 + \sum_{n=1}^{+\infty}\frac{\gamma_n}{n!}\frac{\mathrm{d}^n}{\mathrm{d}x^n}\right)\left(1 + \sum_{m=1}^{+\infty}\frac{\gamma_m}{m!}\frac{\mathrm{d}^m}{\mathrm{d}x^m}\right) \times H(x,y)|_{x=y=0} := \mathcal{I}(H). \tag{5}$$

Here, $H(x, y)$ is the function known as the one-loop Mellin transform, which can be written as

$$H(x,y) = \frac{(1-x-y)\ (1+x)\ (1+y)}{(2+x+y)\ (1-x)\ (1-y)}, \tag{6}$$

where   represents the usual Euler's gamma function. The Dyson–Schwinger Eq. (5) has to be solved together with the RGE, (3). Hence, it is an equation over γ only as advertised.

2.2 Solving the Dyson–Schwinger equation

Our goal is to compute the asymptotic behavior of the solution of (5), and then compute its perturbations. This seems to be quite cumbersome because the Mellin transform is a complicated function. Hence, the idea is to replace one-loop Mellin transform by a suitable truncation, which will encode the contribution of the anomalous dimension to the asymptotic behavior. The point to be noted is that the perturbations can be computed by replacing the truncation with a better one. In order to do so, we will replace $H(x, y)$ by a sum over its poles multiplied by its residues. From the definition of $H(x, y)$, it is clear that it has a pole if and only if the -function of its numerator is a negative integer.

$$H(x,y) \longrightarrow \sum_{k\geq 1}\left[\frac{\mathrm{Res}_{\,x=-k}(H)}{k+x} + \frac{\mathrm{Res}_{\,y=-k}(H)}{k+y} + \frac{\mathrm{Res}_{\,x+y=k}(H)}{k-x-y}\right]. \tag{7}$$

The most natural expression for the residues partially breaks down the $x \leftrightarrow y$ symmetry of H because each of them is a function of either x or y, and only their sum is symmetric under the exchange of x and y. To simplify the computations, we make every term of the sum symmetric under $x \leftrightarrow y$ by taking an analytic extension of the residues.

$$\mathrm{Res}_{\,x=-k}(H) \longrightarrow P_k(xy),$$
$$\mathrm{Res}_{\,y=-k}(H) \longrightarrow P_k(xy),$$
$$\mathrm{Res}_{\,x+y=k}(H) \longrightarrow Q_k(xy).$$

Now, $P_k(X)$ and $Q_k(X)$ are polynomials that coincide with the residues of H when restricted to $x = -k$ (or $y = -k$) for P_k and to $x + y = k$ for Q_k. The general expression for these polynomials is

$$P_k(xy) = \frac{xy}{k(k-1)}\prod_{i=1}^{k}\left(1+\frac{xy}{ki}\right)\prod_{i=1}^{k-2}\left(1+\frac{xy}{ki}\right), \tag{8}$$

$$Q_k(xy) = \frac{xy}{k(k+1)}\prod_{i=1}^{k-1}\left(1-\frac{xy}{i(k-i)}\right), \tag{9}$$

as shown in Ref. [5]. Now, it is easy to see to which order of γ any given term of the expansion of H will contribute. Hence, we will perform a truncation of (7). To see how this works practically, let us look at the simplest case as an example.

Example: asymptotic behavior.
Only the two first poles are needed to compute the asymptotic behavior of γ. Therefore, we will replace H by

$$h(x,y) = (1+xy)\left(\frac{1}{1+x} + \frac{1}{1+y} - 1\right) + \frac{1}{2}\frac{xy}{1-x-y} + \frac{1}{2}xy. \tag{10}$$

It is useful to look at the quantities corresponding to the contribution of the singular parts of H (without the residues) to the γ function through Eq. (5):

$$F(a) = \mathcal{I}\left(\frac{1}{1+x}\right),$$
$$L(a) = \mathcal{I}\left(\frac{1}{1-x-y}\right).$$

Then, the RGE (3) can be used to define equations fulfilled by F and L. The intermediate quantities can also be substituted in the Dyson–Schwinger Eq. (5). After the computations, we get the following system of equations that are required to be solved

$$F = 1 - \gamma(3a\partial_a + 1)F, \tag{11}$$
$$L = \gamma^2 + \gamma(3a\partial_a + 2)L, \tag{12}$$
$$\gamma = 2aF - a - 2a\gamma(F-1) + \frac{1}{2}a(L - \gamma^2). \tag{13}$$

Then, by expanding γ, F, and L in power of a, i.e., $F = \sum f_n a^n$, $L = \sum l_n a^n$, $\gamma = \sum c_n a^n$, assuming that (f_n), (l_n), and (c_n) have a fast growth, and retaining only the most important terms obtained, we get the following after some manipulations

$$f_{n+1} \simeq -(3n+5)f_n, \tag{14}$$
$$l_{n+1} \simeq 3nl_n, \tag{15}$$
$$c_{n+1} \simeq -(3n+2)c_n. \tag{16}$$

This was a result described in Ref. [1]. The next natural step would be to compute the $1/n$ corrections of the recursion for c_n. However, it turns out to be quite tricky, mainly due to the fact that every pole will contribute to the $1/n$ order. Moreover, such procedure is not very easy to implement in a formal computational software.

The idea in Ref. [5] was to use the fact that we know the contributions of F and L to compute the perturbations of the asymptotic behavior of γ. Hence, let us define two sequences as follows:

$$A_{n+1} = -(3n+5)A_n,$$
$$B_{n+1} = 3nB_n,$$

and two formal series

$$A = \sum A_n a^n,$$
$$B = \sum B_n a^n.$$

Because of the definition of sequences (A_n) and (B_n), the formal series have to satisfy two differential equations

$$3a^2\partial_a A = -A - 5aA, \tag{17}$$
$$3a^2\partial_a B = B, \tag{18}$$

up to a term of degree n_0 (the initial condition, which is a part of the formal series that is not defined by the inductive definitions). Hence, n_0 should be greater than the computed order. Now, we can expand γ around the formal series A and B by taking the ansatz

$$\gamma(a) = a[c(a) + d(a)A + e(a)B]. \tag{19}$$

We will also establish a similar ansatz for any intermediate quantities that are needed in the computation. Then, the general procedure is as follows:

- Write the Dyson–Schwinger equation with the truncated Mellin transform.
- Use the ansatz with A and B and the relations in Eq. (17) to get rid of the derivatives of A and B.
- Ignore the mixed terms (AB, A^2, ...). This is a key point and will be studied later in the Borel plane.
- Ask for each of the remaining terms to vanish.
- Solve the equations perturbatively without assuming any fast growth as it was already taken care of in the formal series A and B.

This is a powerful algorithmic procedure. In Ref. [5], this procedure was used for the computation of the fourth order of the perturbations of the asymptotic behavior of γ. Moreover, the results showed remarkable agreement with the numerical data in Ref. [1]. However, this procedure is not entirely satisfactory because of the following reasons:

- It relies on analytic continuation of the residues of the Mellin transform. The computation of

$$H(x,y) = \sum_{k\geq 1}\left[\left(\frac{1}{k+x} + \frac{1}{k+y} - \frac{1}{k}\right)P_k(xy) + \frac{Q_k(xy)}{k-x-y}\right],$$

could not proved to be exact, i.e., an analytic term may be missed in such an expansion over poles. Typically, we observed that a factor of $1/k$ had to be introduced for the derivative to have the correct value at the origin; however, we do not have an in depth understanding for this observation. We checked that there are no missing analytical terms at the computed orders and that an eventual analytic term could not be diagonal (i.e., of the form $(\partial_x)^n(\partial_y)^n H(x,y)|_{x=y=0}$); however, a full analysis has yet to be performed and is quite cumbersome.

- The use of formal series makes the comparison to numerical results quite unnatural.
- Dropping the mixed term is questionable. It is justified by stating that A and B encode very different types of contributions to the asymptotics, and so do not talk to each other. However, this is not a rigorous argument.
- An analysis at every order is very technical. To show that there is no $\zeta(2n)$ in the solution is a highly nontrivial task. We expect that there is no $\zeta(2n)$ in the solution as one can write H as the exponential of a polynomial having odd zetas as coefficients.

Mapping our problem to the Borel plane seems to be relevant to solve (at least) some of these issues.

3 The Borel plane

3.1 Borel transform

We will provide the definition of the Borel transform used here and also describe some of its very basic properties. We see the Borel transform as a ring morphism between two rings of the formal series:

$$\mathcal{B} : a\mathbb{C}[[a]] \longrightarrow \mathbb{C}[[\xi]]$$

$$\tilde{f}(a) = a\sum_{n=0}^{+\infty} c_n a^n \longrightarrow \hat{f}(\xi) = \sum_{n=0}^{+\infty} \frac{c_n}{n!}\xi^n. \tag{20}$$

Even if $\tilde{f}$ is a purely formal series (i.e., with a null radius of convergence), $\hat{f}$ might be convergent. In that case, $\tilde{f}$ can be seen as a function. An interesting aspect of the Borel transform is that it has an inverse called the Laplace transform, which gives a function that takes $\tilde{f}$ as its power series whenever $\tilde{f}$ is convergent and is a summation method for an interesting class of divergent series, which are A and B in our case.

A caveat has to be done here, i.e., in general, this resummation has to be done in sectors of the complex plane bounded by the singularities of the Borel transform. This is called sectorial resummation. When we go from one sector to the other (i.e., cross a line between the origin and a singularity of the Borel transform), the result of the summation changes. This is known as the Stokes phenomenon and it is actively studied in the field of dynamical systems.

An important property of the Borel transform is that the Borel transform of a pointwise product is the convolution product of the Borel transforms:

$$\mathcal{B}(\tilde{f}\tilde{g})(\xi) = \hat{f} \star \hat{g}(\xi) = \int_0^\xi \hat{f}(\xi-\eta)\hat{g}(\eta)\mathrm{d}\eta. \tag{21}$$

The last line is defined if and only if $\hat{f}$ and $\hat{g}$ are two functions. In the following, ξ will be the Borel transform of the fine structure constant a. When working with a, we will say that we are in the physical plane, which is contrary of the Borel plane (where we are when working with ξ).

3.2 Dyson–Schwinger equation in the Borel plane

First, we want to check that the procedure for dealing with the Dyson–Schwinger equation in the physical plane described in the first section is coherent with the approach for the Borel plane. Hence, let us inductively define a sequence

$$A_{n+1} = (\alpha n - \beta)A_n \tag{22}$$

and the corresponding formal series

$$A = \sum A_n a^n. \tag{23}$$

Now, we can compute the Borel transform of this formal series. It is easy to check that

$$\widehat{aA} \underset{\xi\to 1/\alpha}{\sim} (1-\alpha\xi)^{\beta/\alpha} \quad \text{for } \frac{\beta}{\alpha} \notin \mathbb{N},$$
$$\underset{\xi\to 1/\alpha}{\sim} (1-\alpha\xi)^{\beta/\alpha}\ln(1-\alpha\xi) \quad \text{for } \frac{\beta}{\alpha} \in \mathbb{N}.$$

Here, we have written the Borel transform of aA rather than that of A as it is simpler and also because there is no term of the form $a^0 A$ in our ansatz (19) for γ. We proceed similarly for B. Hence, we see that the A and B formal series translate into the Borel plane as singularities of $\hat{\gamma}$ at $\xi = \pm 1/3$. More precisely

$$\hat{\gamma}(\xi) \underset{\xi\to -1/3}{\sim} \left(1+\frac{\xi}{3}\right)^{-5/3} \quad \text{for } \frac{\beta}{\alpha} \notin \mathbb{N},$$
$$\underset{\xi\to 1/3}{\sim} \ln\left(1-\frac{\xi}{3}\right) \quad \text{for } \frac{\beta}{\alpha} \in \mathbb{N}.$$

Now, we have to verify that ignoring the mixed terms like AB or A^2 as in the physical plane is applicable in the Borel plane. Hence, let us take two functions similar to those used in the computation in the physical plane.

$$f(a) = a^n + a^m A,$$
$$g(a) = a^p + a^q A.$$

For correspondingly ignoring the mixed terms similar to that in the physical plane, the following replacement can be performed

$$f(a)g(a) \longrightarrow a^{n+p} + \left[a^{m+p} + a^{q+n}\right] A. \tag{24}$$

It was emphasized that this was a key point in the procedure for the physical plane; however, we could not properly justify it. Hence, it is crucial to check that this procedure is coherent in the Borel plane. Indeed, it can be seen that

$$\hat{f} \star \hat{g}(\xi) \underset{\xi\to 0}{\sim} \mathcal{B}(a^{q+n}A) \\ \underset{\xi\to 1/\alpha}{\sim} \mathcal{B}(\left[a^{m+p} + a^{q+n}\right] A). \tag{25}$$

The first point is trivial since the Borel transform is well-defined for ordinary functions. For the second case, one has distinguish between $\beta/\alpha \notin \mathbb{N}$ and $\beta/\alpha \in \mathbb{N}$. During the computation, it is realized that (for both cases) the singular contribution will be the expected one. The main difficultly is the combinatorial factors obtained during the computation. The fact that they are the same for the two members of the equation is important; if they would not have been the same, then the computation in the physical plane would have to be corrected.

Hence, the procedure in Ref. [5] is strictly equivalent to solving the Dyson–Schwinger equation in the Borel plane in the vicinity of $\xi = \pm 1/3$. Now, the question of what makes those singularities special arises. To answer this question, we will look for other singularities of $\hat{\gamma}$.

3.3 Higher singularities

From now on, we will be even more brief than before. Detailed computations have been presented in [4]. Let us assume that $\hat{\gamma}$ has an algebraic singularity of order α in $\xi = \xi_0$. This means that $\hat{\gamma}$ diverges at $\xi = \xi_0$ and that this divergence is canceled by any polynomial (with non-integer power) of degree that is strictly greater than α. More rigorously, α is the smallest real number such that

$$\forall \varepsilon > 0, \quad \left|(\xi - \xi_0)^{\alpha+\varepsilon}\hat{\gamma}(\xi)\right| \underset{\xi\to\xi_0}{\longrightarrow} 0.$$

We will write $\mathrm{sing}_{\xi_0}(\hat{\gamma}) = \alpha$. The virtue of this definition is that we do not need to worry about logarithms any more. However, the drawback is that if we prove a statement such as "$\hat{\gamma}$ has a singularity of order α in $\xi = \xi_0$", then in $\xi = \xi_0$ we could have $\hat{\gamma}(\xi) \sim (\xi - \xi_0)^\alpha$ or $\hat{\gamma}(\xi) \sim (\xi - \xi_0)^\alpha \ln|\xi - \xi_0|$, or any other function diverging in $\xi = \xi_0$ more slowly than any other polynomial. In fact, it is quite easy to see from the above analysis than logarithms will only occur when α is strictly a negative integer.

The statement that $\hat{\gamma}$ has only algebraic singularities is quite strong. However, it is justified by the physical plane analysis; the asymptotic behavior of γ was encoded into the singularities of its Borel transform at $\xi = \pm 1/3$, which are algebraic singularities. A singularity more serious than any algebraic one (e.g. an exponential singularity) would be expected to contribute to the asymptotic behavior of γ.

The procedure to find the localization of the singularities of $\hat{\gamma}$ is first to assume that it has an algebraic singularity in $\xi = \xi_0$ in the Borel transform of the renormalization group equation. This explains how this singularity is transferred to $\hat{G}$, the Borel transform of the two-point function. We then substitute this result into the Dyson–Schwinger equation, which can be written in the form

$$\hat{\gamma}(\xi) = \mathcal{F}[\hat{G}](\xi), \tag{26}$$

where $\mathcal{F}$ a functional, which it too lengthy to be explained in detail here. However, an important remark is that the one-loop Mellin transform appears in this functional. Then, we check what conditions are needed to be satisfied to have the same kind of singularities in the two members of the equation. The answer is the following

$$\mathrm{sing}_{\xi_0}\left(\mathcal{F}[\hat{G}]\right) = \alpha \Longleftrightarrow \xi_0 = \frac{k}{3}, \quad k \in \mathbb{Z}^*. \tag{27}$$

Actually, with this procedure we just show the implication of the above statement; however, the converse implication is trivial with $\mathcal{F}[\hat{G}](\xi)$.

This shows that the aim of this manuscript is justified, i.e., $\xi = \pm 1/3$ are the two first singularities (nearest to the origin).

Now, much more can be done with roughly the same procedure detailed above. First, we can find the orders of the singularities of $\hat{\gamma}$ by looking at the orders which would be coherent with the Dyson–Schwinger equation. The results can be summarized by the statement that the only singularities of $\hat{\gamma}$ are for $\xi \in \mathbb{Z}^*/3$. They have the following orders

$$\beta_k = -\frac{2}{3}(k-1) \quad \text{for } \xi = -k/3, k \geq 2, \tag{28a}$$

$$\alpha_k = \frac{2}{3}(k-1) \quad \text{for } \xi = +k/3, k \geq 1, \tag{28b}$$

$$\beta_1 = -5/3 \quad \text{for } \xi = -1/3. \tag{28c}$$

Finally, the RGE suggests the correct expansion for an intermediate function and this expansion will allow us to prove that the transcendental content of $\hat{\gamma}$ near its singularities is from odd Riemann zetas only. Moreover, a bound has been set in Ref. [4] on their weight for the two first singularities of $\hat{\gamma}$. This bound is saturated by the results in Ref. [5].

4 Conclusion

We have explained how the Borel plane approach for the Dyson–Schwinger equation of the massless Wess–Zumino model sheds light on a procedure which was otherwise not entirely satisfactory. Furthermore, we have also found all the singularities of $\hat{\gamma}$. This suggests that much more is possible within the Borel plane approach, which is found to be true.

In Ref. [4], we found the relations between the leading coefficients of every singularity. A link between the combinatorics of multiple zeta values and the asymptotic behavior of $\hat{\gamma}$ (away from the real line and for positive real values of ξ) was also found, which has been presented in Ref. [3].

However, some interesting results related to the number in the expansion of $\hat{\gamma}$ around its first singularity could also be found. For example, it is very easy to show in this framework that no $\zeta(2n)$ will occur. A bound on the weight of the $\zeta(2n+1)$ have also been set. These are important results and we are now seeking for similar results for other singularities of $\hat{\gamma}$. Nevertheless, a new complication occurs for higher singularities; the convolution product becomes harder to define since it can whirl around a singularity. Devices have been constructed by Jean Écalle to deal with such technicalities, e.g., the Alien Calculus.

Finally, we would like to emphasize that the Borel plane approach for the Dyson–Schwinger equation allows us to extract a lot of information about its solution. However, it does not allow us to do everything. In particular, numerical analysis is much more involved than that in the physical plane due to the convolution integrals that are very sensitive to numerical instabilities. Importantly, it is very hard to compute the rational factors of the zetas arising in the expansion of the anomalous dimension around its singularities. Such numbers are not very interesting for a mathematician but are crucial to the physicist who wants to compare the results to an experiment. Thus, in that sense, the Borel plane and physical plane approaches are complementary and both deserve to be investigated further.

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