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REVIEW ARTICLE

Photon condensation: A new paradigm for Bose–Einstein condensation

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Bose–Einstein condensation is a state of matter known to be responsible for peculiar properties exhibited by superfluid Helium-4 and superconductors. Bose–Einstein condensate (BEC) in its pure form is realizable with alkali atoms under ultra-cold temperatures. In this paper, we review the experimental scheme that demonstrates the atomic Bose–Einstein condensate. We also elaborate on the theoretical framework for atomic Bose–Einstein condensation, which includes statistical mechanics and the Gross–Pitaevskii equation. As an extension, we discuss Bose–Einstein condensation of photons realized in a fluorescent dye filled optical microcavity. We analyze this phenomenon based on the generalized Planck's law in statistical mechanics. Further, a comparison is made between photon condensate and laser. We describe how photon condensate may be a possible alternative for lasers since it does not require an energy consuming population inversion process.

Keywords Bose–Einstein condensation, photon condensation, magneto-optical trap, Gross–Pitaevskii equation, Planck's radiation law

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1 Introduction

Particles at the elementary level are classified into bosons and fermions. Bosons are particles that obey Bose–Einstein statistics and fermions are particles that follow Fermi-Dirac statistics. These statistics are named after the scientists who proposed them. While Bose and Einstein are the architects of Bose–Einstein statistics, Fermi and Dirac worked out Fermi–Dirac statistics. These statistics are known as quantum statistics since the particles involved are microscopic and they obey the rules of quantum mechanics. These statistics explain the way in which particles are distributed among various energy levels and provide an expression for the average particle number in a state. In statistical mechanics, it is possible to evaluate macroscopic physical quantities from the distribution of elementary particles among the energy levels of a system [1]. In this way, statistical mechanics assist in framing the microscopic theory for macroscopic entities that are governed by the laws of thermodynamics.

While analyzing Bose statistics, Einstein predicted the possibility of Bose–Einstein condensation, wherein atoms accumulate to the ground state devoid of interparticle interaction at ultra-low temperatures above absolute zero [2, 3]. This peculiar state was identified by London as the cause for superfluidity [4]. However, the theory of superfluidity is rather complicated considering the interaction between normal fluid and superfluid [5]. Researchers eventually focused their interest on the study of this state in its pure form. This was made possible by advancements in laser cooling and trapping techniques [6, 7]. Contrary to popular belief that lasers could only be used for demanding applications such as cutting and welding of metals, they were also proved to be useful in cooling a gas of atoms [8]. This is based on the fact that the temperature of a gas depends on the velocity of its atoms. Lasers can be used to cool these atoms by reducing their velocity considerably and thereby reducing the temperature of the gas. Moreover, once the gas is sufficiently cooled and if it has a net magnetic moment, the atoms can be brought to a particular point in space by confining them in a magnetic trap. These ideas helped to achieve the Bose–Einstein condensate (BEC) state in alkali atoms [9, 10].

According to quantum mechanics, elementary particles have an intrinsic angular momentum associated with them called spin [11, 12]. The spin of particles is characterized by their spin quantum number. For elementary particles such as electrons, protons, and neutrons, the spin is $\frac{1}{2}$. In general, fermions have half-integer spin, and bosons have integer spin. Composite particles such as atoms can either be bosons or fermions, depending on their total spin [13]. Accordingly, typical atomic species used for Bose–Einstein condensation, namely, Rb, Na, and Li, are all bosons. In the case of atomic condensates, when the velocity of particles is sufficiently small, the wave nature of the particles becomes conspicuous according to the de Broglie relation. When a condensate is formed, the de Broglie wavelength of adjacent atoms overlaps, and this state can be represented by a single wave function. The progress made in laser cooling and trapping techniques resulted in the successful generation of the BEC state with alkali atoms. This accomplishment has been recognized with two Nobel prizes, one in 1997, to Chu, Cohen–Tannoudji, and Phillips for the laser cooling and trapping techniques, and another in 2001 to Cornell, Ketterle, and Wieman for achieving the Bose–Einstein condensate with alkali atoms. The guiding factor for the selection of these alkali atoms for Bose–Einstein condensation is the atoms' non-zero magnetic moment, which helps in confining them in a magnetic trap. Moreover, the availability of laser frequencies that match with the transitions in the alkali atoms makes them suitable for laser cooling [14].

The remainder of this paper is organized as follows. In Section 2, we illustrate the stepwise approaches commonly employed to generate the Bose–Einstein condensate state for alkali atoms. Next, in Section 3, the theoretical framework used to model the atomic Bose-Einstein condensate is illustrated by a statistical approach and the Gross–Pitaevskii equation. In Section 4, we discuss the physics behind the theory of photon condensation. The experimental scheme that successfully demonstrates the photon condensate is illustrated in Section 5. A comparison is made between photon condensate and lasers in Section 6. The paper is concluded in Section 7.

2 Experimental techniques for BEC

After the prediction of Bose–Einstein condensation by Einstein in 1925, over 70 years elapsed before it was experimentally realized in 1995. This was made possible with advancements in laser cooling and trapping techniques, which culminated in the successful realization of Bose–Einstein condensation with alkali atoms. In this section, we review experimental techniques on laser cooling, magnetic trapping, and evaporative cooling.

2.1 Laser cooling

The first step in the formation of the BEC is the cooling of atoms using a laser, which is known as the laser cooling technique [15]. Once the atoms are sufficiently cooled by reducing their velocity, they are trapped by using a magnetic trap [16]. These two techniques are integrated into a single apparatus known as the magneto-optical trap, which is nowadays used for alkali atoms [17]. The temperature is further reduced by a technique called evaporative cooling to achieve the BEC state [18]. In the laser cooling technique, the radiation pressure of photons from the laser source reduces the velocity of atoms, and this results in cooling of the atoms. In a typical experiment, atoms from an oven in a gaseous state, which is generally referred to as a dilute atomic gas, are cooled by a laser beam. For effective cooling, three pairs of counter propagating laser beams are used in a practical system along the three Cartesian coordinate directions [19].

2.2 Magnetic trapping

The magnetic trap consists of a quadrupole magnetic field produced by a pair of Helmholtz coils carrying current in opposite directions as shown in Fig. 1(a). The current flowing through the coils produces a magnetic field. At the intersection point of these magnetic fields in between the coils, there exists a local minimum for the field [20]. Owing to the presence of a net magnetic moment, these alkali atoms are trapped at this zero point of the field. Since the magnetic trap is a shallow one, atoms with low kinetic energy or those that are sufficiently cooled alone are trapped by this method. The other atoms with high kinetic energy are allowed to leave the trap. Therefore, it is necessary to cool the atoms before they can be trapped. In this way, laser cooling

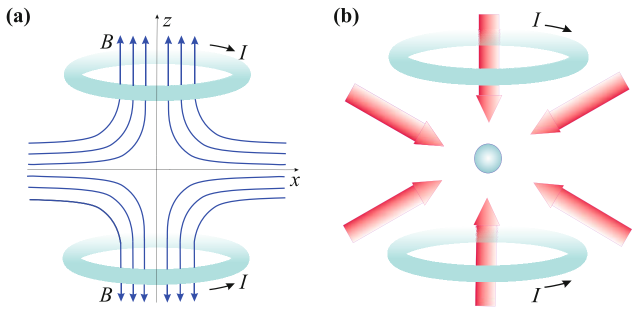

Fig. 1 **(a)** A magnetic quadrupole trap with two Helmholtz coils carrying current in opposite directions for trapping alkali atoms. **(b)** A magneto-optical trap consisting of three pairs of laser beams along Cartesian coordinate directions with two Helmholtz coils carrying current in opposite directions.

and magnetic trapping together facilitate the trapping of atoms in a magneto-optical trap. Magnetic trapping helps in increasing the density of atoms, which is essential for observing the collective behavior of atoms during the BEC phase transition [21]. The schematic diagram in Fig. 1(b) illustrates the magneto-optical trap commonly used for alkali atoms.

2.3 Evaporative cooling

The final step towards realizing the BEC state is the evaporative cooling technique [22]. In this technique, the atoms in a magnetic trap are allowed to cool further into the nano kelvin range, a transition critical temperature required for the formation of the BEC state. This is achieved by reducing the intensity of the magnetic field, which, in turn, lowers the depth of the magnetic trap. As a result, fast moving atoms are allowed to leave the trap. Simultaneously, the temperature of the remaining atoms drops further to reach the critical value. This is analogous to cooling a cup of tea or coffee by evaporation, wherein the high-energy water vapor molecules evaporate, which further reduces the temperature of the remaining liquid. In a similar way, the evaporating cooling technique helps in reaching the BEC phase transition but at the cost of atoms that are lost from the trap.

3 Theoretical framework for BEC

The concept of Bose–Einstein condensation dates back to 1925, when Einstein predicted the existence of such a state for atoms under ultra-cold temperatures above absolute zero [13]. This theory was framed using Bose statistics pioneered by Bose in 1924. Einstein's theory considered atoms in free space where there is no interaction between atoms. However, the experimentally demonstrated Bose-Einstein condensate involves binary interaction between atoms in a magnetic trap. The theory for this interacting condensate is modeled after an effective mean-field theory known as the Gross–Pitaevskii equation [23]. In fact, the Gross–Pitaevskii equation was developed in 1960s for quasi-particles in liquid helium, but it was more suited as a model after the discovery of BEC with alkali atoms in 1995 [24]. In this section, we briefly provide an overview of these theoretical models.

3.1 Physics of Bose–Einstein condensation

Bose invented the Bose statistics, which describes the distribution of bosons in various quantum states of a system, and the same is of the form:

$$f_{\mathrm{BE}} = \frac{1}{\exp\left(\frac{E-\mu}{k_{\mathrm{B}}T}\right) - 1}, \tag{1}$$

which represents an expression for the average number of particles in a quantum state. Here, E is energy, μ is chemical potential, k_{B} is Boltzmann constant, and T is temperature. Bose used this distribution function to derive Planck's blackbody radiation law. Einstein further expanded this scheme for describing the Bose-Einstein condensation of atoms, wherein he considered an ideal gas of N non-interacting bosonic atoms of mass m at temperature T in a volume V of a system [13]. In the system, the number density of atoms is conserved and is given by

$$\frac{N}{V} = \int_0^\infty f_{\mathrm{BE}}\, g(E)\, \mathrm{d}E, \tag{2}$$

where $g(E)$ is the density of states per unit volume. Here, the density of states is given by

$$g(E) = 2\pi \left(\frac{2m}{h^2}\right)^{3/2} E^{1/2}\mathrm{d}E. \tag{3}$$

Substituting Eq. (3) in Eq. (2), gives

$$\frac{N}{V} = 2\pi \left(\frac{2m}{h^2}\right)^{3/2} \int_0^\infty \frac{E^{1/2}}{\exp\left(\frac{E-\mu}{k_B T}\right) - 1}\, \mathrm{d}E. \tag{4}$$

When we analyze Eq. (4), it turns out that the value of the integrand is undefined at $E = 0$. Therefore, treating the zero energy state differently from other states, Eq. (4) is rewritten as

$$\frac{N}{V} = N_0(T) + 2\pi \left(\frac{2m}{h^2}\right)^{3/2} \int_0^\infty \frac{E^{1/2}}{\exp\left(\frac{E-\mu}{k_{\mathrm{B}}T}\right) - 1}\, \mathrm{d}E, \tag{5}$$

where $N_0(T)$ is the number density of atoms in the zero energy state. At a sufficiently low temperature called critical temperature T_c, it becomes impossible to hold all atoms in the higher lying states with $E > 0$. The number density of atoms at this critical temperature is given by

$$\frac{N}{V} = 2\pi \left(\frac{2m}{h^2}\right)^{3/2} \int_0^\infty \frac{E^{1/2}}{\exp\left(\frac{E}{k_B T_c}\right) - 1}\, \mathrm{d}E. \tag{6}$$

Here, we assume the value of the chemical potential to be zero ($\mu = 0$) for all states except the one with $E = 0$. Upon substituting the appropriate expression for T_c from Eq. (6), we can deduce an expression for the number density of atoms in the ground state from Eq. (5) as

$$N_0(T) = \frac{N}{V}\left[1 - \left(\frac{T}{T_c}\right)^{3/2}\right]. \tag{7}$$

Thus, at the critical temperature T_c, a phase transition occurs, wherein a substantial fraction of atoms with zero velocity condense to the ground state. This phenomenon is known as the Bose–Einstein condensation of atoms.

3.2 Gross–Pitaevskii equation

Unlike the ideal Bose gas considered by Einstein, the alkali atoms in a magnetic trap do interact with each other. For a dilute gas of alkali atoms that weakly interact, two body interactions are prominent. However, three body and other many body interactions can be neglected to a great extent. In accordance with the practice in scattering theory, these two body interactions are expressed in terms of an interaction potential, $V(r_1 - r_2)$, which is given by

$$V(r_1 - r_2) = g\ \delta(r_1 - r_2), \tag{8}$$

where $\delta(r_1 - r_2)$ is the Dirac delta-function, and g is the coupling constant related to s-wave scattering length a_s through the relation:

$$g = \frac{4\pi a_s \hbar^2}{m}. \tag{9}$$

For repulsive interactions between atoms in the condensate, scattering length a_s is positive, and for attractive interactions, its value is negative. Moreover, the scattering length can be shifted from positive to negative by varying the external magnetic field near the Feshbach resonances [25, 26]. The effect of these two body interactions can be further simplified by a mean-field potential, which is the potential felt by an atom in the trap due to the interaction with other neighboring atoms [27]. The effective potential V_{eff} is given by

$$V_{eff} = g\ n(r,t), \tag{10}$$

where $n(r,t)$ is the condensate density. The condensate density is related to the macroscopic wave function of the condensate $\psi(r,t)$, and is given by

$$n(r,t) = |\psi(r,t)|^2. \tag{11}$$

In addition to the effective interaction potential V_{eff}, there is also a trapping potential $V(r)$ that arises from the confinement of atoms in a magnetic trap. Coupling both of these potentials, the dynamics of this dilute gas in the ground state can be modeled using the Gross–Pitaevskii (GP) equation, which is given below:

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(r) + g|\psi|^2\right)\psi. \tag{12}$$

The GP equation is in fact a nonlinear Schrödinger equation with a trapping potential $V(r)$, along with a nonlinear term arising from the effective interaction potential $V_{eff} = g|\psi|^2$.

3.3 Soliton dynamics in a trapped condensate

The nonlinear nature of the interaction between atoms necessitates investigation of the various nonlinear phenomena in BEC including solitons. The condensate atoms in a magnetic trap interact with each other. These interatomic interactions are considerably strong and give rise to a nonlinear expression for the interaction potential. It is quite obvious from the GP equation that the stability of the condensate in a magnetic trap is influenced by dispersion and nonlinear interaction between atoms. Dispersion gives rise to spreading of atoms, whereas the attractive nonlinear interactions pave the way for collapse of the condensate. None of these are advantageous for the integrity of the condensate. This dilemma can be solved by balancing these counteracting processes for the formation of a soliton, wherein the dispersion is balanced by the nonlinear attractive interaction to form a stable pulse.

Solitons are a class of localized wave packets that appear as solutions in nonlinear dispersive systems. The peculiarity of these solutions is that they preserve their shape and can be propagated undistorted over a range. Solitons have been investigated in various physical systems such as hydrodynamics, fiber optics, and plasmas [28, 29]. In the case of the Bose-Einstein condensate, bright soliton, which is characterized by a peak in density of the condensate is of particular interest. A bright soliton is formed when there is a balance between nonlinear attractive interaction and dispersion due to the spreading of atoms in the trap. Mathematically, bright solitons are represented by a secant hyperbolic profile and appear as solutions for the GP equation. For a quasi-one

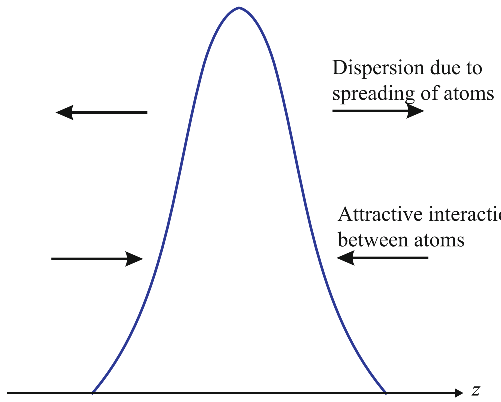

Fig. 2 A sketch of bright soliton formed in a magnetic trap due to the balance between attractive interaction and dispersion due to the spreading of atoms.

dimensional GP equation, the bright soliton solution is given by

$$\psi(z,t) = \eta \mathrm{sech}[\eta(z - vt)] \exp[\mathrm{i}(kz - \omega t)], \tag{13}$$

where η is amplitude, k is wavenumber, ω is frequency, and v is velocity of the soliton [30]. Since the bright soliton helps in preserving the shape of the condensate from spreading out, it is important as far as its applications are concerned. Figure 2 shows a schematic representation of a bright soliton along with the factors influencing its formation.

4 Physics of photon condensation

It is well proved that photons are basically wave packets that represent the energy content of radiation. A clear road map for understanding the particle nature of electromagnetic radiation was laid out by Einstein when he attempted to explain the photoelectric effect, which eventually earned him the Nobel Prize in 1921. In fact, the concept of quantization of energy was brought out by Planck while trying to deduce an expression for the blackbody radiation spectrum, which is referred to as Planck's radiation law [31]. Planck was awarded the Nobel Prize in 1918 for introducing the concept of quanta. Planck is regarded as the father of quantum mechanics, which was later considered as the ultimate theory in physics. Planck's law for blackbody radiation was deduced from a statistical view point by Bose using the statistics that he had formulated. Einstein recognized the importance of Bose's work, and further worked on it to propose the concept of the Bose–Einstein condensation for atoms.

It may be noted that the concept of Bose–Einstein condensation for atoms was put forward by Einstein in 1925. Even though photons come under the category of bosons, experimental conditions under which the photon number could be conserved was not known at that time. This was because of the fact that the well-known phenomenon of blackbody radiation does not provide any room for photon number conservation. This was partially due to the admittance that most of the materials are opto-electronically inactive. These materials are unable to store electronic energy internally. However, there is a class of materials known to be opto-electronically active that can store electronic energy internally. The formulation, which takes into account the case of opto-electronically active materials, is known as the generalized Planck's law for radiation. With this, photon condensation turned out to be feasible in a cavity.

Planck's law provides an expression for spectral energy distribution of radiation from a blackbody. Inside a blackbody, the radiation is in thermal equilibrium with its cavity walls. Hence, the radiation coming out from the cavity corresponds to the spectral energy distribution inside the cavity. In the case of a blackbody that does not support the storage of electronic energy internally, the chemical potential is taken as zero. This parameter denotes the ability of a body to store electronic energy internally. It is defined as the change in internal energy with the addition of a single electronic excitation, which may be brought out by the absorption of a single photon. In the case of an opto-electronically active material, chemical potential can have a non-zero value [32]. With the addition of chemical potential in the Planck's formula, it becomes the generalized Planck's law, and is given by

$$U_\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{\exp\left(\frac{h\nu - \mu}{k_{\mathrm{B}}T}\right) - 1}, \tag{14}$$

where U_ν is the spectral energy density of radiation inside the cavity, ν is frequency, μ is chemical potential, and $k_{\mathrm{B}}T$ is the thermal energy.

In the case of blackbody radiation, the emitted radiation is in thermal equilibrium with the cavity walls. In the case considered here, the radiation coming out of the cavity is in thermal equilibrium with the excited state of opto-electronically active material. To be precise, once an opto-electronically active material is excited by radiation, two quasi-equilibrium states are formed in it. One is the ground state and the other is the excited state. It is also possible to establish a quasi-equilibrium between radiation inside the cavity and the excited state of the material. The latter quasi-equilibrium is instrumental for photon condensation. With an increase in light intensity, it becomes feasible to accumulate the photons for the condensate formation inside the cavity.

5 Photon condensation: Experiment

Photon condensation is realized in an optical microcavity with a cavity length in the micrometer range [33]. This cavity is filled with a fluorescent dye solution, namely, rhodamine 6G in methanol. Second harmonic at 532 nm from an Nd:YAG laser is passed through the cavity that could induce absorption in the fluorescent dye. The laser output is chopped using an acousto–optic modulator to produce the pulsed output [34]. This helps in reducing the triplet state population [35]. After undergoing Stokes shift, fluorescent emission is obtained at a longer wavelength compared to excitation wavelength of the input laser. The fluorescent dye that gets excited by the pump beam goes to a higher electronic state. There it undergoes internal conversion within the vibrational energy levels of the dye and reorients itself towards equilibrium with the solvent before undergoing fluorescence emission [36]. This happens within a time scale of picoseconds, which is within the fluorescent life time in nanosecond range. As a result, the excited dye is in thermal equilibrium before undergoing fluorescence emission [37]. By increasing the intensity of the input laser, the number density of photons in the cavity is increased. Upon increasing the intensity further, a stage is reached beyond which the phase transition occurs in the cavity to form photon condensate. In this way, the photons in the cavity undergo Bose–Einstein condensation beyond a critical pump power of the laser source. The experimental scheme, which was used for realizing photon condensation, is shown in Fig. 3.

6 Comparison: Laser and photon condensate

Lasers are currently a readily available source of coherent radiation. In a laser, radiation is emitted from the excited state of the lasing medium that is held in a population inversion condition. For achieving the population inversion, it is necessary to excite the bulk of the atoms from their ground state to the excited states. Since the laser alters the thermal equilibrium nature of the system due to the population inversion condition, the laser is a typical example of a system that is not in thermal equilibrium [38]. Moreover, the radiation coming from a laser does not correspond to a thermal equilibrium state of a system such as from a blackbody source. In contrast, in the case of photon condensation, the radiation emulates the thermal equilibrium nature of radiation with the excited state of the opto-electronically active material. Further, with an increase in the photon number density of photons inside the cavity, beyond a critical photon number, a photon condensate is formed inside the cavity. The radiation coming out of the cavity at this stage corresponds to this thermal equilibrium state of radiation inside the cavity, similar to that of blackbody radiation.

Moreover, in a laser, longitudinal modes are formed in a cavity such that they satisfy the standing wave condition [39]. However, in the photon condensation experiment, the condensate is found to exist even when the cavity length varies. This does not happen in a laser cavity, where a different cavity length other than that decided by the standing wave condition is not supported. This is again proof for differentiating between a photon condensate and laser. It is a known fact that the conventional atomic Bose–Einstein condensate is formed due to the overlapping of de Broglie matter waves of adjacent atoms at ultra-low temperatures. On the other hand, the photon condensate is the condensate of electromagnetic waves, which are in coherence with each other similar to a laser, though due to a thermalization mechanism mediated by the excited state of the fluorescent dye inside the cavity.

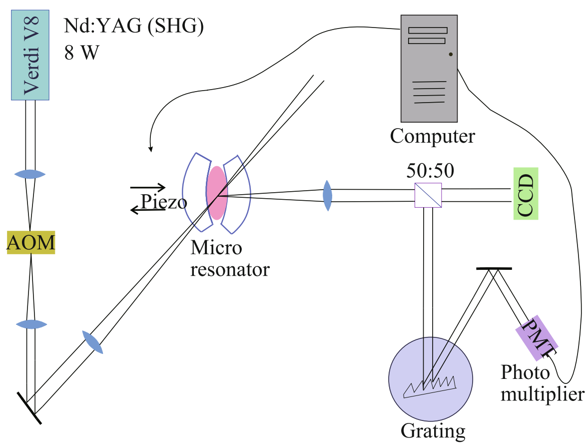

Fig. 3 Experimental scheme used for photon condensation in a fluorescent dye filled optical microcavity.

7 Conclusion

In this paper, we have reviewed the theoretical models and experimental techniques that culminated in the successful realization of the Bose–Einstein condensate for alkali atoms. In the case of atoms, the conditions required for realizing the Bose–Einstein condensate include reducing the temperature down into the nano Kelvin range and increasing the number density of atoms in the trap. This is achieved by the magneto-optical trap for alkali atoms, where laser beams are used to reduce the velocity of atoms and a magnetic trap is used to increase the number density of atoms. As the final step for reaching the critical temperature required for the phase transition, the evaporating cooling technique is employed wherein fast moving atoms are allowed to leave the trap, further reducing the temperature of the remaining atoms in the trap. From a theoretical viewpoint, the statistical mechanics of atomic Bose–Einstein condensation is illustrated on the lines as depicted by Bose and Einstein. Moreover, the mean-field theoretical model called the Gross–Pitaevskii equation is described along with the solitonic solution possible for it, which was realized in the atomic Bose–Einstein condensate. The solitonic propagation is found to make the condensate robust against any external interference, which is a useful technique as far as the applications are concerned.

Photon condensation is a new state of photon discovered recently in an optical microcavity. It was earlier thought to be impossible. However, the breakthrough was brought out by an opto-electronically active material such as fluorescent dye in a microcavity. Similar to blackbody radiation from a cavity, the radiation from a fluorescent dye filled cavity corresponds to the thermal equilibrium state of the radiation inside the cavity that is established between fluorescence emission and the excited state of the dye. By increasing the photon number density inside the cavity, photon condensation is realized inside the cavity above a critical photon number. The theoretical explanation for this phenomenon is supported by the generalized Planck's law, wherein the chemical potential is explicitly included in the Planck's law for radiation. Further, we have compared photon condensation with that of a laser. It turns out that the radiation from a laser source does not correspond to the thermal equilibrium state of the system. However, in the case of photon condensation, the radiation resembles the thermal equilibrium state of radiation inside the cavity. Moreover, photon condensation is not susceptible to variation in cavity length, in contrast to a laser, which requires the cavity length to be an integer multiple of half wavelengths. Thus, photon condensate could be a possible alternative for laser, since there is no requirement for an energy consuming population inversion process.

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