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Research article

Immirzi parameter and quasinormal modes in four and higher spacetime dimensions

Xiang-Dong Zhang[1,2]

[1] *Department of Physics, South China University of Technology, Guangzhou 510641, China*
[2] *Institute for Quantum Gravity, University of Erlangen-Nürnberg, Staudtstraße 7 / B2, 91058 Erlangen, Germany*
E-mail: scxdzhang@scut.edu.cn

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There is a one-parameter quantization ambiguity in loop quantum gravity, which is called the Immirzi parameter. In this paper, we fix this free parameter by considering the quasinormal mode spectrum of black holes in four and higher spacetime dimensions. As a consequence, our result is consistent with the Bekenstein–Hawking entropy of a black hole. Moreover, we also give a possible quantum gravity explanation of the universal $\ln 3$ behavior of the quasinormal mode spectrum.

Keywords Immirzi parameter, quasinormal mode, loop quantum gravity

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1 Introduction

Loop quantum gravity (LQG) is a quantum gravity theory that aims to quantize general relativity (GR) using nonperturbative techniques [1–4]. During past decades, many aspects of LQG have been investigated, particularly those pertaining to loop quantum cosmology, black hole entropy, and spin foam models [5–7]. One of the most interesting features of LQG is the existence of a discrete spectrum of the geometric operators, such as the area and volume operators. For instance, the area spectrum is given by [1, 2]

$$A = 8\pi\gamma\ell_p^2\sqrt{j(j+1)}, \tag{1.1}$$

where j denotes the representation of the $SU(2)$ gauge group and thus takes half-integer values, and γ is an adjustable real number called the Immirzi parameter. Rovelli and Thiemann [8] showed that different choices of the value of γ lead to unitarily inequivalent quantum theories. Thus, it becomes important for the LQG to fix this free parameter γ.

One way to fix the Immirzi parameter γ is to calculate the black hole entropy in the framework of LQG. To recover the famous Bekenstein–Hawking entropy formula

$$S_{\rm BH} = \frac{A}{4\ell_p^2}, \tag{1.2}$$

where $\ell_p = (G\hbar)^{\frac{1}{2}}$ is the Planck length, we can fix the value of γ. However, to date, the value of γ has yet to be uniquely fixed, as different counting methods give different values [7]. Thus, it is desirable to determine whether there is another way to fix its value. Fortunately, Dreyer proposed a novel way of fixing γ using the asymptotic behavior of the quasinormal modes (QNMs) of a four-dimensional Schwarzschild black hole [9]. His idea is as follows. He first notes that the highly damped real part of the QNM frequencies, ω_{QNM}, asymptotically approaches a fixed value

$$\frac{\omega_{\rm QNM}}{T} = \ln 3, \tag{1.3}$$

where $T = \frac{1}{8\pi M}$ denotes the Hawking temperature of a four-dimensional Schwarzschild black hole. This remarkable universal asymptotic behavior of a four-dimensional black hole was first conjectured by Hod [10] and confirmed analytically by Motl [11]. By assuming Bohr's correspondence principle [10], such a universal asymptotic frequency should be associated with one of the quanta induced by the minimum change in the quantized area of the black hole's horizon. Combining this observation with the area spectrum given by Eq. (1.1), Dreyer successfully fixed the value of γ as [9]

$$\gamma = \frac{\ln 3}{2\sqrt{2}\pi}. \tag{1.4}$$

However, this also forces

$$j_{\min} = 1. \tag{1.5}$$

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Because LQG in four dimensions is based on the gauge group $SU(2)$, it is thus more natural to require $j_{\min} = \frac{1}{2}$, which is the minimum half-integer allowed by the $SU(2)$ group representation. To remove this conflict, Dreyer suggests that we should adapt $SO(3)$ as our gauge group rather than $SU(2)$ [9]. There are also other explanations; for example, Corichi argued that one can keep the $SU(2)$ gauge group and still obtain consistency with the QNM [12]. Further, Ling and Zhang proposed that the supersymmetric extension of LQG is preferred by the QNM [13].

To further develop this idea, an interesting question is whether we can generalize Dreyer's original proposal to higher dimensions. This is of course a very difficult problem, because it is well known that the analytical expression of higher-dimensional QNMs is very hard to obtain. However, it is remarkable that the asymptotic behavior of the real part of the QNM of the higher-dimensional Schwarzschild black hole was already obtained analytically by Motl [14] as follows:

$$\frac{\omega_{\rm QNM}}{T} = \ln 3, \tag{1.6}$$

where T is the Hawking temperature of a Schwarzschild black hole in $d+1$ dimensions. This result has also been confirmed numerically for four and five dimensions [15]. This universal asymptotic behavior of the QNM strongly hints that there might be some origin of quantum gravity that has yet to be discovered.

On the other hand, LQG was recently generalized to arbitrary spacetime dimensions by Thiemann *et al.* in a series of papers [16–19] because LQG is based on the connection dynamics formalism of GR. The main idea of Ref. [16] is that for $d+1$-dimensional GR, to obtain a well-defined connection dynamics, one should adopt $SO(d+1)$ connections A_a^{IJ} with $I, J = 1, 2, 3, \ldots, d$ rather than the speculated $SO(d)$ connections. With this higher-dimensional connection dynamics in hand, Thiemann et al. successfully generalized LQG to arbitrary spacetime dimensions. Similar to four-dimensional $SU(2)$ LQG, higher-dimensional LQG also has a discrete area spectrum, which is given by [18, 20]

$$A = 8\pi\gamma\ell_p^{d-1}\sqrt{I(I+d-1)}, \tag{1.7}$$

where I is an integer, and $\ell_p = (G\hbar)^{\frac{1}{d-1}}$ denotes the Planck length in $d+1$ dimensions. The physical meaning of I is that for every edge, we associate a simple representation of $SO(d+1)$, which is labeled by its corresponding highest weight $\Lambda = (I, 0, 0, \ldots)$, where $I = 1, 2, 3, \ldots$ is an integer. This paper thus aims to fix the Immirzi parameter γ appearing in this generalized LQG framework with spacetime dimensions $d \geqslant 3$. Moreover, we also want to give a possible quantum gravity explanation of the universal asymptotic $\ln 3$ behavior found in four- and higher-dimensional QNMs [11, 14, 15].

This paper is organized as follows. After a short introduction, we first fix the Immirzi parameter in four dimensions in Section 2. Then we generalize this result to higher dimensions in Section 3 and explain why the universal asymptotic $\ln 3$ behavior will emerge because of the underlying higher-dimensional LQG. Conclusions are given in the last section.

2 Fixing the Immirzi parameter in four dimensions

Let us first examine the four-dimensional case. The four-dimensional Schwarzschild solution is given as

$$\mathrm{d}s^2 = -(1-\frac{2M}{r})\mathrm{d}t^2 + (1-\frac{2M}{r})^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2, \tag{2.1}$$

where $\mathrm{d}\Omega^2$ is the two-dimensional sphere metric, and M is the mass of the black hole. Using $\omega_{\rm QNM}$, we now can fix the Immirzi parameter. Bohr's correspondence principle states that an oscillatory frequency of a classical system and a transition frequency of the corresponding quantum system should be equal [10]. Thus, the appearance or disappearance of a puncture denoted by the simple representation of $SO(4)$ with integer $I_{\min}$ produces a transition of the quantum black hole. The area of the black hole then changes as follows:

$$\Delta A = A(I_{\min}) = 8\pi\gamma\ell_p^2\sqrt{I_{\min}(I_{\min}+2)}. \tag{2.2}$$

Note that the change in the black hole's mass ΔM is related to its quasinormal frequency $\omega_{\rm QNM}$ by setting

$$\Delta M = \hbar\omega_{\rm QNM} = \frac{\hbar\ln 3}{8\pi M}. \tag{2.3}$$

The mass M and area A of a four-dimensional Schwarzschild black hole are related as

$$A = 16\pi M^2. \tag{2.4}$$

By using Eq. (2.3), it is easy to see that

$$\Delta A = 32\pi M\Delta M = 4\ln 3\ell_p^2. \tag{2.5}$$

Comparing the above equation with Eq. (2.2) leads to the desired value of the Immirzi parameter

$$\gamma = \frac{\ln 3}{2\pi\sqrt{I_{\min}(I_{\min}+2)}}. \tag{2.6}$$

Now our remaining task is to fix $I_{\min}$.

Note that in four dimensions, we adopt $SO(4)$ as our gauge group, and the dimension of a simple representation labeled $\Lambda = (I_{\min}, 0)$, with Λ being the corresponding highest weight, is $2I_{\min} + 1$. Thus, the entropy of a horizon with N punctures is given by

$$S = N \ln(2I_{\min} + 1). \tag{2.7}$$

Here we simply assumed that the dominant contribution to the black hole entropy comes from all the punctures, which have the minimum value of $I_{\min}$. Therefore, we have

$$N = \frac{A}{A(I_{\min})}, \tag{2.8}$$

because we already argued that $\Delta A = A(I_{\min}) = 4\ln 3\ell_p^2$, and thus we have

$$S = \frac{A \ln(2I_{\min} + 1)}{4\ln 3\ell_p^2}. \tag{2.9}$$

Complete agreement with the Bekenstein–Hawking entropy of the black hole requires that

$$I_{\min} = 1. \tag{2.10}$$

Therefore, we completely fix the Immirzi parameter as

$$\gamma = \frac{\ln 3}{2\pi\sqrt{3}}. \tag{2.11}$$

Note that this result is also consistent with a group theoretical argument, because in the four-dimensional case, we adopt $SO(4)$ rather than $SU(2)$; thus, the lowest possible choice of $I_{\min}$ for the $SO(4)$ group is $I_{\min} = 1$. Hence, the unnatural choice of $j_{\min} = 1$ that arises in the $SU(2)$ case in Ref. [9] is no longer a problem in our situation.

3 Generalization to higher dimensions

In this section, we generalize the result obtained in the last section to higher dimensions. First, we write the $(d+1)$-dimensional Schwarzschild metric

$$\mathrm{d}s^2 = -(1 - \frac{m}{r^{d-2}})\mathrm{d}t^2 + (1 - \frac{m}{r^{d-2}})^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega_{d-1}^2, \tag{3.1}$$

where $\mathrm{d}\Omega_{d-1}^2$ is the $(d-1)$-dimensional sphere metric. The Arnowitt–Deser–Misner (ADM) mass of the $(d+1)$-dimensional Schwarzschild black hole is given as

$$M = \frac{(d-1)2\pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2})}\frac{m}{16\pi G}. \tag{3.2}$$

It is easy to see that $r_+ = m^{\frac{1}{d-2}}$ is the radius of the event horizon. The temperature and area of the $(d+1)$-dimensional Schwarzschild black hole are given by

$$T = \frac{(d-2)\hbar}{4\pi} m^{-\frac{1}{d-2}},$$

$$A_{d-1} = \frac{2\pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2})}(r_+)^{d-1} = \frac{2\pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2})} m^{\frac{d-1}{d-2}}, \tag{3.3}$$

respectively. The Bekenstein–Hawking entropy of the black hole is $S = \frac{A_{d-1}}{4\ell_p^{d-1}}$, where $\ell_p = (G\hbar)^{\frac{1}{d-1}}$ is the Planck length of $(d+1)$-dimensional spacetime. These quantities satisfy the first law of black hole thermodynamics,

$$\mathrm{d}S = \frac{\mathrm{d}M}{\hbar T}. \tag{3.4}$$

As in the last section, we first relate the change in the ADM mass of the black hole ΔM with the quasinormal frequency ω_{QNM} as

$$\Delta M = \hbar\omega_{\mathrm{QNM}} = \hbar T \ln 3. \tag{3.5}$$

The area spectrum of $(d+1)$-dimensional spacetime is given by [18]

$$A = 8\pi\gamma\ell_p^{d-1}\sqrt{I(I+d-1)}. \tag{3.6}$$

Thus, with the appearance or disappearance of a puncture with $I_{\min}$, the change in the area of the black hole is given by

$$\Delta A = 8\pi\gamma\ell_p^{d-1}\sqrt{I_{\min}(I_{\min}+d-1)}. \tag{3.7}$$

Combining the expression for the area of the black hole (3.3) and Eq. (3.5), we find that

$$\Delta A = 4\ell_p^{d-1}\frac{\Delta M}{\hbar T} = 4\ell_p^{d-1}\frac{\omega_{\mathrm{QNM}}}{T} = 4\ln 3\ell_p^{d-1}. \tag{3.8}$$

Comparing Eqs. (3.8) and (3.7) gives us the desired value of the Immirzi parameter:

$$\gamma = \frac{\ln 3}{2\pi\sqrt{I_{\min}(I_{\min}+d-1)}}. \tag{3.9}$$

Now our remaining task is to fix $I_{\min}$.

Note that for the $SO(d+1)$ group, the minimum admissible value of $I_{\min}$ is

$$I_{\min} = 1. \tag{3.10}$$

Hence, the Immirzi parameter in $d+1$ dimensions is completely fixed as follows:

$$\gamma = \frac{\ln 3}{2\pi\sqrt{d}}. \tag{3.11}$$

Note that this result is also consistent with the group theoretic argument, because in the $(d+1)$-dimensional case, we adopt $SO(d+1)$ connection dynamics rather than $SU(2)$ connection dynamics; thus, the lowest admissible $I_{\min}$ of the $SO(d+1)$ group is $I_{\min} = 1$. Hence, the unnatural choice of $j_{\min} = 1$ appearing in the $SU(2)$ case in Ref. [9] does not bother us. Thus, it is consistent to take $I_{\min} = 1$ from the perspectives of both QNMs and LQG.

4 Conclusions

In this paper, by relating the asymptotic behavior of the real part of the QNM with the area spectrum of quantum GR, we successfully fixed the Immirzi parameter in $d+1$-dimensional LQG with gauge group $SO(d+1)$. Interestingly, we found that the Immirzi parameter γ is spacetime dependent. Moreover, we also provided a possible quantum gravity mechanism to explain the mysterious universal $\ln 3$ behavior found in the asymptotic QNM.

In Dreyer's previous work, the QNM argument forces $j_{\min} = 1$, whereas $j_{\min} = \frac{1}{2}$ is preferable for LQG because it uses gauge group $SU(2)$. Thus, researchers need to resort to other possibilities, such as supersymmetry [13] or suppression of the contribution of $j = \frac{1}{2}$ [12] to reconcile this apparent discrepancy. However, because we are now working in the $SO(d+1)$ LQG formalism, it is natural to require $I_{min} = 1$ from the perspectives of both QNMs and LQG. We think this can serve as one of the most attractive features of our result. Moreover, the exact value of the Immirzi parameter γ plays a crucial role in phenomenological analysis of LQG and hence becomes quite relevant to subjects such as higher-dimensional LQG [21]. Note, however, that even in four dimensions, some authors already suggested that the minimum irreducible representation may not be dominant for the Immirzi parameter [1, 7, 12]. Hence, if we consider the contributions from other representations, the exact value of the Immirzi parameter may also be corrected accordingly. We will leave this subtle question for future study.

It is worth noting that there are also many other issues deserving further investigation; for example, generalization of our result to the supersymmetric case will be interesting. Furthermore, here we focus only on Schwarzschild black holes. We expect our scheme to be applicable to more general black holes. We hope these topics will be investigated in the near future.

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