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RESEARCH ARTICLE

Multiple teleportation via partially entangled GHZ state

Pei-Ying Xiong[1], Xu-Tao Yu[1], Hai-Tao Zhan[1], Zai-Chen Zhang[2,†]

[1] *State Key Lab of Millimeter Waves, Southeast University, Nanjing 210096, China*
[2] *National Mobile Communications Research Laboratory, Southeast University, Nanjing 210096, China*
Corresponding author. E-mail: [†] *zczhang@seu.edu.cn*

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Quantum teleportation is important for quantum communication. We propose a protocol that uses a partially entangled Greenberger–Horne–Zeilinger (GHZ) state for single hop teleportation. Quantum teleportation will succeed if the sender makes a Bell state measurement, and the receiver performs the Hadamard gate operation, applies appropriate Pauli operators, introduces an auxiliary particle, and applies the corresponding unitary matrix to recover the transmitted state. We also present a protocol to realize multiple teleportation of partially entangled GHZ state without an auxiliary particle. We show that the success probability of the teleportation is always 0 when the number of teleportations is odd. In order to improve the success probability of a multihop, we introduce the method used in our single hop teleportation, thus proposing a multiple teleportation protocol using auxiliary particles and a unitary matrix. The final success probability is shown to be improved significantly for the method without auxiliary particles for both an odd or even number of teleportations.

Keywords auxiliary particle, partially entangled GHZ state, multiple teleportation protocol

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1 Introduction

Quantum teleportation plays a significant role in quantum communication [1–13]. Quantum teleportation usually takes advantage of quantum entanglement, which means that two (or more) particles are in a quantum-mechanically entangled state such that they have an effect on each other even when separated by a long distance. The first teleportation protocol was proposed by Bennett *et al.* in 1993 [1]. Their work showed that with the aid of an EPR (named after Einstein, Podolsky, and Rosen) pair, an unknown quantum state could be teleported to another particle. This was demonstrated experimentally by Bouwmeester *et al.* in 1997 [2]. Their paper is regarded as the pioneer work in the quantum information field. In 1998, Bouwmeester *et al.* presented experiments using parametric down-conversion to produce entangled photon pairs and to perform the Bell state analysis. This realized quantum state teleportation from one photon to another [3].

In recent years, different research schemes have been proposed in the quantum communication domain [14–21]. Wang *et al.* first realized the multiple degrees of freedom in quantum teleportation in 2015. Their work breaks an academic limitation and can be used as a strong basic unit in future quantum networks [20]. Researchers have also done many important experiments to prove the feasibility of quantum teleportation, including the realization of multi-photon entanglement [22–29]. Zhang *et al.* developed a six-photon interferometer and reported the first experimental demonstration of a two-qubit composite system, which presented an important step towards the teleportation of a complex system [24]. In 2010, Jin *et al.* experimented on the non-local aspects of the original teleportation scheme and confirmed the feasibility of space-based experiments [25]. Their work constitutes an important step towards quantum communication applications on a global scale. Because maximally entangled states are difficult to maintain, researchers have extended the scope from maximally entangled states to partially entangled states [30–36]. Yan *et al.* presented a scheme for probabilistic teleportation via a non-maximally entangled Greenberger–Horne–Zeilinger (GHZ) state [30]. In their protocol, the sender makes a generalized Bell state measurement, the cooperator performs a generalized X-basis measurement, and the receiver introduces an auxiliary particle and performs a collective unitary transformation. Man *et al.* presented a scheme for quantum state sharing with a partially entangled GHZ state as the quantum channel [31]. On the other hand, researchers have proposed some effective methods for protecting the

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maximally entangled states, such as entanglement concentration, purification, and amplification [37–42]. Pan *et al.* proposed many schemes and demonstrated them in experiments [37–38]. Their work has reached significant experimental achievement and provides a vital ingredient for long distance quantum communication. Sheng *et al.* described the quantum teleportation protocol of multiple degrees of freedom of a single photon with the help of a hyperentanglement Bell state analysis in both the spatial modes and the polarization degrees of freedom [41].

Quantum information needs to be teleported along multiple nodes in a quantum communication network. Therefore, multiple teleportation is one of the most important applications in the quantum information processing field [43–46]. Wang *et al.* analyzed the multihop teleportation based on arbitrary Bell pairs [43]. Cai *et al.* proposed a quantum bridging method with partially entangled states in a hop-by-hop transmission [44]. Fortes and Rigolin performed quantum teleportation by presenting three protocols and comparing their efficiencies. They also developed new teleportation protocols using multipartite EPR pairs [46].

In this paper, we discuss quantum teleportation using a partially entangled GHZ state. In Section 1, we introduce the general concept of quantum teleportation and some results regarding quantum communication. In Section 2, a protocol is described that uses partially entangled GHZ state for direct quantum teleportation with the help of an auxiliary particle. In Section 3, we present how to realize a multiple teleportation protocol using partially entangled GHZ state. In Section 4, we propose a multiple teleportation protocol with auxiliary particles to realize a more efficient teleportation. In the last section, the conclusions are given.

2 Interaction with an auxiliary particle

Assume that a sender Alice wishes to teleport an unknown state to a receiver Bob. It can be expressed as

$$|\psi\rangle_1 = \alpha|0\rangle_1 + \beta|1\rangle_1, \tag{1}$$

where α and β are satisfying $\alpha^2+\beta^2=1$. Alice and Bob share a partially entangled GHZ state,

$$|GHZ\rangle_{234} = \frac{1}{\sqrt{1+n^2}}(|000\rangle_{234} + n|111\rangle_{234}), \tag{2}$$

where $0<n<1$. The protocol is presented as shown in Fig. 1. Subscripts label the different particles, particles 1 and 2 belong to Alice and particles 3 and 4 belong to Bob. The state of all particles can be written as the state

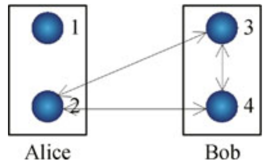

Fig. 1 Alice and Bob share a partially entangled GHZ state as quantum channel.

$$\begin{aligned}|\psi_{total}\rangle &= |\psi\rangle_1 \otimes |GHZ\rangle_{234}\\ &= \frac{1}{\sqrt{1+n^2}}(|000\rangle_{234} + n|111\rangle_{234})\\ &\quad\times(\alpha|0\rangle_1 + \beta|1\rangle_1).\end{aligned} \tag{3}$$

In order to transmit information to Bob, let Alice perform a Bell state measurement on particles 1 and 2. The total state can be rewritten as

$$\begin{aligned}|\psi_{total}\rangle = \frac{1}{\sqrt{2(1+n^2)}}&[|\phi^+\rangle_{12}(\alpha|00\rangle_{34} + n\beta|11\rangle_{34})\\ &+|\phi^-\rangle_{12}(\alpha|00\rangle_{34} - n\beta|11\rangle_{34}) + |\psi^+\rangle_{12}(\alpha|11\rangle_{34}\\ &+n\beta|00\rangle_{34}) + |\psi^-\rangle_{12}(\alpha|11\rangle_{34} - n\beta|00\rangle_{34})],\end{aligned} \tag{4}$$

where

$$|\phi^\pm\rangle_{12} = \frac{1}{\sqrt{2}}(|00\rangle_{12} \pm |11\rangle_{12}), \tag{5}$$

$$|\psi^\pm\rangle_{12} = \frac{1}{\sqrt{2}}(|01\rangle_{12} \pm |10\rangle_{12}). \tag{6}$$

Alice informs Bob of her measurement results via a classical channel. Next, Bob performs the Hadamard gate on particle 3.

$$|0\rangle_3 \to \frac{|0\rangle_3 + |1\rangle_3}{\sqrt{2}}, |1\rangle_3 \to \frac{|0\rangle_3 - |1\rangle_3}{\sqrt{2}}.$$

Then the overall state will be changed to

$$\begin{aligned}|\psi_{total}\rangle = \frac{1}{2\sqrt{1+n^2}}&\{|\phi^+\rangle_{12}[|0\rangle_3(\alpha|0\rangle_4 + n\beta|1\rangle_4)\\ &+|1\rangle_3(\alpha|0\rangle_4 - n\beta|1\rangle_4)] + |\phi^-\rangle_{12}[|0\rangle_3(\alpha|0\rangle_4\\ &-n\beta|1\rangle_4) + |1\rangle_3(\alpha|0\rangle_4 + n\beta|1\rangle_4)]\\ &+|\psi^+\rangle_{12}[|0\rangle_3(n\alpha|1\rangle_4 + \beta|0\rangle_4) - |1\rangle_3(n\alpha|1\rangle_4\\ &-\beta|0\rangle_4)] + |\psi^-\rangle_{12}[|0\rangle_3(n\alpha|1\rangle_4 - \beta|0\rangle_4)\\ &-|1\rangle_3(n\alpha|1\rangle_4 + \beta|0\rangle_4)]\}.\end{aligned} \tag{7}$$

The probability to get $|\phi^\pm\rangle$ or $|\psi^\pm\rangle$ is given by

$$P_{|\phi^+\rangle} = P_{|\phi^-\rangle} = \frac{\alpha^2 + n^2\beta^2}{2(1+n^2)}, \tag{8}$$

$$P_{|\psi^+\rangle} = P_{|\psi^-\rangle} = \frac{n^2\alpha^2 + \beta^2}{2(1+n^2)}. \tag{9}$$

In order to get the original state shown in equation (1), let the Pauli operator act on particle 4 and introduce an auxiliary particle $|0\rangle_5$. Then the state of all particles is given by

$$\begin{aligned}&\{|\phi^+\rangle_{12}[|0\rangle_3(\alpha|0\rangle_4+n\beta|1\rangle_4)+|1\rangle_3\sigma_z(\alpha|0\rangle_4\\&\quad+n\beta|1\rangle_4)]+|\phi^-\rangle_{12}[|0\rangle_3\sigma_z(\alpha|0\rangle_4+n\beta|1\rangle_4)\\&\quad+|1\rangle_3(\alpha|0\rangle_4+n\beta|1\rangle_4)]+|\psi^+\rangle_{12}[|0\rangle_3\sigma_x(n\alpha|0\rangle_4\\&\quad+\beta|1\rangle_4)-|1\rangle_3\sigma_z\sigma_x(n\alpha|0\rangle_4+\beta|1\rangle_4)]\\&\quad+|\psi^-\rangle_{12}[|0\rangle_3\sigma_z\sigma_x(n\alpha|0\rangle_4+\beta|1\rangle_4)-|1\rangle_3\sigma_x(n\alpha|0\rangle_4\\&\quad+\beta|1\rangle_4)]\}\otimes|0\rangle_5. \qquad (10)\end{aligned}$$

When Alice's measurement result is $|\phi^{\pm}\rangle$,

$$|\psi\rangle_{45}=|\psi\rangle_4\otimes|0\rangle_5=(\alpha|0\rangle_4+n\beta|1\rangle_4)\otimes|0\rangle_5. \qquad (11)$$

The unitary matrix that acts on particles 4 and 5 is

$$U=\begin{pmatrix} n & \sqrt{1-n^2} & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\\ \sqrt{1-n^2} & -n & 0 & 0\end{pmatrix}. \qquad (12)$$

After the interaction, the states can be written as

$$U|\psi\rangle_{45}=n(\alpha|0\rangle_4+\beta|1\rangle_4)|0\rangle_5+\sqrt{1-n^2}\alpha|1\rangle_4|1\rangle_5. \qquad (13)$$

Then Bob measures the state of the auxiliary particle 5. If he gets $|0\rangle_5$, the particle 4 collapses to the correct state and the quantum teleportation was successful. If he gets $|1\rangle_5$, the teleportation failed.

In this case, the probability of measuring $|0\rangle_5$ is given by

$$P_{|0\rangle_5}=\frac{n^2}{\alpha^2+n^2\beta^2}. \qquad (14)$$

Then the probability for successfully using the unitary matrix U is given by

$$\begin{aligned}P_U&=P_{|\phi^+\rangle}P_{|0\rangle_5}=P_{|\phi^-\rangle}P_{|0\rangle_5}\\&=\frac{\alpha^2+n^2\beta^2}{2(1+n^2)}\frac{n^2}{\alpha^2+n^2\beta^2}=\frac{n^2}{2(1+n^2)}. \qquad (15)\end{aligned}$$

When Alice's measurement result is $|\psi^{\pm}\rangle$,

$$|\psi\rangle_{45}=|\psi\rangle_4\otimes|0\rangle_5=(n\alpha|0\rangle_4+\beta|1\rangle_4)\otimes|0\rangle_5. \qquad (16)$$

The unitary matrix that acts on particles 4 and 5 is now

$$V=\begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & \sqrt{1-n^2} & n & 0\\ 0 & -n & \sqrt{1-n^2} & 0\end{pmatrix}. \qquad (17)$$

After the interaction, the state can be written as

$$V|\psi\rangle_{45}=n(\alpha|0\rangle_4+\beta|1\rangle_4)|0\rangle_5+\sqrt{1-n^2}\beta|1\rangle_4|1\rangle_5. \qquad (18)$$

Bob measures the state of the auxiliary particle 5. If he gets $|0\rangle_5$, the quantum teleportation was successful. If he gets $|1\rangle_5$, the teleportation failed.

In this case, the probability of measuring $|0\rangle_5$ is given by

$$P_{|0\rangle_5}=\frac{n^2}{n^2\alpha^2+\beta^2}. \qquad (19)$$

Then the probability for successfully using the unitary matrix V is given by

$$\begin{aligned}P_V&=P_{|\psi^+\rangle}P_{|0\rangle_5}=P_{|\psi^-\rangle}P_{|0\rangle_5}\\&=\frac{n^2\alpha^2+\beta^2}{2(1+n^2)}\frac{n^2}{n^2\alpha^2+\beta^2}=\frac{n^2}{2(1+n^2)}. \qquad (20)\end{aligned}$$

Finally, the total probability of success is

$$P=2P_U+2P_V=\frac{2n^2}{1+n^2}. \qquad (21)$$

For convenience, the relationship between the relevant operation on particle 4, Alice's measurement results, and the state of particle 3 is given in Table 1.

Table 1 The relevant operation on particle 4 according to Alice's measurement results and the state of particle 3.

Alice's measurements	The state of particle 3	Operation on particle 4
$\|\phi^+\rangle_{12}$	$\|0\rangle_3$	Do nothing
	$\|1\rangle_3$	σ_z
$\|\phi^-\rangle_{12}$	$\|0\rangle_3$	σ_z
	$\|1\rangle_3$	Do nothing
$\|\psi^+\rangle_{12}$	$\|0\rangle_3$	σ_x
	$\|1\rangle_3$	$\sigma_z\sigma_x$
$\|\psi^-\rangle_{12}$	$\|0\rangle_3$	$\sigma_z\sigma_x$
	$\|1\rangle_3$	σ_x

3 Multiple teleportation

In a quantum communication network, it is very difficult to share entanglement with all the other destination nodes in network. To achieve quantum teleportation between any two nodes, it is necessary to employ a method of teleporting a quantum state in a multihop way from source node to destination node through intermediate nodes. The multiple teleportation is built on a quantum communication network that has multiple nodes in a link/path, and is based on M pairs of partially entangled GHZ particles shared by $M+1$ nodes.

Suppose that we have M pairs of partially entangled GHZ particles that can be described by

$$|GHZ_{n_i}\rangle=\frac{1}{\sqrt{1+{n_i}^2}}(|000\rangle+n_i|111\rangle). \qquad (22)$$

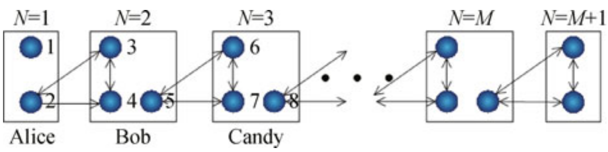

Fig. 2 M pairs of partially entangled GHZ particles shared between $M+1$ nodes. Assume that N is the number of nodes.

Suppose that the first partially entangled GHZ state is shared between the sender Alice and the receiver Bob, and the remaining $M-1$ are shared in the way indicated by Fig. 2. Set $0 < n_i < 1$ and $n_i = n$, where $i = 1, ..., M$. The states of all particles can be written as

$$|\psi\rangle = |\psi\rangle_1 \otimes_{i=1}^{M} |GHZ_{n_i}\rangle = \frac{1}{2\sqrt{1+n^2}} \times\{|\phi^+\rangle_{12}[|0\rangle_3(\alpha|0\rangle_4 + n\beta|1\rangle_4) + |1\rangle_3(\alpha|0\rangle_4 - n\beta|1\rangle_4)] +|\phi^-\rangle_{12}[|0\rangle_3(\alpha|0\rangle_4 - n\beta|1\rangle_4) + |1\rangle_3(\alpha|0\rangle_4 + n\beta|1\rangle_4)] +|\psi^+\rangle_{12}[|0\rangle_3(n\alpha|1\rangle_4 + \beta|0\rangle_4) - |1\rangle_3(n\alpha|1\rangle_4 - \beta|0\rangle_4)] +|\psi^-\rangle_{12}[|0\rangle_3(n\alpha|1\rangle_4 - \beta|0\rangle_4) - |1\rangle_3(n\alpha|1\rangle_4 + \beta|0\rangle_4)]\} \otimes_{i=2}^{M} |GHZ_{n_i}\rangle. \tag{23}$$

After letting the Pauli operator act on particle 4, we obtain

$$|\psi\rangle = |\psi\rangle_1 \otimes_{i=1}^{M} |GHZ_{n_i}\rangle = \frac{1}{2\sqrt{1+n^2}} \times\{|\phi^+\rangle_{12}[|0\rangle_3(\alpha|0\rangle_4 + n\beta|1\rangle_4) + |1\rangle_3\sigma_z(\alpha|0\rangle_4 + n\beta|1\rangle_4)] +|\phi^-\rangle_{12}[|0\rangle_3\sigma_z(\alpha|0\rangle_4 + n\beta|1\rangle_4) + |1\rangle_3(\alpha|0\rangle_4 + n\beta|1\rangle_4)] +|\psi^+\rangle_{12}[|0\rangle_3\sigma_x(n\alpha|0\rangle_4 + \beta|1\rangle_4) -|1\rangle_3\sigma_z\sigma_x(n\alpha|0\rangle_4 + \beta|1\rangle_4)] +|\psi^-\rangle_{12}[|0\rangle_3\sigma_z\sigma_x(n\alpha|0\rangle_4 + \beta|1\rangle_4) -|1\rangle_3\sigma_x(n\alpha|0\rangle_4 + \beta|1\rangle_4)]\} \otimes_{i=2}^{M} |GHZ_{n_i}\rangle. \tag{24}$$

According to Eq. (24), if Alice's measurement result during the first transmission is $|\phi^\pm\rangle$, the state of particle 4 goes to $\alpha|0\rangle + n\beta|1\rangle$. However, if Alice's measurement result is $|\psi^\pm\rangle$, it goes to $n\alpha|0\rangle + \beta|1\rangle$. Hence, after the first teleportation, the probability of successful transmission is $P_{total}^{(1)} = 0$.

Then, the second teleportation is performed by Bob. Now the states that are needed to teleport should be $\alpha|0\rangle_4 + n\beta|1\rangle_4$ and $n\alpha|0\rangle_4 + \beta|1\rangle_4$. For the different possible states of particle 4,

$$(\alpha|0\rangle_4 + n\beta|1\rangle_4) \otimes_{i=2}^{M} |GHZ_{n_i}\rangle = \frac{1}{2\sqrt{1+n^2}} \times\{|\phi^+\rangle_{45}[|0\rangle_6(\alpha|0\rangle_7 + n^2\beta|1\rangle_7) +|1\rangle_6\sigma_z(\alpha|0\rangle_7 + n^2\beta|1\rangle_7)] +|\phi^-\rangle_{45}[|0\rangle_6\sigma_z(\alpha|0\rangle_7 + n^2\beta|1\rangle_7) +|1\rangle_6(\alpha|0\rangle_7 + n^2\beta|1\rangle_7)] +|\psi^+\rangle_{45}[|0\rangle_6\sigma_x n(\alpha|0\rangle_7 + \beta|1\rangle_7) -|1\rangle_6\sigma_z\sigma_x n(\alpha|0\rangle_7 + \beta|1\rangle_7)] +|\psi^-\rangle_{45}[|0\rangle_6\sigma_z\sigma_x n(\alpha|0\rangle_7 + \beta|1\rangle_7) -|1\rangle_6\sigma_x n(\alpha|0\rangle_7 + \beta|1\rangle_7)]\} \otimes_{i=3}^{M} |GHZ_{n_i}\rangle, \tag{25}$$

$$(n\alpha|0\rangle_4 + \beta|1\rangle_4) \otimes_{i=2}^{M} |GHZ_{n_i}\rangle = \frac{1}{2\sqrt{1+n^2}} \times\{|\phi^+\rangle_{45}[|0\rangle_6 n(\alpha|0\rangle_7 + \beta|1\rangle_7) + |1\rangle\sigma_z n(\alpha|0\rangle_7 + \beta|1\rangle_7)] +|\phi^-\rangle_{45}[|0\rangle_6\sigma_z n(\alpha|0\rangle_7 + \beta|1\rangle_7) + |1\rangle_6 n(\alpha|0\rangle_7 + \beta|1\rangle_7)] +|\psi^+\rangle_{45}[|0\rangle_6\sigma_x(n^2\alpha|0\rangle_7 + \beta|1\rangle_7) -|1\rangle_6\sigma_z\sigma_x(n^2\alpha|0\rangle_7 + \beta|1\rangle_7)] +|\psi^-\rangle_{45}[|0\rangle_6\sigma_z\sigma_x(n^2\alpha|0\rangle_7 + \beta|1\rangle_7) -|1\rangle_6\sigma_x(n^2\alpha|0\rangle_7 + \beta|1\rangle_7)]\} \otimes_{i=3}^{M} |GHZ_{n_i}\rangle. \tag{26}$$

To get $\alpha|0\rangle + \beta|1\rangle$ if the first measurement result is $|\phi^\pm\rangle(|\psi^\pm\rangle)$, the second measurement result should be $|\psi^\pm\rangle(|\phi^\pm\rangle)$. The success probability of each measurement is given by Eqs. (8) and (9). Thus, the overall probability of two successful teleportations is given by

$$P_{total}^{(2)} = 8P_{|\psi^+\rangle}P_{|\psi^-\rangle} = 8\frac{\alpha^2 + n^2\beta^2}{2(1+n^2)}\frac{n^2\alpha^2 + \beta^2}{2(1+n^2)} = \frac{2n^2}{(1+n^2)^2}. \tag{27}$$

In this case, the states of unsuccessful teleported particles are $\alpha|0\rangle + n^2\beta|1\rangle$ or $n^2\alpha|0\rangle + \beta|1\rangle$.

Assume that α_i and β_i are the coefficients of Bob's particles before the i-th teleportation. Whenever the measurement result is $|\phi^\pm\rangle$, (α_i, β_i) will change to $(\alpha_i, n\beta_i)$. When the measurement result is $|\psi^\pm\rangle$, (α_i, β_i) will change to $(n\alpha_i, \beta_i)$. Therefore, if we want to realize a successful teleportation, the numbers of $|\phi^\pm\rangle$ and $|\psi^\pm\rangle$ should be equal.

Now assume that $i = 2k-1$ $(k = 1, 2, 3, ...)$. Before the i-th teleportation, unsuccessful teleported particles are described by $n^{i-j-1}\alpha|0\rangle + n^j\beta|1\rangle$, where $j = 0, 1, 2, ..., i-1$ and $(i-j-1 \neq j)$. The system state is

$$(n^{i-j-1}\alpha|0\rangle + n^j\beta|1\rangle) \otimes_{i=2k-1}^{M} |GHZ_{n_i}\rangle = \frac{1}{2\sqrt{1+n^2}} \times\{|\phi^+\rangle[|0\rangle(n^{i-j-1}\alpha|0\rangle + n^{j+1}\beta|1\rangle) +|1\rangle\sigma_z(n^{i-j-1}\alpha|0\rangle + n^{j+1}\beta|1\rangle)] +|\phi^-\rangle[|0\rangle\sigma_z(n^{i-j-1}\alpha|0\rangle + n^{j+1}\beta|1\rangle) +|1\rangle(n^{i-j-1}\alpha|0\rangle + n^{j+1}\beta|1\rangle)] +|\psi^+\rangle[|0\rangle\sigma_x(n^{i-j}\alpha|0\rangle + n^j\beta|1\rangle)$$

$$-|1\rangle\sigma_z\sigma_x(n^{i-j}\alpha|0\rangle+n^j\beta|1\rangle)]$$
$$+|\psi^-\rangle[|0\rangle\sigma_z\sigma_x(n^{i-j}\alpha|0\rangle+n^j\beta|1\rangle)$$
$$-|1\rangle\sigma_x(n^{i-j}\alpha|0\rangle+n^j\beta|1\rangle)]\}$$
$$\otimes_{i=2k}^{M}|GHZ_{n_i}\rangle. \tag{28}$$

We see that teleportation is successful only when $i-j-1=j+1$ or $i-j=j$. Thus, when $i=2k-1$ is odd, the teleportation will be unsuccessful.

Assume that $i=2k$ $(k=1,2,...)$. Before the i-th teleportation, the unsuccessful teleported particles are described by $n^{i-j-1}\alpha|0\rangle+n^j\beta|1\rangle$, $j=0,1,2,...,i-1$. The system state is

$$(n^{i-j-1}\alpha|0\rangle+n^j\beta|1\rangle)\otimes_{i=2k}^{M}|GHZ_{n_i}\rangle=\frac{1}{2\sqrt{1+n^2}}$$
$$\times\{|\phi^+\rangle[|0\rangle(n^{i-j-1}\alpha|0\rangle+n^{j+1}\beta|1\rangle)$$
$$+|1\rangle\sigma_z(n^{i-j-1}\alpha|0\rangle+n^{j+1}\beta|1\rangle)]$$
$$+|\phi^-\rangle[|0\rangle\sigma_z(n^{i-j-1}\alpha|0\rangle+n^{j+1}\beta|1\rangle)$$
$$+|1\rangle(n^{i-j-1}\alpha|0\rangle+n^{j+1}\beta|1\rangle)]$$
$$+|\psi^+\rangle[|0\rangle\sigma_x(n^{i-j}\alpha|0\rangle+n^j\beta|1\rangle)$$
$$-|1\rangle\sigma_z\sigma_x(n^{i-j}\alpha|0\rangle+n^j\beta|1\rangle)]$$
$$+|\psi^-\rangle[|0\rangle\sigma_z\sigma_x(n^{i-j}\alpha|0\rangle+n^j\beta|1\rangle)$$
$$-|1\rangle\sigma_x(n^{i-j}\alpha|0\rangle+n^j\beta|1\rangle)]\}$$
$$\otimes_{i=2k+1}^{M}|GHZ_{n_i}\rangle. \tag{29}$$

We see that the unsuccessful teleported particles are described by $n^{k-1}(\alpha|0\rangle+n\beta|1\rangle)$ or $n^{k-1}(n\alpha|0\rangle+\beta|1\rangle)$ before the i-th teleportation only when $i-j-1=j+1=k$ or $i-j=j=k$. To get the state $\alpha|0\rangle+\beta|1\rangle$ after the i-th teleportation, the number of $|\phi^\pm\rangle$ and $|\psi^\pm\rangle$ should both be $i/2$.

When the numbers of $|\phi^\pm\rangle$ and $|\psi^\pm\rangle$ are equal, we assume that the number of all possible successful sequences of the Bell state measurements is $X(i)$.

$$X(i)=2^i\begin{pmatrix} i \\ i/2 \end{pmatrix}. \tag{30}$$

When i is odd, the success probability after the i-th teleportation is given by

$$P_{total1}^{(i)}=0. \tag{31}$$

When i is even, the success probability after the i-th teleportation is given by

$$P_{total2}^{(i)}=X(i)(P_{|\phi^\pm\rangle})^{i/2}(P_{|\psi^\pm\rangle})^{i/2}$$
$$=2^i\begin{pmatrix} i \\ i/2 \end{pmatrix}(\frac{\alpha^2+n^2\beta^2}{2(1+n^2)})^{i/2}(\frac{n^2\alpha^2+\beta^2}{2(1+n^2)})^{i/2}$$
$$=\begin{pmatrix} i \\ i/2 \end{pmatrix}\frac{n^i}{(1+n^2)^i}. \tag{32}$$

This means that the success probability is

$$P_{total}^{(i)}=0, \qquad i \text{ is odd},$$
$$or \;=\begin{pmatrix} i \\ i/2 \end{pmatrix}\frac{n^i}{(1+n^2)^i}, \qquad i \text{ is even}. \tag{33}$$

4 Multiple teleportation with auxiliary particles

Assume that i is odd. After the i-th teleportation, the unsuccessful teleported particles are described by $\alpha|0\rangle+n^{i-2j}\beta|1\rangle$ or $n^{i-2j}\alpha|0\rangle+\beta|1\rangle$, where $j=0,1,2,...,(i-1)/2$. We complement each case with auxiliary particles.

For the state $\alpha|0\rangle+n^{i-2j}\beta|1\rangle$, the auxiliary particle $|0\rangle_{aux}$ is added to the unsuccessful teleported particles. Then we perform the unitary operation given by Eq. (12) with n replaced by n^{i-2j},

$$U=\begin{pmatrix} n^{i-2j} & \sqrt{1-(n^{i-2j})^2} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ \sqrt{1-(n^{i-2j})^2} & -n^{i-2j} & 0 & 0 \end{pmatrix}, \tag{34}$$

$$U\{(\alpha|0\rangle+n^{i-2j}\beta|1\rangle)\otimes|0\rangle\}$$
$$=n^{i-2j}(\alpha|0\rangle+\beta|1\rangle)|0\rangle_{aux}+\sqrt{1-(n^{i-2j})^2}\alpha|1\rangle|1\rangle_{aux}. \tag{35}$$

In this case, the probability to get $|0\rangle_{aux}$ is given by

$$P_{aux1}=\frac{n^{2(i-2j)}}{\alpha^2+n^{2(i-2j)}\beta^2}. \tag{36}$$

For the state $n^{i-2j}\alpha|0\rangle+\beta|1\rangle$, the auxiliary particle $|0\rangle_{aux}$ is added to the unsuccessful teleported particles. Then we perform the unitary operation given by Eq. (17) with n replaced by n^{i-2j},

$$V=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \sqrt{1-(n^{i-2j})^2} & n^{i-2j} & 0 \\ 0 & -n^{i-2j} & \sqrt{1-(n^{i-2j})^2} & 0 \end{pmatrix}, \tag{37}$$

$$V\{(n^{i-2j}\alpha|0\rangle+\beta|1\rangle)\otimes|0\rangle\}$$
$$=n^{i-2j}(\alpha|0\rangle+\beta|1\rangle)|0\rangle_{aux}+\sqrt{1-(n^{i-2j})^2}\beta|1\rangle|1\rangle_{aux}. \tag{38}$$

In this case, the probability to get $|0\rangle_{aux}$ is given by

$$P_{aux2} = \frac{n^{2(i-2j)}}{n^{2(i-2j)}\alpha^2 + \beta^2}. \tag{39}$$

The success probability of these cases implemented via auxiliary particles is given by

$$\begin{aligned} P^{(i)}_{total1'} = & \sum_{j=0}^{(i-1)/2} \begin{pmatrix} i \\ j \end{pmatrix} 2^i (P_{|\phi^\pm\rangle}P_{|\psi^\pm\rangle})^j (P_{|\phi^\pm\rangle})^{i-2j} P_{aux1} \\ & + \sum_{j=0}^{(i-1)/2} \begin{pmatrix} i \\ j \end{pmatrix} 2^i (P_{|\phi^\pm\rangle}P_{|\psi^\pm\rangle})^j (P_{|\psi^\pm\rangle})^{i-2j} P_{aux2} \\ = & \sum_{j=0}^{(i-1)/2} \begin{pmatrix} i \\ j \end{pmatrix} \frac{2n^{2(i-j)}}{(1+n^2)^i}. \end{aligned} \tag{40}$$

Assume that i is even. After the i-th teleportation, the unsuccessful teleported particles are described by $\alpha|0\rangle + n^{2j}\beta|1\rangle$ or $n^{2j}\alpha|0\rangle + \beta|1\rangle$, where $j = 1,2,...,i/2$. We correct every one of these cases with auxiliary particles.

For the state $\alpha|0\rangle + n^{2j}\beta|1\rangle$, the auxiliary particle $|0\rangle_{aux}$ is added to the unsuccessful teleported particles. Then we perform the unitary operation given by Eq. (12) with n replaced by n^{2j},

$$U = \begin{pmatrix} n^{2j} & \sqrt{1-(n^{2j})^2} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ \sqrt{1-(n^{2j})^2} & -n^{2j} & 0 & 0 \end{pmatrix}, \tag{41}$$

$$\begin{aligned} & U\{(\alpha|0\rangle + n^{2j}\beta|1\rangle) \otimes |0\rangle\} \\ & = n^{2j}(\alpha|0\rangle + \beta|1\rangle)|0\rangle_{aux} + \sqrt{1-(n^{2j})^2}\alpha|1\rangle|1\rangle_{aux}. \end{aligned} \tag{42}$$

In this case, the probability to get $|0\rangle_{aux}$ is given by

$$P_{aux1} = \frac{n^{4j}}{\alpha^2 + n^{4j}\beta^2}. \tag{43}$$

For the state $n^{2j}\alpha|0\rangle + \beta|1\rangle$, the auxiliary particle $|0\rangle_{aux}$ is added to the unsuccessful teleported particles. Then we perform the unitary operation given by Eq. (17) with n replaced by n^{2j},

$$V = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \sqrt{1-(n^{2j})^2} & n^{2j} & 0 \\ 0 & -n^{2j} & \sqrt{1-(n^{2j})^2} & 0 \end{pmatrix}, \tag{44}$$

$$\begin{aligned} & V\{(n^{2j}\alpha|0\rangle + \beta|1\rangle) \otimes |0\rangle\} \\ & \quad = n^{2j}(\alpha|0\rangle + \beta|1\rangle)|0\rangle_{aux} + \sqrt{1-(n^{2j})^2}\beta|1\rangle|1\rangle_{aux}. \end{aligned} \tag{45}$$

In this case, the probability to get $|0\rangle_{aux}$ is given by

$$P_{aux2} = \frac{n^{4j}}{n^{4j}\alpha^2 + \beta^2}. \tag{46}$$

The success probability of these cases implemented via auxiliary particles is given by

$$\begin{aligned} P^{(i)}_{total2'} = & \sum_{j=1}^{i/2} \begin{pmatrix} i \\ \frac{i}{2}-j \end{pmatrix} 2^i (P_{|\phi^\pm\rangle}P_{|\psi^\pm\rangle})^{\frac{i}{2}-j} (P_{|\phi^\pm\rangle})^{2j} P_{aux1} \\ & + \sum_{j=1}^{i/2} \begin{pmatrix} i \\ \frac{i}{2}-j \end{pmatrix} 2^i (P_{|\phi^\pm\rangle}P_{|\psi^\pm\rangle})^{\frac{i}{2}-j} (P_{|\psi^\pm\rangle})^{2j} P_{aux2} \\ = & \sum_{j=1}^{i/2} \begin{pmatrix} i \\ \frac{i}{2}-j \end{pmatrix} \frac{2n^{i+2j}}{(1+n^2)^i}. \end{aligned} \tag{47}$$

Thus, the total probability of success with auxiliary particles is

$$\begin{aligned} P^{(i)}_{total'} & = P^{(i)}_{total} + P^{(i)}_{total1'} + P^{(i)}_{total2'} \\ & = \sum_{j=0}^{(i-1)/2} \begin{pmatrix} i \\ j \end{pmatrix} \frac{2n^{2(i-j)}}{(1+n^2)^i}, \quad i \text{ is odd}, \\ or & = \begin{pmatrix} i \\ i/2 \end{pmatrix} \frac{n^i}{(1+n^2)^i} \\ & \quad + \sum_{j=1}^{i/2} \begin{pmatrix} i \\ \frac{i}{2}-j \end{pmatrix} \frac{2n^{i+2j}}{(1+n^2)^i}, \quad i \text{ is even}, \end{aligned} \tag{48}$$

where $P^{(i)}_{total}$, $P^{(i)}_{total1'}$, $P^{(i)}_{total2'}$, and $P^{(i)}_{total'}$ are the direct, auxiliary-particle-corrected (i odd), auxiliary-particle-corrected (i even), and overall auxiliary-particle-corrected success probabilities of multiple teleportation, respectively.

This multiple teleportation protocol with auxiliary particles is more efficient than that without auxiliary particles. When i is odd, the probability of the multiple protocol introduced in Section 3 is always 0. The final success probability of the improved multiple teleportation protocol introduced in Section 4 is thus increased significantly. When i is even, the success probability of the improved multiple teleportation protocol using auxiliary particles is also larger than that without auxiliary particles.

We present the overall probability of these two protocols for $i = 5$ and $i = 10$ in Figs. 3(a) and (b), respectively. We see that we can always obtain better results for the improved protocol in these cases. For $i = 5$ and $n = 0.8$, $P^{(5)}_{total} = 0$ and $P^{(5)}_{total'} \approx 0.6$. The success probability of the improved multiple teleportation protocol is increased significantly. For $i = 10$ and $n = 0.8$, $P^{(10)}_{total} \approx 0.2$ and $P^{(10)}_{total'} \approx 0.5$. The success probability of

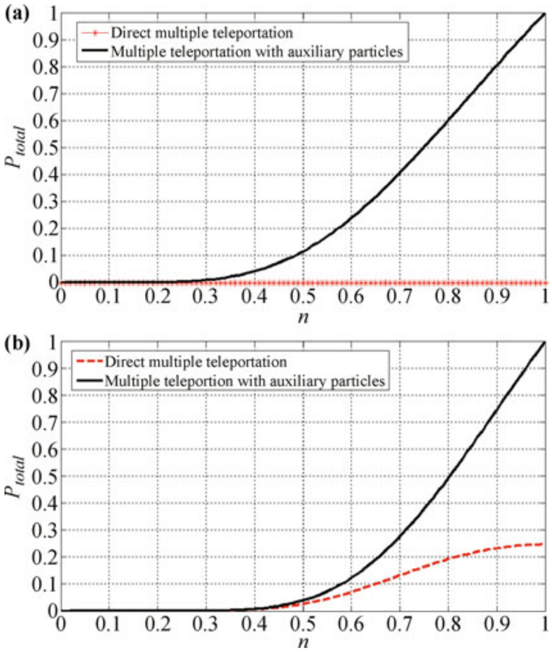

Fig. 3 **(a)** Total success probability of teleportation (i=5). **(b)** Total success probability of teleportation (i=10).

the improved multiple teleportation protocol is 2.5 times larger than the probability of direct multiple teleportation.

5 Conclusion

This paper proposes a single hop teleportation protocol using a partially entangled GHZ state. The sender and receiver share a partially entangled GHZ state. The sender makes a Bell state measurement and the receiver performs the Hadamard gate. Based on the results of Bell state measurement and the state of one of the receiver particles, the receiver then applies the appropriate Pauli operators. Next, the receiver introduces an auxiliary particle and performs the corresponding unitary matrix to recover the transmitted state. We also present a protocol to realize multiple teleportation, where particles are repeatedly teleported along partially entangled GHZ state. The success probability of the teleportation is deduced. The results show that when the number of teleportations is odd, the probability of success is always 0. In order to improve the efficiency of the multiple teleportation, we introduce the method used in our single hop teleportation, thus proposing a multihop teleportation protocol using auxiliary particles. The final success probability is deduced and improved significantly over the case with no auxiliary particles. Regardless of whether the number of teleportations is odd or even, the multiple teleportation protocol with auxiliary particles is more efficient than that without auxiliary particles.

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