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Research article

Chaotic-periodic transition in a two-sided minority game

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Phase transitions are being used increasingly to probe the collective behaviors of social human systems. In this study, we propose a different way of investigating such transitions in a human system by establishing a two-sided minority game model. A new type of agents who can actively transfer resources are added to our artificial bipartite resource-allocation market. The degree of deviation from equilibria is characterized by the entropy-like quantity of market complexity. Under different threshold values, Q_{th}, two phases are found by calculating the exponents of the associated power spectra. For large values of Q_{th}, the general motion of strategies for the agents is relatively periodic whereas for low values of Q_{th}, the motion becomes chaotic. The transition occurs abruptly at a critical value of Q_{th}. Our simulation results were also tested based on human experiments. The results of this study suggest that a chaotic-periodic transition related to the quantity of market information should exist in most bipartite markets, thereby allowing better control of such a transition and providing a better understanding of the endogenous emergence of business cycles from the perspective of quantum mechanics.

Keywords phase transition, minority game, complex adaptive system, random walk, two-sided market, human experiment, entropy-like quantity, market complexity

PACS numbers 89.65.Gh, 89.75.Da, 89.75.-k

1 Introduction

Minority games, which were proposed by Challet and Zhang [1], have been employed to model human collective behaviors and self-organized cooperation in many areas, such as financial markets and other social problems [2–9]. Due to the unsatisfactory performance of this model when resources are biased, Wang *et al.* proposed an extended model called the market-directed resource allocation game [10, 11] by adding agents with heterogeneous preferences. This diversity allows the market to reach an equilibrium even if the resources are highly biased. However, these types of extended minority games only evolve dynamically in one direction from the initial state to equilibrium [12–15]. This behavior is attributable to the fixed amount of resources. Given that resources can actually be movable (often with transfer costs), we propose a two-sided minority game by adding new agents who possess resources and are able to transfer them only under proper market conditions. Therefore, the resources are variable and they can be controlled by their propensities.

Generally, in a two-sided market [16, 17], economic platforms have two distinct participant groups, the respective members of which consistently play the same roles. The two end-participants, who are regarded as buyers and sellers (i.e., consumers and suppliers), mainly trade and make profits from interactions. Many studies have investigated competition, pricing, and strategies in different two-sided markets, such as software markets and credit card markets [17]. In our two-sided minority game, after simplifying and hypothesizing, the two parties are interrelated and mutually dependent on each other. Thus, regardless of whether the interaction is profitable or not, the choice of one must be determined by that of the other, so the market always moves to a dynamically steady state. Due to the win-lose rule in a minority game, if one group of end-participants (e.g., consumers) can arbitrage on one side, then another group (e.g., suppliers) must be able to arbitrage on the other side simultaneously. Therefore, we show that it is useful to interpret this synchronous phenomenon.

In general, phase transitions are observed in various types of natural systems and studies of transitions in social systems are being performed increasingly [18–20]. Using spectrum analysis, we discovered transitions between two predominant phases in this two-sided minority game system. In fact, the critical transition point is controlled by the entropy-like quantity called the complexity of the market, which is introduced to measure the complex relationships between individuals in different groups. In particular, this may be interpreted better by the macroscopic counterpart of quantum mechanics. Based on the assumption that "everyone is rational," optimal strategy mechanics ensures that the proportions of the two sides always fluctuate near the equilibrium in most cases, thereby resulting in a relatively random series. However, in the case when the critical point is exceeded, the behaviors of agents become more convergent and periodic.

In Section 2, we derive the basic analogical relationship from quantum mechanics to macro-social systems. The aim of introducing the quantum analogy is to help analyze the market states. In Section 3, we explain the two-sided resources allocation game by introducing a new type of agent who can control resources. Using the quantum analogy method, we also calculate the market complexity, which is treated based on thresholds. In addition, we analyze the strategies of agents in our model to investigate the phase transition in the game. In Section 4, we describe a human experiment that we conducted to obtain more powerful evidence of this phase transition. In the final section, we give our conclusions regarding the two-sided resources allocation game.

2 Quantum analogy for minority games

The presence of a large number of interacting individuals is a fundamental feature of complex adaptive systems. In quantum physics, we can describe microscopic multi-electron systems using the many-body Schrödinger equation:

$$H\,\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2,\cdots,\boldsymbol{r}_N)=\left[\sum_i^N -\frac{\hbar^2}{2m}\nabla_i^2+\sum_i^N V(\boldsymbol{r}_i)+\sum_{i<j}U(\boldsymbol{r}_i,\boldsymbol{r}_j)\right]\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2,\cdots,\boldsymbol{r}_N), \tag{1}$$

where $\Psi(\boldsymbol{r})$ is the wave function of the N electrons moving around a nucleus with potential V. However, for the macroscopic system of minority games, we are concerned with how the N individuals make better choices under closed circumstances. Similarly, we can write the "Schrödinger equation" for the macroscopic complex system formally as follows:

$$H\,\Psi(1,2,\cdots,N)=\left[\sum_i^N(T_i+V_i)+\sum_{i<j}U(i,j)\right]\times\Psi(1,2,\cdots,N). \tag{2}$$

In this study, we explore an individual's behavior from a different perspective where the wave function $\Psi=\Psi(1,2,\cdots,N)$ represents a possible ensemble of choices. T_i can be taken as a stochastic function similar to the quantum description of the motions of free electrons, while V_i denotes the external resource ratio in the same manner as the potential field. Naturally, the interaction functions between electrons can be treated as interactions between individuals in the game, such as communication (herding behaviors or contrarian behaviors). Due to the limited alternative choices for each individual, the Hamilton operator can actually be reduced to spins in the Ising Model with a single degree of freedom.

In the market-directed resource allocation game, two "rooms" (Room 1 and Room 2) with biased resources can be selected by agents according to their past win-lose information [10]. Therefore, in this game, it is natural for us to predict the normalized solution forms of the wave function as

$$\Psi(1,2,\cdots,N)=\sqrt{\frac{N_1}{N}}|1\rangle+\sqrt{\frac{N_2}{N}}|2\rangle, \tag{3}$$

where N_1 and N_2 denote the numbers of agents who choose Room 1 and Room 2, respectively, and $N_1+N_2=N$. In addition, $|i\rangle$ $(i=1,2)$ actually represent the orthogonal bases to indicate that the agents who choose Room i finally achieve success. Thus, the reduced density matrix ρ for Ψ can be calculated as

$$\rho=|\Psi\rangle\langle\Psi|=\frac{N_1}{N}|1\rangle\langle 1|+\frac{N_2}{N}|2\rangle\langle 2|. \tag{4}$$

This formula indicates that the probabilities of the two choices are $\frac{N_1}{N}$ and $\frac{N_2}{N}$, which are decided by the external "potential field", or resource ratio. Given that the common resource ratio is 1:1 and that the total number N is even, then $N_1=N_2$ makes the system reach equilibrium, which corresponds to the maximal total payoff. If the resource ratio is biased, then N_1 and N_2 would change gradually until N_1/N_2 reaches the resource ratio. In other words, whether the market state reaches the equilibrium is actually related to the numbers of agents in the two rooms as well as the external resource ratio. To generalize this problem, we assume that the resource ratio is controlled by the agents rather than being determined externally, which we introduce in the following section.

3 Two-sided minority game

We modify the market-directed resource allocation game model by adding new agents as suppliers who can control the amount of resources. Suppliers who carry resources across the two-sided market can choose which commodity to produce. As a result, the resource ratio is determined internally, instead of being fixed externally, which makes the model more similar to real markets. Moreover, there are several main assumptions in this model: i) no external information can interfere with this isolated market; ii) the agents interact with each other under free competition; and iii) the two homogeneous commodities, i.e., C_1 and C_2, are traded by the two types of agents, consumers (N) and suppliers (M). In addition, as usual, two major parameters are employed to construct strategies for the agents. One parameter, P, is known as the length of historical information, which is equivalent to the agent's brain size or memory length. The other parameter, S, is the number of part or all of the strategies in the pool 2^P. In fact, P indicates the degree of cognitive ability [10] and S indicates the degree of the ability for deliberate choice [1, 10].

Figure 1 shows a simple outline of our model. In this platform, the two freely traded commodities C_1 and C_2 are alternatives for each other in their own markets (rooms). Unlike the general minority game market [9, 10, 14] where the resources are predefined and unchanging over time, the new model introduces suppliers, thereby leading to continuous changes in the resources on the two markets. As resource holders, suppliers are required to make choices at the appropriate time when positive information is exhausted. However, consumers make new choices whenever changing the present choice would increase their utility. Each type of agent is controlled by the two parameters mentioned above, S and P, during their decision-making processes.

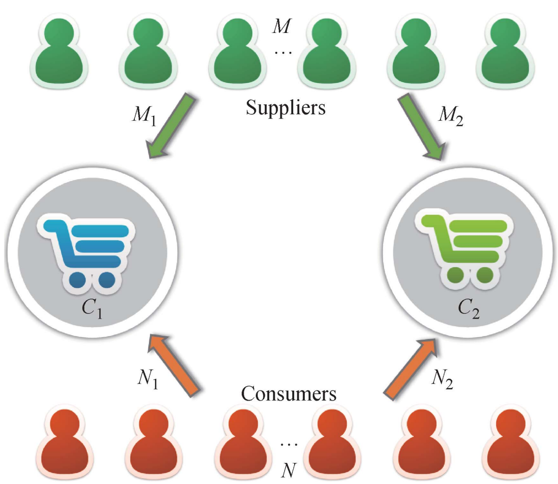

Fig. 1 Simple sketch illustrating the two-sided minority game. Two alternative commodities C_1 and C_2 are freely traded in two rooms (markets). M suppliers and N consumers make choices about the two commodities with $M = M_1 + M_2$ and $N = N_1 + N_2$.

3.1 Win-lose verdict

In this repeated game, during every round, the numbers of suppliers (or consumers) in the two markets, i.e., Room 1 and Room 2, are denoted as M_1 and M_2 (or N_1 and N_2), respectively. Every supplier (consumer) provides (purchases) the same share of commodities, so we can use the simple ratio, M_i/N_i ($i = 1, 2$), to determine which type of agent wins for each commodity. If we assume that at some point, the agents' distribution satisfies $M_1/N_1 > M_2/N_2$, then we can obtain the following results. For consumers, anyone who purchases commodity C_1 wins because there are comparatively more supplies and less demand for commodity C_1 so the "price" of commodity C_1 would be forced down. At the same time, for suppliers, anyone who provides commodity C_2 wins because commodity C_2 can be sold at a relatively higher "price" due to the lower supply and greater demand. This situation is reversed when $M_1/N_1 < M_2/N_2$. Each winner obtains a score of 1 whereas losers receive a score of 0.

3.2 Market complexity

In our model, the two types of individuals can make choices about the two commodities at the same time. The key point is that the consumers and suppliers influence each other. The overall system can be treated as two interrelated subsystems, i.e., N and M.

It should be noted that unlike general quantum entanglement, the macro-states of the two intercoupling subsystems are separable rather than inseparable, which is why quantum mechanics can be treated as analogous to our system. Therefore, our composite system can be represented by multinomial product states. The superposition of product states is identified as a new composite state. By employing physical notations, the Hilbert space of our system can also be described as $H_{NM} = H_N \times H_M$. By considering $|1\rangle$ and $|2\rangle$ as the orthogonal bases of each Hilbert space, then the states of the two subsystems can be written as $|\psi\rangle_M = \sqrt{\frac{M_1}{M}}|1\rangle_M + \sqrt{\frac{M_2}{M}}|2\rangle_M$ and $|\psi\rangle_N = \sqrt{\frac{N_1}{N}}|1\rangle_N + \sqrt{\frac{N_2}{N}}|2\rangle_N$ like Eq. (3). Finally, the general state of the composite system is

$$|\psi\rangle_{NM} = \sum_{i,j} c_{ij}|i\rangle_N \times |j\rangle_M, \tag{5}$$

where c_{ij} is the composite coefficient and $|i\rangle_N$ denotes the basis indicating the winning room state for the N type sub-system. $|i\rangle_N \times |j\rangle_M$ is treated as the composite market state. Given the rule limitation in a minority game, both types cannot actually win in the same room at the same time. Thus, we obtain the state as

$$|\psi\rangle_{NM} = \hat{c_{12}}|1\rangle_N \times |2\rangle_M + \hat{c_{21}}|2\rangle_N \times |1\rangle_M, \tag{6}$$

where the normalized superposition factors, $\hat{c_{12}}$ and $\hat{c_{21}}$, have the following values:

$$\hat{c_{12}} = \sqrt{\frac{N_1M_2}{N_1M_2+N_2M_1}}, \hat{c_{21}} = \sqrt{\frac{N_2M_1}{N_1M_2+N_2M_1}}. \tag{7}$$

Accordingly, the reduced density matrix of $\boldsymbol{\rho}_{NM}$ for any subsystem N (or M) is defined as the partial trace over the other subsystem M (or N), i.e.,

$$\boldsymbol{\rho}_N = \sum_j \langle j|_M(|\psi\rangle_{NM}\langle\psi|_{NM})|j\rangle_M = \mathrm{Tr}_M\boldsymbol{\rho}_{NM}, \tag{8}$$

$$\boldsymbol{\rho}_N = \hat{c_{12}}^2|1\rangle_N\langle 1|_N + \hat{c_{21}}^2|2\rangle_N\langle 2|_N. \tag{9}$$

Since $\hat{c_{12}}^2$ and $\hat{c_{21}}^2$ denote the probabilities of the two complementary composite states, then we can finally obtain an entropy-like quantity to characterize the market complexity, i.e.,

$$Q_N = Q_M = -\mathrm{Tr}(\boldsymbol{\rho}\log_2\boldsymbol{\rho}) = -\sum_{i=1}^{2}\rho_i\log_2\rho_i, \tag{10}$$

where $\boldsymbol{\rho}$ can be any subsystem's reduced density matrix ($\boldsymbol{\rho}_N$ or $\boldsymbol{\rho}_M$) and $\rho_i = \frac{N_i/M_i}{N_1/M_1+N_2/M_2}$. ρ_i can be treated as the deviation rate between the consumer ratio and supplier ratio. We can use Q to quantify the level of market complexity. In our model, the maximum value of Q indicates that the complex system is in a totally mixed and random state, and thus everyone can make choices with a probability of 50%. At this time, the market complexity is extremely high because of the abundance of possible states for the agents. Thus, the arbitrage opportunities become zero. By contrast, the situation with a relatively small value of Q allows some agents to always win because of the non-zero arbitrage opportunities. The possible states then become deficient so the market complexity decreases. Therefore, this quantity of the market complexity plays an important role in regulating the market resource allocation.

From an alternative perspective, Q indicates the gross market information quantity, or the market efficiency level. If the entropy equals 0, the market state is simplex and all of the agents in one group choose the same commodity. If the entropy equals 1, this shows that the market is efficient because the market contains all of the possible information. When the market reaches equilibrium, i.e., $N_1/M_1 = N_2/M_2$, the resources are better distributed and fully consumed by the agents, which maximizes the total payoff. Thus, the market is most steady because of the highest amounts of uncertainty and possible information, while the arbitrage opportunities for the agents are also minimized. Therefore, if the agents want to make profits, they must make new choices so the market will deviate from the equilibrium.

In this study, it is assumed that only suppliers have the ability to know the market complexity level in real-time $[Q(t) = -\sum_{i=1}^{2}\rho_i(t)\log_2\rho_i(t)]$ before making decisions. They can decide to continue providing the current commodity or to transfer their resources to provide the other based on comparisons between the present entropy and their own psychological thresholds. If the distribution of the consumers does not conform to the resource ratio (the distribution of suppliers) in the present round, then chances still remain to obtain excess payoffs. In the balanced state, no arbitrage exists in the market, and thus $Q(t)$ is highest. For simplicity, in this model, we set all of the suppliers with a common psychological warning threshold, Q_{th}. Provided that the complexity level exceeds Q_{th}, the suppliers will use active strategies to change their investment intentions in order to obtain new arbitrage opportunities.

3.3 Strategy analysis

Considering the two types of agents in our model, it is useful for us to treat this multi-player game as a special two-player game. This complex game is transformed into a game between two groups: M-type and N-type agents. We assume that the number of M-type agents (suppliers) is M and the number of N-type agents (consumers) is N. The strategy space for suppliers is $\{(\alpha, 1-\alpha)\}$, where α represents the proportion of agents assigned to provide commodity C_1 (i.e., M_1/M), whereas the strategy space for consumers is $\{(\beta, 1-\beta)\}$, and α or $\beta \in [0, 1]$. In a minority game [1], rational agents will always employ their best strategies to make a choice.

Assuming that the present historical information is $p(t)$ at time t, then we can determine the strategy α and strategy β for the suppliers and consumers, respectively, as

$$\alpha(t) = \frac{1}{N}\sum_i^N c_i(s_i^*(t), p(t)), \tag{11}$$

$$\beta(t) = \frac{1}{M}\sum_j^M c_j(s_j^*(t), p(t)). \tag{12}$$

In these equations, $s_i^*(t)$ denotes the highest scoring strategy for agent i and c_i denotes his choice, i.e., 1 for

Room 1 and 0 for Room 2. If the historical information $p(t)$ is assumed to be a random number from 0 to P, and $M = N$, then it is natural to suggest that the probability of an α-β strategy combination obeys a Gaussian distribution. Thus, it is assumed that the Gaussian-shaped distribution of the strategy combination can be written as $P_{\alpha\beta}\mathrm{d}\alpha\mathrm{d}\beta = k\mathrm{e}^{-(\alpha-m)^2}\mathrm{d}\alpha \cdot \mathrm{e}^{-(\beta-n)^2}\mathrm{d}\beta$. The independent variables α and β are uniformly distributed from 0 to 1, and the mean values are both 0.5 (i.e., $m = n = 0.5$). The probability can be integrated to 1, so the coefficient k is then calculated as 1.175. Finally, we obtain the probability distribution as

$$P_{\alpha\beta} = 1.175\mathrm{e}^{-\left[(\alpha-0.5)^2+(\beta-0.5)^2\right]}. \tag{13}$$

If the strategies α and β are adopted by the two groups together with $\alpha < \beta$, then the suppliers providing commodity C_1 win and the consumers purchasing commodity C_2 win, and the total payoff is $r_{\alpha\beta} = M\alpha + N(1-\beta)$. Assuming that the number of agents is equal in the two groups, i.e., $M = N$, the maximum payoff will be obtained when α is very close to β. If $\alpha > \beta$, the total payoff is $r_{\alpha\beta} = M(1-\alpha) + N\beta$. Thus, the expectation value for the system payoff is

$$r = \sum_{\alpha,\beta} r_{\alpha\beta}P_{\alpha\beta}. \tag{14}$$

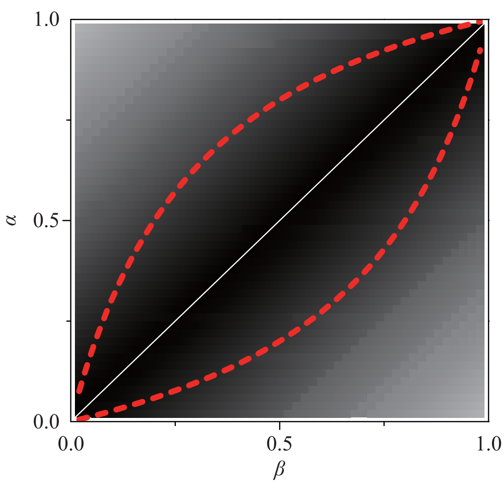

Fig. 2 Payoff map showing the total payoffs for combinations of strategy α by suppliers and strategy β by consumers. α and β represent the proportions of agents who choose commodity C_1 for suppliers and consumers, respectively. The gray level in the background indicates the value of the total payoff $r_{\alpha\beta}$. According to the win-lose rule, the strategy combinations under the $\alpha = \beta$ line (*white solid line*) lead to a result where suppliers of commodity C_1 win, whereas combinations above the line lead to consumers of commodity C_1 winning. Based on the definition of Q_{th}, we calculated the relationship between α and β. The two red dashed lines are drawn for $Q_{th} = 0.72$. After the strategies employed by the suppliers move down toward the "trap" between the two lines, they find a new strategy far from the equilibrium in order to make more excess profits.

Furthermore, Fig. 2 presents a payoff map to show the payoffs for different combinations of α and β. The gray levels in this map indicate various values for the payoffs, where larger values are shown in dark gray and smaller values in light gray. In the case where $\alpha = \beta$, we can see that this is the best strategy for the equilibrium according to Pareto optimality, as well as the best for the Nash equilibrium. Thus, the system will fluctuate dynamically up and down around the equilibrium line. However, as mentioned earlier, the market is most efficient at the equilibrium with almost no chances for arbitrage by individuals. Given the common threshold of market complexity Q_{th}, we can solve the relationship between α and β, which is shown by the red dotted line in Fig. 2.

We have indicated that the real-time market complexity can show the possible chances for arbitrage by suppliers. Figures 3(a) and (c) depict how the α strategy varies with time under two different situations where Q_{th} is lower or higher. With a lower psychological warning threshold Q_{th}, suppliers tend to stay in one market for a longer time until the real-time entanglement $Q(t)$ drops below Q_{th} [see Fig. 3(b)]. The suppliers do not alter their investment intentions while the consumers move continually toward the equilibrium. It is known that once the α strategy reaches the threshold "trap" line, it will change its location to a new place far from the "trap". Thus, the strategy track is drawn as a solid horizontal line. However, when Q_{th} is relatively high, suppliers will reselect

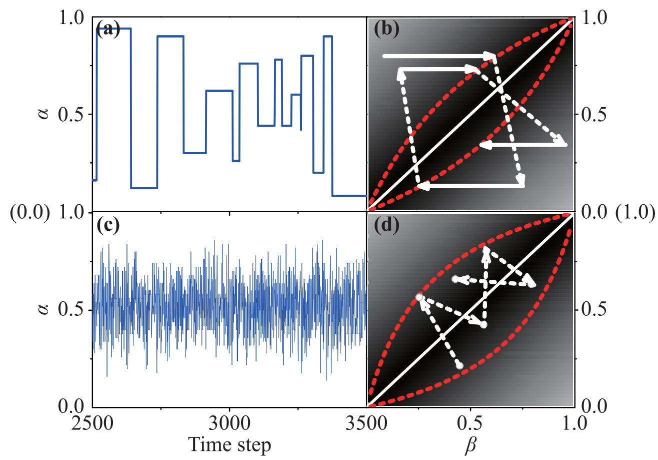

Fig. 3 **(a)** and **(b)** show the α time series and possible track (not exact) for the α strategy in the payoff map for a relatively high entanglement threshold ($Q_{th} = 0.88$); and **(c)** and **(d)** show those for a relatively low threshold ($Q_{th} = 0.72$). For interpretation purposes, only one section of the time series is shown from time step 2500 to step 3500. In (b) and (d), we plot the possible tracks to illustrate the variations in the strategies (*white solid arrows*). If the strategy is located in the "trap", it will jump out and move to a point far from the "trap" (*white dashed arrows*). For the periodic phase in (a), the track of α (*line*) is mainly horizontal. However, for the phase in (c), the track (*dots*) is more chaotic.

commodities continuously because of their dissatisfaction with the chances of arbitrage [see Fig. 3(d)]. The strategy track mainly comprises dots instead of lines because these dots are always located at the threshold "trap". Therefore, we find that for different values of Q_{th}, the tracks of the α strategy have different shapes. Obviously, the threshold of the entropy for market information Q_{th}, which determines the behaviors of agents in this complex adaptive system, requires a new understanding.

Provided that $Q(t)$ exceeds the threshold, suppliers will reselect to move the complexity far from its maximum. It is known that a piece of noise represented by a pseudo-random series, such as fractional Brownian motion or fractional Gaussian noise, has a power spectrum density of $1/f^{\gamma}$, where f denotes the frequency and the exponent γ specifically determines the color of a signal [21, 22]. The value of the exponent γ has a decisive effect on the signal pattern. The series behaves mainly as a signal of fractional Brownian motion if $\gamma > 1$, but as a signal of fractional Gaussian noise if $\gamma < 1$. After calculating the power spectrum exponent in our simulations, we found that γ varied in an upward direction with increasing values of Q_{th} [Fig. 4(a)], where this tendency can be divided into two phases: one phase with a large value of γ is non-stationary (Brownian motion) whereas the other phase is stationary (Gaussian noise) [corresponding to Figs. 3(a) and (c)]. Similarly, calculating the diffusive exponent using the mean squared displacement method [23, 24] also suggests that two basic phases exist: a stationary phase and a non-stationary phase. We refer to the former phase as the periodic phase because the time series exhibits distinct periodicity, and the latter as the chaotic phase because of its stochasticity. Therefore, a phase transition region exists near $\gamma = 1$. In fact, in the periodic phase, agents do not readily fall into the relatively small "trap", thereby leading to a relatively stable periodic motion [Fig. 3(b)]. This may indicate the emergence of business cycles, including crises, from an endogenous perspective [25]. However, if the "trap" is stretched wide, most of the agents in the chaotic phase would never stop changing their choices to pursue profits [Fig. 3(d)], which makes the market become more chaotic [26].

4 Evidence from a human experiment

Given the assumptions of our model, we devised a web game and we invited 38 students from Fudan University to participate in the game. The students received payments based on their overall performance and we also offered attractive rewards to the top three players.

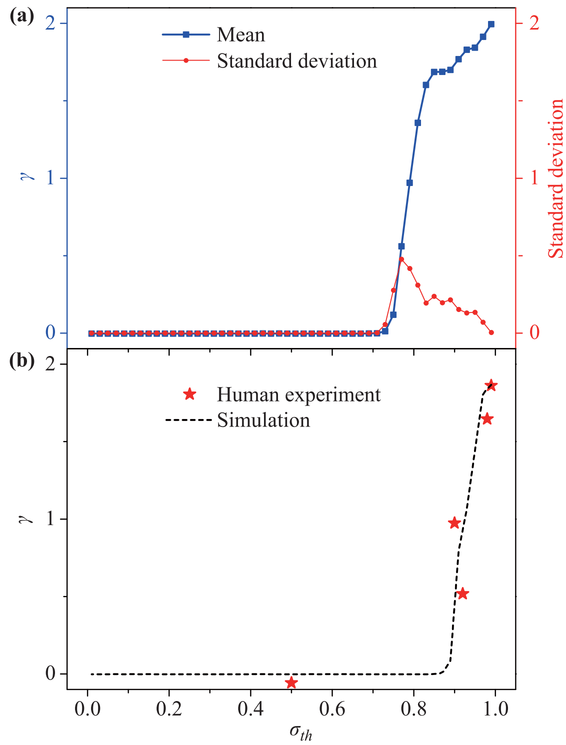

Fig. 4 **(a)** The time series for the α strategy behaves differently with various values of the entanglement threshold Q_{th}. If the threshold is larger than a critical value, the power spectrum exponent γ is close to 2, which means that the series exhibits more Brownian-like motion (periodic phase). If the threshold is smaller than the critical value, γ is near 0, which means that the series becomes more chaotic and similar to Gaussian noise (chaotic phase). **(b)** The five red stars denote the exponent γ obtained from the five groups of human experiments. The dotted line shows one of the possible underlying curves based on the simulations, where the parameters $S = 4$ and $P = 32$.

Five groups of market complexity thresholds ($Q_{th} = 0.5, 0.9, 0.92, 0.98, 0.99$) were tested, where the participants were assigned randomly as suppliers and consumers in each. Before each round, the suppliers were given the ability to know the current market complexity $Q(t)$. Provided that the market complexity exceeded the level of the threshold Q_{th}, the suppliers were allowed to make new choices to make more possible profits. Otherwise, the choices remained the same as their own previous choices. Each group of experiments was conducted over 30 rounds. All of the other designs were the same as those employed in the simulations.

We also calculated the exponents for the M_1/M (strategy α) time series produced by the five values of Q_{th} [Fig. 4(b)]. When the threshold was set close to 1, the suppliers appeared to prefer providing the same resource ratio instead of making frequent changes because the chance of arbitrage still remained. When we calculated the power spectrum exponent, we found that it was close to 2, which indicates totally Brownian motion, as mentioned above. However, the situation was different when the value of Q_{th} was smaller. The exponent decreased to 0 when Q_{th} was less than about 0.9. Compared with the simulation results obtained using $S = 4$ and $P = 32$, we found that the behavior was analogous in our human experiment, with two clear phases: a periodic phase for large values of Q_{th} and a chaotic phase for small values of Q_{th}. The transition region corresponded to the values of Q_{th} where γ increased very rapidly. In this region, the two phases were mixed and they appeared simultaneously.

5 Conclusion

In this study, we established a two-sided minority game model, which involved two alternative commodities and two corresponding trading markets. By adding a new type of agent as suppliers, we obtained variable resource ratios instead of the commonly used unchanged resource ratios [10, 12–14]. We characterized the unbalanced market bias analogically by general quantum mechanics, where the entropy-like quantity of the market complexity indicated the composite states of the two types of agents in the two markets. By investigating the power spectrum properties of the simulation results, we found that there was a transition from a chaotic state (Gaussian noises) to a periodic state (Brownian motion) with respect to the market complexity threshold. Broadly speaking, the latter is related to the endogenous emergence of business cycles [25], which is a topic that remains poorly understood in economics and finance. In addition, to complement our simulations, we conducted human-controlled experiments [27–29] for verification purposes. It would be interesting to perform more in depth studies of this two-sided minority game in future research.

The results of this study suggest that most bipartite markets should exhibit a chaotic to periodic transition, which will be related to their correlation under appropriate conditions. This study provides insights into the better control of these correlations and understanding the endogenous emergence of business cycles in economics and finance from the perspective of quantum mechanics.

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