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Research article

Enhanced phase sensitivity of an SU(1,1) interferometer with displaced squeezed vacuum light

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We study the phase sensitivity of an SU(1,1) interferometer with two input beams in the displaced squeezed vacuum state and the coherent state, respectively. We find that there exists an optimal squeezing fraction of the displaced squeezed vacuum state that optimizes the phase sensitivity. We also examine the effects of some factors, including the loss, mean photon number of the input beams and amplitude gain of the optical parameter amplifiers, on the optimal squeezing fraction so that we can choose the optimal values to enhance the phase sensitivity.

Keywords phase sensitivity, SU(1,1) interferometer, displaced squeezed vacuum state, optimal squeezing fraction

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1 Introduction

Quantum phase estimation is widely researched in terms of both its fundamental and technological implications. Optical and atomic interferometers are extremely useful measuring tools for their high sensitivity to a small change in the phase [1–3]. The Mach–Zehnder interferometer (MZI) is the most-studied optical interferometer, and with solely classical resources, its phase sensitivity is limited by the standard quantum limit (SQL). In 1981, Caves [4] showed that the SQL can be beaten by using the squeezed vacuum (SV) state. Subsequently, nonclassical states of light such as squeezed states [5], Fock states [6], or entangled states [7–9] were proposed for higher precision. One of the most prominent states is the N00N state, which can in principle beat the SQL and reach the ultimate lower limit, known as the Heisenberg limit (HL) [8, 10–12]. However, these states are extremely fragile [13–16] and are very hard to prepare using current technology [17], which constrains their application. We know that coherent states are easily produced using lasers and are also robust to photon losses; therefore, it is practical and meaningful to consider a coherent light as one of the inputs of an interferometer. It has been proved that when one input port of an interferometer is a coherent state, the best state to put into the other port is an SV state with a given fixed mean number of photons [18].

Another class of interferometers, which are classified as belonging to the group SU(1,1), was proposed by Yurke *et al.* [19]. Their structure is like that of an MZI except that the beam splitters are replaced by active elements such as four-wave mixers or parametric amplifiers. This type of interferometer improves the phase sensitivity mainly by amplifying the signal through parametric processes. It has been shown that when only coherent light is used, the SQL can be beaten by an SU(1,1) interferometer [20], whereas it cannot be beaten by an MZI. After that, Ref. [21] showed that an SU(1,1) interferometer performs better with coherent and SV light as inputs than with two coherent lights as inputs.

We see that the SV state performs better than the coherent state as one input of these two types of interferometers, with a coherent state as the other input. This is true in ideal conditions but not when there is photon loss. Although the SV state has advantages over the coherent state because of its nonclassical properties, it lacks robustness. To combine the advantages of the coherent state and the SV state, in this paper, we consider the displaced squeezed vacuum (DSV) state $|\psi\rangle = D(\alpha)S(r)\,|0\rangle$, where $S(r) = \exp\{(1/2)(r^*a^2 - ra^{+2})\}$ is the squeezing operator, and $D(\alpha) = \exp\{(\alpha a^+ - \alpha^* a)\}$ is the displacement operator. For a given mean photon number, we can adjust the proportions of squeezing and displacement by changing the values of α and r. Specifically, when $\alpha=0$, it is an SV state, and when $r=0$, it is a coherent state. We analyze the phase sensitivity of an SU(1,1) interferometer with the DSV state as one of the inputs and then look for the optimal state by choosing the appropriate proportions of squeezing and displacement. Compared with the SV and coherent states, the optimal DSV state performs better under both ideal and lossy conditions. In addition, it is a practical scheme, as the DSV state is relatively easy to produce in the laboratory and has been used for phase estimation [2, 22, 23].

This paper is arranged as follows. In Section 2, we study the phase sensitivity of an ideal SU(1,1) interferometer with homodyne detection and identify the optimal DSV state. We then compare our scheme with previous work. In Section 3, we study the effects of photon loss. In Section 4, we discuss the factors that affect the optimal states. Conclusions are presented in the last section.

2 Ideal SU(1,1) interferometer

2.1 Model

The structure of the SU(1,1) interferometer is shown in Fig. 1. It is similar to that of a traditional MZI except that the two 50:50 beam splitters are replaced by optical parameter amplifiers (OPAs). The parametric process requires the use of a strong pump light in addition to two input beams. In the parametric process, photons are transferred between the pump beam and the two input beams; thus, the total photon number of the input light is amplified. After the first OPA, one output beam accumulates a phase shift ϕ. Then the two beams recombine in the second OPA, and we can obtain the phase information by measuring the output fields.

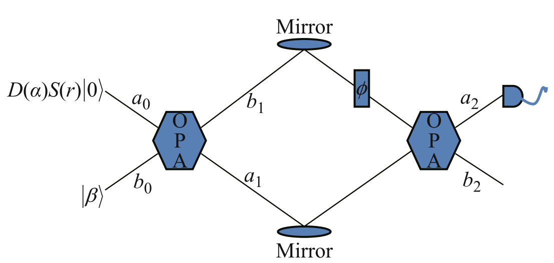

Fig. 1 Schematic diagram of the ideal SU(1,1) interferometer.

We use a ($a^\dagger$) and b ($b^\dagger$) to label the annihilation (creation) operators for the two modes. The input–output operator relation for the interferometer is given by [24, 25]

$$a_2=\mu a_0-\nu b_0^\dagger,\quad b_2=\exp(\mathrm{i}\phi)(\mu b_0-\nu a_0^\dagger),\tag{1}$$

where $\mu=\cosh g_1\cosh g_2+\exp[\mathrm{i}(\theta_2-\theta_1-\phi)]\sinh g_1\cdot\sinh g_2$, $\nu=\exp(\mathrm{i}\theta_1)\sinh g_1\cosh g_2+\exp[\mathrm{i}(\theta_2-\phi)]\cosh g_1\cdot\sinh g_2$. Further, g_1 (g_2) and θ_1 (θ_2) describe the amplitude gain and phase shift introduced by the OPA in the first (second) place, respectively. The interferometer is typically studied in a balanced situation, i.e., $g=g_1=g_2$, and $\theta_2-\theta_1=\pi$. In the balanced situation and when $\phi=0$, one has $a_2=a_0$ and $b_2=b_0$; that is, the output beams of the interferometer are equal to the input beams if there is no phase difference between the two paths of the interferometer. The following study focuses on the balanced situation.

There are several detection methods for extracting phase information from the output fields, e.g., intensity detection, homodyne detection, and parity detection. As shown in Ref. [21], the phase sensitivity of an SU(1,1) interferometer with homodyne detection is better than that of one with intensity detection. Parity detection is relatively complex and has been used in linear interferometers, and for Gaussian states, parity detection can be efficiently simulated with heterodyne detection [26]. Here we use homodyne detection, which is experimentally preferred for simplicity and accuracy. The detected variable is the amplitude quadrature X, which can be written as

$$X=(a_2+a_2^\dagger)/\sqrt{2}.\tag{2}$$

Then the phase sensitivity of the SU(1,1) interferometer can be calculated according to the error propagation formula:

$$(\Delta\phi)^2=\langle(\Delta X)^2\rangle/\left|\partial\langle X\rangle/\partial\phi\right|^2.\tag{3}$$

2.2 Optimal displaced squeezed vacuum state

Now we study the phase sensitivity of the SU(1,1) interferometer with $D(\alpha)S(r)\left|0\right\rangle_a\otimes\left|\beta\right\rangle_b$ as inputs. After calculation, the phase sensitivity is given by

$$\begin{aligned}(\Delta\phi)^2=&[-(\mu^2+\mu^{*2})\sinh r\cosh r+2\left|\mu\right|^2\cosh^2 r-1]\\&\times\{2\left|\alpha\right|\left|\beta\right|\sinh^3 g\cosh g[\mathrm{e}^{-\mathrm{i}(2\phi+\theta_\beta-\theta_1-\theta_\alpha)}+\mathrm{e}^{\mathrm{i}(2\phi+\theta_\beta-\theta_1-\theta_\alpha)}-\mathrm{e}^{\mathrm{i}(\theta_\beta-\theta_1+\theta_\alpha)}-\mathrm{e}^{-\mathrm{i}(\theta_\beta-\theta_1+\theta_\alpha)}]\\&-\left|\alpha\right|^2\sinh^4 g[\mathrm{e}^{-\mathrm{i}(\phi-\theta_\alpha)}-\mathrm{e}^{\mathrm{i}(\phi-\theta_\alpha)}]^2+\left|\beta\right|^2\sinh^2 g\cosh^2 g[2-\mathrm{e}^{2\mathrm{i}(\phi+\theta_\beta-\theta_1)}-\mathrm{e}^{-2\mathrm{i}(\phi+\theta_\beta-\theta_1)}]\}^{-1},\end{aligned}\tag{4}$$

where $\alpha = |\alpha| \exp(\mathrm{i}\theta_\alpha)$, $\beta = |\beta| \exp(\mathrm{i}\theta_\beta)$, and r is a real number.

To optimize the phase sensitivity, we should adjust θ_α, θ_β, and the phase shift ϕ to appropriate values. We find that when $\phi = 0, \theta_1 = 0$, and $\theta_\alpha = \theta_\beta = \pi/2$, the phase sensitivity is optimal and can be written as

$$(\Delta\phi_o)^2 = \frac{(\cosh r - \sinh r)^2}{4\sinh^2 g(|\alpha| \sinh g + |\beta| \cosh g)^2}. \tag{5}$$

The mean photon number of the DSV state is $N_a = |\alpha|^2 + \sinh^2 r$, where $|\alpha|^2$ and $\sinh^2 r$ describe the mean photon number of the displacement and the squeezing component, respectively. We introduce the parameter $\eta = \sinh^2 r/N_a$ to label the squeezing fraction of the mean photon number. When $\eta = 0$, the state is a coherent state, $|\alpha\rangle$, and when $\eta = 1$, it is an SV state, $S(r)|0\rangle$. Next we study the influence of η on the phase sensitivity given a fixed N_a. With η and N_a, Eq. (5) can be rewritten as

$$(\Delta\phi_o)^2 = \frac{(\sqrt{\eta N_a + 1} - \sqrt{\eta N_a})^2}{4\sinh^2 g[\sqrt{(1-\eta)N_a} \sinh g + |\beta| \cosh g]^2}. \tag{6}$$

The total photon number inside the SU(1,1) interferometer is N_{total}, i.e., $\langle a_1^+ a + b_1^+ b\rangle$ [19]. In our scheme, we have

$$N_{total} = (N_a + |\beta|^2)(2\sinh^2 g + 1) + 2\sinh^2 g + 4\sqrt{(1-\eta)N_a}\,|\beta| \sinh g \cosh g. \tag{7}$$

Then we can obtain the HL ($\frac{1}{N_{total}}$) and the SQL ($\frac{1}{\sqrt{N_{total}}}$).

Figure 2 shows the phase sensitivity $\Delta\phi_o$ versus η. We can see that the minimum value of $\Delta\phi_o$ does not appear at $\eta = 1$. That is, for the SU(1,1) interferometer, when one input is a coherent state, the optimal state of the other input is not the SV state. In Fig. 2, the smallest value of $\Delta\phi_o$ is about $\eta = 0.665572$, which corresponds to the optimal phase sensitivity. We denote it by η_o and call it the optimal squeezing fraction, which corresponds to the optimal DSV state optimizing the phase sensitivity. Figure 2 also compares the phase sensitivity with the HL, and we see that it approaches the HL in some regions.

We compare our scheme with an optimal DSV state in mode a and a coherent state in mode b with the schemes in Refs. [20] and [21]. In Ref. [20], both modes are coherent states, and in Ref. [21], mode a is an SV state, and mode b is a coherent state. Figure 3 shows that the optimal DSV light greatly improves the phase sensitivity. We also compare the optimal phase sensitivity of our scheme with the HL, as shown in Fig. 3.

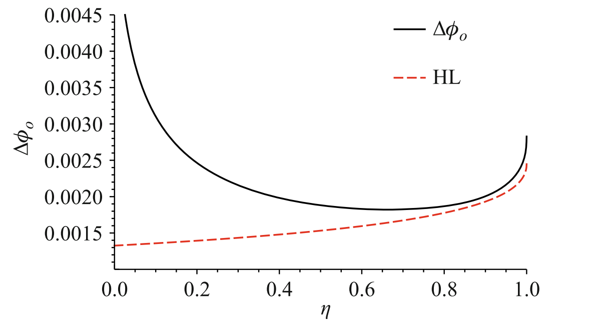

Fig. 2 The phase sensitivity $\Delta\phi_o$ of the SU(1,1) interferometer versus the squeezing fraction η of the DSV state, with $N_a = 10$, $N_b = 4, g = 2$.

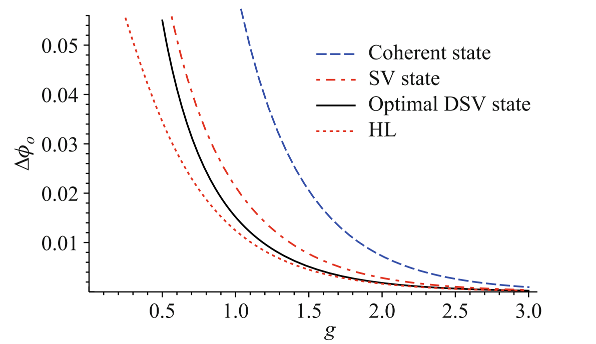

Fig. 3 The phase sensitivity $\Delta\phi_o$ as a function of g for the three schemes and the HL, $N_a = 10$, $N_b = 4$.

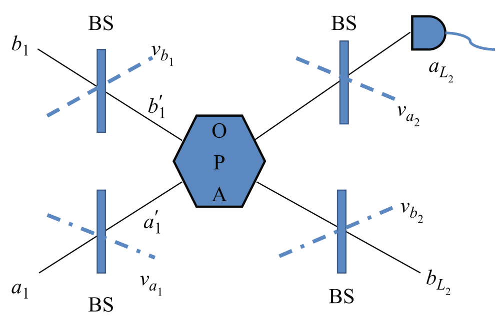

Fig. 4 Photon losses of the SU(1,1) interferometer that take place before and after the second OPA.

3 Lossy SU(1,1) interferometer

It has been shown that photon loss limits the achievable precision of phase measurement [27–29]. Now we study the effects of loss on the phase sensitivity of our scheme. As shown in Fig. 4, we use fictitious beam splitters to model photon losses. We generally call the losses before (after) the second OPA the internal (external) losses. Further, r_1 and r_1' denote the internal loss rates in modes a and b, and r_2 and r_2' denote the external loss rates in modes a and b. The mode transform of the light fields after a fictitious beam splitter is given by an example, a_1, where $a_1' = \sqrt{1-r_1}a_1 + \sqrt{r_1}v_{a_1}$. In addition, this transform relation can be generalized to other modes of light fields, as shown in Fig. 4. The final expression of the phase sensitivity $\Delta\phi_L$ is

$$(\Delta\phi_L)^2 = [(\sqrt{1-r_1'}\sinh^2 g - \sqrt{1-r_1}\cosh^2 g)^2(1-r_2)(\cosh 2r - 2\sinh r\cosh r) + (\sqrt{1-r_1'} - \sqrt{1-r_1})^2(1-r_2)\sinh^2 g\cosh^2 g + (r_1\cosh^2 g + r_1'\sinh^2 g)(1-r_2) + r_2] \times [4\sinh^2 g(|\alpha|\sinh g + |\beta|\cosh g)^2(1-r_1')(1-r_2)]^{-1}. \tag{8}$$

We plot $\Delta\phi_L$ versus η in the presence of only the internal loss ($r_1, r_1' \neq 0$, $r_2, r_2' = 0$). Figure 5 shows that the phase sensitivity is more sensitive to the loss in mode b than to that in mode a. The results indicate that trying to decrease the loss in mode b, which carries the phase shifter, is more effective in the experiment. For the given loss rate, the phase sensitivity surpasses the SQL, as shown in Fig. 5. In addition, Eq. (8) is not relevant to the external loss rate r_2' in mode b because we make the detection only in mode a.

Now we study the optimal DSV state in the presence of loss. For simplicity, we consider the situation in which the two arms of the interferometer have the same internal and external loss rates, i.e., $r_1 = r_1' = R_1$ and $r_2 = r_2' = R_2$. Further, the results can be generalized to a general case. In this case, Eq. (8) can be rewritten as

$$(\Delta\phi_L')^2 = (\Delta\phi_o)^2 + [R_1(1-R_2)(\cosh^2 g + \sinh^2 g) + R_2] \times \left\{4(1-R_1)(1-R_2)\sinh^2 g[\sqrt{(1-\eta)N_a}\sinh g + |\beta|\cosh g]^2\right\}^{-1}, \tag{9}$$

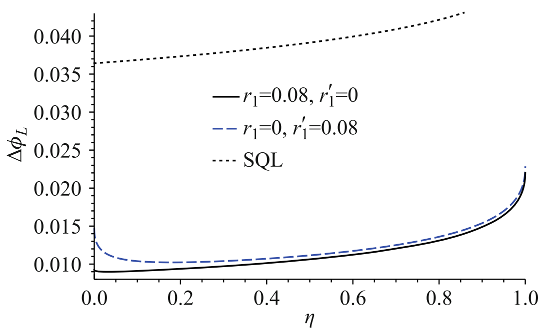

Fig. 5 The phase sensitivity $\Delta\phi_L$ versus η in the presence of internal losses with $N_a = 10, g = 2, |\beta| = 2, r_2 = r_2' = 0$.

where $(\Delta\phi_o)^2$ is given by Eq. (6).

We plot the phase sensitivity $\Delta\phi_L'$ as a function of η in the presence of the internal and external losses in Figs. 6(a) and (b), respectively. We can see from Fig. 6 that the phase sensitivity deteriorates as the loss rates increase. In addition, the lowest point of each curve from the bottom to the top gradually moves to the left, indicating that the optimal squeezing fraction η_o decreases gradually with increasing loss. It also indicates that, when the loss increases, the coherent fraction should be increased to compensate for the loss. In Fig. 6, we can also see that the phase sensitivity is far below the SQL.

We also compare the proposed scheme with those in Refs. [20] and [21], as we did for the ideal case. Figure 7 shows that the optimal DSV state performs best with loss rates below 0.5. Actually, for any loss rate, the optimal DSV state is the best but at a very small η; i.e., the optimal DSV state approaches a coherent state when the loss rate is very large. Figure 7 also shows that when the loss is not large, the phase sensitivity can surpass the SQL.

4 Effects of N_a, g, N_b on the optimal squeezing fraction

We also examine the effects of the parameters N_a, g, and N_b ($N_b = |\beta|^2$) on the optimal squeezing fraction η_o of the DSV state. Figure 8 shows the results for the ideal case. Figure 8(a) shows that with increasing N_a, the

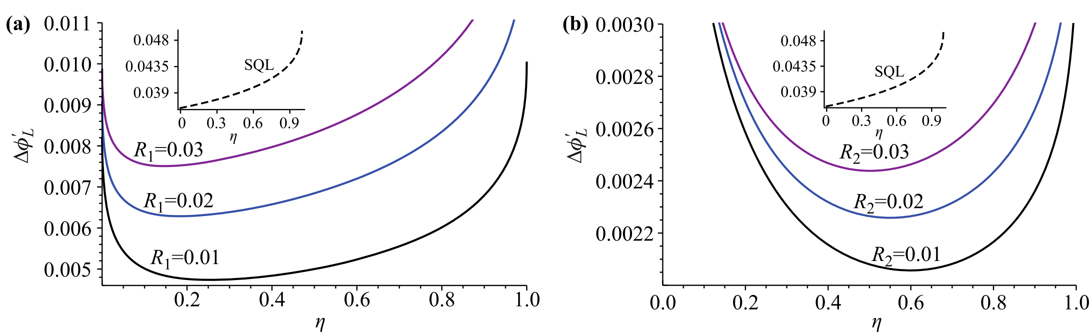

Fig. 6 The phase sensitivity $\Delta\phi_L'$ versus the squeezing fraction η for different loss rates with $N_a = 10, N_b = 4, g = 2$.

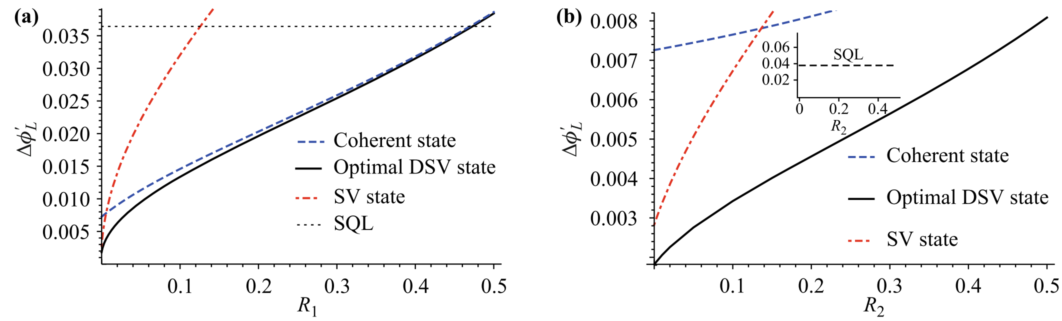

Fig. 7 The phase sensitivity $\Delta\phi'_L$ versus the loss rate R_1 and R_2 for the three schemes and the SQL, $N_a = 10, N_b = 4, g = 2$.

optimal squeezing fraction η_o decreases. Thus, when the energy of the DSV light increases, it is better to increase the coherent amplitude rather than the squeezing.

Similarly, we study the effects of the amplitude gain g. As shown in Fig. 8(b), the optimal squeezing fraction η_o decreases as a function of g. This means when the amplitude gain g of the OPA increases, it is appropriate to increase the coherent amplitude rather than the squeezing.

Furthermore, we find that η_o increases with increasing $|\beta|$, as shown in Fig. 8(c). If the energy of the coherent light in mode b increases, the squeezing fraction of the DSV light should also be increased to enhance the phase sensitivity.

The results for the lossy case are shown in Fig. 9. The effects of N_a, g, and N_b are in accordance with those in the ideal case. In addition, by comparing the two curves in each panel of Fig. 9, we find that the internal loss has a much greater effect than the external loss. This can be understood because the phase sensitivity of the SU(1,1) interferometer is limited mainly by the internal loss and that it is more sensitive to the internal loss than to the external loss [25].

5 Conclusion

In conclusion, we propose a practical scheme to enhance the precision of phase estimation. When the optimal DSV state is used, the phase sensitivity of the SU(1,1) interferometer is greater than that of former schemes. We also find that the photon losses in the two arms of the SU(1,1) interferometer have different effects on the phase sensitivity. More effort should be devoted to decreasing the loss in the arm that carries the phase shifter. The optimal squeezing fraction of the DSV state is affected by the energy of the input DSV light N_a, the loss rate R, the gain factor g, and the mean photon number N_b of

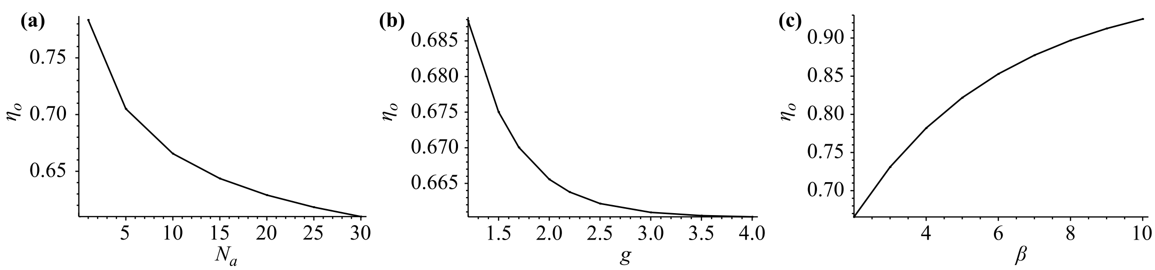

Fig. 8 The optimal squeezing fraction η_o versus N_a, g, and $|\beta|$ respectively. **(a)** $N_b = 4, g = 2$; **(b)** $N_a = 10, N_b = 4$; **(c)** $N_a = 10, N_b = 4$.

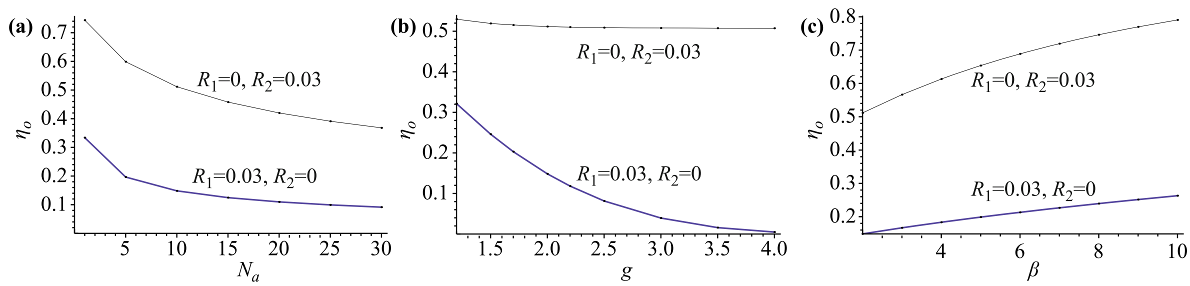

Fig. 9 The optimal squeezing fraction η_o versus N_a, g, and $|\beta|$ respectively in the presence of loss. **(a)** $N_b = 4, g = 2$; **(b)** $N_a = 10, N_b = 4$; **(c)** $N_a = 10, g = 2$.

the coherent state in mode b. We can change the squeezing fraction of the DSV state to optimize the phase sensitivity.

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