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Research article

Effects of the dust size distribution in one-dimensional quantum dusty plasma

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The effects of the dust size distribution in ultracold quantum dusty plasmas are investigated. The amplitude φ_m and width ω of quantum dust acoustic waves are studied with different dust size distributions in the system. The φ_m and ω of the quantum dust acoustic waves are found to increase as the total number density increases. The φ_m and ω are greater for unusual dusty plasmas than for typical dusty plasmas. Moreover, as the Fermi temperature of the dust grains increases, the φ_m of the wave decreases. The ω of quantum dust acoustic waves increases as the speed u_0 of the wave increases.

Keywords dusty plasma, electrostatic waves

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1 Introduction

In the last two decades, the study of dusty plasma has developed rapidly because of the relevance and diverse applications of dusty plasmas in space, the earth's environment, and laboratory plasmas [1–5]. Dusty plasmas or complex plasmas consist of electrons, ions, and extremely massive, highly charged micrometer-sized dust particles. The collective dusty plasma interactions have been studied both theoretically and experimentally [6, 7]. Dust acoustic waves (DAW) were first reported theoretically in unmagnetized dusty plasmas by Rao *et al.* [2]. On the other hand, at higher frequencies, Shukla and Silin showed the existence of dust ion acoustic waves (DIAW) [3]. Laboratory experiments on dusty plasmas have confirmed the existence of both DAW and DIAW [8–10].

Recently, the quantum effects in dusty plasmas have received much attention [11–16]. Quantum mechanical effects become important when $\lambda_{Bj} > \lambda_{Dj}$, where $\lambda_{Bj}(\lambda_{Dj})$ is the thermal de Broglie wavelength of the jth species. For $\lambda_{Bj} < \lambda_{Dj}$, the plasma particles are treated classically and are assumed to be point like. In recent years, there has been a growing interest in studying the physics of quantum plasmas not only in astrophysical systems [17] but also in the manufacturing of microscale and nanoscale objects [18–23] as well as in intense laser-solid density plasma experiments [24]. Generally, the quantum hydrodynamics (QHD) model is employed to study quantum plasma systems instead of the classical fluid model [25–28]. The QHD model is based on the introduction of a quantum correction term, the so-called "Bohm Potential", to the classical fluid equations.

Although there have been many investigations on the quantum effects of dusty plasmas, most of these studies have focused on mono-sized quantum dusty plasmas [13–16]. In fact, a dusty plasma consists of many different dust grains with different sizes [29–32], which has been reported in both space plasmas and laboratory experiments. It has been widely accepted that the dust size distribution can be described by a power law distribution in space plasmas [33, 34] and by a Gaussian distribution in laboratory plasmas [29]. However, in general, the dust size distribution function does not fully satisfy a power law distribution or a Gaussian distribution. The distribution function depends on the environment in space plasmas and the experimental conditions in the laboratory. Therefore, it is important to investigate the role of an arbitrary dust size distribution in quantum dusty plasmas. An arbitrary dust size distribution can contain both the power law distribution and Gaussian distribution [32, 35]. Recently, the dust size distribution effect has been studied in a magnetized quantum dusty plasma [36, 37]. The Landau damping phenomenon of the dusty plasma, which considers the dust size distribution effect, has also been studied [38].

The present paper will study arbitrary dust size distribution effects in unmagnetized quantum dusty plasmas. We consider a polynomially expressed distribution of dust particles in a quantum dusty plasma. For dust grains with radius r in a given range $[r_{min}, r_{max}]$, where r_{min} is the lower limit and r_{max} is the upper limit, the differential polynomially expressed distribution function is of the form

$$n(r)dr = [a_0 + a_1 r + a_2 r^2 + a_3 r^3 + \cdots]\mathrm{d}r \tag{1}$$

This function satisfies the following equation:

$$N_{tot} = \int_{r_{min}}^{r_{max}} n(r)\mathrm{d}r$$

where r is the radii of dust grains, $a_0, a_1, a_2, a_3 \cdots$ are constants, and N_{tot} is the total number density of dust grains. Outside the limits $r < r_{min}$ and $r > r_{max}$, we use $n(r) = 0$.

In this paper, we consider the role of dust size distribution and employ the QHD model. We study the dust size distribution effects for quantum dust acoustic waves in an unmagnetized, collisionless, and ultracold quantum dusty plasma.

This paper is organized as follows. The basic set of QHD equations are presented in Section 2. The dust size distribution effects are studied for quantum dust acoustic waves in Section 3. The conclusions are presented in Section 4.

2 Basic equations

We consider a one-dimensional QHD model to describe the dynamics of quantum dust acoustic waves in a three-component quantum dusty plasma consisting of electrons, ions, and charged dust grains, known as an e-d-i plasma. We assume that the plasma particles obey the following equation [39]:

$$p_s = \frac{m_s V_{F_s}^2}{3n_{s0}^2} n_s^3, \tag{2}$$

where s equals e for electrons, i for ions, and d for dust grains, m_s is the mass, $V_{F_s} = \sqrt{2K_\mathrm{B}T_{Fs}/m_s}$ is the Fermi speed, K_B is the Boltzmann constant, T_{Fs} is the Fermi temperature, and n_s is the number density with its equilibrium value n_{s0}.

In the following, we assume that there are N different dust grains with different sizes. We assume that the jth $(j = 1, 2, 3, \cdots, N)$ dust grain pressure in a one-dimensional zero-temperature Fermi gas obeys the following law

$$p_{dj} = \frac{m_{dj} V_{F_{dj}}^2}{3n_{dj0}^2} n_{dj}^3, \tag{3}$$

where m_{dj} $(j = 1, 2, 3, \cdots, N)$ is the mass of jth dust grain, $V_{F_{dj}} = \sqrt{2K_\mathrm{B}T_{Fdj}/m_{dj}}$ is the jth dust grain Fermi thermal speed (each dust grain is assumed to have equal temperature), and T_{Fdj} is the jth dust grain Fermi temperature, n_{dj} is the jth dust grain number density with its equilibrium value n_{dj0}, p_{dj} is the jth dust grain pressure.

At equilibrium, the charge neutrality condition is

$$n_{i0} = \sum_{j=1}^{N} Z_{d0j} n_{d0j} + n_{e0},$$

where n_{i0}, n_{e0}, and n_{d0j} are the number densities of unperturbed ions, electrons, and the jth dust grain, respectively. Here, Z_{d0j} is the unperturbed number of charges residing on the jth dust grain measured in units of electron charge.

The dynamics of the low-phase velocity $(V_{Fd} \ll V_P \ll V_{Fe,i})$ and low frequency $(\nu_{dn} \ll\mid \partial_t \mid\ll \nu_{e,in},\ \mid \partial_t \mid\ll (V_{Fe,i}^2/\nu_{e,in})\partial_x^2)$, where ν_{sn} is the normalized charged particle-neutral collision frequency, in such a quantum dusty plasma with N different dust grain species is governed by the following set of equations:

$$\frac{\partial u'_{dj}}{\partial t'} + u'_{dj}\frac{\partial u'_{dj}}{\partial x'} = \frac{Z'_{dj} e}{m'_{dj}}\frac{\partial \varphi'}{\partial x'} - \frac{1}{m'_{dj} n'_{dj}}\frac{\partial p_{dj}}{\partial x'} + \frac{\hbar^2}{2m'^2_{dj}}\frac{\partial}{\partial x'}\left(\frac{\frac{\partial^2}{\partial x'^2}\sqrt{n'_{dj}}}{\sqrt{n'_{dj}}}\right), \tag{4}$$

$$\frac{\partial n'_{dj}}{\partial t'} + \frac{\partial}{\partial x'}(n'_{dj} u'_{dj}) = 0, \tag{5}$$

$$\frac{\partial^2 \varphi'}{\partial x'^2} = 4\pi e\Big[\sum_{j=1}^{N} n'_{dj} Z'_{dj} + n'_e - n'_i\Big] \tag{6}$$

where u'_{dj} is the fluid velocity in the x-direction of the jth dust grain, Z'_{dj} is the charge number of the jth dust grain, and φ' is the electrostatic potential. Eqs. (3)–(–) constitute the QHD model for a three-component e-d-i quantum dusty plasma.

We now introduce the following normalizations: $u_{dj} = u'_{dj}/c_d$, $n_{dj0} = n'_{dj0}/N_{tot}$, $t = t'\omega_{pd}$, $Z_{dj} = Z'_{dj}/\bar{z}_{d0}$, $m_{dj} = m'_{dj}/\bar{m}_d$, $\varphi = e\varphi'/F_0$, and $n_j = n'_j/n_{j0}$ for $(j = e, i)$ as well as $c_d = \sqrt{(2\bar{z}_{do}K_B T_{Fi})/\bar{m}_d}$, and $\omega_{pd} = \sqrt{(4\pi\bar{z}_{do}^2 e^2 N_{tot})/\bar{m}_d}$, where $\lambda_D = \sqrt{(2K_B T_{Fi}/4\pi\bar{z}_{d0}e^2 N_{tot})}$ is the Debye length. Therefore, we obtain the following set of dimensionless equations:

$$\frac{\partial u_{dj}}{\partial t}+u_{dj}\frac{\partial u_{dj}}{\partial x}=\frac{Z_{dj}}{m_{dj}}\frac{\partial\varphi}{\partial x}$$
$$-\delta\frac{1}{n_{dj0}^2}\frac{n_{dj}}{m_{dj}}\frac{\partial n_{dj}}{\partial x}+\frac{H_d^2}{2m_{dj}}\frac{\partial}{\partial x}(\frac{\frac{\partial^2}{\partial x^2}\sqrt{n_{dj}}}{\sqrt{n_{dj}}}), \tag{7}$$

$$\frac{\partial n_{dj}}{\partial t}+\frac{\partial}{\partial x}(n_{dj}u_{dj})=0, \tag{8}$$

$$\frac{\partial^2\varphi}{\partial x^2}=\sum_{j=1}^{N}n_{dj}Z_{dj}+\mu_e n_e-\nu_i n_{i4}, \tag{9}$$

where $F_0=c_d^2\bar{m}_d/\bar{z}_{d0}$, $\delta=T_{Fd}/T_{Fi}\bar{z}_{d0}$, $H_d=\sqrt{\hbar^2\omega_{pd}^2/\overline{m}_d c_d^4}$, $\mu_e=n_{e0}/\overline{z}_{d0}N_{tot}$, $\mu_i=n_{i0}/\overline{z}_{d0}N_{tot}$, $\mu_e=1/(\mu-1)$, and $\mu_i=\mu/(\mu-1)$. Here, n_e and n_i are given as follows:

$$n_e=[1+\frac{2\varphi}{\sigma}+\frac{H_e^2}{\sigma}(1+\frac{2\varphi}{\sigma})^{-\frac{1}{4}}\frac{\partial^2}{\partial x^2}(1+\frac{2\varphi}{\sigma})^{\frac{1}{4}}]^{\frac{1}{2}}, \tag{10}$$

$$n_i=[1-2\varphi+H_i^2(1-2\varphi)^{-\frac{1}{4}}\frac{\partial^2}{\partial x^2}(1-2\varphi)^{\frac{1}{4}}]^{\frac{1}{2}}, \tag{11}$$

where $\sigma=T_{Fe}/T_{Fi}$ and where $H_e=\sqrt{(\overline{z}_{d0}\hbar^2\omega_{pd}^2)/m_e\overline{m}_d c_d^4}$ and $H_i=\sqrt{(\overline{z}_{d0}\hbar^2\omega_{pd}^2)/m_i\overline{m}_d c_d^4}$ are quantum parameters for electron and ion, respectively.

We now study the dispersion relation of the system by assuming $n_{dj}=n_{dj0}+\tilde{n}_j e^{i(kx-\omega_0 t)}$, $u_{dj}=\tilde{u}_j e^{i(kx-\omega_0 t)}$, $\varphi=\tilde{\varphi}e^{i(kx-\omega_0 t)}$, where k and ω_0 are the wave number and the frequency, respectively. Moreover, $\tilde{n}_j e^{i(kx-\omega_0 t)}$, $\tilde{u}_j e^{i(kx-\omega_0 t)}$, and $\tilde{\varphi}e^{i(kx-\omega_0 t)}$ are small perturbations in equilibrium state. We assume that the fluid velocity and electrostatic potential equal zero at the unperturbed equilibrium state. Substituting these expansions into Eqs. (7)–(9), we obtain the dispersion relation as follows:

$$\sum_{j=1}^{N}\frac{4k^2Z_{dj}^2m_{dj}n_{dj0}}{4k^2m_{dj}\delta+k^4H_d^2-4m_{dj}^2\omega_0^2}=-[k^2+\mu_e(\frac{1}{\sigma}$$
$$-k^2\frac{H_e^2}{4\sigma^2})+\mu_i(1-k^2\frac{H_i^2}{4})]. \tag{12}$$

3 Korteweg-de Vries (KdV) equation for a quantum dusty plasma with N different dust grain species

For small but finite amplitude waves, we use the traditional perturbation method [40], which has been used extensively in previous studies. For an arbitrary dust size distribution of dusty plasma, we introduce the following stretched variables:

$$\xi=\varepsilon(x-v_0t),\quad \tau=\varepsilon^3 t \tag{13}$$

where ε is a small parameter characterizing the strength of the nonlinearity and v_0 is the phase speed of the waves. The variables can be expanded as follows:

$$n_{dj}=n_{d0j}+\varepsilon^2n_{d1j}+\varepsilon^4n_{d2j}+\cdots,$$
$$u_{dj}=\varepsilon^2u_{d1j}+\varepsilon^4u_{d2j}+\cdots\quad (j=1,\cdots,N),$$
$$\varphi=\varepsilon^2\varphi_1+\varepsilon^4\varphi_2+\cdots, \tag{14}$$

Substituting Eqs. (13) and (14) into Eqs. (7)–(9) and collecting the lowest order terms of ε, we find

$$n_{d1j}=\frac{n_{doj}}{v_0}u_{d1j}, \tag{15}$$

$$u_{d1j}=\frac{Z_{dj}v_0}{\delta-m_{dj}v_0^2}\varphi_1, \tag{16}$$

$$\sum_{j=1}^{N}n_{d1j}Z_{dj}=-(\frac{\mu_e}{\sigma}+\mu_i)\varphi_1, \tag{17}$$

Using Eqs. (15)–(17), we obtain the velocity of the soliton as follows:

$$\sum_{j=1}^{N}\frac{Z_{dj}^2n_{d0j}}{\delta-m_{dj}v_0^2}=-(\frac{\mu_e}{\sigma}+\mu_i). \tag{18}$$

We collect the next higher order of ϵ from the equation of continuity, the equation of motion, and Poisson's equation. We obtain

$$(-v_0)\frac{\partial n_{d2j}}{\partial\xi}+\frac{\partial n_{d1j}}{\partial\tau}+u_{d1j}\frac{\partial n_{d1j}}{\partial\xi}$$
$$+n_{d1j}\frac{\partial u_{d1j}}{\partial\xi}+n_{d0j}\frac{\partial u_{d2j}}{\partial\xi}=0, \tag{19}$$

$$(-v_0)\frac{\partial u_{d2j}}{\partial\xi}+\frac{\partial u_{d1j}}{\partial\tau}+u_{d1j}\frac{\partial u_{d1j}}{\partial\xi}$$
$$-\frac{Z_{dj}}{m_{dj}}\frac{\partial\varphi_2}{\partial\xi}+\delta\frac{1}{n_{dj0}^2m_{dj}}(n_{d0j}\frac{\partial n_{d2j}}{\partial\xi}+n_{d1j}\frac{\partial n_{d1j}}{\partial\xi})$$
$$-\frac{H_d^2}{4m_{dj}^2}\frac{1}{n_{d0j}}\frac{\partial^3n_{d1j}}{\partial\xi^3}=0, \tag{20}$$

$$(1-\frac{H_e^2\mu_e}{4\sigma^2}-\frac{H_i^2\mu_i}{4})\frac{\partial^2\varphi_1}{\partial\xi}-\sum_{j=1}^{N}n_{d2j}Z_{dj}$$
$$-(\frac{\mu_e}{\sigma}+\mu_i)\varphi_2+\frac{1}{2}(\frac{\mu_e}{\sigma^2}-\mu_i)\varphi_1^2=0. \tag{21}$$

Now, using Eqs. (15)–(18) and eliminating n_{d2j}, u_{d2j}, and φ_2 from Eqs. (19)–(21), we obtain the KdV equation:

$$\frac{\partial\varphi_1}{\partial\tau}+A\varphi_1\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial^3\varphi_1}{\partial\xi^3}=0, \tag{22}$$

with the coefficients

$$A=\frac{\sum_{j=1}^{N}\frac{3Z_{dj}^3n_{d0j}v_0^2m_{dj}+Z_{dj}^3n_{d0j}\delta}{(\delta-m_{dj}v_0^2)^3}+\frac{\mu_e}{\sigma^2}-\mu_i}{\sum_{j=1}^{N}\frac{2Z_{dj}^2n_{d0j}m_{dj}v_0}{(\delta-m_{dj}v_0^2)^2}}, \tag{23}$$

and

$$B = \frac{1 - \frac{H_e^2\mu_e}{4\sigma^2} - \frac{H_i^2\mu_i}{4} - \sum_{j=1}^{N} \frac{H_d^2 Z_{dj}^2 n_{d0j}}{4m_{dj}(\delta - m_{dj}v_0^2)^2}}{\sum_{j=1}^{N} \frac{2Z_{dj}^2 n_{d0j} m_{dj} v_0}{(\delta - m_{dj}v_0^2)^2}}. \quad (24)$$

We transform the independent variables ξ and τ to $\eta = \xi - u_0\tau$ and $\tau = \tau$, where u_0 is a constant speed normalized by c_d. The possible stationary solution of the KdV equation (22) is given as

$$\varphi_1 = \varphi_m \mathrm{Sech}^2\left(\frac{\eta}{W}\right), \quad (25)$$

where

$$\varphi_m = \frac{3u_0}{A} \quad (26)$$

and

$$W = \sqrt{\frac{4B}{u_0}} \quad (27)$$

are the amplitude and width of the quantum dust acoustic waves, respectively.

The charge and mass of the jth dust grain can be expressed as follows [29]: $Z_{dj} = k_z r_j$, $m_{dj} = k_m r_j^3$, where $k_z \approx 4\pi\varepsilon_0 V_0/e$ and $k_m \approx 4\pi\rho_d/3$. Here, ε_0 is the vacuum permittivity, V_0 is the electric surface potential at equilibrium, and ρ_d is the mass density of the dust grains (assumed to be constant and equal for all grains). We assume that the dust size distribution is given by Eq. (1). Therefore, after substituting Eq. (1) into Eqs. (23) and (24), we obtain

$$A = \frac{\int_{r_1}^{r_2} \frac{3k_z^3 k_m v_0^2(a_0r^6 + a_1r^7) + k_z^3(a_0r^3 + a_1r^4)\delta}{(\delta - k_m r^3 v_0^2)^3} \mathrm{d}r + \frac{\mu_e}{\sigma^2} - \mu_i}{\int_{r_1}^{r_2} \frac{2k_z^2 k_m v_0 (a_0r^5 + a_1r^6)}{(\delta - k_m r^3 v_0^2)^2} \mathrm{d}r}, \quad (28)$$

$$B = \frac{1 - \frac{H_e^2\mu_e}{4\sigma^2} - \frac{H_i^2\mu_i}{4} - \int_{r_1}^{r_2} \frac{H_d^2 k_z^2 (a_0r^2 + a_1r^3)}{4k_m r^3(\delta - k_m r^3 v_0^2)^2} \mathrm{d}r}{\int_{r_1}^{r_2} \frac{2k_z^2 k_m v_0 (a_0r^5 + a_1r^6)}{(\delta - k_m r^3 v_0^2)^2} \mathrm{d}r}. \quad (29)$$

In our numerical simulation, we choose previously reported parameters [15, 41, 42]: $n_{i0} = 2.0 \times 10^{30}\mathrm{m}^{-3}$, $n_{e0} = 5 \times 10^{29}\mathrm{m}^{-3}$, $Z_{d0} = 10^3$, $m_i \doteq m_p$, $T_{Fe} = 100\mathrm{K}$, $m_d = 10^{-17}\mathrm{kg}$, $\rho_d \sim 1\mathrm{g/cm}^3$, and $a \sim 0.1 - 1\mathrm{um}$.

4 Results and discussion

Figure 1 shows how the dust distribution function affects the amplitude and width of quantum dust acoustic waves. For generality and simplicity, we first let $n(r) = a_0$, and we find that the wave amplitude φ_m and width W depend on the parameter a_0. As a_0 increases, φ_m and W increase as shown in Fig. 1. Because a_0 stands for the magnitude of the total number density of dust particles, which should always be positive, we conclude that the wave amplitude and width increase as the total number density of dust particles increases.

For the given system parameters, a larger amplitude results in a narrower width of the KdV solitary wave. Because the amplitude and width of the solitary wave only depend on the value of u_0, the coefficients of both A and B are constants. However, for different system parameters such as a_0 or dust grain density with a given u_0, the results are different. Figure 1 shows that the amplitude and width increase as a_0 increases because the coefficient A decreases while B increases as a_0 increases.

In Fig. 2, we let $n(r) = a_0 + a_1 r$ and $a_0 = 0.9$. We note that the wave amplitude φ_m and width W depend on the parameter of a_1. It is observed that φ_m and W increase as a_1 increases. As we showed above, for the case of $a_1 > 0$, the number density of larger dust particles is greater than that of smaller dust particles. At the same time, for the case of $a_1 < 0$, which is usually found both in space plasmas and in the laboratory, the number density of larger dust particles is smaller than that of smaller dust particles. Therefore, we conclude

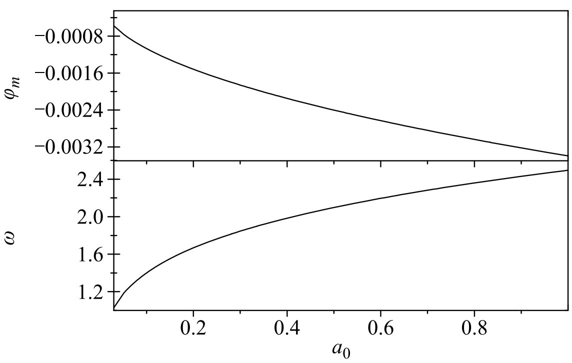

Fig. 1 The variation of φ_m and W with respect to a_0. The other parameter values are $\mu = 4.0$, $\sigma = 10$, $H_i = 6.494 \times 10^{-3}$, $H_e = 0.2782$, $H_d = 2.656 \times 10^{-9}$, $\delta = 5 \times 10^{-8}$, $k_z = 1$, $k_m = 4$, $r_{min} = 0.01$, $r_{max} = 1$, $u_0 = 1.0$, and $a_1 = a_2 = \cdots = 0$.

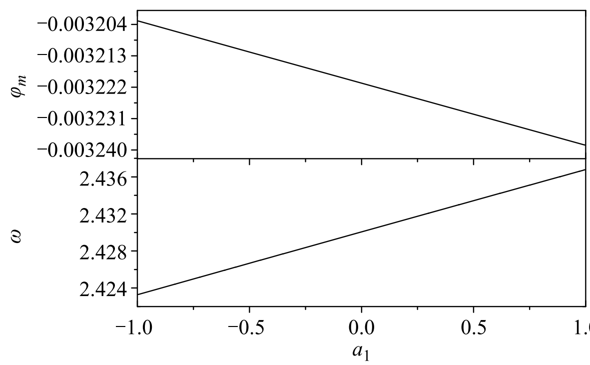

Fig. 2 The variation of φ_m and W with respect to a_1. The other parameter values are $\mu = 4.0$, $\sigma = 10$, $H_i = 6.494 \times 10^{-3}$, $H_e = 0.2782$, $H_d = 2.656 \times 10^{-9}$, $\delta = 5 \times 10^{-8}$, $k_z = 1$, $k_m = 4$, $r_{min} = 0.01$, $r_{max} = 1$, $u_0 = 1.0$, $a_0 = 0.9$, and $a_2 = a_3 = \cdots = 0$.

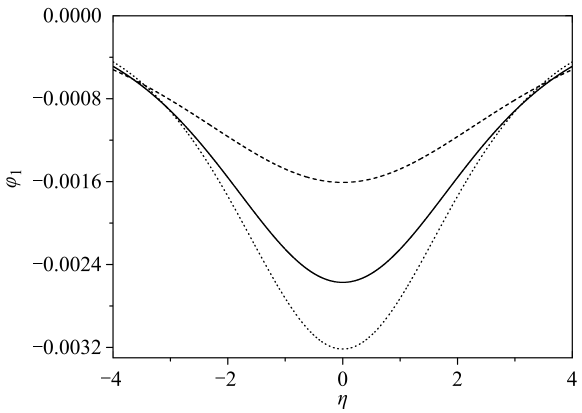

Fig. 3 The electrostatic potential φ_1 is plotted against $\eta = \xi - u_0\tau$, keeping different values of speed $u_0 = 1.0$ (*dotted line*), $u_0 = 0.8$ (*solid line*), and $u_0 = 0.5$ (*dashed line*). Other parameter values are $\mu = 4.0$, $\sigma = 10$, $H_i = 6.494 \times 10^{-3}$, $H_e = 0.2782$, $H_d = 2.656 \times 10^{-9}$ $\delta = 5 \times 10^{-8}$, $k_z = 1$, $k_m = 4$, $r_{min} = 0.01$, $r_{max} = 1$, $a_0 = 0.9$, $a_1 = -0.3$, and $a_2 = a_3 = \cdots = 0$.

Figure 3 shows the variation of the electrostatic potential φ_1 as a function of η for different values of speed u_0. From Fig. 3, the wave amplitude φ_m clearly increases as the speed u_0 increases, considering the effect of dust size distribution. Thus, as the speed u_0 becomes faster, the wave amplitude φ_m increases.

The previous studies [32, 37] have focused on classical plasmas, while the investigation in Ref. [36] studies quantum dust acoustic waves in magnetized plasma. The present investigation studies quantum dust acoustic waves in unmagnetized plasma.

5 Conclusions

In this study, we obtain a KdV equation by using the reductive perturbation method in an unmagnetized, collisionless, and ultracold quantum dusty plasma. The amplitude and width of the quantum dust acoustic waves are studied, accounting for dust size distribution. For an arbitrary dust size distribution in quantum dusty plasmas, we choose a polynomially expressed distribution function to investigate how the dust size distribution influences the amplitude and width of quantum dust acoustic waves.

From the numerical results, we conclude that the amplitude φ_m and width W of quantum dust acoustic waves increase as the total number density increases. The amplitude and width are greater for unusual dusty plasmas than for the usual dusty plasma. This conclusion is similar to that for classical dusty plasmas in which the quantum effect is neglected [32]. We can see the amplitude of the wave φ_m increases as the speed u_0 increases. Finally, as the Fermi temperature of the dust grains increases, the amplitude of the wave decreases.

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