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RESEARCH ARTICLE

Superconductivity and superfluidity as universal emergent phenomena

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Superconductivity (SC) or superfluidity (SF) is observed across a remarkably broad range of fermionic systems: in BCS, cuprate, iron-based, organic, and heavy-fermion superconductors, and in superfluid helium-3 in condensed matter; in a variety of SC/SF phenomena in low-energy nuclear physics; in ultracold, trapped atomic gases; and in various exotic possibilities in neutron stars. The range of physical conditions and differences in microscopic physics defy all attempts to unify this behavior in any conventional picture. Here we propose a unification through the shared symmetry properties of the emergent condensed states, with microscopic differences absorbed into parameters. This, in turn, forces a rethinking of specific occurrences of SC/SF such as high-T_c SC in cuprates, which becomes far less mysterious when seen as part of a continuum of behavior shared by a variety of other systems.

Keywords superconductivity, superfluidity

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Superconductivity (SC) and superfluidity (SF) are collective phenomena owing their existence to many-body interactions; the corresponding *emergent states* are not related perturbatively to the parent state. Thus, characterization of SC and SF through microscopic properties of the parent system fails on two levels: i) It cannot provide a unified view, as microscopic physics differs fundamentally between fields. ii) The transition from the microscopic parent state to the collective emergent state is not analytic; thus, it is conjecture to assume that the microscopic tendencies of the parent state are related directly to the collective properties of the emergent state.

Conventional understanding of SC and SF is built on the idea of a *Fermi liquid*, in which single-particle states of the interacting system are in one-to-one correspondence with those of the non-interacting system. SC is assumed to develop from a Fermi liquid parent through the *Cooper instability*, in which two fermions outside a filled Fermi sea can form a bound state for a vanishingly small attraction [1]. In the solid state, the weak attraction is assumed conventionally to arise from interaction of electrons with lattice vibrations.

The Cooper instability was developed into a many-body theory by the Bardeen–Cooper–Schrieffer (BCS) postulate that the SC state is a coherent superposition of fermion pairs in a weak coupling limit [2]; this was generalized to Eliashberg theory, which removed the weak-coupling restrictions. The BCS idea was soon adapted to applications in nuclear physics [3], with pairs bound by attractive nucleon–nucleon forces.

BCS theory in condensed matter and nuclear physics involves quite different interactions operating on energy and distance scales differing by many orders of magnitude. However, emergent SC/SF properties were unified by sharing the same form for the BCS wavefunction, which implied a common pseudospin symmetry of the effective Hamiltonians that could be expressed elegantly in terms of an SU(2) Lie algebra [4]. Thus, similarities between these fields could be understood through a common algebraic structure, whereas differences could be viewed as primarily *parametric* and not fundamental.

By the early 1970s, all cases of fermionic SC and SF were thought to be understood in these conventional BCS terms. This unity was shattered by a series of dis-

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coveries beginning with ^{3}He SF in 1972 [5], followed by SC in heavy-fermion compounds in 1979 [5], SC in various organics in 1980 [8], SC in copper oxides at high temperatures in 1986 [7], high-temperature SC in various iron-based compounds in 2008 [9], and in the past decade, direct observations suggesting proton SC and neutron SF in neutron stars [10], and SF of ultracold fermionic atoms [11–13].

SC or SF in all these systems is thought to result from condensation of Cooper pairs in parent states that may not be Fermi liquids, through interactions that may not be mediated by phonons and may differ from the s-wave form of conventional BCS theory (*unconventional pairing*). This calls into question whether the BCS paradigm, even generalized to accommodate unconventional pairing, can describe the diversity of SC and SF behavior. The issue is how the Cooper instability emerges from a variety of parent states that need not be Fermi liquids, enabled by highly diverse fermion–fermion correlations.

Let us begin with a brief survey of SC and SF behavior. Our aim is to highlight the simultaneous microscopic diversity but emergent-level unity of SC and SF. Tests of conventional BCS are well known, so we shall emphasize more complex behavior, with BCS viewed as a limit of this more complex behavior.

Phase diagrams for cuprate superconductors are rather universal, with features similar to those of Fig. 1(a). A striking feature is the proximity of the superconducting phase to the antiferromagnetic (AF) phase. The microscopic pairing structure is believed to be dominated by a single band near the Fermi surface and to have $d_{x^2-y^2}$ orbital geometry.

High-temperature SC in FeAs and FeSe compounds [18] indicates that cuprate phenomenology such as Cu–O planes, d-wave pairing, 2D SC, and Mott insulator parentage is not essential to high-T_c SC. A typical phase diagram is shown in Fig. 1(b). It is similar to the cuprate diagram in Fig. 1(a), with adjacent AF and superconducting phases. The SC and associated pairing in these systems seem more varied and complex than those in the cuprates. For example, Fe valence-orbital degeneracy suggests that multiple bands contribute and that several orbital geometries may be important for pairing. Thus, the Fe-based compounds give compelling evidence that high-T_c SC is compatible with a range of microscopic structures (a result foreshadowed well before the discovery of Fe-based SC [19]).

A phase diagram for a heavy-fermion superconductor is displayed in Fig. 1(c). An AF phase lies adjacent to the superconducting phase, as in the cuprate and Fe-based phase diagrams. The SC is thought to be unconventional and to involve pairs of electrons with effective masses hundreds of times that of normal electrons.

A phase diagram for an organic superconductor is displayed in Fig. 1(d). It has many similarities to that of the cuprates, with pressure replacing doping as the control parameter. The spin density waves at lower pressure are indicative of AF correlations. This and many other organic superconductors appear to be unconventional.

A generic nuclear correlation energy diagram at zero temperature is shown in Fig. 1(e). It is schematic, as nuclei have a finite valence space, and "phases" are mixed by fluctuations. Comparison with Figs. 1(a)–(d) suggest a strong analogy, with pairing playing a similar role in both cases and nuclear quadrupole deformation being the analog of condensed-matter AF correlations (an analogy that is elaborated in [22]).

A theory accounting for this diversity of behavior must exhibit several emergent-state properties: i) A robust Cooper instability arising in both Fermi-liquid and other contexts, depending on microscopic physics only through parameters. ii) Accommodation of SC/SF and other

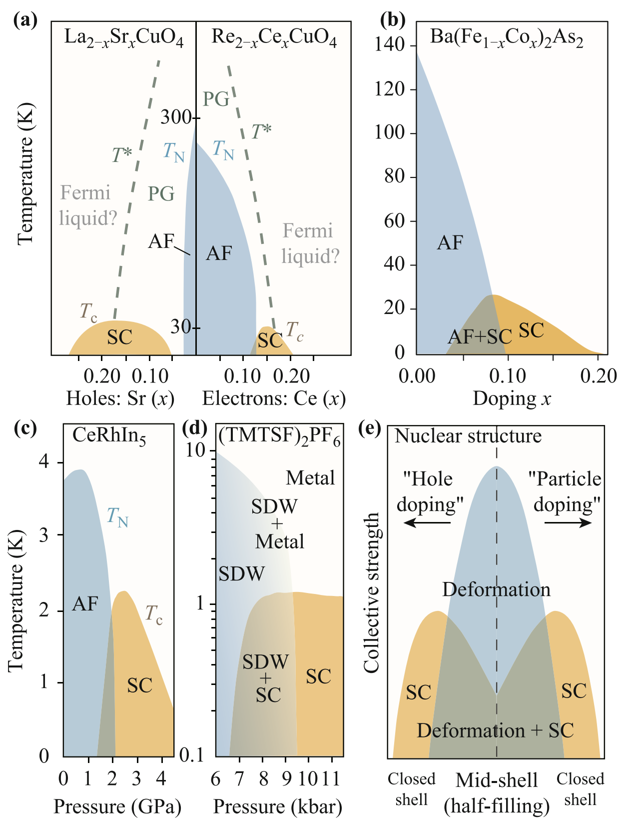

Fig. 1 **(a)** Phase diagram for hole- and electron-doped cuprates [14]. Superconducting (SC), antiferromagnetic (AF), and pseudogap (PG) regions are labeled, as are Néel (T_N), SC critical (T_c), and PG (T^*) temperatures. **(b)** Phase diagram for Fe-based SC [15]. **(c)** Phase diagram for heavy-fermion SC [16]. **(d)** Phase diagram for an organic superconductor [17] (SDW denotes spin density waves). **(e)** Generic correlation-energy diagram for nuclear structure at $T = 0$.

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emergent modes, with quantum phase transitions among these modes. iii) Limits corresponding to pure SC and to the pure collective modes that compete with SC. iv) Limits corresponding to conventional BCS. v) Spontaneous breaking of gauge and possibly other symmetries in the emergent state.

Unless we assume the great similarity of SC/SF across many disciplines to be mere coincidence, the data suggest that SC and SF must have a description that can be approximately separated into two parts: i) A *universal part* describing the essential emergent properties of SC/SF that is largely independent of microscopic specifics for the weakly interacting parent systems. ii) A *system-specific part* that can vary from case to case and parameterizes the quantitative differences between SC/SF cases without altering substantially the essence of the emergent properties.

The distinction is similar to that between a class and instances of that class in object-oriented computer programming. The class has a generic description specifying the essence of the class that may include parameters having unspecified values; various instances of that class then correspond to specific implementations (instantiations) of the class with different parameter data sets. Then different instances all inherit the same generic properties of the class but may differ from each other quantitatively because they have different parameter values.

As a simple example of this concept, consider a class specified by the minimal definition of a 2D sphere, with properties corresponding to the radius, location, and color defined but with unspecified values. Then multiple instances may correspond to spheres having different locations, radii, and colors. The instances differ, yet in a deep sense they are the same, as intuitively the specific values of the instance parameters for color, radius, and location are secondary to the essence of being a sphere.

A theory embodying these features cannot be based directly on microscopic properties, as these differ essentially between fields. The only properties that these systems share are that i) SC/SF involves Cooper pairs of fermions, possibly occurring in the presence of other collective modes, and ii) the normal system has many degrees of freedom, but the SC/SF state is phenomenologically simple and so must have only a few effective degrees of freedom.

This implies that the SC/SF state results from an enormous truncation of the Hilbert space to a simple collective subspace. The similarity of SC/SF implies that this subspace is in some sense the *same subspace* across these varied disciplines. The observed similarities across diverse systems can be ensured if the collective subspaces corresponding to SC/SF have the same symmetries of the Hamiltonian (dynamical symmetries). Then matrix elements (observables) can be similar across fields because they are determined by the symmetry, even if the microscopic content of the wavefunctions and operators (not observables) has little similarity among disciplines.

Thus, we propose that all fermionic SC and SF results from a spontaneous reorganization of the Hilbert space that transcends the microscopic details of the normal system. The generic structure of this reorganized space accounts for SC and SF in its myriad forms. Normal-state physics influences the reorganized space, but only parametrically. The pair condensate competes for Hilbert space with other emergent modes, suggesting that conventional BCS states are highly *atypical*, representing limiting cases where the space factorizes and SC/SF decouples from other emergent modes.

The most powerful means of implementing this space truncation is to identify dynamical symmetries expressed through Lie algebras and associated Lie groups [19–30]. Such methods have been applied extensively in nuclear [20, 27], elementary particle [28], molecular [29], and condensed matter physics [22]. They have exact many-body solutions for special ratios of coupling strengths and approximate solutions for all coupling strengths using generalized coherent state methods. There is no reason to expect these dynamical symmetries to be directly related to symmetries of the weakly interacting system, as the properties of emergent collective modes cannot be obtained by power series expansion from the parent system.

Such theories are designed to describe the low-energy collective states and are likely to fail outside that domain. Furthermore, on physical grounds, the effective interactions should vary smoothly with control parameters such as doping, so rapid local fluctuations reflecting inadequacies of the dynamical symmetry simplification may not always be captured. Thus, such approaches are best suited to providing simple descriptions of global behavior for highly collective states. However, that is precisely what is required for our hypothesized universal part of the SC/SF description.

One might fear that such dynamical symmetry methods imply non-unique candidate Lie algebras, but their "quantized" nature (algebras close only for certain generator numbers) and the generic properties of SC and SF states severely restrict the options. Bound states imply compact groups, and the number of low-dimensional compact Lie groups is small. (Physically, the collective degrees of freedom fit together consistently only in highly constrained ways.) Furthermore, a pair condensate is required, so the algebra must contain both fermion particle–particle and particle–hole operators, constraining the possibilities further.

A lower limit on the generator number follows from counting the physical operators. SC requires spin singlet (or triplet) pairs. However, collective modes carrying angular momentum (magnetism or quadrupole fields) mix pairs of different spin. In condensed matter, this implies both singlet and triplet pairs, and a minimum of 8 generators (creation and annihilation operators with spin degeneracy). In nuclear physics, this corresponds minimally to 12 generators, counting total angular momentum $J = 0$ and $J = 2$ pairs. An AF field adds 3 generators, a quadrupole field adds 5, and conservation of charge and spin or total angular momentum implies 4 additional generators. Adding up, we require minimally 15 generators for condensed matter and 21 for nuclear physics applications. An upper limit may be estimated by noting that previous applications to topics as complex as high-T_c SC and the structure of heavy nuclei required no more than 28 generators [20, 22, 24].

The *only candidate algebras* meeting these conditions with more than 10 and less than 35 generators are SO(8), Sp(6), SO(7), and SU(4). The highest symmetries needed for prior nuclear or condensed matter applications have been SO(8) or Sp(6), with SO(7) and SU(4) as subalgebras. Thus, we conjecture further that all fermionic SC or SF derives from SO(8) or Sp(6) dynamical symmetries. This last simplification is not essential to our argument but is consistent with present knowledge.

We have outlined a universal classification of superconducting and superfluid behavior, but we also require matrix elements for observables. Calculation of observables is documented extensively in the references, but here we give representative examples from condensed

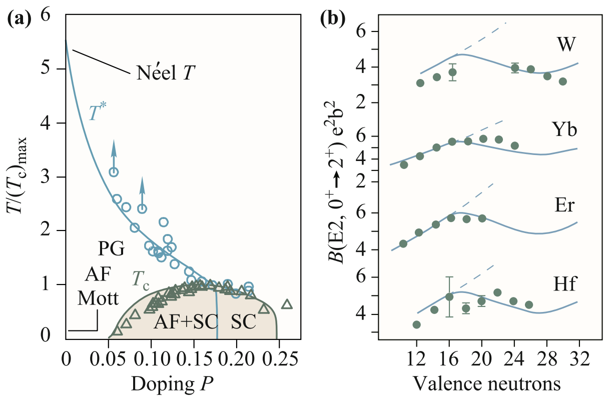

Fig. 2 **(a)** Calculated SU(4) cuprate phase diagram [23]. PG temperature is T^*, and SC transition temperature is T_c. Dominant correlations in each region are indicated by labels SC and AF. Data from [31, 32]. **(b)** Quadrupole transition rates for rare earth isotopes. Data from Ref. [33]. Dashed blue curves correspond to approximations of Cooper pairs as bosons; clearly they are bosons only for low valence-space occupancy.

matter and nuclear physics. Figure 2(a) compares a cuprate phase diagram with SU(4) model calculations [23]. The calculated phase diagram agrees quantitatively with data. In Fig. 2(b), we use the fermion dynamical symmetry model to calculate transition rates between the ground and first excited states in rare earth nuclei [20]. Again, agreement with data is quite good. Thus, fermion dynamical symmetries provide both a universal classification and methods of calculating observables within specific fields for superconducting and superfluid behavior, in possible competition with other collective modes.

The highest symmetries having multiple dynamical symmetry subchains imply competing ground states and *quantum phase transitions*. The SU(4) model of cuprate SC illustrates this. Because of SU(4) symmetry, the SC order parameter Δ satisfies

$$\left.\frac{\partial \Delta}{\partial x}\right|_{x=0} = \frac{1}{4}\left.\frac{x_q^{-1} - 2x}{[x(x_q^{-1} - x)]^{1/2}}\right|_{x=0} = \infty, \tag{1}$$

where x is the doping, and x_q is a critical doping predicted by the theory; the undoped AF Mott state is unstable against condensing pairs with infinitesimal doping for finite attractive pairing [25], as illustrated in Fig. 3(a). SU(4) symmetry also implies a second fundamental instability: the AF order parameter Q must satisfy

$$\left.\frac{\partial Q}{\partial x}\right|_{x=x_q} = -\frac{1}{4}\left.\frac{x_q + x_q^{-1} - 2x}{[(x_q - x)(x_q^{-1} - x)]^{1/2}}\right|_{x=x_q} = -\infty, \tag{2}$$

and a small change in the doping x causes the AF correlations to diverge if $x \simeq x_q$, as illustrated in Fig. 3(b). In addition, *critical dynamical symmetries*, which generalize a quantum critical point to an entire critical phase and enable various types of emergent complexity, have been observed when dynamical symmetries compete in condensed matter and nuclear systems [26, 30].

Condensed matter SO(8) ⊃ SU(4) and nuclear physics SO(8) and Sp(6) symmetries reduce to conventional or unconventional BCS SC in the limit where the non-pairing order is neglected [19, 20]. Fig. 4 illustrates the condensed matter case. The essential point is not whether SC is conventional or unconventional, as that influences only the pairing form factor, and dynamical symmetries are often compatible with a variety of form factors [19, 24]. It is the symmetry of the truncated Hilbert space that is central to understanding SC and SF, not the pairing geometry.

Our proposal has an abstract similarity to general relativity, where gravity is a universal consequence of spacetime structure, not of interactions between particles in

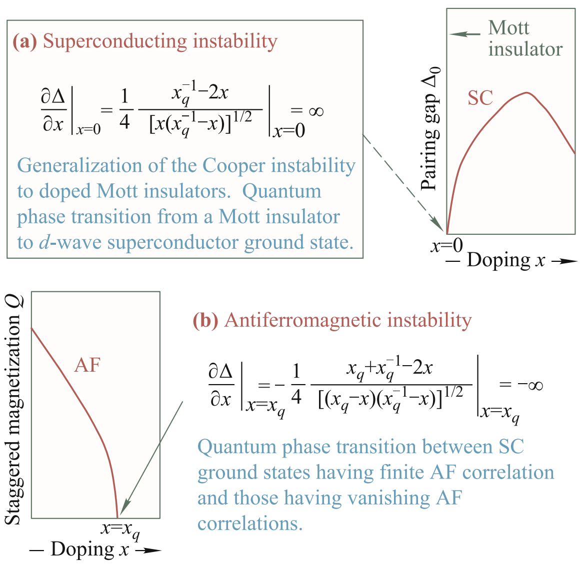

Fig. 3 **(a)** Cooper instability in a Mott insulator. SU(4) symmetry requires the ground state at half-filling to be an AF Mott insulator, which for infinitesimal hole doping becomes unstable against a quantum phase transition to a d-wave SC state if the pairing interaction is finite. **(b)** AF instability of the d-wave superconductor. Because of SU(4) symmetry, as the doping x approaches the critical doping x_q, the system becomes unstable with respect to a quantum phase transition between a superconducting state perturbed by AF correlations for $x < x_q$ and a pure superconductor with no AF correlations for $x > x_q$.

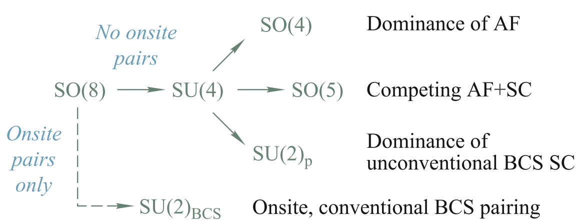

Fig. 4 Recovery of BCS states in a condensed-matter superconductor. Both $SU(2)_{BCS}$ and $SU(2)_p$ subgroups imply BCS-like states. They differ in that the pairs are onsite for $SU(2)_{BCS}$ and bondwise for $SU(2)_p$, and in that $SU(2)_{BCS}$ is consistent with conventional pairing, but $SU(2)_p$ can have unconventional pairing.

spacetime. Similarly, the universality observed in SC and SF across disciplines derives from the structure of a common Hilbert subspace selected by dynamical symmetries.

There also is an analogy with renormalization group flow, as the dynamical symmetries distinguish between "relevant operators" characterizing the collective subspace and "irrelevant operators" that differentiate microscopic systems but enter only parametrically into the collective behavior. The "flow" is in the dimensionality of the generator space; as it is decreased from that of the full Hilbert space toward that of the collective subspace, the influence of irrelevant operators falls away, leaving only relevant operators to define the SC/SF Hilbert subspace. Universality is implied because differences between fields are represented by irrelevant operators, but the relevant operators define SC/SF subspaces having common algebraic structures across fields.

Finally, we note that the global view advocated here may illuminate specific occurrences of SC and SF in particular subfields. For example, high-T_c cuprate SC becomes far less mysterious when viewed as part of a continuum of behavior shared with many other systems. The question of why cuprate SC differs so much from conventional BCS SC becomes inverted in the present view: it is the *conventional BCS superconductors* that should more properly be viewed as anomalous, in that they represent only a special limit where we may neglect other collective modes competing with SC.

In summary, a unified understanding of SC and SF cannot focus on microscopic properties of the normal state, which are not connected analytically to properties of the emergent state and may differ radically between disciplines. Nor can it focus on Fermi liquid instabilities, as these phenomena do not require Fermi liquid parentage. A common algebraic structure for the matrix elements is arguably the only framework that can unify at the emergent level but depend only parametrically on microscopic details in such diverse systems. We propose that all fermionic SC and SF results from a generalized Cooper instability manifested through fermion dynamical symmetries. All cases examined thus far in condensed matter and nuclear physics derive from the two highest symmetries, SO(8) or Sp(6), suggesting that an economical unification is possible.

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References

1. L. N. Cooper, Bound electron pairs in a degenerate Fermi gas, *Phys. Rev.* 104(4), 1189 (1956)
2. J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Theory of superconductivity, *Phys. Rev.* 108(5), 1175 (1957)
3. A. Bohr, B. R. Mottelson, and D. Pines, Possible analogy between the excitation spectra of nuclei and those of the superconducting metallic state, *Phys. Rev.* 110(4), 936 (1958)
4. P. W. Anderson, Random-phase approximation in the theory of superconductivity, *Phys. Rev.* 112(6), 1900 (1958)
5. D. D. Osheroff, R. C. Richardson, and D. M. Lee, Evidence for a new phase of solid He_3, *Phys. Rev. Lett.* 28(14), 885 (1972)
6. J. Steglich, J. Aarts, C. D. Bredl, W. Lieke, D. Meschede, W. Franz, and H. Schäfer, Superconductivity in the presence

of strong pauli paramagnetism: $CeCu_2Si_2$, *Phys. Rev. Lett.* 43(25), 1892 (1979)

7. J. G. Bednorz and K. A. Müller, Possible high T_c superconductivity in the Ba-La-Cu-O system, *Z. Phys. B* 64, 189 (1986)
8. D. Jérome, Organic superconductors: When correlations and magnetism walk in, arXiv: 1201.5796 (2012)
9. Y. Kamihara, T. Watanabe, M. Hirano, and H. Hosono, Iron-based layered superconductor $La[O_{1-x}F_x]FeAs$ (x = 0.05–0.12) with T_c = 26 K, *J. Am. Chem. Soc.* 130(11), 3296 (2008)
10. D. Page, J. M. Lattimer, M. Prakash, and A. W. Steiner, Stellar superfluids, arXiv: 1302.6626 (2013)
11. M. W. Zwierlein, J. R. Abo-Shaeer, A. Schirotzek, C. H. Schunck, and W. Ketterle, Vortices and superfluidity in a strongly interacting Fermi gas, *Nature* 435(7045), 1047 (2005)
12. Q. Chen and J. Wang, Pseudogap phenomena in ultracold atomic Fermi gases, *Front. Phys.* 9(5), 539 (2014)
13. W. Yi, W. Zhang, and X. L. Cui, Pairing superfluidity in spin-orbit coupled ultracold Fermi gases, *Sci. China-Phys. Mech. Astron.* 58(1), 014201 (2015)
14. M. R. Norman, Unconventional superconductivity, arXiv: 1302.3176 (2013)
15. L. Fang, H. Q. Luo, P. Cheng, Z. S. Wang, Y. Jia, G. Mu, B. Shen, I. I. Mazin, L.Shan, C. Ren, and H.-H. Wen, Roles of multiband effects and electron-hole asymmetry in the superconductivity and normal-state properties of $Ba(Fe_{1-x}Co_x)_2As_2$, *Phys. Rev. B* 80, 140508(R) (2009)
16. G. Knebel, D. Aoki, and J. Flouquet, Magnetism and superconductivity in $CeRhIn_5$, arXiv: 0911.5223 (2009)
17. N. Kang, B. Salameh, P. Auban-Senzier, D. Jérome, C. R. Pasquier, and S. Brazovskii, Domain walls at the spin-density-wave endpoint of the organic superconductor $(TMTSF)_2PF_6$ under pressure, *Phys. Rev. B* 81, 100509(R), (2010)
18. J. Tao, S. Li, X. Y. Zhu, H. Yang, and H. H. Wen, Growth and transport properties of $CaFeAsF_{1-x}$ single crystals, *Sci. China-Phys. Mech. Astron.* 57(4), 632 (2014)
19. M. W. Guidry, Y. Sun, and C. L. Wu, Mott insulators, no double occupancy, and non-Abelian superconductivity, *Phys. Rev. B* 70(18), 184501 (2004)
20. C. L. Wu, D. H. Feng, and M. W. Guidry, The fermion dynamical symmetry model, *Adv. Nucl. Phys.* 21, 227 (1994)
21. C. L. Wu, D. H. Feng, X. G. Chen, J. Q. Chen, and M. W. Guidry, Fermion dynamical symmetries and the nuclear shell model, *Phys. Lett. B* 168(4), 313 (1986)
22. M. W. Guidry, L. A. Wu, Y. Sun, and C. L. Wu, SU(4) model of high-temperature superconductivity and antiferromagnetism, *Phys. Rev. B* 63(13), 134516 (2001)
23. Y. Sun, M. W. Guidry, and C. L. Wu, Pairing gaps, pseudogaps, and phase diagrams for cuprate superconductors, *Phys. Rev. B* 75(13), 134511 (2007)
24. M. W. Guidry, Y. Sun, and C. L. Wu, A unified description of cuprate and iron arsenide superconductors, *Front. Phys. China* 4(4), 433 (2009)
25. M. Guidry, Y. Sun, and C.-L.Wu, Generalizing the Cooper pair instability to doped Mott insulators, *Front. Phys. China* 5(2), 171 (2010)
26. M. W. Guidry, Y. Sun, and C. L. Wu, Inhomogeneity, dynamical symmetry, and complexity in high-temperature superconductors: Reconciling a universal phase diagram with rich local disorder, *Chin. Sci. Bull.* 56(4–5), 367 (2011)
27. F. Iachello and A. Arima, The Interacting Boson Model, Cambridge: Cambridge University Press, 1987
28. R. Bijker, F. Iachello, and A. Leviatan, Algebraic models of hadron structure (I): Nonstrange baryons, *Ann. Phys.* 236(1), 69 (1994)
29. F. Iachello and R. D. Levine, Algebraic Theory of Molecules, Oxford: Oxford University Press, 1995
30. W. M. Zhang, D. H. Feng, and J. N. Ginocchio, Geometrical interpretation of SO(7): A critical dynamical symmetry, *Phys. Rev. Lett.* 59(18), 2032 (1987)
31. P. Dai, H. A. Mook, S. M. Hayden, G. Aeppli, T. G. Perring, R. D. Hunt, and F. Doğan, The magnetic excitation spectrum and thermodynamics of high-T_c superconductors, *Science* 284(5418), 1344 (1999)
32. J. C. Campuzano, H. Ding, M. R. Norman, H. M. Fretwell, M. Randeria, A. Kaminski, J. Mesot, T. Takeuchi, T. Sato, T. Yokoya, T. Takahashi, T. Mochiku, K. Kadowaki, P. Guptasarma, D. G. Hinks, Z. Konstantinovic, Z. Z. Li, and H. Raffy, Electronic spectra and their relation to the (π,π) collective mode in high-T_c superconductors, *Phys. Rev. Lett.* 83(18), 3709 (1999)
33. S. Raman, C. H. Malarkey, W. T. Milner, P. H. Jr Nestor, and P. H. Stelson, Transition probability, B(E2)$\uparrow$, from the ground to the first-excited 2^+ state of even-even nuclides, *At. Data Nucl. Data Tables* 36, 1 (1987)