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RESEARCH ARTICLE

Conditions for ponderomotive resonances in the Kapitza–Dirac effect

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By applying a nonperturbative quantum electrodynamic theory, we study ponderomotive resonances when an electron beam is scattered by a standing photon wave. Our study shows that the ponderomotive parameter u_p, the ponderomotive energy per laser-photon energy, for each of the two traveling laser modes possesses a minimum value $\hbar\omega/(m_e c^2)$. Ponderomotive resonances occur only when the ratio of the laser photon energy to the electron rest-mass energy is a fraction, where the denominator is twice the square of a positive integer and the numerator is the total ponderomotive number, which is also a positive integer.

Keywords Kapitza–Dirac effect, strong laser physics, nonperturbative quantum electrodynamics

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1 Introduction

In 1933, Kapitza and Dirac (KD) [1] predicted that electrons could be reflected by standing-wave light. In their prediction, the reflection of an electron was caused by absorption of a photon in one traveling mode of the standing-wave light followed by a stimulated emission of a photon in the other traveling mode. Thus, the electron would experience a momentum change as much as that of two photons in either direction of the two traveling waves of light. The originally predicted electron incident angle and reflection angle were subject to Bragg's law governing X-ray scattering in crystal lattices, in which the half wavelength of the standing-wave light played a role of the lattice spacing constant.

Shortly after the invention of the laser, several unsuccessful experimental attempts were made to observe this effect. The early experiments [2–4] only observed some electron recoil in a standing laser wave, but did not confirm the predicted electron scattering, which should obey Bragg's law. The deep splitting of photoelectron angular distributions in standing-wave multiphoton ionization (MPI) observed by Bucksbaum *et al.* [5] was the only evidence supporting KD's conjecture. They interpreted the effect as the intense-field KD effect. More than a decade ago, Batelaan *et al.* [6] observed a resolved diffraction pattern formed by electrons passing through a standing-wave light beam, which consisted of two opposite travelling waves. The diffraction pattern did not obey Bragg's law.

Early papers on the theory of the KD effect [7–12] are largely based on perturbative quantum mechanics, with classical field treatments. Early quantum-field descriptions of the KD effect started with Guo and Drake's treatment [14] of the angular splitting observed by Bucksbaum *et al.* [5], which can be called the half KD effect. The treatment [14] is based on the nonperturbative quantum electrodynamic (NPQED) scattering theory developed by Guo, Åberg, and Crasemann (GAC) [15], where the quantum-field Volkov states play a role as intermediate states. In the early descriptions, the NPQED theory predicted some quantum-field effects, which have not been and can hardly be predicted by classical-field theories. In Guo and Drake's interpretation of the angular splitting observed by Bucksbaum *et al.*, a multiple splitting was implied by their formulas owing to the ponderomotive decay of an integer number of antinode photons. A notable success of NPQED theory was the resolution of the final-state problem in MPI.

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Several classical-field theories claimed that the electron final state was a Volkov state [16, 17], i.e., the well-known Keldysh ansatz. The nonperturbative scattering theory of GAC asserted that the Volkov states can only be the intermediate states while the true final state in MPI is an electron photon separated plane wave. In Guo and Drake's analysis, GAC's assertion was verified by Bucksbaum *et al.*'s experiment which also showed the half KD effect. The half KD effect disproved the Keldysh ansatz. It is not possible, at least up to now, to use a classical-field theory to justify the Keldysh ansatz. However, the NPQED theory with experimental measurements was able to achieve this. In addition to observing possible quantum-field effects, another notable advantage of using the NPQED theory is that the entire interacting system is treated as a closed system, where step-by-step energy and momentum conservation can be achieved. In contrast to the classical-field treatment, in the NPQED theory, the radiation field is treated as an internal field. Recent developments in the NPQED theory, which include the exact solutions to driven N-level atoms [18] and modification of the cutoff law in high harmonic generation [19–21], further manifest this advantage.

The first theoretical indication of ponderomotive resonances was from the NPQED treatment of above-threshold ionization (ATI) [15], where, with the single-mode assumption of the radiation field, the ATI rate vanished when the ponderomotive parameter u_p, the ponderomotive energy per laser-photon energy, was not an integer. With the inclusion of spontaneously emitted modes [22], the ATI rate is nonzero when u_p is not an integer, while it is infinite when u_p is an integer. In these two treatments, the ratios of the ATI rate in the u_p integer case to that in the u_p non-integer case are all infinite. Thus, we call the phenomenon ponderomotive resonance. Ponderomotive resonance can drastically increase the MPI rates. The produced electron beams can be used as beam sources of a table-top accelerator to perform collision experiments, which are done in huge accelerator laboratories; and directly be used in industrial techniques such as the lithography technique to fabricate integrated circuits. Since ponderomotive resonance was predicted theoretically, there is no experimental verification of this effect. Hence, it is necessary to re-examine the conditions for ponderomotive resonance. The new condition presented in this paper may offer more possibilities for experimentalists to observe this predicted effect.

In this paper, following the nonperturbative scattering approach [14, 15, 23] with a refined treatment of ponderomotive energy and momentum in the KD effect, we study the conditions for ponderomotive resonances in a standing-wave case.

2 Theory of electron scattering in a photon standing wave

We assume that a scattering state satisfies the Lippmann-Schwinger equation [24, 25]

$$\Psi_i^+ = \phi_i + \frac{1}{\mathcal{E}_i - H_0 + \mathrm{i}\varepsilon} V \Psi_i^+, \tag{1}$$

where H_0 is the non-interaction Hamiltonian, which includes the energy of the free electron and the energy of the free photons, V is the interaction energy due to the interaction between the electron and the standing photon wave, and ϕ_i is an eigenstate of H_0. i represents the initial state and f represents final state. By projecting Ψ_i^+ onto a final plane wave ϕ_f, we have

$$\Omega_{fi} \equiv \langle \phi_f | \Psi_i^+ \rangle = \delta_{fi} + \frac{1}{\mathcal{E}_i - \mathcal{E}_f + \mathrm{i}\varepsilon} \langle \phi_f | V | \Psi_i^+ \rangle, \tag{2}$$

i.e.,

$$\begin{aligned} \Omega_{fi} - \delta_{fi} = & -\mathrm{i}\pi\delta(\mathcal{E}_f - \mathcal{E}_i)\langle \phi_f | V | \Psi_i^+ \rangle \\ & + \mathcal{P}\frac{1}{\mathcal{E}_i - \mathcal{E}_f}\langle \phi_f | V | \Psi_i^+ \rangle, \end{aligned} \tag{3}$$

where Ω_{fi} is the Møller operator matrix element and $\mathcal{P}$ means taking the principal value. By limiting transitions on the energy shell, one gets

$$\begin{aligned} & \Omega_{fi} - \delta_{fi} = -\mathrm{i}\pi\delta(\mathcal{E}_f - \mathcal{E}_i) T_{fi}, \\ & T_{fi} \equiv \langle \phi_f | V | \Psi_i^+ \rangle, \quad (\text{for} \quad \mathcal{E}_i = \mathcal{E}_f). \end{aligned}$$

The transition rate in the momentum space, $\mathrm{d}W/\mathrm{d}\boldsymbol{P}_f$, is obtained by

$$\frac{\mathrm{d}W}{\mathrm{d}\boldsymbol{P}_f} = \frac{4}{T}|\Omega_{fi} - \delta_{fi}|^2 \tag{4}$$

where T is the total interaction time. According to GAC's theory, the scattering state Ψ_i^+ can be expanded using the complete set of eigenstates of the full Hamiltonian H. Thus, by using only the leading term, one gets

$$\Psi_i^+ = \sum_{(\mu, \mathcal{E}_\mu = \mathcal{E}_i)} | \Psi_\mu \rangle \langle \Psi_\mu | \phi_i \rangle \tag{5}$$

where Ψ_μ represents the eigenstates of H. The Møller operator matrix element in this kind of scattering states has the form

$$\Omega_{fi} = \sum_{(\mu, \mathcal{E}_\mu = \mathcal{E}_i)} \langle \phi_f | \Psi_\mu \rangle \langle \Psi_\mu | \phi_i \rangle. \tag{6}$$

For deriving Eq. (4), we need a formula to treat the square of the energy δ-function. In later sections, we need to treat the squares of momentum δ-functions. For this, we use the following formulas [23]:

$$\left[\frac{2\pi}{T}\delta(\mathcal{E}_f-\mathcal{E}_i)\right]^2=\frac{2\pi}{T}\delta(\mathcal{E}_f-\mathcal{E}_i),$$
$$\left[\frac{(2\pi)^3}{V_e}\delta(\boldsymbol{P}_i-\boldsymbol{P}_f)\right]^2=\frac{(2\pi)^3}{V_e}\delta(\boldsymbol{P}_i-\boldsymbol{P}_f). \quad (7)$$

In the experiments of Bucksbaum *et al.* [5] and Batelaan *et al.* [6], the standing waves were made by two opposite propagating laser beams. In our theory, the vector potential of the standing-wave field is constructed using two anti-propagating laser modes and is given by

$$\boldsymbol{A}(-\boldsymbol{k}\cdot\boldsymbol{r})=\boldsymbol{A}_1(-\boldsymbol{k}\cdot\boldsymbol{r})+\boldsymbol{A}_2(\boldsymbol{k}\cdot\boldsymbol{r}), \quad (8)$$

where

$$\boldsymbol{A}_i(-\boldsymbol{k}\cdot\boldsymbol{r})=g_i(\boldsymbol{\epsilon}_i\mathrm{e}^{\mathrm{i}\boldsymbol{k}_i\cdot\boldsymbol{r}}a_i+\boldsymbol{\epsilon}_i^*\mathrm{e}^{-\mathrm{i}\boldsymbol{k}_i\cdot\boldsymbol{r}}a_i^\dagger), \quad (i=1,2)$$

represents the quantized vector potentials of propagating modes, with $g_1=g_2=g=(2V_\gamma\omega)^{-\frac{1}{2}}$ and V_γ being the normalization volume of the photon field. When the two modes have the spatially same polarization, the polarization vectors $\boldsymbol{\epsilon}_i$, $i=1,2$, are defined by

$$\boldsymbol{\epsilon}_1=\boldsymbol{\epsilon}_2=[\boldsymbol{\epsilon}_x\cos(\xi/2)+\mathrm{i}\boldsymbol{\epsilon}_y\sin(\xi/2)], \quad (9)$$

and satisfy

$$\boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}^*=1, \quad \boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}=\cos\xi, \quad \boldsymbol{\epsilon}^*\cdot\boldsymbol{\epsilon}^*=\cos\xi,$$

where the angle ξ is the polarization degree. In the case of circular polarization, the polarization vector is simplified as

$$\boldsymbol{\epsilon}=\frac{1}{\sqrt{2}}(\boldsymbol{\epsilon}_x+\mathrm{i}\boldsymbol{\epsilon}_y).$$

The Hamiltonian for a non-relativistic electron in a quantized standing-wave field constructed according to the minimum-coupling principle is

$$H=\frac{(-\mathrm{i}\nabla)^2}{2m_e}+\frac{e\boldsymbol{A}(-\boldsymbol{k}\cdot\boldsymbol{r})^2}{2m_e}+\omega(N_{a_1}+N_{a_2})$$
$$-\frac{e}{2m_e}[(-\mathrm{i}\nabla)\cdot\boldsymbol{A}(-\boldsymbol{k}\cdot\boldsymbol{r})+\boldsymbol{A}(-\boldsymbol{k}\cdot\boldsymbol{r})\cdot(-\mathrm{i}\nabla)]. \quad (10)$$

When the two beams are circularly polarized with the same spatial angular momentum, the quantum-field Volkov states and their energy eigenvalues are derived as [23]

$$\Psi_\mu=V_e^{-1/2}\mathrm{e}^{\mathrm{i}[\boldsymbol{P}_\mu-\boldsymbol{k}(N_{a_1}-N_{a_2})]\cdot\boldsymbol{r}}\sum_j|n_1$$
$$+j,n_2\rangle_c X_{-j}(z),$$
$$\mathcal{E}_\mu=\frac{\boldsymbol{P}_\mu^2}{2m_e}+(n_1+1/2)\omega+(n_2+1/2)\omega+2u_p\omega. \quad (11)$$

where $2u_p\omega$ is the total ponderomotive energy with

$$u_p\equiv U_p/\omega=\frac{|e|^2\Lambda^2}{m_e\omega},$$

and 2Λ is the classical amplitude for the vector potential of each photon mode. The other terms in the above equation are defined as follows. The photon states $|n_1,n_2\rangle_c$ are defined by

$$|n_1,n_2\rangle_c=\frac{(c_1^\dagger)^{n_1}}{\sqrt{n_1!}}\frac{(c_2^\dagger)^{n_2}}{\sqrt{n_2!}}|0,0\rangle. \quad (12)$$

The photon operators c_1 and c_2 represent the two normal modes recombined by the original modes as

$$c_1=\frac{1}{\sqrt{2}}(a_1+a_2),$$
$$c_2=\frac{1}{\sqrt{2}}(a_1-a_2). \quad (13)$$

The notations N_{a_1} and N_{a_2} stand for the number operators of the original photon modes defined by

$$N_{a_i}=\frac{1}{2}(a_ia_i^\dagger+a_i^\dagger a_i), \quad (i=1,2).$$

To calculate the transition matrix element, we use the large-photon-number (LPN) limit. In the limit, any field photon number, e.g., n, tends to infinity and the interaction coefficient g is infinitesimal such that $g\sqrt{n}\rightarrow\Lambda$. From the expression

$$u_p=\frac{e^2\Lambda^2}{m_e\omega}=\frac{2\pi e^2I}{m_e\hbar c\omega^3},$$

one can relate Λ to u_p and the laser-field intensity I. The phased Bessel functions are defined by [26]

$$X_j(z)\equiv J_j(|z|)\mathrm{e}^{\mathrm{i}j\arg(z)}. \quad (14)$$

Here, the complex argument z is given by

$$z=\frac{2\sqrt{2}|e|\Lambda}{m_e\omega}\boldsymbol{P}_\mu\cdot\boldsymbol{\epsilon}.$$

This wave function has a unique advantage because its momentum phase contains an operator difference, $\boldsymbol{k}(N_{a_1}-N_{a_2})$, allowing an arbitrarily large momentum transfer between the two laser modes. For the original classical-field Volkov wave functions obtained by Volkov, all the modes propagated in one direction; hence, the standing-wave case was not considered. Thus far, the wave function shown in Eq. (11) has no classical-field correspondence [27]. The first overlap factor in the Møller operator matrix element is

$$\langle\Psi_\mu\mid\phi_i;l_1,l_2\rangle=\frac{(2\pi)^3}{V_e}\delta(\boldsymbol{P}_i-\boldsymbol{P}_\mu+l_1\boldsymbol{k}-l_2\boldsymbol{k})$$
$$\times\sum_j\langle n_1+j,n_2|_c l_1,l_2\rangle X_{-j}(z)^*.$$

The photon transition factor was evaluated in the LPN limit as follows (for more details, see the Appendix of Ref. [14]:

$$\langle n_1+j, n_2 |_c l_1, l_2\rangle = 2^{-l}\left[\begin{pmatrix} 2l-n_2 \\ l-n_2/2 \end{pmatrix}\begin{pmatrix} n_2 \\ n_2/2 \end{pmatrix}\right]^{\frac{1}{2}} \times(-1)^{n_2/2}\cos(\Delta_i\gamma)\delta_{l_1+l_2,n_1+n_2+j}. \quad (15)$$

where we assume that the total initial photon number, l_1+l_2, is an even number for convenience:

$$2l = l_1+l_2, \quad \Delta_i \equiv l_2 - l_1 \ll l, \quad \gamma = \arccos\left(\frac{2l-n_2}{2l}\right)^{1/2}.$$

The other overlap factor in the Møller operator matrix element is the Hermit conjugate of the first one, where i is replaced by f, l_1 and l_2 is replaced by m_1 and m_2, the final laser photon numbers.

The Møller operator matrix element has an explicit form:

$$\Omega_{fi} = \frac{(2\pi)^3}{V_e}\delta(\boldsymbol{P}_i - \boldsymbol{P}_f + \Delta_f\boldsymbol{k} - \Delta_i\boldsymbol{k}) \times\sum_{j,s} X_{-j}(z)^* X_{-s}(z) F_{j-s}, \quad (16)$$

where the factor F is

$$F_{j-s} \equiv \sum_{n_2}\langle m_1, m_2 \mid n_1+s, n_2\rangle_c\langle n_1+j, n_2 |_c l_1, l_2\rangle = \frac{1}{\pi}\left\{\frac{\sin[(\Delta_f+\Delta_i)\pi/2]}{\Delta_f+\Delta_i} + \frac{\sin[(\Delta_f-\Delta_i)\pi/2]}{\Delta_f-\Delta_i}\right\} \times\delta_{l_1+l_2-m_1-m_2,j-s}. \quad (17)$$

Here, δ-symbol gives the value of the net transferred photon number $j-s$, with j being the absorbed antinode-photon number when the electron enters the radiation field and s being the emitted antinode-photon number when it leaves the field. Thus,

$$j = l_1+l_2-n_1-n_2, \quad s = m_1+m_2-n_1-n_2.$$

Next, we determine the values for Δ_i and Δ_f, and set the constraints for j, s, and the incident angle θ_i. We consider the entry process first. The energy conservation in the entry process gives

$$\frac{\boldsymbol{P}_\mu^2}{2m_e} + (2u_p - j)\omega = \frac{\boldsymbol{P}_i^2}{2m_e}. \quad (18)$$

By combining the equation above with the momentum conservation determined by the momentum δ function,

$$\boldsymbol{P}_\mu = \boldsymbol{P}_i - \Delta_i\boldsymbol{k}, \quad (19)$$

we obtain a quadratic equation for Δ_i:

$$\Delta_i^2 - 2\Delta_i|\boldsymbol{P}_i|\omega^{-1}\cos\theta_i + 2m_e\omega^{-1}(2u_p - j) = 0, \quad (20)$$

with two solutions

$$\Delta_i^{(1,2)} = \{|\boldsymbol{P}_i|\cos\theta_i \mp [\boldsymbol{P}_i^2\cos^2\theta_i - 2m_e(2u_p-j)\omega]^{1/2}\}/\omega. \quad (21)$$

The existence condition for Δ_i from the equations above is

$$(2u_p - j)\omega \leqslant \frac{\boldsymbol{P}_i^2}{2m_e}\cos^2\theta_i. \quad (22)$$

Hence, we can obtain the constraint between j and the incident angle θ_i. Within the constraint, there are still three cases defined according to the values of j: (a) $j > 2u_p$, extra-absorption case or accelerating case; (b) $0 \leqslant j \leqslant 2u_p$, absorption case or decelerating case; (c) $j < 0$, emission case or extra-decelerating case.

In the experiment conducted by Batelaan *et al.* [6], the ponderomotive parameter $2u_p$ is much less than 1 and the incident angle θ_i is approximately 90°. From the equation above, we must have $2u_p - j < 0$. To guarantee an entry, the electron has to absorb at least one photon to gain the required kinetic energy, such that $j = 1, 2, 3, \cdots$. Thus, we consider case (a). In this case, when the electron enters the field, it absorbs extra photons beyond the number needed for obtaining the ponderomotive energy. The extra energy $(j - 2u_p)\omega$ turns into electron kinetic energy. The electron is then accelerated, i.e., $|\ \boldsymbol{P}_\mu\ |>|\ \boldsymbol{P}_i\ |$. The electron experiences a big deflection due to the extra antinode-photon absorption corresponding to a background photon number change $\Delta_i = \pm\sqrt{2m_e\omega^{-1}(j-2u_p)}$. In the experiment conducted by Batelaan *et al.*, $2u_p$ is around the order of magnitude 0.000567 and can be ignored in the equation, which guides $\Delta_i = \pm\sqrt{2m_e/\omega}$, with one extra-photon absorption, of ±666.2. In the four-momentum space, the absorption of one antinode photon in the energy direction causes a photon number change $\sqrt{2m_e/\omega}$ in the photon $\boldsymbol{k}$ direction. For $j = 2, 3, \cdots$, the background photon number change will be much greater. The electron with such a large deflection may exit the field directly to form two jth-order sidebands in the electron momentum spectrum or exit the field with a s-antinode-photon emission to form two qth-order (noting that $q \equiv j - s$) sidebands in contrast to the central band observed by Batelaan *et al.*

Now, we consider the exit process. It is the reverse of the entry process, which has been treated in earlier papers [14]. Here, we list all the necessary equations:

$$\Delta_f^2 - 2\Delta_f|\boldsymbol{P}_f|\omega^{-1}\cos\theta_f + 2m_e\omega^{-1}(2u_p - s) = 0.$$

$$\frac{\boldsymbol{P}_\mu^2}{2m_e} + (2u_p - s)\omega = \frac{\boldsymbol{P}_f^2}{2m_e},$$

$$\boldsymbol{P}_\mu = \boldsymbol{P}_f - \Delta_f \boldsymbol{k}. \tag{23}$$

The momentum and energy conservation relations between the initial and the final states are

$$\begin{aligned} &\Delta^2 - 2\Delta|\boldsymbol{P}_i|\omega^{-1}\cos\theta_i - 2m_e\omega^{-1}(j-s) = 0,\\ &\frac{\boldsymbol{P}_i^2}{2m_e} = \frac{\boldsymbol{P}_f^2}{2m_e} + (s-j)\omega,\\ &\boldsymbol{P}_i = \boldsymbol{P}_f + \Delta\boldsymbol{k}, \end{aligned} \tag{24}$$

where $\Delta = \Delta_i - \Delta_f$. These relations determine the value of Δ_f directly from parameters of the initial state.

From Eq. (24), in the $j = s$ case, we have either $\Delta_f = \Delta_i$, the penetration case, or $(\Delta_f - \Delta_i) = -2 \mid \boldsymbol{P}_i \mid \omega^{-1}\cos\theta_i$, the reflection case, satisfying Bragg's law. In the case where $j = 1, s = 0$, with a small value of $2u_p$ (e.g., approximately 0.0005), $\Delta_f - \Delta_i$ satisfies almost the same quadratic algebraic equation satisfied by Δ_i; hence, $\Delta_f = 0$ and each sideband can be irresolved as one bright line.

From the preceding discussion, we conclude that in the case of a strictly anti-parallel standing light wave, an electron beam passing through the standing wave (injected perpendicularly) will not produce diffraction peaks in the central band; however, it will produce sidebands. The sidebands are located at two sides far away from the central position. For an electron beam not injected perpendicularly, Bragg's scattering angles are enforced to guarantee a stimulated emission [1]. The observed electron Bragg scattering by Freimund and Batelaan also confirms Bragg's law [28, 29].

3 Minimum quantum of the ponderomotive number

By analyzing either the exit or the entry process, we can determine the minimum quantum of the ponderomotive number u_p. Considering the entry process, the energy conservation for electron entry without absorption of an extra antinode photon is

$$(\boldsymbol{P}_i^2/2m_e) - (\boldsymbol{P}_\mu^2/2m_e) = 2u_p\omega. \tag{25}$$

To form a Volkov state in which the electron momentum is perpendicular to the photon momentum $\boldsymbol{k}$, the electron has to enter the field at the appropriate Bragg's angle. The minimum background photon number change is 2. Using momentum conservation and the Pythagorean theorem in the momentum space, we obtain

$$\boldsymbol{P}_i^2 - \boldsymbol{P}_\mu^2 = (\Delta_i\boldsymbol{k})^2, \quad (\Delta_i = \pm 2, \pm 4, \pm 6, \cdots), \tag{26}$$

where Δ_i is the change of the background photon number. It can only be an even number if there is no extra antinode-photon absorption and emission. In this case, the total photon number (see Eq. (15)) remains the same as in the entry process. Thus, all photon number changes should be an even number. Combining the two above equations, we obtain the value of the ponderomotive parameter for one laser beam:

$$u_p(d) = d^2\frac{\omega}{m_e}, \tag{27}$$

where $d = |\Delta_i|/2 = 1, 2, 3\cdots$. When $d = 1$, one obtains the minimum quantum of the ponderomotive parameter, $u_p(1) = \omega/m_e$, which corresponds to the first pair of diffraction peaks in Batelaan *et al.*'s experiment. If this pair is the only one observed, the critical laser-beam intensity can be obtained:

$$I_c = \frac{\omega^4}{2\pi e^2}, \tag{28}$$

using the formula $I = u_p m_e \omega^3/2\pi e^2$. When the laser beam intensity falls below this critical value, no electron diffraction peaks can be formed.

Table 1 The critical laser intensity for KD diffraction effect corresponding to the laser wavelength.

λ (nm)	267	532	800	1064	1800
$I(10^8\mathrm{W/cm^2})$	63.39	4.022	0.7866	0.2514	0.03069

4 Conditions for ponderomotive resonances

When the ponderomotive parameter $2u_p \equiv 2U_p/\hbar\omega$ is an integer, ponderomotive resonances may occur. The ponderomotive resonances for the single-mode case have been discussed [22]. In the standing-wave case, since there is a minimum ponderomotive energy requirement (Eq. (27)), when the total ponderomotive parameter is a positive integer n, i.e.,

$$2u_p = n, \quad (n = 1, 2, 3, \cdots) \tag{29}$$

we have

$$n = 2d^2\frac{\omega}{m_e}, \tag{30}$$

and

$$\frac{\hbar\omega}{m_e c^2} = \frac{n}{2d^2}, \tag{31}$$

with writing the Planck constant $\hbar$ and the speed of light c explicitly. The left side of above equation is the ratio between the laser photon energy and the electron mass energy. Thus, we reach an amazing property of the photon-electron interaction: ponderomotive resonances can occur only when the ratio of the laser photon energy to the electron rest-mass energy is a fraction, where the denominator is twice the square of a positive integer and the numerator is the total ponderomotive parameter, which is also a positive integer.

Now, we consider a special case, the first-order ponderomotive resonance, where $n = 1$. The condition for this case can be expressed as

$$\sqrt{\frac{2m_e c^2}{\hbar\omega}} = 2d = \mid \Delta_i \mid, \tag{32}$$

which is a positive even number.

As an application, we consider an experimental case where the conditions are similar to those described by Batelaan *et al.* [6]. In the Batelaan *et al.*'s experiment, the laser wavelength is 532 nm. From Eq. (32), we obtain

$$\Delta_i = 662.212\,89,$$

which does not satisfy the even-number condition. To observe first-order ponderomotive resonance, where $2u_p = 1$, the laser frequency should be tuned such that

$$\Delta_1 = 662,$$

which corresponds to the laser wavelength

$$\lambda_1 = 531.658 \text{ nm}.$$

At this wavelength, the laser beam intensity to guarantee the occurrence of ponderomotive resonance is

$$I = 8.836 \times 10^{13} \text{ W/cm}^2.$$

5 Conclusions

We conclude that when an electron is scattered by a photon standing wave, i) the ponderomotive number for each beam has a minimum $u_p(1) = \hbar\omega/(m_e c^2)$, corresponding to the first pair of electron diffraction peaks, and other small ponderomotive numbers $u_p(d) = d^2\hbar\omega/m_e c^2$, $(d = 2, 3, 4, \cdots)$, corresponding to other pairs of the diffraction peaks; ii) ponderomotive resonances occur only when the ratio of the laser photon energy to the electron rest-mass energy, as the minimum value of the ponderomotive parameter of one beam, is a fraction number, where the denominator is twice the square of a positive integer and the numerator is the total ponderomotive number, which is also a positive integer.

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