Front. Phys. 10, 100305 (2015)
DOI 10.1007/s11467-015-0470-4

RESEARCH ARTICLE

A possible interplay between electron beams and magnetic fluxes in the Aharonov–Bohm effect

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Received November 5, 2014; accepted March 4, 2015

Most studies on the magnetic Aharonov–Bohm (A–B) effect focus on the action exerted by the magnetic flux on the electron beam, but neglect the back-action exerted by the electron beam on the magnetic flux. This paper focuses on the latter, which is the electromotive force ΔU across the solenoid induced by the time-dependent magnetic field of the electron beam. Based on the back-action analysis, we observe that the magnetic A–B effect arises owing to the interaction energy between the magnetic field of the electron beam and the magnetic field of the solenoid. We also demonstrate that the interpretation attributing the magnetic A–B effect to the vector potential violates the uncertainty principle.

Keywords Aharonov–Bohm effect, uncertainty principle, superconductivity, superconducting quantum interference device (SQUID)

PACS numbers 03.65.Vf, 03.65.Ta, 85.25.Dq

1 Introduction

Quantum-mechanical measurement is quite different from the classical one. In classical physics, the disturbance on the measured object can be considered arbitrarily small and the measurement's precision is virtually unlimited. One can simultaneously measure the position and momentum of an object with arbitrarily high precision. However, in quantum mechanics it is impossible to measure an object without exerting some disturbance on it, a concept expressed in Heisenberg's uncertainty principle. The more precise the measurement is, the stronger is the disturbance exerted on the measured object [1, 2]. If the Aharonov–Bohm (A–B) effect is considered from the viewpoint of quantum-mechanical measurement, some new perspectives must be introduced.

In 1959, Aharonov and Bohm [3] predicted that the phase of an electron could be affected by the electromagnetic potentials describing the excluded electromagnetic fields, even though they always move in the region where both the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$ are zero. This effect is today known as the A–B effect. The A–B effect includes the electric A–B effect and the magnetic A–B effect. The existence of the magnetic A–B effect has been supported by some experiments [4], especially the experiments by Tonomura *et al.* that were reported in 1986 [5]. The interpretations of the A–B effect can be classified into two categories. The first interpretation asserts that the effect is owing to the electromagnetic potentials describing the excluded electromagnetic fields [3, 6, 7], i.e., the electromagnetic potentials can result in some observable quantum-mechanical phenomena, although they are only mathematical fields in classical physics. In this paper, this interpretation will be called "the interpretation of electromagnetic potentials." The second interpretation asserts that the effect arises owing to the interaction energy between the electron's electromagnetic fields and the excluded electromagnetic fields [8–10]. In what follows, this interpretation will be called "the interpretation of interaction energy." This paper will only discuss the magnetic A–B effect, because the electric A–B effect has been discussed elsewhere [11].

Up to date, most studies on the magnetic A–B effect focused on the action exerted by the magnetic flux on the electron beam, and only several studies [12, 13] discussed the back-action exerted by the electron beam on the magnetic flux (or on the current through the solenoid that produces this magnetic flux). In Ref. [12], it was proposed that a which-way experiment using this back-action would destroy the interference pattern. Recently, in Ref. [13], it was proposed that this back-action will result in entanglements between the electron and the potential's source.

Here, we focus on the back-action on the magnetic flux, which is the electromotive force ΔU across the solenoid induced by the time-dependent magnetic field of the electron beam. We demonstrate that the action exerted by the magnetic flux on the electron beam decreases to zero if the back-action exerted by the electron beam on the magnetic flux is reduced to zero. In this situation, the A–B effect will not be observed, even though the vector potential still exists in space. Further, we show that the interpretation of electromagnetic potentials violates Heisenberg's uncertainty principle.

2 Back-action on the magnetic flux

The A–B effect states that the phase difference $\Delta\varphi$ between two electron beams is proportional to the magnetic flux Φ enclosed by their paths, i.e.,

$$\Delta\varphi = \frac{2\pi\Phi}{h/e}. \tag{1}$$

Thus, by knowing the phase difference between these two electron beams, we can measure the magnetic flux Φ produced by a solenoid. The question is: what is the disturbance on the magnetic flux Φ during this measurement process? In other words, what is the back-action exerted by the electron beams on the magnetic flux?

First, this disturbance on the magnetic flux must exist, because it is indispensable for explaining the wave-particle duality, the uncertainty principle and the wave-packet reduction according to the Copenhagen interpretation [14, 15]. Second, this disturbance should be electromagnetic interaction. Thus, a natural answer to this question is that the magnetic field generated by the electron beams will interact with the magnetic field produced by the solenoid and thus will affect the measured magnetic flux.

For a more detailed discussion, in Fig. 1 we assume that a beam of electrons generates the magnetic field $\boldsymbol{B}_1(\boldsymbol{r})$ around it and a long, straight solenoid produces a static field $\boldsymbol{B}_2(\boldsymbol{r})$ inside it. When the electron beam approaches the solenoid, the magnetic field $\boldsymbol{B}_1(\boldsymbol{r})$ in the solenoid increases; however, when the electron beam moves away from the solenoid, the magnetic field $\boldsymbol{B}_1(\boldsymbol{r})$ in the solenoid decreases. The change in the magnetic field $\boldsymbol{B}_1(\boldsymbol{r})$ will induce an electromotive force ΔU across the solenoid [12, 13]. Obviously, this electromotive force ΔU will change the current through the solenoid and thus, disturb the magnetic flux Φ. This is the back-action exerted by the electron beams on the magnetic flux. This analysis is consistent with the conclusions of a quantum circuit, in which the magnetic flux Φ through the solenoid and the voltage U across the solenoid are non-commuting, i.e., the measurement of the magnetic flux Φ through the solenoid must be accompanied by an inevitable perturbation of the voltage U across this solenoid (see Appendix A of this paper).

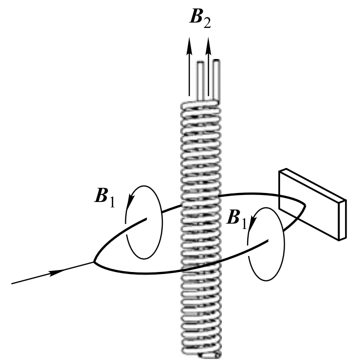

Fig. 1 The magnetic interaction between the moving electron and the magnetic flux in the A–B effect.

The mutual interaction between the electron beam and the long solenoid can be discussed in detail as follows.

The total magnetic field $\boldsymbol{B}(\boldsymbol{r})$ in space is the sum of the fields $\boldsymbol{B}_1(\boldsymbol{r})$ and $\boldsymbol{B}_2(\boldsymbol{r})$:

$$\boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{B}_1(\boldsymbol{r}) + \boldsymbol{B}_2(\boldsymbol{r}). \tag{2}$$

The energy of the total magnetic field is given by

$$\begin{aligned} E &= \frac{1}{2\mu_0}\int [\boldsymbol{B}_1(\boldsymbol{r}) + \boldsymbol{B}_2(r)]^2 \mathrm{d}r^3 \\ &= \frac{1}{2\mu_0}\int \boldsymbol{B}_1^2(\boldsymbol{r})\mathrm{d}r^3 + \frac{1}{2\mu_0}\int \boldsymbol{B}_2^2(\boldsymbol{r})\mathrm{d}r^3 \\ &\quad + \frac{1}{\mu_0}\int \boldsymbol{B}_1(\boldsymbol{r})\cdot\boldsymbol{B}_2(\boldsymbol{r})\mathrm{d}r^3 \\ &= E_1 + E_2 + E'. \end{aligned} \tag{3}$$

The first term $E_1 = \frac{1}{2\mu_0}\int \boldsymbol{B}_1^2(\boldsymbol{r})\mathrm{d}r^3$ is the energy of the magnetic field $\boldsymbol{B}_1(\boldsymbol{r})$ produced by the electron; the second term $E_2 = \frac{1}{2\mu_0}\int \boldsymbol{B}_2^2(\boldsymbol{r})\mathrm{d}r^3$ is the energy of the magnetic field $\boldsymbol{B}_2(\boldsymbol{r})$ produced by the solenoid. The third term $E' = \frac{1}{\mu_0}\int \boldsymbol{B}_1(\boldsymbol{r})\cdot\boldsymbol{B}_2(\boldsymbol{r})\mathrm{d}r^3$ is the interaction energy between the magnetic fields $\boldsymbol{B}_1(\boldsymbol{r})$ and $\boldsymbol{B}_2(\boldsymbol{r})$, which describes the interaction between the electron beam and the solenoid. Obviously, the interaction energy E' is time-dependent in this experiment. In Ref. [10] it was shown that the interaction energy E' is related to the vector potential $\boldsymbol{A}_2$ describing the magnetic field $\boldsymbol{B}_2(\boldsymbol{r})$ that is produced by the solenoid:

$$E' = \int \frac{1}{\mu_0}\boldsymbol{B}_1\cdot\boldsymbol{B}_2\mathrm{d}r^3 = \boldsymbol{A}_2(\boldsymbol{x})\cdot q\boldsymbol{v}, \tag{4}$$

where $q, \boldsymbol{v}$, and $\boldsymbol{x}$ are the charge, velocity, and position of the electron, respectively, and $\boldsymbol{A}_2(\boldsymbol{x})$ is given by

$$\boldsymbol{A}_2(\boldsymbol{x}) = \frac{1}{4\pi}\int_{\Omega} \frac{\boldsymbol{B}_2 \times (\boldsymbol{x}-\boldsymbol{r})}{|\boldsymbol{x}-\boldsymbol{r}|^3}\mathrm{d}r^3. \tag{5}$$

It is easy to show that $\boldsymbol{A}_2(\boldsymbol{x})$ is the vector potential describing the magnetic field $\boldsymbol{B}_2(\boldsymbol{r})$ of the solenoid.

As demonstrated in Ref. [7] (on pages 25 and 26), when the electron beam passes near but outside the solenoid, its magnetic field $\boldsymbol{B}_1(\boldsymbol{r})$ in the solenoid changes with time. The time-dependent $\boldsymbol{B}_1(\boldsymbol{r})$ induces an electromotive force ΔU across the solenoid. The electromotive force ΔU can change the current through the solenoid and do work against the current. During this process, the sum of the time-dependent interaction energy E' and the energy of the solenoid remains constant, where the energy of the solenoid includes the energy E_2 of the magnetic field $\boldsymbol{B}_2$ and the energy of the electrical source connecting to the solenoid. On the other hand, both the kinetic energy $\frac{1}{2}mv^2$ of the electron beam and the energy of its magnetic field $\boldsymbol{B}_1$ remain constant. Thus, the electron's velocity v remains constant, and the electron beam experiences no force from the magnetic field $\boldsymbol{B}_2$ produced by the solenoid. (It should be noted that in Refs. [16, 17] it was postulated that the electron beam experiences a non-Lorentzian force $F' = -\nabla W'$, based upon Eq. (4). However, this postulation of non-Lorentzian force is wrong and has been ruled out by experiments [18].)

The analysis above shows that the interaction energy interpretation of the A–B effect is consistent with the principles of quantum-mechanical measurements. In what follows, we will show that the interpretation of electromagnetic potentials violates the uncertainty principle.

3 Disadvantages of the electromagnetic potentials interpretation

Let us now consider the experiment in Fig. 2. There is a long, straight solenoid in space that is coated by a long, straight superconducting cylinder with thickness d much larger than the magnetic penetration depth λ. A coherent beam of electrons is split into two parts, each moving towards opposite sides of the superconducting cylinder, but avoiding it. Then, these two beams are brought together and interfere with each other. In this experiment, the electron's speed is less than 10^6m/s and the superconducting cylinder can completely exclude the electron's magnetic field outside it, owing to the Meissner effect [19]. In this experiment, the electron's speed is much lower than in the experiment of Tonomura *et al.*

When the superconducting cylinder is cooled into the superconducting state, the magnetic flux enclosed by it should be quantized in units of $\Phi_0 = h/(2e)$ [20, 21].

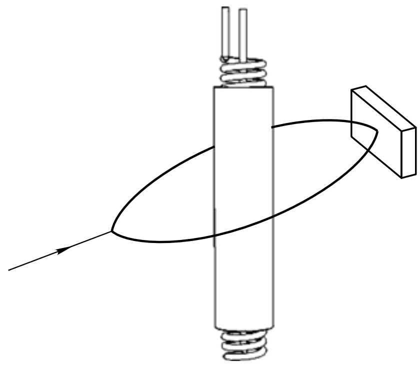

Fig. 2 An A–B effect experiment with the solenoid coated by a superconducting cylinder.

Thus, for simplicity, at the beginning of the experiment, we assume that the magnetic flux Φ_2 generated by the solenoid is $n\Phi_0$ when the superconducting cylinder is in its normal state. In this situation, after the cylinder is cooled into the superconducting state, no current is formed on the inner surface of the superconducting cylinder and the current through the solenoid remains unchanged. Thus, the vector potential $\boldsymbol{A}_2$, describing the field $\boldsymbol{B}_2(\boldsymbol{r})$ inside the solenoid, remains unchanged in space. For a single-turn closed loop outside the superconducting cylinder, $\oint \boldsymbol{A}_2 \cdot \mathrm{d}\boldsymbol{l} = \Phi_2 = n\Phi_0$. If the electromagnetic potentials interpretation of the A–B effect is correct, the A–B effect should arise from the vector potential, implying that the A–B effect should still be observed in the situation described above. According to Eq. (1), if $\Phi_2 = (2n+1)\Phi_0$, the relative phase shift $\Delta\phi$ between the two electron beams should be $(2n+1)\pi$; however, if $\Phi_2 = 2n\Phi_0$, the shift $\Delta\phi$ should be $2n\pi$. This implies that the interference pattern of the electron beams with $\Phi_2 = (2n+1)\Phi_0$ is different from that of the beams with $\Phi_2 = 2n\Phi_0$. Thus, by measuring the interference pattern of the electron beams outside the superconducting cylinder, it is possible to determine whether the magnetic flux through the solenoid enclosed by the superconducting cylinder is an odd or an even multiple of Φ_0. Obviously, the above procedure is a measurement of the magnetic flux through the solenoid enclosed by the superconducting cylinder. However, in this process, the magnetic field produced by the electron beam has been excluded from the superconducting cylinder, because some shielding currents appear on the outer surface of the superconducting cylinder owing to the Meissner effect. Obviously, the electric field of the electron has also been shielded by the superconducting cylinder. Thus, the magnetic and electric fields of the electron beams cannot affect the solenoid enclosed by the superconducting cylinder, because these fields cannot penetrate into the superconducting cylinder (since $d \gg \lambda$). However, in quantum mechanics a measurement without back-action on the measured object is impossible, because it would violate the uncertainty principle. Thus, the electromagnetic potentials interpretation of the A–B effect is wrong.

It could be argued that the electron beam could exhibit an "A–B effect-type" interaction with the solenoid, i.e., the electron beam could affect the phase of the electrons in the solenoid. A simple analysis shows that it is impossible. Suppose the magnetic field $\boldsymbol{B}_1(\boldsymbol{r})$ that is produced by the electron beam is described by the vector potential $\boldsymbol{A}_1(\boldsymbol{r})$, i.e., $\nabla \times \boldsymbol{A}_1(\boldsymbol{r}) = \boldsymbol{B}_1(\boldsymbol{r})$. Because $\boldsymbol{B}_1(\boldsymbol{r}) = 0$ in the region enclosed by the superconducting cylinder, where the solenoid is located, for any closed loop in this region, we have $\oint \boldsymbol{A}_1 \cdot \mathrm{d}\boldsymbol{l} = \int \boldsymbol{B}_1 \cdot \mathrm{d}\boldsymbol{s} = \boldsymbol{0}$. Thus, the electron beam outside the superconducting cylinder cannot affect the phase of the electrons in the solenoid enclosed by the superconducting cylinder. Consequently, the electron beam cannot affect the solenoid at all.

Thus, the electromagnetic potentials interpretation of the A–B effect should be revised.

4 The interaction energy interpretation

The above contradiction does not exist in the interaction energy interpretation, which attributes the A–B effect to the interaction energy between the magnetic fields, but not to the vector potential $\boldsymbol{A}$. After the superconducting cylinder is cooled into the superconducting state, it can completely exclude the magnetic field produced by the electron beam outside the cylinder. Consequently, the magnetic field produced by the electron beam cannot be superimposed on the static magnetic field in the solenoid, and the interaction energy W' between the two fields becomes zero. Thus, the A–B effect cannot be observed, i.e., the interference pattern between the electron beams should be the same for $\Phi_2 = (2n+1)\Phi_0$ and for $\Phi_2 = 2n\Phi_0$. Therefore, an observer cannot measure the magnetic flux through the solenoid by determining the interference pattern of the electron beams outside the superconducting cylinder. Obviously, this conclusion is consistent with the theoretical underpinnings of quantum-mechanical measurement. Furthermore, this conclusion has also been reached by using the quantum-mechanical method (see Appendix A in Ref. [10]).

5 Analysis of the Tonomura *et al.*'s experiment

It should be pointed out that the interaction energy interpretation does not contradict the experiment reported by Tonomura *et al.* [5].

In the experiment by Tonomura *et al.*, tiny toroidal magnets covered with superconductors were fabricated. An electron wave was incident on the tiny toroidal sample, and the phase difference $\Delta\phi$ between two waves passing through the hole and outside of the toroid was measured from interferograms. Many samples were measured when the temperature was below the critical temperature of the superconducting layer. It was found that if the magnetic flux Φ surrounded by the superconductor was $(2n+1)\Phi_0$, the phase difference $\Delta\phi$ was π (modulo 2π); however, for $\Phi = 2n\Phi_0$, the value of $\Delta\phi$ was 0 (modulo 2π) [22]. This implies that, by measuring the interference pattern of the electron beams outside the superconducting film, the observer could determine whether the magnetic flux Φ surrounded by the superconducting film is an odd or an even multiple of Φ_0. This experimental result apparently contradicts the interaction energy interpretation. However, a detailed analysis can demonstrate that this conclusion is wrong.

In Ref. [10] it was asserted that the superconducting layer in this experiment could only confine the magnetic flux within it, but could not shield the magnetic field generated by the electron beams. The main reason for this was that the electron beams in this experiment were too fast (electron speed of approximately 2.2×10^8 m/s [23]). In this experiment, an electron beam could be regarded as a system of successive wave packets, and the coherence length Δl of each wave packet was approximately 3–5 μm. The wave packets were separated from each other, and the quantum interference that occurred while forming an electron hologram involved only one electron at a time [24]. Each wave packet generated a magnetic field $\boldsymbol{B}_1(\boldsymbol{r})$ at the tiny toroidal magnet. The magnetic field $\boldsymbol{B}_1(\boldsymbol{r})$ generated a very short pulse at the tiny magnet. The width Δt of this pulse was equal to the length Δl of the wave packet divided by its velocity v, i.e., $\Delta t \approx \Delta l/v \approx 2\times10^{-14}$ s. When the Fourier transform was introduced, the main frequency of this pulse was approximately $\nu \approx 5\times10^{13}$ Hz, i.e., $h\nu \approx 2\times10^{-2}$ eV, which was much larger than kT_C of the Nb film (approximately 7.9×10^{-4} eV). Obviously, the Nb film could not shield such high-frequency variations of the magnetic field [25]. Therefore, the magnetic field $\boldsymbol{B}_1(\boldsymbol{r})$ generated by the electron beam could still penetrate into the superconducting film and form a superposition with the static magnetic field in the tiny magnet. Consequently, the interaction energy between these two magnetic fields was not zero. For this reason, the A–B effect was observed in the experiments by Tonomura *et al.* (i.e., the interference pattern with $\Phi = 2n\Phi_0$ was different from that with $\Phi = (2n+1)\Phi_0$). Because of this, many physicists have been misled by these experimental observations into adopting the interpretation of electromagnetic potentials. If the speed of the electron beam is lowered and the superconducting layers can shield the magnetic fields produced by the electron beams, the interference patterns will be the same for all samples, whether or not the magnetic flux Φ surrounded by the superconducting film is $2n\Phi_0$ or $(2n+1)\Phi_0$.

To account for the disadvantages in the experiment of Tonomura *et al.*, a novel experimental scheme based on SQUID (Superconducting Quantum Interference Device) has been proposed in Ref. [10].

6 Conclusions

In summary, a correct interpretation of the A–B effect should not violate the uncertainty principle. The analysis presented in this paper shows that only the interaction energy interpretation is consistent with the uncertainty principle. Given this, the A–B effect could be explained as follows.

The concept of "energy" plays a fundamental role in quantum mechanics, replacing the concept of "force" in classical physics. Consequently, the electromagnetic potentials ϕ and $\boldsymbol{A}$, which describe the interaction energy between a charge and electromagnetic fields, appear in the Hamiltonian of Schrödinger's equation, instead of the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$, which describe the force acting on the charge in the electromagnetic field. Although the A–B effect is related to the electromagnetic potential through the differential equation, it does not arise from the electromagnetic potential, but from the interaction energy described by the electromagnetic potentials. Only this interpretation of the A–B effect is consistent with Heisenberg's uncertainty principle.

Acknowledgements The author thanks Mrs. Chang-Yu Deng for her encouragement. This work was supported by the Fundamental Research Funds for the Central Universities (Grant No. 2009JBM102).

Appendix A

In a quantum circuit [26], for a lossless LC (inductance-capacitance) quantum circuit, which is composed of inductance L and capacitance C, the charge q on the capacitance and the variable p satisfy the commutation relation $[q,p]=\mathrm{i}\hbar$, where p is given by $p(t)=L\frac{\mathrm{d}q}{\mathrm{d}t}$. Because the magnetic flux through the inductance is $\Phi(t)=L\frac{\mathrm{d}q}{\mathrm{d}t}=p$ and the voltage across the inductance (or the capacitance) is $U=q/C$, the commutation relation between U and Φ is $C[U,\Phi]=\mathrm{i}\hbar$. (Here, q, p, U, and Φ are quantum-mechanical operators, while C and L are constants). This implies that any measurement of the magnetic flux Φ through a solenoid must be considered with a perturbation on its voltage U.

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