Front. Phys. 10, 100501 (2015)
DOI 10.1007/s11467-014-0438-9

RESEARCH ARTICLE

Effects of correlation time between noises on the noise enhanced stability phenomenon in an asymmetric bistable system

Chun Li[1], **Zheng-Lin Jia**[2], **Dong-Cheng Mei**[1,2,3,†]

[1] *Department of Computer Science, Puer College, Puer 665000, China*
[2] *Department of Physics, Yuxi Normal University, Yuxi 653100, China*
[3] *Department of Physics, Yunnan University, Kunming 650091, China*
Corresponding author. E-mail: [†] *meidch@ynu.edu.cn*

Received May 20, 2014; accepted July 5, 2014

The effects of the correlation time τ between noises on the noise-enhanced stability (NES) phenomenon in an asymmetric bistable system driven by cross-correlated noise are investigated. The expressions for the average escape time from the left metastable state T_L and from the right metastable state T_R are derived. The results indicate that: i) The NES effect is suppressed as the correlation time τ increases for two metastable states; ii) The increase in τ speeds up the escape process from the right state for positively correlated noise, whereas its role is reverses for negatively correlated; iii) In the escape process from the left state, the role of τ is opposite to that in escape from the right state.

Keywords asymmetric bistable system, noise, correlation time, noise enhanced stability

PACS numbers 05.45.-a, 02.50.-r

1 Introduction

It is well known that nonlinear systems in the presence of random fluctuations produced by noise sources can show quite counterintuitive dynamics. Prominent examples are stochastic resonance [1], resonant activation [2], and noise enhanced stability (NES) [3, 4]. The problem of noise-induced phenomena is of special interest in nonequilibrium natural systems covering a broad class of examples ranging from physics and chemistry to the biological sciences. Recent theoretical investigations have shown that the average escape time from metastable states in fluctuating or static potentials exhibits nonmonotonic behavior as a function of the noise intensity [5, 6]. This implies that the stability of metastable states can be enhanced by noise; i.e., the lifetime of a metastable state can be prolonged by the presence of noise. This noise-induced resonance-like effect is known as the NES phenomenon [3]. Non-monotonic dependence of the mean first-passage time (MFPT) on the noise intensity with the presence of a maximum is a typical signature of the NES phenomenon [6]. Enhancement of the stability of metastable states by noise has been observed in various systems such as ecological systems [7], biological systems [8], chemical systems [9], magnetic systems [10], Josephson junctions [11], and tunnel diodes [12] (see Ref. [13] for a review). Moreover, the effects of the noise correlation time [14], damping parameter [15], and time delay [16, 17] on the NES phenomenon in metastable systems have been investigated. In particular, the NES effect has also been observed and investigated in the double-well potential model [17–20], which is widely used to investigate noise activated phenomena. Most previous studies considered a symmetric bistable system. However, real-world manifestations of these systems are usually asymmetric [21–24]. The simplest way to obtain asymmetry in a bistable system is to introduce a small constant term into the Langevin equation or, equivalently, a linear term into the symmetric potential [21, 22]. Following this route, the effects of asymmetry on the properties of bistable systems have been extensively investigated [21–26]. We recently studied the transient properties of an asymmetric bistable system driven by cross-correlated noise in the context of the NES phenomenon [27]. We found that the combination of noise correlation and asymmetry can cause meaningful modifications of the NES effect induced by multiplicative noise,

Frontiers of Physics

and that additive noise cannot produce the NES effect in this system. However, the case of a nonzero correlation time between additive and multiplicative noises has not been considered in the context of the NES phenomenon. Physically, the correlation time of a real noise, although it may be small, is never strictly equal to zero. For noise with a zero correlation time, the power spectral distribution, which is given by the Fourier transform of its correlation function, is independent of the frequency. Thus, the total power dissipated at all frequencies is infinite, but the actual power dissipated would be somewhat less than infinite. In other words, it appears as an idealization that is valid only when the time scale for the correlation is much shorter than the time scale for the relaxation of the driven process. On the other hand, the assumption that the correlation time is zero is usually adopted as a first step in studying a system driven by noise. Later, it is reasonable to relax this condition and include the finite correlation time [28]. Thus, we think that the transient dynamics of the asymmetric bistable system driven by cross-correlated noise with a nonzero cross-correlation time between noises would be an interesting issue to investigate. In particular, the effects of the cross-correlation time between noises on the NES phenomenon deserve further discussion.

In this study, as a continuation of our previous work [27], we examine the mean escape time in an asymmetric bistable system driven by cross-correlated noise for a nonzero correlation time between noises. We focus on the effects of the correlation time between noises on the NES phenomenon. This paper is arranged as follows. In Section 2, the Novikov theorem, Fox approach, and Hänggi Ansatz are applied to obtain the approximate Fokker–Planck equation of the system, and then the expression of the mean escape time is obtained using the steepest-descent approximation. In Section 3, the impacts of the correlation time between noises on the average escape time are analyzed by numerical computations. Finally, a brief conclusion ends the paper.

2 The average escape time of the system

Consider an asymmetric bistable system driven by cross-correlated multiplicative and additive noises with a nonzero correlation time between the multiplicative and additive noises. Its Langevin equation reads

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -r + x - x^3 + x\xi(t) + \eta(t) \tag{1}$$

where r is the asymmetry parameter, which measures the deviation from the symmetric bistable potential. $\xi(t)$ and $\eta(t)$ are Gaussian white noises with zero mean and have the following statistical properties

$$\langle \xi(t)\xi(t')\rangle = 2D\delta(t-t') \tag{2}$$

$$\langle \eta(t)\eta(t')\rangle = 2Q\delta(t-t') \tag{3}$$

and

$$\langle \xi(t)\eta(t')\rangle = \langle \eta(t)\xi(t')\rangle = \frac{\lambda\sqrt{\alpha D}}{\tau}\exp[-|t-t'|/\tau] \rightarrow 2\lambda\sqrt{\alpha D}\delta(t-t') \quad as \quad \tau\rightarrow 0 \tag{4}$$

Here D and Q are the multiplicative and additive noise intensities, respectively; λ ($|\lambda| < 1$) denotes the cross-correlation intensity between $\xi(t)$ and $\eta(t)$; τ is the correlation time between $\xi(t)$ and $\eta(t)$. The deterministic potential corresponding to Eq. (1) reads

$$V(x) = -\frac{1}{2}x^2 + \frac{1}{4}x^4 + rx \tag{5}$$

and has two metastable states $x_{\pm}$ and an unstable state x_u under the condition of $|r| < \frac{2\sqrt{3}}{9}$ [29]; namely, the system is bistable. The metastable and unstable states of the system are determined by

$$x_+ = \frac{2\sqrt{3}}{3}\cos\left(\frac{1}{3}\arccos(-\frac{3\sqrt{3}}{2}r)\right)$$

$$x_- = -\frac{2\sqrt{3}}{3}\cos\left(\frac{1}{3}\arccos(-\frac{3\sqrt{3}}{2}r)-\frac{\pi}{3}\right)$$

$$x_u = -\frac{2\sqrt{3}}{3}\cos\left(\frac{1}{3}\arccos(-\frac{3\sqrt{3}}{2}r)+\frac{\pi}{3}\right) \tag{6}$$

The deterministic potential $V(x)$ is plotted as a function of x for various values of the asymmetry parameter r in Fig. 1. Figure 1 shows that the right potential well goes up whereas the left one goes down as r increases. When r exceeds $\frac{2\sqrt{3}}{9}$, the right potential well disappears, and the system becomes a monostable system. Moreover, r varies, the two sides of the potential barrier separating

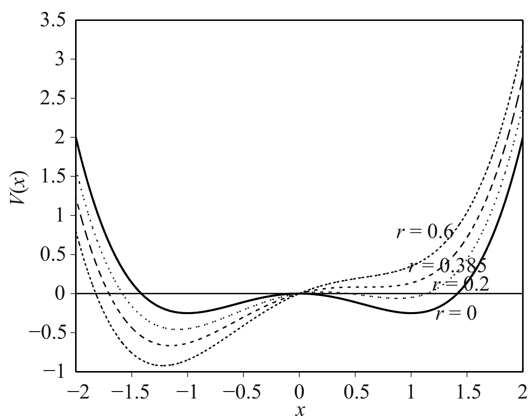

Fig. 1 $V(x)$ as a function of x with $r = 0, 0.2, \frac{2\sqrt{3}}{9} \simeq 0.385$ and 0.6.

the metastable states $x_\pm$ become asymmetric.

By using the Novikov theorem [30], Fox approach [31], and Hänggi Ansatz [32], the approximate Fokker–Planck equation corresponding to Eq. (1) with Eqs. (2)–(4) can be obtained as [33, 34]

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}\left(-r + x - x^3 + Dx + \frac{\lambda}{1+2\tau}\sqrt{QD}\right) P(x,t) + \frac{\partial^2}{\partial x^2}\left(Dx^2 + \frac{2\lambda}{1+2\tau}\sqrt{QD}x + Q\right)P(x,t) \quad (7)$$

The steady-state probability distribution function of the system can be derived from Eq. (7) and is given by

$$P_{st}(x) = N\left(Dx^2 + \frac{2\lambda}{1+2\tau}\sqrt{QD}x + Q\right)^{-1/2} \times \exp\left[-\frac{U(x)}{D}\right] \quad (8)$$

where N is the normalization constant, and the generalized potential $U(x)$ is

$$\begin{aligned} U(x) = {} & \frac{x^2}{2} - \frac{2\lambda}{1+2\tau}\sqrt{\frac{Q}{D}}x + \left\{\frac{Q}{2D}\left[4\left(\frac{\lambda}{1+2\tau}\right)^2 - 1\right] - \frac{1}{2}\right\} \\ & \times \ln\left|Dx^2 + \frac{2\lambda}{1+2\tau}\sqrt{QD}x + Q\right| - \frac{\frac{\lambda}{1+2\tau}}{\sqrt{1-\left(\frac{\lambda}{1+2\tau}\right)^2}} \\ & \times \left\{\frac{Q}{D}\left[4\left(\frac{\lambda}{1+2\tau}\right)^2 - 3\right] - 1 - \frac{r}{\frac{\lambda}{1+2\tau}}\sqrt{\frac{D}{Q}}\right\} \\ & \times \arctan\frac{\sqrt{D/Q}x + \frac{\lambda}{1+2\tau}}{\sqrt{1-(\frac{\lambda}{1+2\tau})^2}} \end{aligned} \quad (9)$$

Note that the correlation time τ must be zero when the strength λ of the correlation between noises is zero. However, Eq. (8) is valid when $\tau = 0$.

In this study, we focus on the transient dynamics of the system, i.e., the average escape time of a particle from one metastable state to the other, as measured by the MFPT. The exact expression for the MFPT for a particle to reach the final point $x_\mp$, from the initial state $x_\pm$ is given by [35, 36]

$$T(x_\pm \to x_\mp) = \int_{x_\pm}^{x_\mp} \frac{\mathrm{d}x}{B(x)P_{st}(x)} \int_{\pm\infty}^{x} P_{st}(y)\mathrm{d}y \quad (10)$$

in which $B(x) = Dx^2 + \frac{2\lambda}{1+2\tau}\sqrt{QD}x + Q$. When the noise intensities D and Q are small enough in comparison with the energy barrier height $\Delta U = |U(x_u) - U(x_\pm)|$, by using the steepest-descent approximation [37] to Eq. (10), $T(x_\pm \to x_\mp)$ can be approximated by

$$T(x_\pm \to x_\mp) \approx \frac{2\pi}{\sqrt{|V''(x_\pm)V''(x_u)|}} \times \exp\left[\frac{U(x_u) - U(x_\pm)}{D}\right] \quad (11)$$

where the double prime denotes the second derivative with respect to x; $V(x)$ and $U(x)$ are given by Eqs. (5) and (9), respectively. From Eq. (11), explicit expressions of the average escape times from $x_\mp$ to $x_\pm$ for small D and Q are obtained as

$$T_L = T(x_- \to x_+) \approx \frac{2\pi}{\sqrt{|(3x_-^2 - 1)(3x_u^2 - 1)|}} \times \exp\left[\frac{U(x_u) - U(x_-)}{D}\right] \quad (12)$$

and

$$T_R = T(x_+ \to x_-) \approx \frac{2\pi}{\sqrt{|(3x_+^2 - 1)(3x_u^2 - 1)|}} \times \exp\left[\frac{U(x_u) - U(x_+)}{D}\right] \quad (13)$$

When the noise intensities D and Q are small enough in comparison with the energy barrier height $\Delta U = |U(x_u) - U(x_\pm)|$, the steepest-descent approximation is valid. In Ref. [27], the validity of the steepest-descent approximation for $\tau = 0$ is confirmed by stochastic simulations. The difference in the expression for T (or $U(x)$) between the cases of $\tau \neq 0$ and $\tau = 0$ is only $\frac{\lambda}{1+2\tau}$ instead of λ, therefore, the steepest-descent approximation is also valid for $\tau \neq 0$.

3 Results and discussion

By numerical evaluation of the expressions for the MFPT in Eqs. (12) and (13), the effects of the cross-correlation time τ on the average escape times T_L and T_R can be discussed.

First, we examine the effect of the cross-correlation time τ between noises on the NES phenomenon. To this end, we plot T_L and T_R as a function of the multiplicative noise intensity D for different values of τ in Figs. 2 and 3, respectively. Figure 2 shows that the average escape time from the left metastable state T_L as a function of D exhibits non-monotonic behavior with the presence of a maximum for positively correlated noises (i.e., $\lambda = 0.7$), which is the identifying characteristic of the

NES phenomenon for the left state. The maximum of T_L versus D decreases and its position shifts toward smaller values of D as τ increases. This implies that increasing τ can suppress the NES phenomenon. The τ-induced suppression of the NES phenomenon can be physically understood as a result of the symmetry-breaking effect caused by the cross-correlation between noises. That is, the right potential minimum can be favored over the left one with increasing τ for $\lambda > 0$, giving rise to a decrease in the local stability of the left state. Moreover, we argue that the τ-induced shift of the peak position of T_L is due to the memory effect caused by the cross-correlation between noises with a finite cross-correlation time. A similar behavior induced by the noise correlation time has also been observed in a cubic potential system driven by colored noise [14]. Figure 3 shows the effect of τ on the NES effect for the right state. A critical value of D exists at which T_R as a function of D attains its maximum for negatively correlated noise terms (i.e., $\lambda = -0.7$). This means that the NES effect for the right state can be observed when $\lambda < 0$. A comparison with the effect of τ in the NES phenomenon for the left state reveals that increases in τ play the same role in the NES effect for the right state. However, the effect of τ on the NES

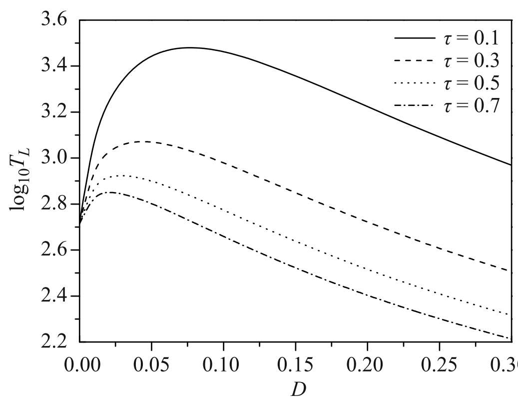

Fig. 2 The MFPT T_L as a function of D for different values of τ. The parameters are $r = 0.2$, $Q = 0.1$ and $\lambda = 0.7$.

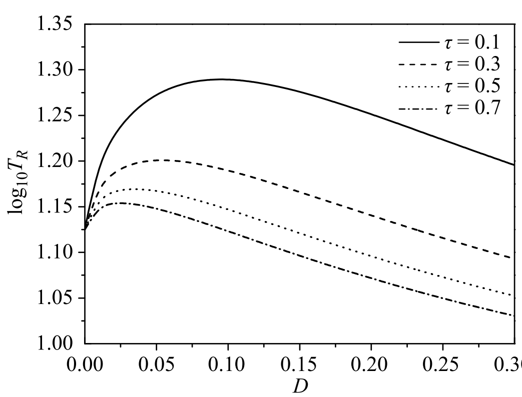

Fig. 3 The MFPT T_R as a function of D for different values of τ. The parameters are $r = 0.2$, $Q = 0.1$ and $\lambda = -0.7$.

phenomenon is opposite to that of the cross-correlation intensity λ between noise terms, which is in agreement with observations in other systems where λ and τ have opposite effects [34]. Moreover, we observe a narrowing of the NES region with increasing τ. Note that the dependence of the NES effect on the self-correlation time of colored noise has been investigated in a metastable system [14]. It was found that increasing the noise correlation time can shift the maximum of the average escape time toward higher noise intensity values and increase the value of this maximum. Thus, a comparison of these results with our present observations indicates that the effects of the cross-correlation time between noises on the NES phenomenon are opposite to those of the noise self-correlation time.

In the next step, we analyze how the other parameters (i.e., the cross-correlation intensity λ, asymmetry parameter r, and additive noise intensity Q) affect the dependence of the MFPT on τ. Figure 4 shows the average escape times T_L and T_R as a function of τ for different values of λ when the other parameters are fixed. One can see from Figs. 4(a) and (b) that increasing τ affects T_R differently when $\lambda > 0$ and $\lambda < 0$. T_R decreases monotonously with increasing τ for $\lambda < 0$, whereas T_R increases monotonously with increasing τ for $\lambda > 0$. This implies that increasing τ speeds up the escape process from the right state when $\lambda < 0$, but slows it down when $\lambda > 0$. Moreover, as $|\lambda|$ increases, the effect of τ on T_R can be restrained for both $\lambda > 0$ and $\lambda < 0$. However, as shown in Figs. 4(c) and (d), the effect of τ on T_L is opposite to that on T_R owing to the asymmetry of the system. That is, increasing τ speeds up the escape process from the left state when $\lambda > 0$, but slows it down when $\lambda < 0$. Increasing $|\lambda|$ can also suppress the effect of τ on T_L.

In Fig. 5, we depict the effect of the asymmetry parameter r on T_L and T_R as a function of τ when the other parameters are fixed. With increasing r, T_R decreases [see Figs. 5(a) and (b)] and T_L increases [see Figs. 5(c) and (d)]. However, the effect of τ on both T_L and T_R does not change with variation in r.

In Fig. 6, we plot T_L and T_R as a function of τ for different values of Q when the other parameters are fixed. Increasing Q weakens the effect of τ on both T_L and T_R, but it does not change the qualitative behavior of T_L and T_R with respect to τ.

4 Conclusions

In this study, we examined the effects of the cross-correlation time on the transient behavior of an

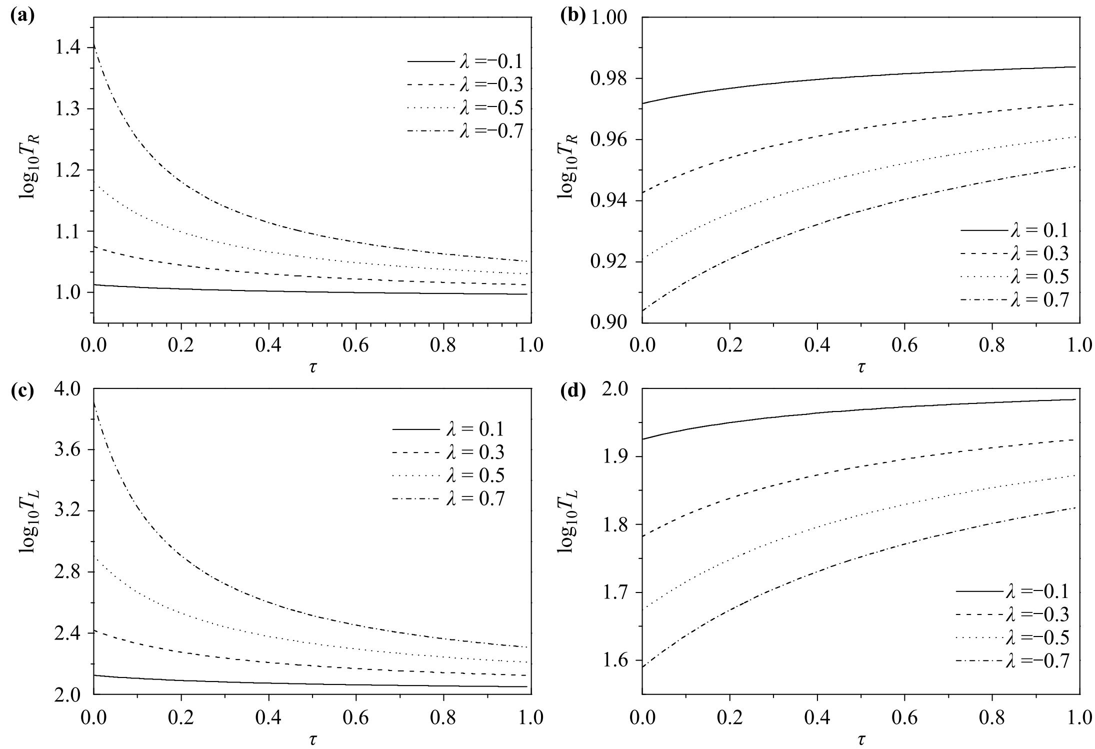

Fig. 4 The MFPT (T_R, T_L) as a function of τ for different values of λ when $D = 0.2$, $Q = 0.1$ and $r = 0.2$. **(a)** T_R versus τ for $\lambda < 0$; **(b)** T_R versus τ for $\lambda > 0$; **(c)** T_L versus τ for $\lambda > 0$; (d) T_L versus τ for $\lambda < 0$.

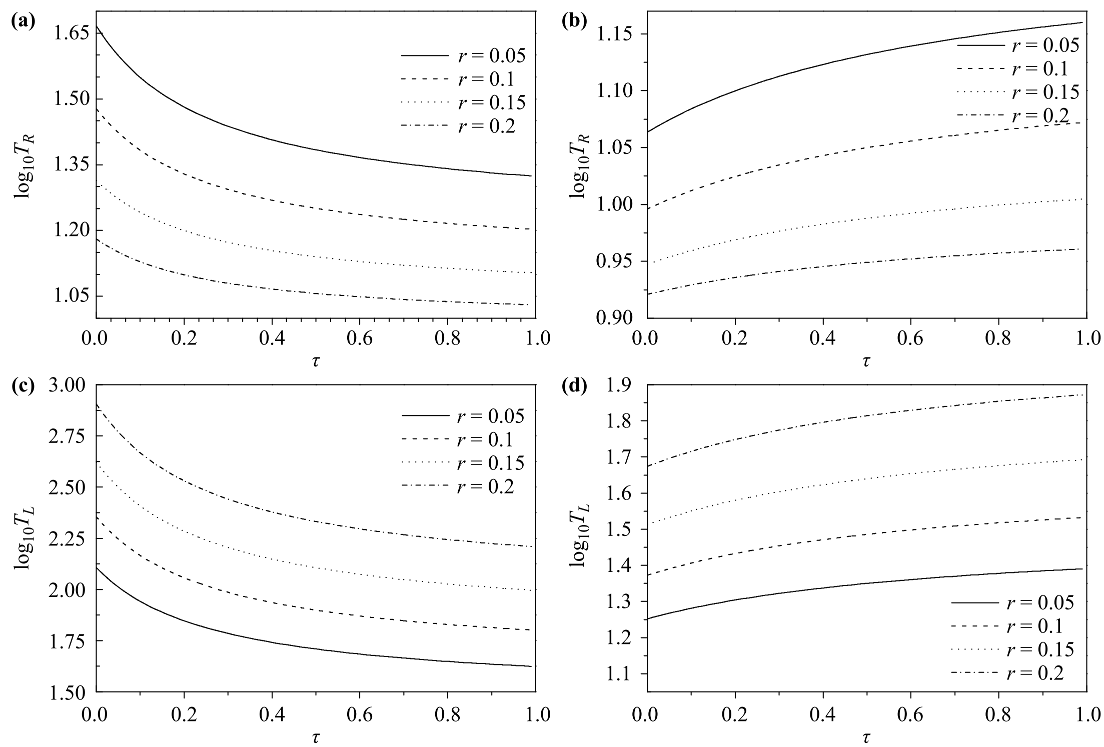

Fig. 5 The MFPT (T_R, T_L) as a function of τ for different values of r when $D = 0.2$ and $Q = 0.1$. **(a)** T_R versus τ for $\lambda = -0.5$; **(b)** T_R versus τ for $\lambda = 0.5$; **(c)** T_L versus τ for $\lambda = 0.5$; **(d)** T_L versus τ for $\lambda = -0.5$.

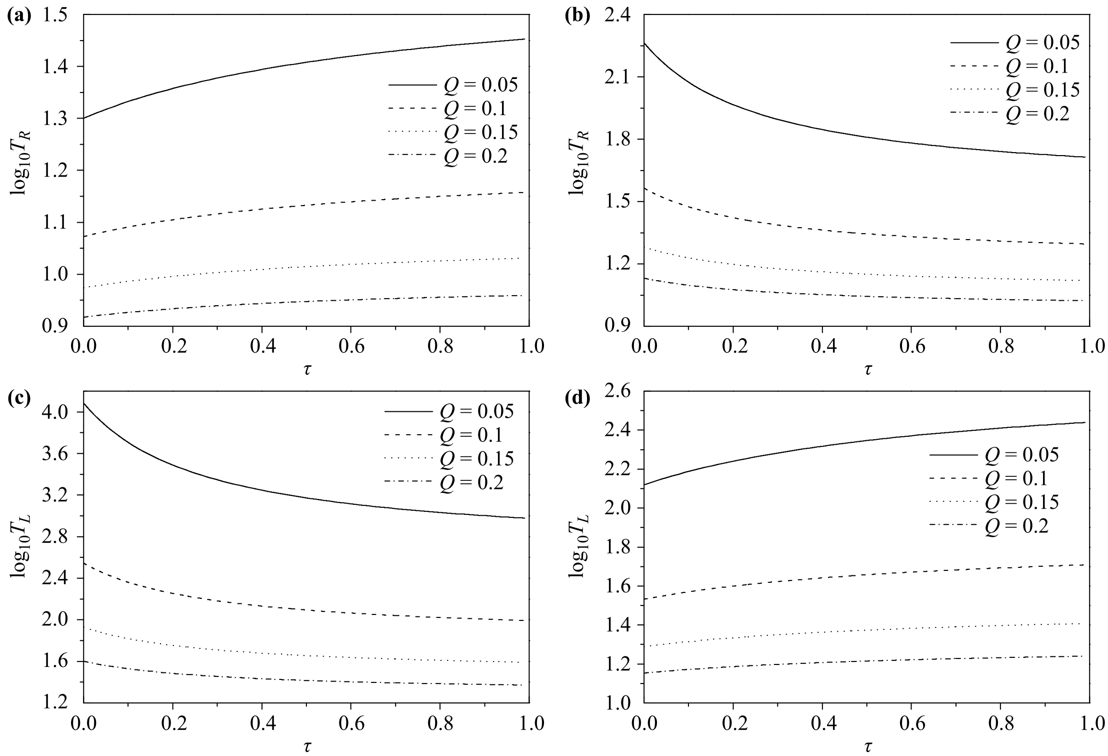

Fig. 6 The MFPT (T_R, T_L) as a function of τ for different values of Q when $D = 0.1$ and $r = 0.1$. **(a)** T_R versus τ for $\lambda = 0.5$; **(b)** T_R versus τ for $\lambda = -0.5$; **(c)** T_L versus τ for $\lambda = 0.5$; **(d)** T_L versus τ for $\lambda = -0.5$.

asymmetric bistable system by introducing an asymmetry parameter into a symmetric bistable system. By means of the Novikov theorem [30], Fox approach [31], and Hänggi Ansatz [32], the average escape times from the left and right metastable states are obtained analytically by the steepest-descent approximation [37]. Our study shows that increasing the correlation time between noises weakens the NES effect produced by multiplicative noise for the left and right states, which is opposite to the effect of the cross-correlation strength on the NES phenomenon [27]. We also observe that the peak position of the MFPT shifts toward smaller values of the noise intensity and the NES region becomes narrow with increasing the correlation time between the noises. In addition, our results indicate that by increasing the correlation time between noises, the escape process from the right state can be speeded up for negatively correlated noise terms, whereas it can be slowed down for positively correlated noise terms. However, for the escape process from the left state, the dependence of the role of the correlation time on the sign of the cross-correlation intensity is reversed. Moreover, the effect of the correlation time on the MFPT can be suppressed by increasing the cross-correlation intensity or additive noise intensity, but variation in the asymmetry parameter does not affect the contribution of the correlation time to the MFPT.

We hope that our results will be helpful for understanding the complex behavior of some practical stochastic systems. Additionally, real noise sources in experiments are correlated with a finite correlation time. As a result, the NES effect can be observed at smaller noise intensities than in the zero correlation time case. The suppression and the shift of the NES region toward smaller values of the noise intensity, allow us to control experimentally the NES effect by using only a suitable correlation time between noises.

Acknowledgements This work was supported by the National Natural Science Foundation of China (Grant No. 11165016), the Key Project of Research Fund of Education Department of Yunnan Province (Grant No. 2013Z097) and the Natural Science Foundation of Yunnan Province of China (Grant No. 2012FB176).

References

1. L. H. Gammaitoni, P. Hänggi, P. Jung, and F. Marchesoni, Stochastic resonance, *Rev. Mod. Phys.* 70(1), 223 (1998)
2. C. R. Doering and C. J. Gadoua, Resonant activation over a fluctuating barrier, *Phys. Rev. Lett.* 69(16), 2318 (1992)
3. B. Spagnolo, N. Agudov, and A. Dubkov, Noise enhanced stability, *Acta Phys. Pol. B* 35(4), 1419 (2004)
4. J. E. Hirsch, B. A. Huberman, and D. J. Scalapino, Theory of intermittency, *Phys. Rev. A* 25(1), 519 (1982)
5. A. A. Dubkov, N. V. Agudov, and B. Spagnolo, Noise-enhanced stability in fluctuating metastable states, *Phys.*

Rev. E 69(6), 061103 (2004)

6. A. Fiasconaro, B. Spagnolo, and S. Boccaletti, Signatures of noise-enhanced stability in metastable states, *Phys. Rev. E* 72(6), 061110 (2005)
7. P. D'Odorico, F. Laio, and L. Ridolfi, Noise-induced stability in dryland plant ecosystems, *Proc. Natl. Acad. Sci. USA* 102(31), 10819 (2005)
8. A. Fiasconaro, B. Spagnolo, A. Ochab-Marcinek, and E. Gudowska-Nowak, Co-occurrence of resonant activation and noise-enhanced stability in a model of cancer growth in the presence of immune response, *Phys. Rev. E* 74(4), 041904 (2006)
9. M. Yoshimoto, H. Shirahama, and S. Kurosawa, Noise-induced order in the chaos of the Belousov–Zhabotinsky reaction, *J. Chem. Phys.* 129(1), 014508 (2008)
10. M. Trapanese, Noise enhanced stability in magnetic systems, *J. Appl. Phys.* 105, 07D313 (2009)
11. G. Z. Sun, N. Dong, G. F. Mao, J. Chen, W. W. Xu, Z. M. Ji, L. Kang, P. H. Wu, Y. Yu, and D. Y. Xing, Thermal escape from a metastable state in periodically driven Josephson junctions, *Phys. Rev. E* 75(2), 021107 (2007)
12. R. N. Mantegna and B. Spagnolo, Noise enhanced stability in an unstable system, *Phys. Rev. Lett.* 76(4), 563 (1996)
13. B. Spagnolo, A. A. Dubkov, A. L. Pankratov, E. V. Pankratova, A. Fiasconaro, and A. Ochab-Marcinek, Lifetime of metastable states and suppression of noise in interdisciplinary physical models, *Acta Phys. Polo. B* 38(5), 1925 (2007)
14. A. Fiasconaro and B. Spagnolo, Stability measures in metastable states with Gaussian colored noise, *Phys. Rev. E* 80(4), 041110 (2009)
15. A. Fiasconaro, J. J. Mazo, and B. Spagnolo, Noise-induced enhancement of stability in a metastable system with damping, *Phys. Rev. E* 82(4), 041120 (2010)
16. Z. L. Jia, Numerical investigation of noise enhanced stability phenomenon in a time-delayed metastable system, *Chin. Phys. Lett.* 25(4), 1209 (2008)
17. Z. L. Jia and D. C. Mei, Effects of linear and nonlinear time-delayed feedback on the noise-enhanced stability phenomenon in a periodically driven bistable system, *J. Stat. Mech.* 2011(10), P10010 (2011)
18. C. W. Xie and D. C. Mei, Mean first-passage time of a bistable kinetic model driven by multiplicative coloured noise and additive white noise, *Chin. Phys. Lett.* 20(6), 813 (2003)
19. A. Mielke, Noise induced stability in fluctuating, bistable potentials, *Phys. Rev. Lett.* 84(5), 818 (2000)
20. B. Dybiec and E. Gudowska-Nowak, Quantifying noise induced effects in the generic double-well potential, *Acta Phys. Pol. B* 38(5), 1759 (2007)
21. A. R. Bulsara, M. E. Inchiosa, and L. Gammaitoni, Noise-controlled resonance behavior in nonlinear dynamical systems with broken symmetry, *Phys. Rev. Lett.* 77(11), 2162 (1996)
22. M. E. Inchiosa, A. R. Bulsara, and L. Gammaitoni, Higher-order resonant behavior in asymmetric nonlinear stochastic systems, *Phys. Rev. E* 55(4), 4049 (1997)
23. S. Bouzat and H. S. Wio, Stochastic resonance in extended bistable systems: The role of potential symmetry, *Phys. Rev. E* 59(5), 5142 (1999)
24. L. Gammaitoni and A. R. Balsara, Noise activated nonlinear dynamic sensors, *Phys. Rev. Lett.* 88(23), 230601 (2002)
25. J. H. Li, Effect of asymmetry on stochastic resonance and stochastic resonance induced by multiplicative noise and by mean-field coupling, *Phys. Rev. E* 66(3), 031104 (2002)
26. F. Long, L. C. Du, and D. C. Mei, Asymmetric effects on the associated relaxation time and the correlation function of a bistable system with correlated noises, *Phys. Scr.* 79(4), 045007 (2009)
27. D. C. Mei, Z. L. Jia, and C. J. Wang, Combined effects of asymmetry and noise correlation on the noise-enhanced stability phenomenon in a bistable system, *Phys. Scr.* 84(4), 045012 (2011)
28. A. J. R. Madureira, P. P. Hänggi, and H. S. Wio, Giant suppression of the activation rate in the presence of correlated white noise sources, *Phys. Lett. A* 217(4–5), 248 (1996)
29. M. I. Dykman, D. G. Luchinsky, P. V. E. McClintock, N. D. Stein, and N. Stocks, Stochastic resonance for periodically modulated noise intensity, *Phys. Rev. A* 46(4), R1713 (1992)
30. E. A. Novikov, Functionals and the random-force method in turbulence theory, *Zh. Eksp. Teor. Fiz.* 47, 1919 (1964) (*Sov. Phys. JETP* 20(5), 1290 (1965))
31. R. F. Fox, Uniform convergence to an effective Fokker–Planck equation for weakly colored noise, *Phys. Rev. A* 34(5), 4525 (1986)
32. P. Hänggi, T. T. Mroczkowski, F. Moss, and P. V. E. McClintock, Bistability driven by colored noise: Theory and experiment, *Phys. Rev. A* 32(1), 695 (1985)
33. D. J. Wu, L. Cao, and S. Z. Ke, Bistable kinetic model driven by correlated noises: Steady-state analysis, *Phys. Rev. E* 50(5), 2496 (1994)
34. D. C. Mei, G. Z. Xie, L. Cao, and D. J. Wu, Mean first-passage time of a bistable kinetic model driven by cross-correlated noises, *Phys. Rev. E* 59(4), 3880 (1999)
35. K. Lindenberg and B. J. West, The first, the biggest, and other such considerations, *J. Stat. Phys.* 42, 201 (1986)
36. J. Masoliver, B. J. West, and K. Lindenbergerg, Bistability driven by Gaussian colored noise: First-passage times, *Phys. Rev. A* 35(7), 3086 (1987)
37. C. W. Gardiner, Handbook of Stochastic Methods, Springer Series in Synergetics, Vol. 13, Berlin: Springer-Verlag, 1983