

# Entanglement concentration for a non-maximally entangled four-photon cluster state

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We present a scheme for locally concentrating a non-maximally entangled four-photon cluster state into a maximally-entangled four-photon cluster state. This scheme has a high success probability. The controlled-NOT (CNOT) gate is a crucial ingredient in this scheme, and we use a nearly deterministic CNOT gate, which is similar with that first introduced by Nemoto *et al.* (*Phys. Rev. Lett.*, 2004, 93: 250502). This CNOT gate has a simple structure and does not need the strong nonlinearity.

**Keywords** cluster state, entanglement concentration, controlled-NOT gate

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## 1 Introduction

Quantum entanglement is an important concept in quantum mechanics. It is a key ingredient in the field of quantum information processing (QIP) [1–7]. For many tasks in QIP, the quantum channels should be preferred in maximally entangled states, so one needs to convert non-maximally entangled states into pure maximally entangled ones. Generally speaking, the non-maximally entangled states usually include two different types. The first type is the mixed state and the second type is the pure less-entangled state. Both of which will make the fidelity of quantum teleportation degraded, quantum dense coding failed, and the quantum cryptography protocol insecure. Many schemes have been proposed to extract maximally-entangled states from non-maximally entangled states, such as entanglement concentration (for pure non-maximally entangled states) [8–10], entanglement purification or distillation (for mixed entangled states) [11–14].

The concentration schemes for two-particle state have been well studied. In a three-particle system [7, 15–17], there are two classes of tripartite-entangled states. They are the Greenberger–Horne–Zeilinger (GHZ) state and the W state. The entanglement concentration for two-particle state is usually suitable for the case of multipartite GHZ state [18–20]. However, these kinds of entanglement concentration cannot deal with the case of a

less-entangled W state. In 2003, Cao *et al.* presented an entanglement concentration for a W class state with the help of joint unitary transformation [21]. In 2007, Zhang *et al.* presented an entanglement concentration based on the Bell-state measurement [22]. In 2010, Wang *et al.* presented an entanglement concentration for a W state with linear optics [23]. In 2012, Sheng *et al.* presented two two-step practical entanglement concentration protocols for concentrating an arbitrary three-particle less-entangled W state into a maximally entangled W state assisted with single photons [24] and so on.

Recently, more and more attention has been focused on the cluster states. The cluster states are harder to be destroyed by local operations than two-particle states and three-particle states [25]. A few schemes begin to mention the concentration for cluster states [26–28]. In 2013, Si *et al.* presented an efficient three-step entanglement concentration for an arbitrary four-photon cluster state [26]. Zhao *et al.* presented two-step entanglement concentration for arbitrary electronic cluster state [27], and Choudhury *et al.* presented an entanglement concentration protocol for cluster states [28].

In the present article, we present a scheme for locally concentrating a non-maximally entangled four-photon cluster state into a maximally-entangled four-photon cluster state. We can obtain a maximally-entangled cluster state via local operations and classical communication. The scheme needs some controlled-NOT (CNOT) gates [29] to assist the concentration. It has a high success

probability and a simple structure. It only needs the help of the linear optical elements and weak cross-Kerr nonlinearity.

This paper is organized as follows. In Section 2 we introduce the cross-Kerr nonlinear interaction between two light modes and use a nearly deterministic CNOT gate, which is similar with that first introduced by Nemoto *et al.* [30]. In Section 3 we present a scheme for the entanglement concentration of a non-maximally entangled four-photon cluster state. Section 4 includes discussion and conclusions.

## 2 Cross-Kerr nonlinearity and CNOT gate

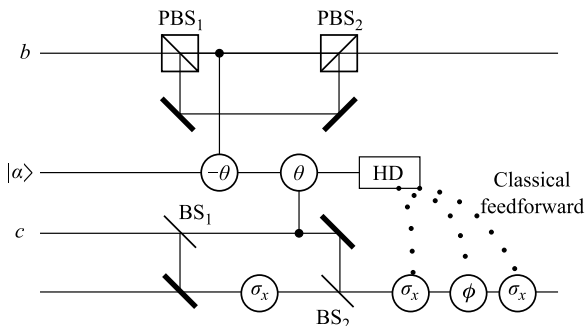
We consider the cross-Kerr nonlinear interaction between a signal beam  $S$  and a probe beam  $P$ . The Hamiltonian for the interaction is [30]

$$\hat{H} = -\hbar\chi\hat{n}_s\hat{n}_p \quad (1)$$

where  $\chi$  is the coupling strength,  $\hat{n}_s$  and  $\hat{n}_p$  are the photon number operators of the signal mode and the probe mode, respectively. If the signal mode is in a Fock state  $|n\rangle_s$  and the probe mode is in a coherent state  $|\alpha\rangle_p$ , then the combined system (signal mode + probe mode) evolves as follows:

$$e^{i\chi\hat{n}_s\hat{n}_p t}|n\rangle_s|\alpha\rangle_p \rightarrow |n\rangle_s|\alpha e^{i\theta}\rangle_p \quad (2)$$

where  $\theta = \chi t$  and  $t$  is the interaction time. From Eq. (2) we can see that the state of the signal mode is unaffected, while the state of the probe mode picks up a phase shift which is proportional to the number of photons in the signal mode [31]. The phase shift of the probe mode can be measured by homodyne detection [32]. In this way, we can determine the state of the signal mode without destroying it by measuring the probe mode. This is a kind of non-destructive measurements.



**Fig. 1** The controlled-NOT gate. PBS: The polarizing beam splitter which is used to transmit the horizontally and to reflect the vertically polarized photons.  $\theta, -\theta$ : Cross-Kerr nonlinearity. HD: Homodyne detection. The probe light is initially in a coherent state  $|\alpha\rangle$ .

The CNOT gate with weak cross-Kerr nonlinearity was first introduced by Nemoto *et al.* [30]. Here we use a similar form of it. The CNOT gate is shown in Fig. 1. We suppose that the input photons are prepared in the state

$$|\varphi_1\rangle_{bc} = a|HH\rangle_{bc} + b|HV\rangle_{bc} + c|VH\rangle_{bc} + d|VV\rangle_{bc} \quad (3)$$

where  $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$ ,  $H$  and  $V$  represent horizontal and vertical polarization, respectively. The controlling photon  $b$  passes through a balanced Mach-Zehnder (M-Z) interferometer made up of two PBSs, while the target photon  $c$  passes through a balanced M-Z interferometer made up of two BSs. The Pauli operator  $\sigma_x$  acts on one arm. The input photons  $b$  and  $c$  interact with the probe mode (in a coherent state  $|\alpha\rangle$ ) via the cross-Kerr nonlinearities respectively, then the state evolves as follows:

$$\begin{aligned} |\varphi_1\rangle_{bc} \otimes |\alpha\rangle &= (a|HH\rangle_{bc} + b|HV\rangle_{bc} \\ &+ c|VH\rangle_{bc} + d|VV\rangle_{bc}) \otimes |\alpha\rangle \rightarrow |\phi\rangle \\ &= \sqrt{\frac{R_1 T_2}{2}}(a|HH\rangle_{bc} + b|HV\rangle_{bc})|\alpha e^{-i\theta}\rangle \\ &+ \sqrt{\frac{T_1 R_2}{2}}(c|VV\rangle_{bc} + d|VH\rangle_{bc})|\alpha e^{i\theta}\rangle \\ &+ [\sqrt{\frac{T_1 R_2}{2}}(a|HV\rangle_{bc} + b|HH\rangle_{bc}) \\ &+ \sqrt{\frac{R_1 T_2}{2}}(c|VH\rangle_{bc} + d|VV\rangle_{bc})] \otimes |\alpha\rangle \end{aligned} \quad (4)$$

where  $R$  and  $T$  represent reflection and transmission coefficients, respectively. We perform the  $X$  homodyne detection on the probe mode. If we find that the probe mode is in the coherent state  $|\alpha e^{\pm i\theta}\rangle$ , we do a feed-forward operation on photon  $c$  and obtain the following state:

$$\begin{aligned} |\varphi_2\rangle_{bc} &= \sqrt{R_1 T_2}(a|HH\rangle_{bc} + b|HV\rangle_{bc}) \\ &+ \sqrt{T_1 R_2}(c|VV\rangle_{bc} + d|VH\rangle_{bc}) \end{aligned} \quad (5)$$

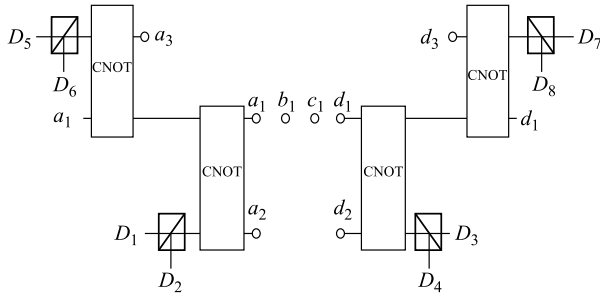
On the other hand, if we find that the probe mode is in the coherent state  $|\alpha\rangle$ , we carry out a single photon operation  $\sigma_x$  on photon  $c$  and obtain the the following state:

$$\begin{aligned} |\varphi_3\rangle_{bc} &= \sqrt{T_1 R_2}(a|HH\rangle_{bc} + b|HV\rangle_{bc}) \\ &+ \sqrt{R_1 T_2}(c|VV\rangle_{bc} + d|VH\rangle_{bc}) \end{aligned} \quad (6)$$

When the conditions  $\sqrt{T_1 R_2} = \sqrt{R_1 T_2}$  and  $T_1 = R_2 = \frac{1}{2}$  are satisfied, we obtain a nearly deterministic CNOT gate, the success probability is  $p = 4|T_1 R_2| = 1$ .

### 3 Entanglement concentration of a non-maximally entangled four-photon cluster state

In the following, we present the scheme for the concentration process of a non-maximally entangled four-photon cluster state in detail. The schematic diagram is shown in Fig. 2. In the scheme, besides some CNOT gates to assist the concentration, there are also some other ingredients, for example, PBSs and single-photon detectors.



**Fig. 2** Schematic diagram of entanglement concentration for a non-maximally entangled four-photon cluster state. CNOT: Controlled-NOT gate.  $D_i$ : Single-photon detector.

We suppose that a non-maximally entangled four-photon cluster state in polarization is described as

$$|\varphi\rangle_{a_1 b_1 c_1 d_1} = (a|HHHH\rangle + b|HHVV\rangle + c|VVHH\rangle - d|VVVV\rangle)_{a_1 b_1 c_1 d_1} \quad (7)$$

where  $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$ . First, we use a photon pair in the state

$$|\varphi\rangle_{a_2 d_2} = (a|HH\rangle + b|HV\rangle + c|VH\rangle + d|VV\rangle)_{a_2 d_2} \quad (8)$$

Let photons  $a_1$  and  $a_2$  pass through the first CNOT gate, and photons  $d_1$  and  $d_2$  pass through the second CNOT gate, then the following transformation can be accomplished:

$$\begin{aligned} |\varphi\rangle_{a_1 b_1 c_1 d_1} \otimes |\varphi\rangle_{a_2 d_2} \rightarrow & (ab|HHHHHV\rangle + ab|HHHVVV\rangle + cd|VHVHHV\rangle \\ & - cd|VHVVVV\rangle)_{a_1 a_2 b_1 c_1 d_1 d_2} + (ac|HVHHHH\rangle + ac|VVVHHH\rangle + bd|HVVHVV\rangle \\ & - bd|VVVVVH\rangle)_{a_1 a_2 b_1 c_1 d_1 d_2} + (bc|HVHVVV\rangle + bc|VVVHHV\rangle + ad|HVVHHV\rangle \\ & - ad|VVVVVV\rangle)_{a_1 a_2 b_1 c_1 d_1 d_2} + (a^2|HHHHHH\rangle + b^2|HHHVHV\rangle) + c^2|VHVHHH\rangle \\ & - d^2|VHVVVH\rangle)_{a_1 a_2 b_1 c_1 d_1 d_2} \end{aligned} \quad (9)$$

After photons  $a_2$  and  $d_2$  pass through PBS, photon  $a_2$  is detected by single photon detectors  $D_1$  and  $D_2$ , while photon  $d_2$  is detected by single photon detectors  $D_3$  and  $D_4$ . The above state will collapse into different states depending on which detectors click:

*Case 1.* If  $D_1$  and  $D_4$  click, the state will collapse into

$$|\varphi_1\rangle_{a_1 b_1 c_1 d_1} = \frac{1}{\sqrt{2|ab|^2 + 2|cd|^2}} [ab(|HHHH\rangle + |HHVV\rangle) + cd(|VVHH\rangle - |VVVV\rangle)]_{a_1 b_1 c_1 d_1} \quad (10)$$

with a success probability of  $p_1 = 2|ab|^2 + 2|cd|^2$ .

*Case 2.* If  $D_2$  and  $D_3$  click, the state will collapse into

$$|\varphi_2\rangle_{a_1 b_1 c_1 d_1} = \frac{1}{\sqrt{2|ac|^2 + 2|bd|^2}} [ac(|HHHH\rangle + |VVHH\rangle) + bd(|HHVV\rangle - |VVVV\rangle)]_{a_1 b_1 c_1 d_1} \quad (11)$$

with a success probability of  $p_2 = 2|ac|^2 + 2|bd|^2$ .

*Case 3.* If  $D_2$  and  $D_4$  click, the state will collapse into

$$|\varphi_3\rangle_{a_1 b_1 c_1 d_1} = \frac{1}{\sqrt{2|bc|^2 + 2|ad|^2}} [bc(|HHVV\rangle + |VVHH\rangle) + ad(|HHHH\rangle - |VVVV\rangle)]_{a_1 b_1 c_1 d_1} \quad (12)$$

with a success probability of  $p_3 = 2|bc|^2 + 2|ad|^2$ .

*Case 4.* If  $D_1$  and  $D_3$  click, the state will collapse into

$$|\varphi_4\rangle_{a_1 b_1 c_1 d_1} = \frac{1}{\sqrt{|a|^4 + |b|^4 + |c|^4 + |d|^4}} (a^2|HHHH\rangle + b^2|HHVV\rangle + c^2|VVHH\rangle - d^2|VVVV\rangle)_{a_1 b_1 c_1 d_1} \quad (13)$$

with a success probability of  $p_4 = |a|^4 + |b|^4 + |c|^4 + |d|^4$ .

Now, let us discuss the concentration process when we have the four-photon state  $|\varphi_1\rangle_{a_1 b_1 c_1 d_1}$ . We use a photon pair in the polarization state

$$|\varphi\rangle_{a_3 d_3} = \frac{1}{\sqrt{2|ab|^2 + 2|cd|^2}} [ab(|HH\rangle + |HV\rangle) + cd(|VH\rangle + |VV\rangle)]_{a_3 d_3} \quad (14)$$

Let the photons  $a_1$  and  $a_3$  pass through the third CNOT gate, photons  $d_1$  and  $d_3$  pass through the fourth CNOT gate, then the following transformation can be accomplished:

$$\begin{aligned} |\varphi_1\rangle_{a_1 b_1 c_1 d_1} \otimes |\varphi\rangle_{a_3 d_3} \rightarrow & \frac{1}{2|ab|^2 + 2|cd|^2} [(ab)^2(|HHHHHV\rangle + |HHHVVV\rangle) + (cd)^2|VHVHHV\rangle \\ & - (cd)^2|VHVVVV\rangle]_{a_1 a_3 b_1 c_1 d_1 d_3} + \frac{abcd}{2|ab|^2 + 2|cd|^2} (|HVVHHH\rangle + |VVVHHH\rangle + |HVVHVV\rangle) \end{aligned}$$

$$\begin{aligned}
 & -|VVVVVH\rangle_{a_1 a_3 b_1 c_1 d_1 d_3} + \frac{abcd}{2|ab|^2 + 2|cd|^2} (|HVHVVV\rangle + |VVVHHV\rangle + |HVHHHV\rangle) \\
 & -|VVVVVV\rangle_{a_1 a_3 b_1 c_1 d_1 d_3} + \frac{1}{2|ab|^2 + 2|cd|^2} [(ab)^2(|HHHHHH\rangle + |HHHVVV\rangle) \\
 & + (cd)^2(|VHVHHH\rangle - |VHVVVH\rangle)]_{a_1 a_3 b_1 c_1 d_1 d_3}
 \end{aligned} \tag{15}$$

After photons  $a_3$  and  $d_3$  pass through PBS, if  $D_6$  and  $D_7(D_8)$  click, then the original four-photon state is left in the maximally entangled cluster state.

$$\begin{aligned}
 |\varphi\rangle_{cluster} = & \frac{1}{2}(|HHHH\rangle + |HHVV\rangle \\
 & + |VVHH\rangle - |VVVV\rangle)_{a_1 b_1 c_1 d_1}
 \end{aligned} \tag{16}$$

with a success probability of

$$p_{cluster} = p_1 \otimes \frac{2|abcd|^2}{(|ab|^2 + |cd|^2)^2} = \frac{4|abcd|^2}{|ab|^2 + |cd|^2} \tag{17}$$

If  $D_5$  and  $D_7(D_8)$  clicks, then the original four-photon state is left in

$$\begin{aligned}
 |\varphi_{11}\rangle_{a_1 b_1 c_1 d_1} = & \frac{1}{\sqrt{2|ab|^4 + 2|cd|^4}} [(ab)^2(|HHHH\rangle \\
 & + |HHVV\rangle) + (cd)^2(|VVHH\rangle
 \end{aligned}$$

$$\begin{aligned}
 P_1^a = & \frac{4|abcd|^2}{|ab|^2 + |cd|^2}, \quad P_2^a = p_{11} \otimes \frac{2|abcd|^4}{(|ab|^4 + |cd|^4)^2} = \frac{4|abcd|^4}{(|ab|^2 + |cd|^2)(|ab|^4 + |cd|^4)} \\
 P_3^a = & p_{11} \otimes \frac{|ab|^8 + |cd|^8}{(|ab|^4 + |cd|^4)^2} \otimes \frac{2|abcd|^8}{(|ab|^8 + |cd|^8)^2} = \frac{4|abcd|^8}{(|ab|^2 + |cd|^2)(|ab|^4 + |cd|^4)(|ab|^8 + |cd|^8)} \\
 P_n^a = & \frac{4|abcd|^{2^n}}{(|ab|^2 + |cd|^2)(|ab|^4 + |cd|^4)(|ab|^8 + |cd|^8) \cdots (|ab|^{2^n} + |cd|^{2^n})}
 \end{aligned} \tag{21}$$

The entanglement concentration processes for the state  $|\varphi_2\rangle_{a_1 b_1 c_1 d_1}$  and  $|\varphi_3\rangle_{a_1 b_1 c_1 d_1}$  are similar as that for the state  $|\varphi_1\rangle_{a_1 b_1 c_1 d_1}$ , so with reiteration of the entanglement concentration process  $n$  times, the success probability of  $|\varphi_2\rangle_{a_1 b_1 c_1 d_1}$  and  $|\varphi_3\rangle_{a_1 b_1 c_1 d_1}$  concentrated into the maximally entangled cluster state are

$$\begin{aligned}
 P_n^b = & \frac{4|abcd|^{2^n}}{(|ac|^2 + |bd|^2)(|ac|^4 + |bd|^4)(|ac|^8 + |bd|^8) \cdots (|ac|^{2^n} + |bd|^{2^n})} \\
 P_n^c = & \frac{4|abcd|^{2^n}}{(|bc|^2 + |ad|^2)(|bc|^4 + |ad|^4)(|bc|^8 + |ad|^8) \cdots (|bc|^{2^n} + |ad|^{2^n})}
 \end{aligned} \tag{23}$$

The state  $|\varphi_4\rangle_{a_1 b_1 c_1 d_1}$  has a similar form with the state  $|\varphi\rangle_{a_1 b_1 c_1 d_1}$ , we only need replace  $\frac{a^2}{\sqrt{a^4+b^4+c^4+d^4}}$ ,  $\frac{b^2}{\sqrt{a^4+b^4+c^4+d^4}}$ ,  $\frac{c^2}{\sqrt{a^4+b^4+c^4+d^4}}$  and  $\frac{d^2}{\sqrt{a^4+b^4+c^4+d^4}}$  with  $a'$ ,  $b'$ ,  $c'$  and  $d'$ . Therefore, after reiteration of the entanglement concentration process  $n$  times, the success proba-

$$-|VVVV\rangle_{a_1 b_1 c_1 d_1} \tag{18}$$

with a success probability of

$$p_{11} = p_1 \otimes \frac{|ab|^4 + |cd|^4}{(|ab|^2 + |cd|^2)^2} = \frac{2(|ab|^4 + |cd|^4)}{|ab|^2 + |cd|^2} \tag{19}$$

Moreover, we can concentrate the four-photon into the maximally entangled cluster state from the less-entangled four-photon state  $|\varphi_{11}\rangle_{a_1 b_1 c_1 d_1}$  in the next round. With repeating the entanglement concentration process  $n$  times, the total success probability of the state  $|\varphi_1\rangle_{a_1 b_1 c_1 d_1}$  concentrated into the maximally entangled cluster state is

$$P^a = \sum_{i=1}^n P_i^a \tag{20}$$

where

$$\begin{aligned}
 P^b = & \sum_{i=1}^n P_i^b \\
 P^c = & \sum_{i=1}^n P_i^c
 \end{aligned} \tag{22}$$

respectively, where

bility of the state  $|\varphi_4\rangle_{a_1 b_1 c_1 d_1}$  concentrated into the maximally entangled cluster state is

$$P^d = \sum_{i=1}^n P_i^d \tag{24}$$

where

$$\begin{aligned}
 P_n^d = & \sum_{n=1}^{\infty} \frac{1}{(|a|^4 + |b|^4 + |c|^4 + |d|^4) \cdots (|a|^{2^{n+1}} + |b|^{2^{n+1}} + |c|^{2^{n+1}} + |d|^{2^{n+1}})} \\
 & \times \left\{ \frac{4|abcd|^{2^{2n}}}{(|ab|^{2^{n+1}} + |cd|^{2^{n+1}})(|ab|^{2^{n+2}} + |cd|^{2^{n+2}}) \cdots (|ab|^{2^{2n}} + |cd|^{2^{2n}})} \right. \\
 & + \frac{4|abcd|^{2^{2n}}}{(|ac|^{2^{n+1}} + |bd|^{2^{n+1}})(|ac|^{2^{n+2}} + |bd|^{2^{n+2}}) \cdots (|ac|^{2^{2n}} + |bd|^{2^{2n}})} \\
 & \left. + \frac{4|abcd|^{2^{2n}}}{(|bc|^{2^{n+1}} + |ad|^{2^{n+1}})(|bc|^{2^{n+2}} + |ad|^{2^{n+2}}) \cdots (|bc|^{2^{2n}} + |ad|^{2^{2n}})} \right\} \quad (25)
 \end{aligned}$$

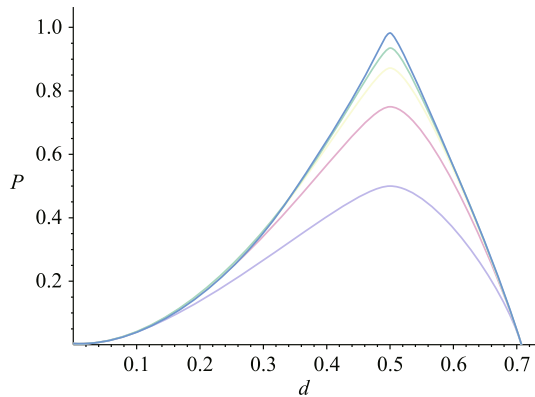
Therefore, by combining above steps, we obtain the total success probability

$$P_{total} = P^a + P^b + P^c + P^d \quad (26)$$

In Eq. (26), if  $a = b = c = d = \frac{1}{2}$ , then

$$\begin{aligned}
 P_{total} &= P^a + P^b + P^c + P^d \\
 &= \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = 1 \quad (27)
 \end{aligned}$$

We have plotted the total success probability of this entanglement concentration process for the iterating times  $n = 1, 2, 3, 4, 5$ , as shown in Fig. 3. We can see that a high success probability can be obtained after reiterating the entanglement concentration process 5 times.



**Fig. 3** The total success probability  $P$  of obtaining a maximally-entangled cluster state after iterating entanglement concentration process  $n = 1, 2, 3, 4, 5$  times (in the bottom-up order). Here  $a = b = \frac{1}{2}$ ,  $d \in (0, \frac{\sqrt{2}}{2})$ .

### 4 Discussion and conclusions

By far, we have fully presented our scheme. Let us discuss its feasibility. Our scheme is based on the cross-Kerr nonlinearity. We know that the natural cross-Kerr nonlinearities are extremely weak. However, Nemoto *et al.* [30] have shown that weak cross-Kerr nonlinearities can be used for quantum information processing. After their work, a series of studies have been done on the cross-Kerr

nonlinearity [33–34]. In our scheme, we need to differentiate the coherent states  $|\alpha e^{i\theta}\rangle$  and  $|\alpha e^{-i\theta}\rangle$ . However, in general, they are not completely orthogonal, and cannot be distinguished determinately [35]. For a measurement of the momentum quadrature, the probability of misidentifying them is given by  $P_{error} = \text{erfc}(\sqrt{2}\alpha \sin \theta)$ .  $P_{error}$  is small when  $\alpha \sin \theta$  is large. For a small  $\theta$ , as long as  $\alpha$  is large enough, we can differentiate the states  $|\alpha e^{i\theta}\rangle$  and  $|\alpha e^{-i\theta}\rangle$  with a small error probability. For example, for  $\alpha \sin \theta = \frac{\pi}{2}$ , the error probability will be in the order of  $10^{-3}$ , and  $P_{error}$  decreases rapidly with increasing  $\alpha \sin \theta$ . This scheme is also influenced by the quantum controlled-NOT gates, and we have used a nearly deterministic controlled-NOT gate.

There is a work by another group (Si *et al.* [26]) which also reports on a concentration scheme for cluster states in an optical set-up. This work uses a single photon resource to assist the protocol and also uses linear optical elements and a weak cross-Kerr nonlinearity. Let us discuss the differences between our scheme from their’s and the advantages of our scheme. We use an almost deterministic controlled-NOT gate with weak cross-Kerr nonlinearity without Toffoli gates which are not completely deterministic. In addition, we use two-photon less-entangled states to assist the entanglement concentration. All of this can simplify the process of entanglement concentration. It has a high success probability which is larger than that in Si *et al.*’s scheme.

In summary, we have proposed a scheme for local concentrating a non-maximally entangled four-photon cluster state into a maximally-entangled four-photon cluster state. The scheme is based on linear optical elements, cross-Kerr nonlinearity, and homodyne measurement. The key ingredient of the scheme is a controlled-NOT gate, and we use a nearly deterministic CNOT gate which is similar with that first introduced by Nemoto *et al.* [30]. And we can accomplish concentrating process only via local operations and classical communication.

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## References

- H. Jeong and M. S. Kim, Efficient quantum computation using coherent states, *Phys. Rev. A*, 2002, 65(4): 042305
- T. C. Ralph, A. Gilchrist, G. J. Milburn, W. Munro, and S. Glancy, Quantum computation with optical coherent states, *Phys. Rev. A*, 2003, 68(4): 042319
- S. J. van Enk and O. Hirota, Entangled coherent states: Teleportation and decoherence, *Phys. Rev. A*, 2001, 64(2): 022313
- H. Jeong, M. S. Kim, and J. Lee, Quantum-information processing for a coherent superposition state via a mixed-entangled coherent channel, *Phys. Rev. A*, 2001, 64(5): 052308
- D. Gottesman and J. Preskill, Secure quantum key distribution using squeezed states, *Phys. Rev. A*, 2001, 63(2): 022309
- N. J. Cerf, M. Lévy, and G. Assche, Quantum distribution of Gaussian keys using squeezed states, *Phys. Rev. A*, 2001, 63(5): 052311
- W. Dür, G. Vidal, and J. I. Cirac, Three qubits can be entangled in two inequivalent ways, *Phys. Rev. A*, 2000, 62(6): 062314
- C. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. Smolin, and W. Wootters, Purification of noisy entanglement and faithful teleportation via noisy channels, *Phys. Rev. Lett.*, 1996, 76(5): 722
- Z. Zhao, J. W. Pan, and M. S. Zhan, Practical scheme for entanglement concentration, *Phys. Rev. A*, 2001, 64(1): 014301
- L. Ye and G. C. Guo, Scheme for entanglement concentration of atomic entangled states in cavity QED, *Phys. Lett. A*, 2004, 327(4): 284
- C. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. Smolin, and W. Wootters, Purification of noisy entanglement and faithful teleportation via noisy channels, *Phys. Rev. Lett.*, 1996, 76(5): 722
- M. Yang and Z. L. Cao, Entanglement distillation for W class states, *Physica A*, 2004, 337(1-2): 141
- M. Yang, W. Song, and Z. L. Cao, Entanglement distillation for atomic states via cavity QED, *Physica A*, 2004, 341: 251
- J. W. Pan, C. Simon, C. Brukner, and A. Zeilinger, Entanglement purification for quantum communication, *Nature*, 2001, 410(6832): 1067
- H. F. Wang, S. Zhang, and K. H. Yeon, Linear optical scheme for entanglement concentration of two partially entangled three-photon W states, *Eur. Phys. J. D*, 2010, 56(2): 271
- L. L. Sun, H. F. Wang, S. Zhang, and K. H. Yeon, Entanglement concentration of partially entangled three-photon W states with weak cross-Kerr nonlinearity, *J. Opt. Soc. Am. B*, 2012, 29(4): 630
- Y. B. Sheng, L. Zhou, and S. M. Zhao, Efficient two-step entanglement concentration for arbitrary W states, *Phys. Rev. A*, 2012, 85(4): 042302
- C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher, Concentrating partial entanglement by local operations, *Phys. Rev. A*, 1996, 53(4): 2046
- Z. Zhao, J. W. Pan, and M. S. Zhan, Practical scheme for entanglement concentration, *Phys. Rev. A*, 2001, 64(1): 014301
- Y. B. Sheng, F. G. Deng, and H. Y. Zhou, Nonlocal entanglement concentration scheme for partially entangled multipartite systems with nonlinear optics, *Phys. Rev. A*, 2008, 77(6): 062325
- Z. L. Cao and M. Yang, Entanglement distillation for three-particle W class states, *J. Phys. B*, 2003, 36(21): 4245
- L. H. Zhang, M. Yang, and Z. L. Cao, Entanglement concentration for unknown W class states, *Physica A*, 2007, 374(2): 611
- H. F. Wang, S. Zhang, and K. H. Yeon, Linear optical scheme for entanglement concentration of two partially entangled three-photon W states, *Eur. Phys. J. D*, 2010, 56(2): 271
- Y. B. Sheng, L. Zhou, and S. M. Zhao, Efficient two-step entanglement concentration for arbitrary W states, *Phys. Rev. A*, 2012, 85(4): 042302
- W. Dür and H. J. Briegel, Stability of macroscopic entanglement under decoherence, *Phys. Rev. Lett.*, 2004, 92(18): 180403
- B. Si, S. L. Su, L. L. Sun, L. Y. Cheng, H. F. Wang, and S. Zhang, Efficient three-step entanglement concentration for an arbitrary four-photon cluster state, *Chin. Phys. B*, 2013, 22(3): 030305
- S. Y. Zhao, J. Liu, L. Zhou, and Y. B. Sheng, Two-step entanglement concentration for arbitrary electronic cluster state, *Quantum Inf. Process.*, 2013, 12(12): 3633
- B. S. Choudhury and A. Dhara, An entanglement concentration protocol for cluster states, *Quantum Inf. Process.*, 2013, 12(7): 2577
- Q. Lin and J. Li, Quantum control gates with weak cross-Kerr nonlinearity, *Phys. Rev. A*, 2009, 79(2): 022301
- K. Nemoto and W. J. Munro, Nearly deterministic linear optical controlled-NOT gate, *Phys. Rev. Lett.*, 2004, 93(25): 250502
- P. Kok, W. J. Munro, K. Nemoto, T. C. Ralph, J. P. Dowling, and G. J. Milburn, Linear optical quantum computing with photonic qubits, *Rev. Mod. Phys.*, 2007, 79(1): 135
- B. Yurke, Wideband photon counting and homodyne detection, *Phys. Rev. A*, 1985, 32(1): 311
- J. H. Shapiro, Single-photon Kerr nonlinearities do not help quantum computation, *Phys. Rev. A*, 2006, 73: 062305
- J. H. Shapiro and M. Razavi, Continuous-time cross-phase modulation and quantum computation, *New. J. Phys.*, 2007, 9: 16
- W. J. Munro, Kae Nemoto, T. P. Spiller, S. D. Barrett, P. Ieter Kok, and R. G. Beausoleil, Efficient optical quantum information processing, *J. Opt. B*, 2005, 7(7): S135