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RESEARCH ARTICLE

A generalized two-mode entangled state: Its generation, properties, and applications

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Using the technique of integration within an ordered product of operators we construct a generalized two-mode entangled state, which can be generated by an asymmetrical beam splitter (BS). Some important properties of this state, such as orthogonality and Schmidt decomposition, are also discussed by deriving the expression of BS operator in coordinate representation. As its applications, to conjugate state, obtain operator identities, generate new squeezing operators (squeezed state) are also presented. It is shown that the fidelity of quantum teleportation can be enhanced under certain case by using the asymmetrical new squeezed state as entangled resource.

Keywords entangled state, beam splitter (BS), quantum teleportation

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1 Introduction

Quantum mechanical representation plays an important role in quantum optics and quantum statistics [1]. How to construct a useful representation is still full of challenge for both theoretical and experimental physicists, especially multipartite entangled states. For example, for Einstein–Podolsky–Rosen' (EPR) entanglement [2], Fan *et al.* [3, 4] constructed a two-mode entangled state and a parametrized entangled state in Fock space, later Hu and Fan [5–7] proposed the three-mode CV entangled state which can be prepared by using an asymmetric BS [8] and a parametric down-conversion (PDC) instrument [9–14]. Both two kinds of entangled states are proved to make up new quantum mechanical representations.

As Dirac said [15], "*When one has a particular problem to work out in quantum mechanics, one can minimize the labor by using a representation in which the representatives of the more important abstract quantities occurring in that problem are as simple as possible*". For instance, the appropriate entangled state representation $|\zeta\rangle$ [see below Eq. (1)] is used to tackle quantum teleportation [16], controlled dense coding, the evolution issue from pure states to mixed states, and the correspondence between classical optics and quantum optics [17]. It is shown that this entangled state is very convenient and effective. In recent year, there are an increasing number of physicists studying quantum entangled states since more and more potential uses of them in quantum communication and quantum optics were discovered [18–21].

In fact, the entanglement was first revealed in EPR pioneer paper by explicitly writing the wave function of two particles with their relative position $Q_1 - Q_2$ being x_0 and their total momentum $P_1 + P_2$ being p_0, i.e., $\psi(x_1, x_2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}p e^{\mathrm{i}p(x_1-x_2+x_0)}$. In Refs. [4, 22, 23], it is found that the simultaneous eigenstate $|\zeta\rangle$ of commutative operators $(Q_1 + Q_2, P_2 - P_1)$ in the two-mode Fock space is

$$|\zeta\rangle = \exp\left\{-\frac{1}{2}|\zeta|^2 + \zeta b^\dagger + \zeta^* a^\dagger - a^\dagger b^\dagger\right\}|00\rangle \tag{1}$$

where $\zeta = q + ip$ is complex number; $|00\rangle$ is the two-mode vacuum state; $a, b(a^\dagger, b^\dagger)$ are Bose annihilation (creation) operators in Fock space; and $Q_1 = (a + a^\dagger)/\sqrt{2}, Q_2 = (b + b^\dagger)/\sqrt{2}, P_1 = (a - a^\dagger)/(\sqrt{2}\mathrm{i}), P_2 = (b - b^\dagger)/(\sqrt{2}\mathrm{i})$. The $|\zeta\rangle$ obeys the eigenvector equations

$$(Q_1 + Q_2)|\zeta\rangle = \sqrt{2}q|\zeta\rangle, (P_2 - P_1)|\zeta\rangle = \sqrt{2}p|\zeta\rangle \tag{2}$$

as well as the orthogonal property $\langle\zeta|\zeta'\rangle = \pi\delta(\zeta -$

$\zeta')\delta(\zeta^* - \zeta^{'*})$. $|\zeta\rangle$ can be built when a coordinate state and a momentum state are imported in mode a and mode b of a 50:50 beam splitter (BS) respectively. Exchanging the imported states, the conjugate state of $|\zeta\rangle$ can also be generated at the output of the BS. Notcing that the communicative relation $[\mu Q_1 + \nu Q_2, \mu P_2 - \nu P_1] = 0$, where μ, ν are any number, then four questions naturally arise: what is the common eigenvector? how to generate it experimentally? what are the properties? what applications are obtained by using this state? As far as we are concerned, there is no report about the common eigenvector of $\mu Q_1 + \nu Q_2$ and $\mu P_2 - \nu P_1$, its generation, as well as its appliactions.

In this paper, we will give the answer: a generalized entangled state denoted as $|\zeta\rangle_{\mu,\nu}$. Similarly, if we exchange the imported states, the conjugate state of $|\zeta\rangle_{\mu,\nu}$ will be found at output. By controlling the value of μ, ν, we can obtain a large number of entangled states needed. The work is arranged as follows. In Section 2, we derive the explicit form of the generalized entangled state $|\zeta\rangle_{\mu,\nu}$ by the technique of integration within an ordered product of operators (IWOP) [4, 24]. Then we derive the eigenvalue equations of $|\zeta\rangle_{\mu,\nu}$ in Section 3. In Section 4, we present an experiment scheme for generating the $|\zeta\rangle_{\mu,\nu}$. In Section 5, we analyze its main properties. At last, we discuss some important applications of $|\zeta\rangle_{\mu,\nu}$ in quantum optics in Section 6.

2 Theoretical method to get a generalized entangled state

In this section, we shall propose the analytical expression of the generalized entangled state by using the IWOP technique and then derive its eigenvalue equations.

2.1 The entangled state obtained by the IWOP technique

It is well known that if a quantum state can span a complete space, then its completeness can be written as normally ordered Gaussian integration. For instance, the coherent state's completeness can be written as

$$\int \frac{\mathrm{d}^2\alpha}{\pi}|\alpha\rangle\langle\alpha| = \int \frac{\mathrm{d}^2\alpha}{\pi}{:}\,\mathrm{e}^{-(\alpha-a)(\alpha^*-a^\dagger)}{:} = 1 \tag{3}$$

where $|\alpha\rangle$ is the coherent state, and the symbol $::$ denotes the normally ordering.

Eq. (3) implies that under the frame of normally ordered form, we can construct miscellaneous normally ordered Gaussian integration operators, which are unity operators, and decompose the integration of operators in normal order, then we may derive new quantum mechanical representations with the characteristics of completeness relation. For example, from the normally ordered Gaussian form of unity

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}q}{\sqrt{\pi}}{:}\exp[-(q-Q)^2]{:} = 1 \tag{4}$$

where $Q = (a + a^\dagger)/\sqrt{2}$ is the coordinate operator, by using the normal ordering of vacuum state projector $|0\rangle\langle 0| = {:}\exp(-a^\dagger a){:}$, we can obtain the coordinate state $Q|q\rangle = q|q\rangle$, where

$$|q\rangle = \frac{1}{\pi^{1/4}}\exp\left(-\frac{1}{2}q^2 + \sqrt{2}qa^\dagger - \frac{a^{\dagger 2}}{2}\right)|0\rangle \tag{5}$$

Motivated by the above spirit, now we construct a unity of the Gaussian operators integration within normal ordering as follows:

$$1 = \int_{-\infty}^{\infty}\frac{\mathrm{d}^2\zeta}{\pi}{:}\exp\left\{-\left[\zeta - \frac{(\mu Q_1+\nu Q_2)+\mathrm{i}(\mu P_2-\nu P_1)}{\lambda}\right]\right.$$
$$\left.\cdot\left[\zeta^* - \frac{(\mu Q_1+\nu Q_2)-i(\mu P_2-\nu P_1)}{\lambda}\right]\right\}{:} \tag{6}$$

where μ, ν are two independent parameters, $\lambda = \sqrt{\mu^2+\nu^2}$, $\zeta = q + ip$ is complex number. Using the normal product form of the two-mode vacuum projector $|00\rangle\langle 00| = {:}\exp[-a^\dagger a - b^\dagger b]{:}$ [25], by decomposing the unity of equation (6), we can rewrite Eq. (6) as

$$\int \frac{d^2\zeta}{\pi}|\zeta\rangle_{\mu,\nu\,\mu,\nu}\langle\zeta| = 1 \tag{7}$$

where $|\zeta\rangle_{\mu,\nu}$ is called as a generalized entangled state and it reads as

$$|\zeta\rangle_{\mu,\nu} = \exp\left\{-\frac{1}{2}|\zeta|^2 + \frac{1}{\sqrt{2}\lambda}[(\mu+\nu)\zeta - (\mu-\nu)\zeta^*]b^\dagger\right.$$
$$+\frac{1}{\sqrt{2}\lambda}[(\mu-\nu)\zeta + (\mu+\nu)\zeta^*]a^\dagger$$
$$\left.+\frac{1}{2\lambda^2}(\mu^2-\nu^2)(b^{\dagger 2}-a^{\dagger 2}) - \frac{2\mu\nu}{\lambda^2}a^\dagger b^\dagger\right\}|00\rangle \tag{8}$$

which is a new entangled state representation (see discussion below). Eq. (7) is just the completeness relation of $|\zeta\rangle_{\mu,\nu}$. In particular, when $\mu = \nu$, Eq. (8) just reduces to Eq. (1), and when $\mu = -\nu$, Eq. (8) becomes

$$|\zeta\rangle = \exp\left(-\frac{1}{2}|\zeta|^2 + \zeta a^\dagger - \zeta^* b^\dagger + a^\dagger b^\dagger\right)|00\rangle \tag{9}$$

which is just the common eigenvector of communicative operators $Q_1 - Q_2, P_2 + P_1$ satisfying $[Q_1 - Q_2, P_2 + P_1] = 0$ [23, 24].

2.2 The eigenvalue equations of $|\zeta\rangle_{\mu,\nu}$

In order to obtain the eigenvalue equations of $|\zeta\rangle_{\mu,\nu}$, using the bosnic commutative relation $[a_i, a_j] = \delta_{i,j}$ and $[a_i, :f(a_i, a_i^\dagger):] =: \frac{\partial}{\partial a_i^\dagger} f(a_i, a_i^\dagger):$, we have

$$a|\zeta\rangle_{\mu,\nu} = \left[\frac{\sqrt{2}}{\lambda}(\mu q - \mathrm{i}p\nu) - \frac{1}{\lambda^2}(\mu^2 - \nu^2)a^\dagger - \frac{2\mu\nu}{\lambda^2}b^\dagger\right]|\zeta\rangle_{\mu,\nu} \tag{10}$$

$$b|\zeta\rangle_{\mu,\nu} = \left[\frac{\sqrt{2}}{\lambda}(q\nu + \mathrm{i}p\mu) + \frac{1}{\lambda^2}(\mu^2 - \nu^2)b^\dagger - \frac{2\mu\nu}{\lambda^2}a^\dagger\right]|\zeta\rangle_{\mu,\nu} \tag{11}$$

Combining Eqs. (10) and (11), we can obtain the eigenvalue equations

$$(\mu Q_1 + \nu Q_2)|\zeta\rangle_{\mu,\nu} = \lambda q|\zeta\rangle_{\mu,\nu} \tag{12}$$

$$(\mu P_2 - \nu P_1)|\zeta\rangle_{\mu,\nu} = \lambda p|\zeta\rangle_{\mu,\nu} \tag{13}$$

from which we can see that the generalized entangled state $|\zeta\rangle_{\mu,\nu}$ is the common eigenvector of $(\mu Q_1 + \nu Q_2)$ and $(\mu P_2 - \nu P_1)$ due to the communicative relation $[\mu Q_1 + \nu Q_2, \mu P_2 - \nu P_1] = 0$.

In particular, when we take $\mu = \mu_1, \nu = \mu_2$, satisfying $\mu_1 + \mu_2 = 1$, then Eqs. (12), and (13) reduce to

$$(\mu_1 Q_1 + \mu_2 Q_2)|\zeta\rangle_{\mu,\nu} = \sqrt{\mu_1^2 + \mu_2^2}\, q|\zeta\rangle_{\mu,\nu} \tag{14}$$

$$(\mu_1 P_2 - \mu_2 P_1)|\zeta\rangle_{\mu,\nu} = \sqrt{\mu_1^2 + \mu_2^2}\, p|\zeta\rangle_{\mu,\nu} \tag{15}$$

which is just the common eigenstates of two particles' center-of-mass coordinates and mass-weighted relative momentum [22]. So the states $|\zeta\rangle_{\mu,\nu}$ can be considered as the generalized entangled state representation.

3 Generation of the $|\zeta\rangle_{\mu,\nu}$ by using an asymmetric beam splitter

The beam splitter (BS) is an extraordinary important role in generating some quantum states. For instance, a coherent-entangled state can be generated by asymmetric beam splitter [6]; in Ref. [26], the superposition states $C_0|0\rangle + C_1|1\rangle + C_2|2\rangle$ can also be generated by an asymmetric BS. In this section, we will show how the generalized entangled state $|\zeta\rangle_{\mu,\nu}$ can be produced by using an asymmetric BS.

When a quantum state pass through BS, the function of BS can be regarded as unity operator, and the role of an asymmetric BS operator with free-phase can be expressed by [27]

$$B(\theta) = \exp[\theta(ab^\dagger - a^\dagger b)] \tag{16}$$

the transformarion relation unsder the BS operator are given by

$$B(\theta)aB^\dagger(\theta) = a\cos\theta + b\sin\theta \tag{17}$$

$$B(\theta)bB^\dagger(\theta) = b\cos\theta - a\sin\theta \tag{18}$$

We consider a coordinate state and a momentum state as import states of BS in mode a and mode b respectively,

$$|q\rangle_a = \left(\frac{1}{\pi}\right)^{1/4} \exp\left(-\frac{1}{2}q^2 + \sqrt{2}qa^\dagger - \frac{a^{\dagger 2}}{2}\right)|0\rangle \tag{19}$$

$$|p\rangle_b = \left(\frac{1}{\pi}\right)^{1/4} \exp\left(-\frac{1}{2}p^2 + \mathrm{i}\sqrt{2}pb^\dagger + \frac{b^{\dagger 2}}{2}\right)|0\rangle \tag{20}$$

then according to Eqs. (17)–(20), the exporting state is given by

$$\begin{aligned}
&B\left(\theta = \arccos\frac{\mu}{\lambda}\right)|q\rangle_a \otimes |p\rangle_b \\
&= \frac{1}{\pi^{1/2}} \exp\left\{-\frac{1}{2}|\zeta|^2 + \frac{\sqrt{2}}{\lambda}(q\nu + \mathrm{i}p\mu)b^\dagger + \frac{\sqrt{2}}{\lambda}(\mu q - \mathrm{i}p\nu)a^\dagger \right. \\
&\quad \left. + \frac{1}{2\lambda^2}(\mu^2 - \nu^2)(b^{\dagger 2} - a^{\dagger 2}) - \frac{2\mu\nu}{\lambda^2}a^\dagger b^\dagger\right\}|00\rangle \\
&= \frac{1}{\pi^{1/2}}|\zeta\rangle_{\mu,\nu}
\end{aligned} \tag{21}$$

which indicates that the generalized entangled state $|\zeta\rangle_{\mu,\nu}$ can be implemented via the following way

$$|\zeta\rangle_{\mu,\nu} = \sqrt{\pi}B\left(\theta = \arccos\frac{\mu}{\lambda}\right)|q\rangle_a \otimes |p\rangle_b \tag{22}$$

where $\cos\theta = \mu/\lambda, \sin\theta = v/\lambda, \lambda = \sqrt{\mu^2 + \nu^2}$. Here we should point out that the above method is simple and convenient from the view of theoretical point to construct new quantum mechanical representation, which will be seen clearly in next section.

4 Main properties of $|\zeta\rangle_{\mu,\nu}$

In this section, we shall discuss the main properties of $|\zeta\rangle_{\mu,\nu}$ such as the orthogonality and the entanglement by deriving Schmidt decomposition of $|\zeta\rangle_{\mu,\nu}$.

4.1 Orthogonality of $|\zeta\rangle_{\mu,\nu}$

Instead of using Eqs. (12) and (13), we employ Eq. (22) to obtain the orthogonality of $|\zeta\rangle_{\mu,\nu}$. Notice that the orthogonal relation of coordinate states and momentum states read

$$\langle q|q'\rangle = \delta(q - q') \tag{23}$$

$$\langle p|p'\rangle = \delta(p - p') \tag{24}$$

and $B^{\dagger}(\theta)B(\theta)=1$, thus from Eq. (22) we can get

$$\begin{aligned}{}_{\mu,\nu}\langle\zeta'|\zeta\rangle_{\mu,\nu}&=\pi_b\langle p'|\otimes_a\langle q'|B^{\dagger}B|q\rangle_a\otimes|p\rangle_b\\&=\pi\delta(q-q')\delta(p-p')\\&=\pi\delta^{(2)}(\zeta'-\zeta)\end{aligned}\tag{25}$$

where $\zeta'=q'+\mathrm{i}p'$, and $\zeta=q+\mathrm{i}p$. It is obvious that $|\zeta\rangle_{\mu,\nu}$ possesses the orthogonality. Actually, we can check the relation Eq. (25) by executing additive and subtractive operation to Eqs. (12) and (13).

4.2 The Schmidt decomposition of $|\zeta\rangle_{\mu,\nu}$

In order to clarify the entanglement property of $|\zeta\rangle_{\mu,\nu}$, here we examine its Schmidt decomposition. For this purpose, we first present the representation of BS operator in coordinate representation.

4.2.1 Representation of BS operator in coordinate representation

From the transformation relation in Eqs. (17) and (18), we can see that

$$B(\theta)\begin{pmatrix}Q_1\\Q_2\end{pmatrix}B^{\dagger}(\theta)=\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}\begin{pmatrix}Q_1\\Q_2\end{pmatrix}\tag{26}$$

which leads to

$$B(\theta)Q_1=(Q_1\cos\theta+Q_2\sin\theta)B(\theta)\tag{27}$$

$$B(\theta)Q_2=(Q_2\cos\theta-Q_1\sin\theta)B(\theta)\tag{28}$$

Operating Eqs. (27) and (28) on the two-mode coordinate states $|q_1,q_2\rangle=|q_1\rangle_a\otimes|q_2\rangle_b$, we can get

$$(Q_1\cos\theta+Q_2\sin\theta)B(\theta)|q_1,q_2\rangle=q_1B(\theta)|q_1,q_2\rangle\tag{29}$$

$$(Q_2\cos\theta-Q_1\sin\theta)B(\theta)|q_1,q_2\rangle=q_2B(\theta)|q_1,q_2\rangle\tag{30}$$

From Eqs. (29) and (30) we know that the quantum states $B(\theta)|q_1,q_2\rangle$ are the common eigenstate of operator $(Q_1\cos\theta+Q_2\sin\theta)$ and $(Q_2\cos\theta-Q_1\sin\theta)$. The corresponding matrix form is given by

$$\begin{aligned}&\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}\begin{pmatrix}Q_1\\Q_2\end{pmatrix}B(\theta)|q_1,q_2\rangle\\&=\begin{pmatrix}q_1\\q_2\end{pmatrix}B(\theta)|q_1,q_2\rangle\end{aligned}\tag{31}$$

It then follows

$$\begin{aligned}&\begin{pmatrix}Q_1\\Q_2\end{pmatrix}\{B(\theta)|q_1,q_2\rangle\}\\&=\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}^{-1}\begin{pmatrix}q_1\\q_2\end{pmatrix}\{B(\theta)|q_1,q_2\rangle\}\end{aligned}\tag{32}$$

which indicates that

$$B(\theta)|q_1,q_2\rangle=|\begin{pmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{pmatrix}\begin{pmatrix}q_1\\q_2\end{pmatrix}\rangle\tag{33}$$

where $|\begin{pmatrix}q_1\\q_2\end{pmatrix}\rangle\equiv|q_1,q_2\rangle$.

Using the completeness relation of the coordinate state,

$$\int_{-\infty}^{\infty}\mathrm{d}q_1\mathrm{d}q_2|q_1,q_2\rangle\langle q_1,q_2|=1\tag{34}$$

then BS operator in coordinate representation can be expressed as

$$B(\theta)=\int_{-\infty}^{\infty}\mathrm{d}q_1\mathrm{d}q_2|\begin{pmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{pmatrix}\begin{pmatrix}q_1\\q_2\end{pmatrix}\rangle\langle\begin{pmatrix}q_1\\q_2\end{pmatrix}|\tag{35}$$

which is just the coordinate representation of BS operator.

In a similar way, we can derive the BS operator in momentum representation as

$$B(\theta)=\int_{-\infty}^{\infty}\mathrm{d}p_1\mathrm{d}p_2|\begin{pmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{pmatrix}\begin{pmatrix}p_1\\p_2\end{pmatrix}\rangle\langle\begin{pmatrix}p_1\\p_2\end{pmatrix}|\tag{36}$$

where $|\begin{pmatrix}p_1\\p_2\end{pmatrix}\rangle\equiv|p_1\rangle\otimes|p_2\rangle$.

4.2.2 Schmidt decomposition of $|\zeta\rangle_{\mu,\nu}$

On the basis of the coordinate (momentum) representation of BS operator, it is simple to derive the Schmidt decomposition of $|\zeta\rangle_{\mu,\nu}$. Note the overlap between the momentum state $|p\rangle$ and the coordinate state $|q\rangle$,

$$\langle q|p\rangle=\frac{1}{\sqrt{2\pi}}\mathrm{e}^{\mathrm{i}pq}\tag{37}$$

and combining Eqs. (22) and (35), we can derive

$$\begin{aligned}|\zeta\rangle_{\mu,\nu}&=\pi^{1/2}B\left(\theta=\arccos\frac{\mu}{\lambda}\right)|q\rangle_a\otimes|p\rangle_b\\&=\pi^{1/2}\int_{-\infty}^{\infty}\mathrm{d}q_1\mathrm{d}q_2|\begin{pmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{pmatrix}\begin{pmatrix}q_1\\q_2\end{pmatrix}\rangle\\&\quad\cdot\langle\begin{pmatrix}q_1\\q_2\end{pmatrix}||q\rangle_a|p\rangle_b\end{aligned}$$

$$= \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} \mathrm{d}q_2 | \frac{1}{\lambda} \begin{pmatrix} \mu & -\nu \\ \nu & \mu \end{pmatrix} \begin{pmatrix} q \\ q_2 \end{pmatrix} \rangle \mathrm{e}^{\mathrm{i}pq_2}$$
$$= \int_{-\infty}^{\infty} \mathrm{d}q_2 | \frac{1}{\lambda} (\mu q - \sqrt{2}\nu q_2) \rangle_a \cdot | \frac{1}{\lambda} (\nu q + \sqrt{2}\mu q_2) \rangle_b \mathrm{e}^{\mathrm{i}\sqrt{2}pq_2} \quad (38)$$

which is actually the Schmidt decomposition of $|\zeta\rangle_{\mu,\nu}$ in coordinate representation.

In the similar way, using the BS operator in momentum representation [Eq. (36)], the Schmidt decomposition of $|\zeta\rangle_{\mu,\nu}$ in momentum representation can be calculated as

$$|\zeta\rangle_{\mu,\nu} = \int_{-\infty}^{\infty} \mathrm{d}p_1 | \frac{1}{\lambda} (\sqrt{2}\mu p_1 - \nu p) \rangle_a \cdot | \frac{1}{\lambda} (\sqrt{2}\nu p_1 + \mu p) \rangle_b \mathrm{e}^{-\mathrm{i}\sqrt{2}p_1 q} \quad (39)$$

From Eq. (38) or Eq. (39), one can draw a conclusion that $|\zeta\rangle_{\mu,\nu}$ is an entangled state [28].

5 Applications of $|\zeta\rangle_{\mu,\nu}$

5.1 The conjugate state of $|\zeta\rangle_{\mu,\nu}$

Taking a complex Fourier transformation of $|\zeta\rangle_{\mu,\nu}$, we can see

$$|\eta\rangle_{\mu,\nu} = \frac{1}{2} \int_{-\infty}^{\infty} \mathrm{d}^2\eta \exp\left[\frac{1}{2}(\zeta\eta^* - \zeta^*\eta)\right] |\zeta\rangle_{\mu,\nu}$$
$$= \exp\left\{ -\frac{|\eta|^2}{2} + \frac{1}{\sqrt{2}\lambda}[(\mu+\nu)\eta + (\mu-\nu)\eta^*]b^\dagger + \frac{1}{\sqrt{2}\lambda}[(\mu-\nu)\eta - (\mu+\nu)\eta^*]a^\dagger + \frac{1}{2\lambda^2}(\mu^2-\nu^2)(a^{\dagger 2} - b^{\dagger 2}) + \frac{2\mu\nu}{\lambda^2}a^\dagger b^\dagger \right\} |00\rangle \quad (40)$$

where we have used the following integration formula [29]

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}^2\eta}{\pi} \exp(-a|\eta|^2 + b\eta + c\eta^* + d\eta^2 + f\eta^{*2}) = \frac{1}{\sqrt{a^2-4df}} \exp\left(\frac{abc + dc^2 + fb^2}{\sqrt{a^2-4df}}\right) \quad (41)$$

Using the bosnic commutative relation $[a_i, a_j^\dagger] = \delta_{i,j}$ and $[a_i, :f(a_i, a_i^\dagger):] =: \frac{\partial}{\partial a_i^\dagger} f(a_i, a_i^\dagger):$, we have

$$a|\eta\rangle_{\mu,\nu} = \left[\frac{\sqrt{2}}{\lambda}(\mathrm{i}p'\mu - \nu q') + \frac{1}{\lambda^2}(\mu^2 - \nu^2)a^\dagger + \frac{2\mu\nu}{\lambda^2}b^\dagger\right] |\eta\rangle_{\mu,\nu} \quad (42)$$

$$b|\eta\rangle_{\mu,\nu} = \left[\frac{\sqrt{2}}{\lambda}(\mu q' + \mathrm{i}p'\nu) - \frac{1}{\lambda^2}(\mu^2 - \nu^2)b^\dagger + \frac{2\mu\nu}{\lambda^2}a^\dagger\right] |\eta\rangle_{\mu,\nu} \quad (43)$$

where $\eta = q' + \mathrm{i}p'$. Combining Eqs. (42) and (43), the eigenvalue equations are

$$(\mu Q_2 - \nu Q_1)|\eta\rangle_{\mu,\nu} = \lambda q' |\eta\rangle_{\mu,\nu}$$
$$(\mu P_1 + \nu P_2)|\eta\rangle_{\mu,\nu} = \lambda p' |\eta\rangle_{\mu,\nu} \quad (44)$$

It is shown that $|\eta\rangle_{\mu,\nu}$ is the common eigenvector of communicative operators $\mu Q_2 - \nu Q_1$ and $\mu P_1 + \nu P_2$, $[\mu Q_2 - \nu Q_1, \mu P_1 + \nu P_2] = 0$. Note $[\mu Q_2 - \nu Q_1, \mu P_2 - \nu P_1] = \mathrm{i}\lambda^2$, and $[\mu Q_1 + \nu Q_2, \mu P_1 + \nu P_2] = \mathrm{i}\lambda^2$, thus we can call $|\eta\rangle_{\mu,\nu}$ as the conjugated state of $|\zeta\rangle_{\mu,\nu}$.

In foregoing section, on the other hand, we know that the way of generating $|\zeta\rangle_{\mu,\nu}$ is importing a coordinate state and a momentum state in mode a and mode b respectively. Contrast with the generation of $|\zeta\rangle_{\mu,\nu}$, if we exchange the imported states in mode a and mode b, then the exporting state will be the conjugate state of $|\zeta\rangle_{\mu,\nu}$,

$$\sqrt{\pi}B\left(\theta = \arccos\frac{\mu}{\lambda}\right) |p'\rangle_a \otimes |q'\rangle_b = |\eta\rangle_{\mu,\nu} \quad (45)$$

Thus similar to deriving Eq. (39), the Schmidt decomposition of $|\eta\rangle_{\mu,\nu}$ in momentum representation is

$$|\eta\rangle_{\mu,\nu} = \int_{-\infty}^{\infty} \mathrm{d}p_2 | \frac{1}{\lambda}(\mu p' - \sqrt{2}\nu p_2) \rangle_a \cdot | \frac{1}{\lambda}(\nu p' + \sqrt{2}\mu p_2) \rangle_b \mathrm{e}^{-\mathrm{i}\sqrt{2}p_2 q'} \quad (46)$$

So the the inner product between $|\eta\rangle_{\mu',\nu'}$ and $|\zeta\rangle_{\mu,\nu}$ with different parameter values μ, ν can be calculated as

$${}_{\mu',\nu'}\langle\eta|\zeta\rangle_{\mu,\nu} = \frac{\lambda\lambda'}{2(\nu\nu' + \mu\mu')} \exp\left(-\mathrm{i}\frac{\lambda\mu' p' q + \lambda'\nu p q}{\lambda'\mu}\right) \cdot \exp\left\{\mathrm{i}\frac{\lambda\lambda'(\mu'\nu - \mu\nu')\mu p' q'}{\lambda\lambda'\mu(\nu\nu' + \mu\mu')}\right\} \cdot \exp\left\{\mathrm{i}\frac{\lambda^2\lambda'^2\mu p q' + \lambda^2(\mu'\nu - \mu\nu')\cdot\nu' p' q + \lambda\lambda'\lambda^2\nu' p q}{\lambda\lambda'\mu(\nu\nu' + \mu\mu')}\right\} \quad (47)$$

In particular, when $\mu = \mu', \nu = \nu'$, we have

$${}_{\mu,\nu}\langle\eta|\zeta\rangle_{\mu,\nu} = \frac{1}{2}\exp\left[\frac{1}{2}(\zeta\eta^* - \eta\zeta^*)\right] \quad (48)$$

as expected. From Eq. (47), one can clearly see that the overlap between $|\eta\rangle_{\mu',\nu'}$ and $|\zeta\rangle_{\mu,\nu}$ is actually a complex Fourier transformation kernel, which implies that one can introduce a generalized complex Fourier transform [17].

5.2 Some operator identities

In fact, quantum mechanical representation has its application in deriving some operator identities and integration formulas. Here we use the generalized entangled state representation and the IWOP technique to realize this purpose.

Using the completeness of generalized entangled state representation [Eq. (7)], we have

$$\begin{aligned}(\mu Q_1+\nu Q_2)^n &= \int_{-\infty}^{\infty}\frac{\mathrm{d}^2\zeta}{\pi}(\mu Q_1+\nu Q_2)^n|\zeta\rangle_{\mu,\nu\,\mu,\nu}\langle\zeta| \\ &= \int_{-\infty}^{\infty}\frac{\mathrm{d}^2\zeta}{\pi}(\lambda q)^n|\zeta\rangle_{\mu,\nu\,\mu,\nu}\langle\zeta| \\ &= \int_{-\infty}^{\infty}\frac{\mathrm{d}q}{\sqrt{\pi}}(\lambda q)^n \\ &\quad :\exp\left\{-\left(q-\frac{\mu Q_1+\nu Q_2}{\lambda}\right)^2\right\}: \end{aligned} \tag{49}$$

On the other hand, we can calculate

$$\begin{aligned}&(\mu Q_1+\nu Q_2)^n \\ &= \frac{\partial^n}{\partial\tau^n}\exp\{\tau\mu Q_1+\tau\nu Q_2\}\Big|_{\tau=0} \\ &= \frac{(-\mathrm{i}\lambda)^n\partial^n}{2^n\partial(\frac{-\mathrm{i}\tau\lambda}{2})^n}:\exp\left\{-\left(\frac{-\mathrm{i}\tau\lambda}{2}\right)^2\right. \\ &\quad \left.+2\frac{-\mathrm{i}\tau\lambda}{2}\frac{(\mu Q_1+\nu Q_2)}{-\mathrm{i}\lambda}\right\}:\Big|_{\tau=0} \\ &= \frac{\lambda^n}{(2\mathrm{i})^n}:H_n\left[\mathrm{i}\frac{(\mu Q_1+\nu Q_2)}{\lambda}\right]: \end{aligned} \tag{50}$$

where $H_n(x)$ is the single-variable Hermite polynomial, and we have used the following formula

$$\mathrm{e}^{\lambda Q} =:\exp\left(\lambda Q+\frac{\lambda^2}{4}\right): \tag{51}$$

$$H_n(x) = \frac{\partial^n}{\partial\tau^n}\exp(-\tau^2+2x\tau) \tag{52}$$

Note that the right hand side of Eqs. (49) and (50) are in the normally ordering form, thus comparing them we can obtain operator identity

$$\begin{aligned}&\int_{-\infty}^{\infty}\frac{\mathrm{d}q}{\sqrt{\pi}}(\lambda q)^n:\exp\left\{-\left(q-\frac{\mu Q_1+\nu Q_2}{\lambda}\right)^2\right\}: \\ &= \frac{\lambda^n}{(2\mathrm{i})^n}:H_n\left[\mathrm{i}\frac{1}{\lambda}(\mu Q_1+\nu Q_2)\right]: \end{aligned} \tag{53}$$

and the integration formula

$$\int_{-\infty}^{\infty}\frac{\mathrm{d}q}{\sqrt{\pi}}q^n\exp\{-(q-x)^2\} = \frac{1}{(2\mathrm{i})^n}H_n[\mathrm{i}x] \tag{54}$$

In the similarly way, using the Eq. (7) we can also derive the normally ordered expansion of operator $\mathrm{e}^{f(\mu Q_1+\nu Q_2)^2}$,

$$\mathrm{e}^{f(\mu Q_1+\nu Q_2)^2}=\frac{1}{\sqrt{1-f\lambda^2}}:\exp\left\{\frac{f(\mu Q_1+\nu Q_2)^2}{(1-f\lambda^2)}\right\}: \tag{55}$$

where $\lambda=\sqrt{\mu^2+\nu^2}$, and we have used the integration formula

$$\int_{-\infty}^{\infty}\mathrm{d}x\exp(-ax^2+bx) = \sqrt{\frac{\pi}{a}}\exp(\frac{b^2}{4a}) \tag{56}$$

5.3 Generating new squeezing operators

From the preceding part, we know that $|\zeta\rangle_{\mu,\nu}$ is complete. Similar to generating the single-mode squeezing operator, $S=\int_{-\infty}^{\infty}\frac{\mathrm{d}q}{\sqrt{\mu}}|\frac{q}{\mu}\rangle\langle q|$, we can construct the following ket-bra integration,

$$S=\int_{-\infty}^{\infty}\frac{\mathrm{d}^2\zeta}{\pi\kappa}|\zeta/\kappa\rangle_{\mu,\nu\,\mu,\nu}\langle\zeta| \tag{57}$$

Using the IWOP technique and Eq. (7), we can directly perform the integration in Eq. (57) to obtain

$$\begin{aligned}S &= \exp\left\{-\frac{1}{2\lambda^2}[(\mu^2-\nu^2)(a^{\dagger 2}-b^{\dagger 2})+4\mu\upsilon a^\dagger b^\dagger]\tanh r\right\} \\ &\quad\cdot\exp\{(a^\dagger a+b^\dagger b+1)\ln\operatorname{sech} r\} \\ &\quad\cdot\exp\left\{\frac{1}{2\lambda^2}[(\mu^2-\nu^2)(a^2-b^2)+4\mu\upsilon ab]\tanh r\right\}\end{aligned} \tag{58}$$

where $\kappa=e^r$, $\operatorname{sech} r=\frac{2\kappa}{\kappa^2+1}$, $\tanh r=\frac{\kappa^2-1}{\kappa^2+1}$, and r is squeezing parameter. From Eq. (58), we can see (i) because of $\frac{2\mu\nu}{\mu^2+\nu^2}\leqslant 1$, when $\mu=\upsilon$, it is maximal-squeezed, and Eq. (58) reduces to the common two-mode squeezing operator:

$$S_{\mu=\upsilon}=\mathrm{e}^{-a^\dagger b^\dagger\tanh r}\mathrm{e}^{(a^\dagger a+b^\dagger b+1)\ln\operatorname{sech} r}\mathrm{e}^{ab\tanh r} \tag{59}$$

(ii) when $\mu=1,\upsilon=0$, we have

$$S=\mathrm{e}^{-\frac{1}{2}(a^{\dagger 2}-b^{\dagger 2})\tanh r}\mathrm{e}^{(a^\dagger a+b^\dagger b+1)\ln\operatorname{sech} r}\mathrm{e}^{\frac{1}{2}(a^2-b^2)\tanh r} \tag{60}$$

which is the direct product of two single-mode squeezing operator.

Defining operators

$$A^\dagger=-\frac{1}{2\lambda^2}[(\mu^2-\nu^2)(a^{\dagger 2}-b^{\dagger 2})+4\mu\upsilon a^\dagger b^\dagger] \tag{61}$$

$$B=\frac{a^{\dagger}a+b^{\dagger}b+1}{2} \tag{62}$$

these two operators accord with the $SU(1,1)$ Lie algebra relation:

$$[B,A^{\dagger}]=A^{\dagger},[B,A]=-A,[A,A^{\dagger}]=2B \tag{63}$$

which leads to

$$\begin{aligned}S&=\exp(A^{\dagger}\tanh r)\exp(2B\ln \mathrm{sech}r)\exp(-A\tanh r)\\&=\exp\{r(A^{\dagger}-A)\}\end{aligned} \tag{64}$$

Thus using Eq. (64) and the Baker–Hausdroff formula [30]

$$\mathrm{e}^{A}B\mathrm{e}^{-A}=B+[A,B]+\frac{1}{2!}[A,[A,B]]+\cdots \tag{65}$$

we can derive the transformation relations as follows

$$S^{\dagger}aS=a\cosh r-\left(\frac{\mu^2-\upsilon^2}{\lambda^2}a^{\dagger}+\frac{2\mu\upsilon}{\lambda^2}b^{\dagger}\right)\sinh r \tag{66}$$

$$S^{\dagger}bS=b\cosh r+\left(\frac{\mu^2-\upsilon^2}{\lambda^2}b^{\dagger}-\frac{2\mu\upsilon}{\lambda^2}a^{\dagger}\right)\sinh r \tag{67}$$

In addition, acting the upper squeezing operator on vacuum state $|00\rangle$, we can obtain a new squeezed state,

$$\begin{aligned}S|00\rangle=\exp\Bigg\{&-\frac{1}{2\lambda^2}[(\mu^2-\nu^2)(a^{\dagger 2}-b^{\dagger 2})\\&+4\mu\upsilon a^{\dagger}b^{\dagger}]\tanh r\Bigg\}|00\rangle\end{aligned} \tag{68}$$

which will be used as entanglement resource to teleport a coherent state in next subsection.

5.4 Quantum teleportation via the squeezed vacuum state (68)

Now we consider the quantum teleportation (QT) for single-mode coherent-input states $|\alpha\rangle$ by using the squeezed vacuum state [Eq. (65)] as entangled resource. The fidelity of QT is often used as a measurement to quantify the performance of a QT protocol, which expressed as $F=\mathrm{tr}(\rho_{in}\rho_{out})$, where ρ_{in} is a pure input state and ρ_{out} is the output state. For a continuous variable quantum system, the fidelity can be given in terms of the characteristic functions (CFs) of the input state and the entangled resource [31].

For the two-mode system ρ, the characteristic function (CF) is defined as $\chi_{AB}(\eta_1,\eta_2)=tr[\rho D_1(\eta_1)D_2(\eta_2)]$, where $D_{1,2}(\eta)$ are the displacement operators. We can calculate

$$\begin{aligned}\chi_{AB}(\eta_1,\eta_2)&=\mathrm{Tr}\{S|00\rangle\langle 00|S^{\dagger}D_1(\eta_1)D_2(\eta_2)\}\\&=\langle 00|S^{\dagger}\exp\{\eta_1a^{\dagger}-\eta_1^*a+\eta_2b^{\dagger}-\eta_2^*b\}S|00\rangle\\&=\langle 00|D_1(\overline{\eta_1})D_2(\overline{\eta_2})|00\rangle\\&=\exp\left\{-\frac{1}{2}|\overline{\eta_1}|^2-\frac{1}{2}|\overline{\eta_2}|^2\right\}\end{aligned} \tag{69}$$

where we have used Eqs. (66), (67), and set

$$\overline{\eta_1}=\eta_1\cosh r+\eta_1^*\frac{\mu^2-\upsilon^2}{\lambda^2}\sinh r+\eta_2^*\frac{2\mu\upsilon}{\lambda^2}\sinh r \tag{70}$$

$$\overline{\eta_2}=\eta_2\cosh r-\eta_2^*\frac{\mu^2-\upsilon^2}{\lambda^2}\sinh r+\eta_1^*\frac{2\mu\upsilon}{\lambda^2}\sinh r \tag{71}$$

In particular, when $\eta_1=\eta,\eta_2=\eta^*$, we can obtain

$$\chi_{AB}(\eta,\eta^*)=\exp\left\{-|\eta|^2\left(\cosh 2r+\frac{2\mu\upsilon}{\lambda^2}\sinh 2r\right)\right\} \tag{72}$$

Note that the fidelity is independent of amplitude of the coherent state, thus, for simplicity we take $\alpha=0$; then we have only to calculate the fidelity of the vacuum input state with the CF $\chi_{in}(\eta)=\exp[-|\eta|^2/2]$. The fidelity of QT of continuous variable quantum system reads [31]

$$F=\int\frac{\mathrm{d}^2\eta}{\pi}\chi_{in}(-\eta)\chi_{in}(\eta)\chi_{AB}(\eta,\eta^*) \tag{73}$$

Substituting $\chi_{in}(\eta)$ and Eq. (72) into Eq. (73) yields

$$\begin{aligned}F&=\int\frac{\mathrm{d}^2\eta}{\pi}\chi_{in}(-\eta)\chi_{in}(\eta)\chi_{AB}(\eta,\eta^*)\\&=\int\frac{\mathrm{d}^2\eta}{\pi}\exp\left\{-|\eta|^2\left(2\cosh^2 r+\frac{2\mu\upsilon}{\lambda^2}\sinh 2r\right)\right\}\\&=\frac{1}{2\cosh^2 r+\frac{2\mu\upsilon}{\lambda^2}\sinh 2r}\end{aligned} \tag{74}$$

where we have used the integration formula Eq. (41).

In Fig. 1, for several different squeezing parameter values, we plot the graph of the fidelity as the function of ν for some given values of $\mu=1,0.6,-0.2,-0.7$. Form Fig. 1, we can see that the fidelity of teleporting the coherent state have the maximum value when $\mu=-\nu$, which corresponds to the maximum entangled states shown in Eq. (1); and the fidelity increases with the increasing squeezing parameter r at the left of point A, which is contrast with the right of point A. In another word, there exists a tri-angle type region, in which the fidelity teleporting coherent state has bigger value for a smaller squeezing parameter than that for a bigger squeezing parameter with the increasing (or decreasing) value of ν. Here we should point out that, in this region, the fidelity is more than 0.5 which implies the quantum property. Thus using the new squeezed state generated by an asymmetric BS as an entanglement resource (which can be considered as a real case), for a smaller squeezing case, the correspon-

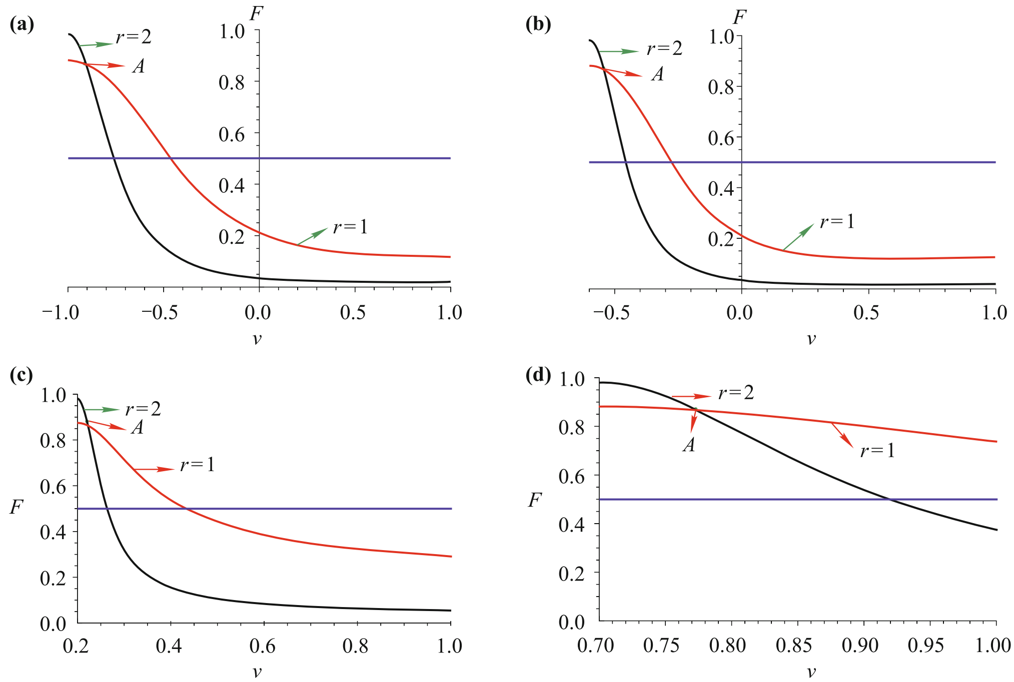

Fig. 1 The fidelity of teleportation of coherent states using the new two-mode squeezed vacuum as an entangled resource. **(a)** $\mu = 1.0$; **(b)** $\mu = 0.6$; **(c)** $\mu = -0.2$; **(d)** $\mu = -0.7$.

ding fidelity for teleporting coherent state is over 0.5 and more superior to the case of bigger squeezing value under a certain region of ν.

6 Conclusions

In this paper, we introduce the generalized entangled state $|\zeta\rangle_{\mu,\nu}$ by decomposing a unity of the Gaussian integration within normal ordering, which is the the common eigenstates of communicative operators $\mu Q_1 + \nu Q_2$ and $\mu P_2 - \nu P_1$, and the common eigenvector of two particles' center-of-mass coordinates and mass-weighted relative momentum can be considered as its a special case. Then the experiment scheme of generating the generalized entangled state was presented and the specific protocol is that a coordinate state and a momentum state are imported in mode a and mode b of an asymmetric beam splitter respectively. Furthermore, we discuss some main properties of the $|\zeta\rangle_{\mu,\nu}$, i.e., orthogonality, and the Schmidt decomposition of $|\zeta\rangle_{\mu,\nu}$. In addition, as its applications, we used the new state to obtain some new operator identities, integration formulas and normal ordering of some complicated operators. In particular, using the new squeezed state as an entanglement resource, we discussed the fidelity of teleporting a coherent state. It is shown that the fidelity of quantum teleportation can be enhanced under certain case by using the asymmetrical new squeezed state as entangled resource.

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