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RESEARCH ARTICLE

A unified dynamic scaling property for the unified hybrid network theory framework

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In this article, we present a new type of unified dynamic scaling property for synchronizability, which can describe the scaling relationship between dynamic synchronizability and four hybrid ratios under the unified hybrid network theory framework (UHNTF). Our theory results can not only be applied to judge and analyze dynamic synchronizability for most of complex networks associated with the UHNTF, but also we can flexibly adjust and design different hybrid ratios and scaling exponent to meet actual requirement for the dynamic characteristics of the UHNTF.

Keywords dynamic scaling property, unified hybrid network theory framework (UHNTF), synchronizability, hybrid ratios

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1 Introduction

Synchronization is one kind of universal and important phenomenon in nature and society, and has been a significant issue for complex network research. A lot of works and experiments focus on it. Nevertheless the synchronization dynamic research of complex networks has still been a challenging task [1–6]. Since Watts and Strogatz found small-world effect in 1998 [7], and Barabási and Albert proposed scale-free property in 1999 [8], all kinds of complex network models have been proposed to research different types of questions. Lots of synchronization research works addressed completely regular network, randomly coupled network, small-world network, scale-free network and other models [9–13], meanwhile the relationships between some complex network characteristics and synchronization have been studied, such as degree distribution, clustering coefficient, average shortest distance, and so on [14–16]. But some models do not have universality, so correlative conclusions can only be used to explain some special problems. Our group proposed a unified hybrid network theory framework (UHNTF) for network science by trilogy models, which has universality and reveals some new phenomena and findings. How to analyze the dynamic synchronization of the UHNTF? Does a unified relationship exist between synchronizability and hybrid ratios or other network characteristics? These are the motivations of this article.

Our network group has analyzed main network science models in all aspects, proposed a unified hybrid network theory framework (UHNTF) for network science by trilogy models: Harmonious Unifying Hybrid Preferential Model (HUHPM) [17–24], Large Unifying Hybrid Network Model (LUHNM) [25–32] and Large Unifying Hybrid Variable Growing Model (LUHVGM) [33–36], as shown in Fig. 1. The UHNTF is constructed by four kinds of different hybrid ratios. A merit of hybrid form is to have a wide range of practical basis in nature and human society, in line with the natural, social, physical, and technical as well as the lives of the majority to seek an answer to this question and the corresponding solutions and means, so the UHNTF may have preferable understanding in revealing complicated transition relationship between simplicity-universality and complexity-diversity. For example, they have been applied to study China high-tech enterprise networks [37–39], nuclear science technology networks [40–41], and so on. Some results have shown the potential application of the UHNTF.

We have theoretically analyzed and derived some scale properties of the UHNTF: 3 power laws for degree distribution, weight distribution, strengthen distribution and so on. These works have basically formed a relatively complete theoretical framework besides the dynamic synchronization property. So here raises a question: does

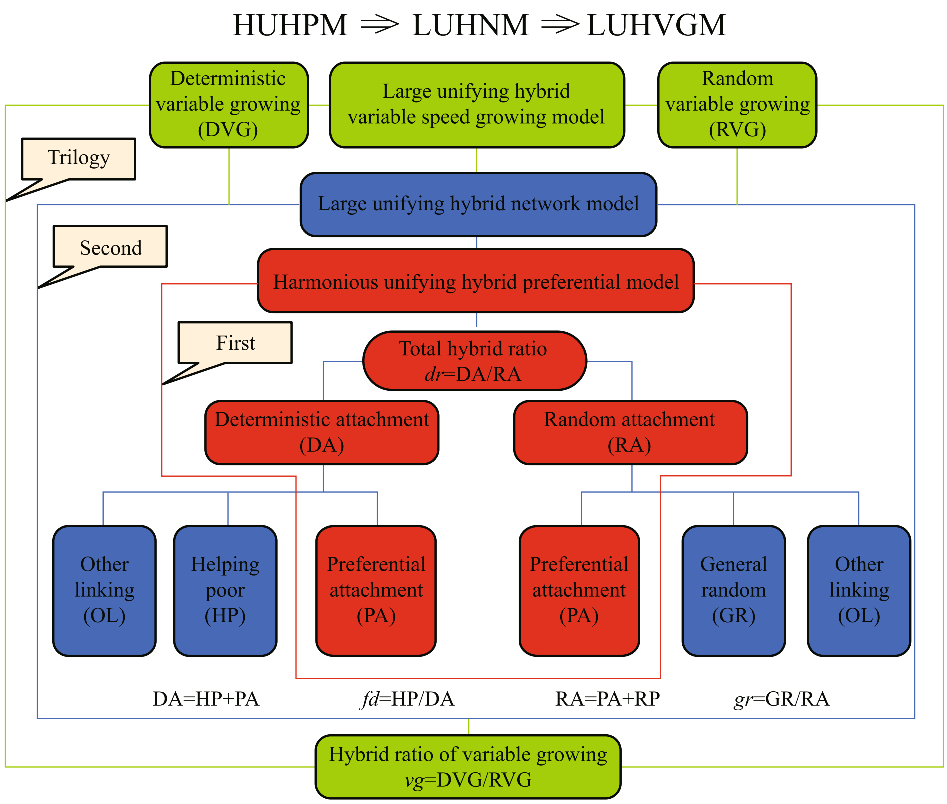

Fig. 1 Unified hybrid network theory framework diagrams.

synchronizability scaling law exist in the UHNTF? Our answer is sure in this article. We analyzed and derived the unified scaling relationship between dynamic synchronizability and system parameters (i.e., four hybrid ratios (dr, fd, gr, α)) for the UHNTF. The emergence of synchronization patterns in complex networks has been shown to be closely related to the underlying topology. In this article, we proposed a unified dynamic scaling relation based on different exponent β, which can not only be applied to judge and analyze the dynamic synchronizability for most of complex networks associated with the UHNTF in theory, but also can be used as a further supplement of the UHNTF. We can flexibly adjust and design different hybrid ratios and the scaling exponent to meet actual requirement for the dynamic characteristics of the UHNTF.

2 Some theoretical analytical results for the UHNTF

Let us recall, the main ideas of the UHNTF are shown in Fig. 1. Three hybrid ratios and variable growing exponent (dr, fd, gr, α) are introduced respectively by [17–36]

$$\left.\begin{cases} dr = DPA/RPA & HUHPM \\ \begin{cases} fd = HPA/(HPA+DPA) \\ gr = GRA/(GRA+RPA) \end{cases} & LUHNM \\ vg = DVG/RVG & LUHVGM \end{cases}\right\rangle \quad (1)$$

The UHNTF consists of three level models (HUHPM, LUHNM, LUHVGM). For the HUHPM model, we have deduced the function relationship between index γ power law and total hybrid ratio dr in line with following Eq. (10) [17–22]:

$$\gamma = \frac{A_1}{\exp\left[\left(\dfrac{dr}{A_2}\right)^{A_3}\right]} + A_4 \quad (2)$$

Also we can obtain

$$\gamma = \frac{A_1}{\exp\left[\left(\dfrac{\sqrt[\beta]{R/x_0}}{A_2}\right)^{A_3}\right]} + A_4 \quad (3)$$

The function relationship of degree-degree correlation r_c versus fd and gr for the LUHNM with other parameters, we have [25–30]

$$r_c = a_1 gr + b_1 \tag{4}$$

$$r_c = \begin{cases} c_1 e \dfrac{fd}{c_2} + c_3, & 0 \leqslant fd \leqslant 0.9 \\ b_1 + \dfrac{2b_2 b_3}{\pi[4(fd - b_4)^2 + b_5^2]}, & 0.9 \leqslant fd \leqslant 1 \end{cases} \tag{5}$$

Further, we can obtain

$$r_c = a_1 \sqrt[\beta]{\frac{R}{x_0}} + b_1 \tag{6}$$

$$r_c = \begin{cases} c_1 e \sqrt[\beta]{\dfrac{R}{x_0}} / c_2 + c_3, & 0 \leqslant fd \leqslant 0.9 \\ b_1 + \dfrac{2b_2 b_3}{\pi \left[4 \left(\sqrt[\beta]{\dfrac{R}{x_0}} - b_4 \right)^2 + b_5^2 \right]}, & 0.9 \leqslant fd \leqslant 1 \end{cases} \tag{7}$$

The cumulative degree distribution $P(k)$ of the LUHVGM is [34, 35]

$$P(k) \sim k^{\gamma f(dr,a)} \mathrm{e}^{\frac{k}{const} g(dr,a)} \tag{8}$$

We then can theoretically derive the new form of Eq. (8) as

$$P(k) \sim k^{-\gamma f(dr, \sqrt[\beta]{R/x_0})} \mathrm{e}^{\frac{k}{const} g(dr, \sqrt[\beta]{R/x_0})} \tag{9}$$

The main topological properties above for the UHNTF are summarized theoretically, all β above is the scaling exponent shown in Eq. (10).

3 A unified scaling property for dynamic synchronizability of the UHNTF

In this article, we further explored the unified dynamic scaling relationships between four hybrid ratios and dynamic synchronization in the UHNTF. This issue has been a very difficult question for theoretical analysis, so we can use the numerical simulation method to reveal some results.

In order to explore the relationships between dynamic synchronizability and four hybrid ratios (dr, fd, gr, α) of the UHNTF, we firstly define a general unified dynamic scaling relation for the synchronizability as follows:

$$y = x_0 \cdot x^{\beta} \tag{10}$$

where y is R, x may be one of α, dr, fd and gr, β is the dynamic scaling exponent. When x_0 is positive, if $\beta > 0$, y is an increasing function, the synchronizability is enhanced; if $\beta = 0$, y is a constant, the synchronizability is in a critical state; if $\beta < 0$, y is a decreasing function, the synchronizability is reduced.

As well-known, Pecora and Carroll proposed main stable function method in the 1990s [2, 3] and gave the algebraic criteria condition R ($\frac{\lambda_N}{\lambda_2}$) for the existence of a linearly stable synchronous state. What are similar and equivalent results for the UHNTF?

For three level models (HUHPM, LUHNM, LUHVGM), three corresponding types of numerically calculating results are shown in Figs. 2–4, and corresponding scaling characteristics are obtained from Eq. (10) for all hybrid ratios. All the dynamic scaling exponents β are given in Table 1 and Figs. 2–4. In Figs. 2–4, the dotted lines denote R curve, the solid lines are the results which are calculated by Eq. (10).

For the HUHPM model dr is total hybrid ratio which is the first level ratio, DPA is time interval for the deterministic preferential attachment, RPA is time interval for the random preferential attachment. We can see from Fig. 2 that the algebraic criteria condition R falls when dr ratio increases, accordingly the value of the dynamic scaling exponent β is negative. According to Eq. (1), we know that the increasing of dr ratio means the increasing of the deterministic preferential attachment or decreasing of the random preferential attachment, so it will cause new nodes connecting to old nodes in the network and the synchronizability decreasing.

Table 1 Dynamic scaling exponent based on Eq. (10) under the different hybrid ratios for the UHNTF.

x	Hybrid ratios	β	x_0	R
dr	$fd=0,\ gr=0,\ \alpha=0$	–0.18685	255.7545	increasing
gr	$fd=0.5,\ dr=1$	0.77625	285.7143	decreasing
fd	$gr=0.5,\ dr=1$	0.44657	255.1020	decreasing
α	$dr=1/999,\ fd=0.1$	–0.02728	534.7594	increasing
α	$dr=1/999,\ gr=0.1$	–0.16362	709.2199	increasing
α	$dr=999,\ fd=1$	–0.46978	190.1141	increasing
α	$dr=999,\ gr=1$	–0.76574	185.8736	increasing

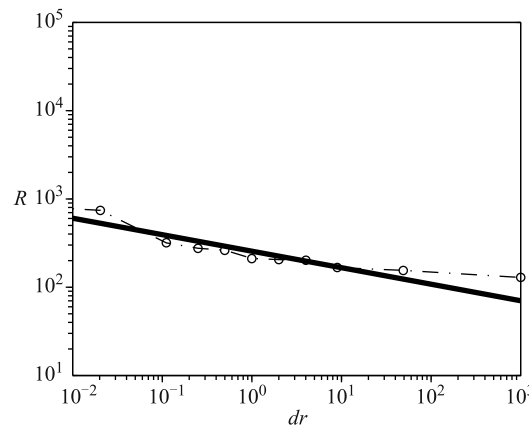

Fig. 2 The relationship between dr ratio and synchronizability for the HUHPM.

The LUHNM is the second level for the UHNTF which includes two hybrid ratios: one is helping poverty ratio fd, the other is general random hybrid ration gr. HPA is time interval for the helping poverty attachment, GRA is time interval for the general random attachment. In Fig. 3 the curves of R rise when gr or fd ratio rises, and the values of the dynamic scaling exponent β are positive in Table 1. According to above results, we know that increasing deterministic preferential attachment DPA can enhance synchronizability, but increasing random preferential attachment RPA can weaken synchronizability, and increasing deterministic helping poor attachment HPA will weaken synchronizability.

The third level of the UHNTF is the LUHVGM, in which the most important idea is the variable growing exponent α. We can see the relationship between the variable growing exponent α and synchronizability in Figs. 4(a)–(d), the curves of R for different gr or fd (0.1, 0.3, 0.5, 0.7, 1.0) keep linearity in semilogarithmic coordinates when $dr = 1/999$ in Figs. 4(a) and (b). We

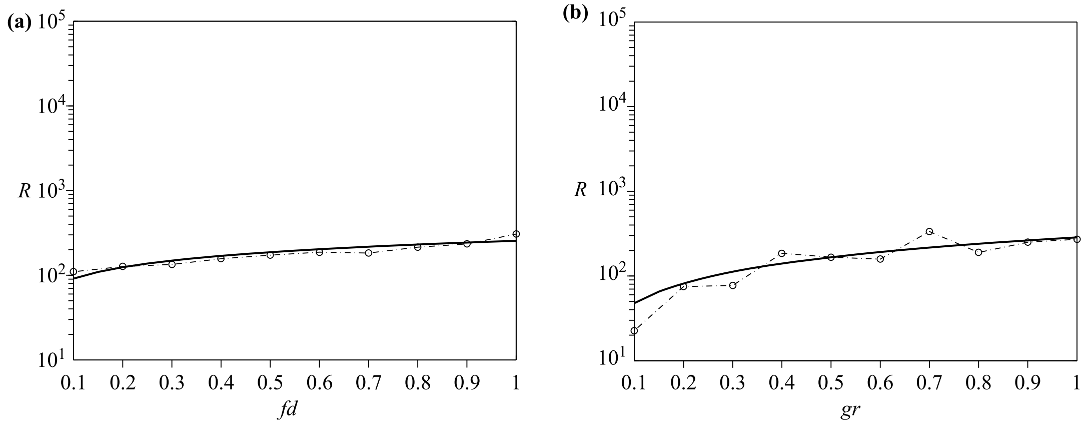

Fig. 3 The relationship between gr or fd ratio and synchronizability for the LUHNM, **(a)** $gr = 0.5$ and $dr = 1$, **(b)** $fd = 0.5$ and $dr = 1$.

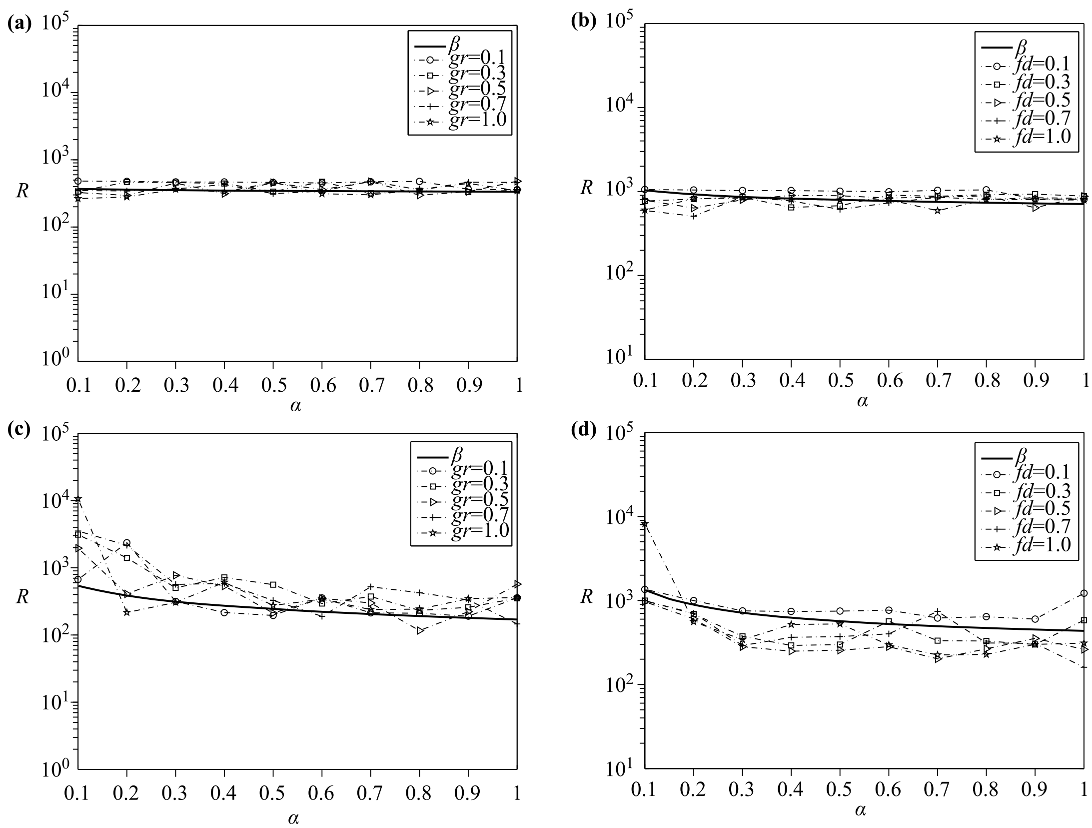

Fig. 4 The relationship between the variable growing exponent α and synchronizability for the LUHVGM, **(a)** $dr = 1/999$, $fd = 0.1$; **(b)** $dr = 1/999$, $gr = 0.1$; **(c)** $dr = 999$, $fd = 1$; **(d)** $dr = 999$, $gr = 1$.

also know from Figs. 4(c) and (d) that the curves of R fall. Table 1 shows the results of the dynamic scaling exponent β based on Eq. (10) under the different hybrid ratios. When $dr = 1/999$ (randomness leading), the dynamic scaling exponent β for the LUHVGM is approximately equal to 0, so $y \approx x$, and y is similar to a constant, the synchronizability keeps invariant with α changing. When $dr = 999$ (determinacy leading), the dynamic scaling exponent $\beta < 0$, so under different hybrid ratios (gr or fd), the synchronizability increases with α increasing.

4 Conclusions

The UHNTF theory has been used to construct and analyze some actual networks, and has a wide application, so the comprehensive analysis of various network characteristics is very important. The synchronizability is one of most important dynamical characteristics of the UHNTF, so we analyzed it from the theoretical and numerical aspects in this article. As shown in Table 1 and Figs. 2–4, it is found that the unified scaling relation Eq. (10) and its exponent β can reveal the relationship between four hybrid ratios dr, gr, fd or α and synchronizability, as well as they can be applied to judge and analyze the dynamic synchronizability for most of complex networks. The theoretical results show that if the deterministic preferential attachment dr and ratio α are increased, when $\beta < 0$, which can enhance the synchronizability of the UHNTF. But increasing the deterministic helping poor attachment fd and random attachment gr will weaken the synchronizability of complex networks since $\beta > 0$. According to above numerical results and theoretical derivation, we basically obtain a relatively complete theoretical framework of the UHNTF. Thus people can flexibly adjust and design different hybrid ratios (dr, fd, gr or α) to reach the scaling exponent β and meet actual requirement for the dynamic characteristics of the UHNTF. So the unified dynamic scaling properties for synchronizability in the UHNTF studied in this article cannot only be used to perfect the UHNTF from theoretical angle, but also may play a very important role to expand the application scope of the UHNTF model.

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