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Review article

Production representation of partial wave scattering amplitudes and the $f_0(600)$ particle

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The establishment of production representation of partial wave scattering amplitudes is reviewed in the context of quantum field theory. Its relation to the production representation, or Ning Hu representation in quantum mechanical scattering theory is pointed out. One of the most important application of the production representation is the physics of the $f_0(600)$ and $K(700)$ scalar hadron resonances, on which we also give a brief review. It is emphasized that all evidences accumulated so far are consistent with the picture that the $f_0(600)$ meson is the chiral partner of the Nambu–Goldstone bosons in a linear realization of chiral symmetry.

Keywords scalar mesons, chiral symmetry, unitarization

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Contents

In this review we first briefly introduce in Section 1 the history related to the σ meson, and demonstrate how dispersion techniques can be used to confirm the existence of the $f_0(600)$ particle. Then in Section 2 we discuss in detail how to establish the production representation (we also call it the PKU representation) in the context of quantum field theory, and compare it with the Ning Hu representation in quantum mechanical scattering theory. At last, in Section 3, we discuss the physics related to the $f_0(600)$ and $K(800)$ resonances and argue that they behave very much like chiral partners of the pseudo-goldstone bosons of QCD, in a linear realization of chiral symmetry.

1 Confirmation of the $f_0(600)$ meson using dispersion techniques

1.1 A brief review on the history related to the σ meson

A scalar − iso-scalar quanta exchange in inter − nucleon interactions has firstly been proposed in 1955 by Johnson and Teller [1]. A few years later the celebrated linear σ model has been introduced by Gell-Mann and Levy [2]. In such a model, the σ meson develops a non-vanishing vacuum expectation value which plays a crucial role by breaking the global chiral symmetry of QCD spontaneously, and pions emerge as the (pseudo-)goldstone bosons associated with the spontaneous chiral symmetry breaking (SχSB). In the linear σ model the σ meson fills in the linear chiral multiplet together with pions. The σ and the pion fields transform and mix with each other under chiral rotations, and before SχSB the σ field and the π field were essentially the same. To summarize, the

σ meson – if exists – plays a crucial role in low energy strong interaction physics.

On the other hand, it had been a long and painful history in the struggle to confirm the existence of such a light scalar resonance from experimental data, not to mention clarifying the profound role it may play in theory aspect. It is actually remarkable that in Ref.[1] the authors cautiously wrote: "*The scalar neutral meson need not be an elementary particle in any sense of the word. It may be a virtual state composed of other mesons. It may be even a superposition of such virtual states. It may decay into π mesons so quickly that it cannot be observed. It may be related to mesons like a sound-quantum is related to electrons and nuclei. In any case, we assume that nuclear interactions follow in first approximation from a linear coupling with the meson field.*" This quotation actually foresaw the difficulties and confusions that occur in the late era.

Early studies on nuclear physics require the σ in order to cancel the large πN scattering lengths caused by the π nucleon Born term. However these studies are based on models with linearly realized chiral symmetry. In such models, there must exist a large cancelation between the σ contribution and the π contribution at low energies in order to obey various constraints from soft pion theorems. One example of the latter is the Adler zero condition.

Non-linear realization of chiral symmetry was later discovered [3, 4], which satisfies all low energy theorems induced by PCAC [5, 6]. Hence it is suggested that the σ is not necessary for chiral symmetry breaking. Chiral perturbation *theory* (χPT) is established based on the non-linear realization of chiral symmetry [7, 8]. It is a model independent approach and successfully describes strong interaction physics at (very) low energies. Furthermore, it was shown that the renormalizable (hence a toy) linear σ model is not QCD at low energies [8].

On the other side, efforts have been made by physicists who insist on the existence of σ meson, which led to the return of the σ meson in PDG after disappearing for more than 30 years [9], even though most of these previous studies are rather model dependent, and are hence less convincing. On experimental side, it is also very difficult to claim the discovery of the σ pole from experimental data, since the σ meson, if exists, has to be a very broad resonance. In other words, s wave $I=0$ channel $\pi\pi$ interaction at low energies is of highly non-perturbative, strong interaction nature. It is not easy to extract the σ pole from the background contributions[1]. As a consequence, previous conclusions in supporting the existence of the σ meson are not convincing.

1.2 The σ meson must exist to adjust χPT to experiments

The situation still seemed to be rather confusing: chiral perturbation theory does not seem to support the existence of the σ meson. On the other hand, see Fig. 1, the steady rise of the $I,J=0,0$ channel $\pi\pi$ scattering phase shift data below 1 GeV, provided by the old CERN-Munich collaboration, left a big puzzle for imagination.

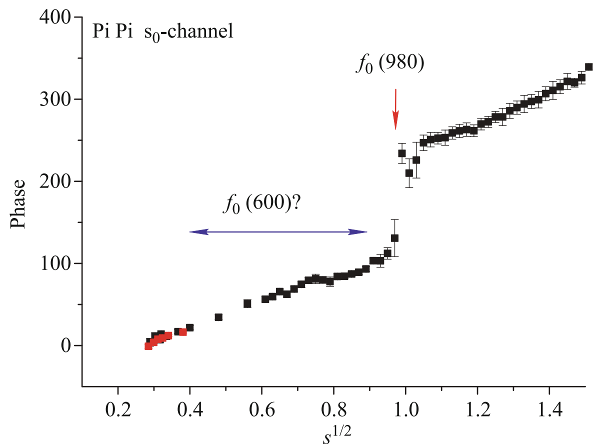

Fig. 1 $IJ=00$ channel $\pi\pi$ scattering phase shift data from Refs. [11–15].

In my opinion, the clearest way to reveal the very existence of a light and broad resonance hidden behind the low energy phase shift data in Fig. 1 may be to write down a dispersion relation for the $\sin(2\delta_0^0)$, where δ_0^0 is the phase shift in the $IJ=00$ channel of $\pi\pi$ elastic scattering [16, 17]. Define

$$F(s)\equiv\frac{1}{2\mathrm{i}\rho}\left[S(s)-\frac{1}{S(s)}\right] \tag{1}$$

it is easy to understand that $F(s)$ is analytic across the elastic cut of $\pi\pi$ elastic scattering amplitude and

$$\sin(2\delta_\pi)=\rho F \tag{2}$$

where $\rho=\sqrt{\frac{s-4m_\pi^2}{s}}$. One assumes a high energy Regge behavior for the partial wave amplitude, hence $F(s)$ satisfies a once-subtracted dispersion relation [16, 17],

$$F(s)=\alpha+\sum\text{poles}+\frac{1}{\pi}\int_L\frac{\mathrm{Im}_LF(s')}{s'-s}\mathrm{d}s' \\ +\frac{1}{\pi}\int_R\frac{\mathrm{Im}_RF(s')}{s'-s}\mathrm{d}s' \tag{3}$$

where α is a subtraction constant and the rest contri-

[1] The major difficulties in accepting the σ resonance, and how can they be overcome, have been previously reviewed in Ref. [10].

bution to $F(s)$ come either from poles or cuts. For $\pi\pi$ scatterings $L = (-\infty, 0]$ and R starts from first inelastic threshold, practically $R = [4m_K^2, \infty)$.

The right hand cut can be estimated using experimental data and it is found that its contribution is not important at low energies. The nearby left hand cut can be estimated using χPT. As seen in Fig. 2, the left hand cut contribution to the phase shift is negative and concave whereas the experimental data curve on $\sin(2\delta_0^0)$ is positive and convex. Compared with Fig. 1, Fig. 2 clearly demonstrates that it is essential to include the σ meson to adjust chiral perturbation theory to experiments, according to Eq. (3). Evidences for the existence of σ and κ resonances are also reported in recent years' production experiments [18–23].

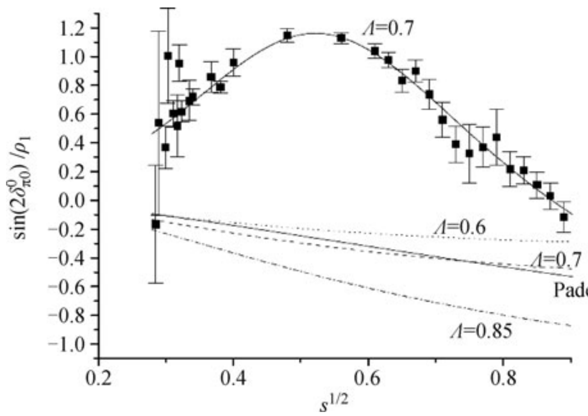

Fig. 2 Experimental data and fit curve of $\sin(2\delta_0^0)$. Various estimates on left hand cut using χPT and its Padé approximant are also drawn.

2 Production representation for partial wave scattering amplitudes

Because the physics related to the σ meson and the κ meson in s wave $I = 1/2$ channel πK scatterings are of highly non-perturbative nature, a dispersive analysis is hence needed to extract the physical information in a model independent way. In this section we review how a new dispersive approach can be developed for the study of pole structure, from low energy phase shift data.

2.1 Dispersion relations without elastic cut

It is observed that the method used in deriving Eq. (3) can also be applied to the following function [24]:

$$\tilde{F} \equiv \frac{1}{2}\left(S + \frac{1}{S}\right) \tag{4}$$

It is easy to demonstrate that $\tilde{F}$ has no discontinuity across the real axis when $0 < s < 16m_\pi^2$, since

$$\begin{aligned} \tilde{F}(s - \mathrm{i}\epsilon) &= \frac{1}{2}\left[S(s-\mathrm{i}\epsilon) + \frac{1}{S(s-\mathrm{i}\epsilon)}\right] \\ &= \frac{1}{2}\left[S^{II}(s+\mathrm{i}\epsilon) + \frac{1}{S^{II}(s+\mathrm{i}\epsilon)}\right] \\ &= \frac{1}{2}\left[\frac{1}{S^{I}(s+\mathrm{i}\epsilon)} + S^{I}(s+\mathrm{i}\epsilon)\right] \\ &= \tilde{F}(s+\mathrm{i}\epsilon) \end{aligned} \tag{5}$$

and the left hand cut it contains starts from $-\infty$ to 0. The cut structure of $\tilde{F}$ is the same as the cut structure of function F studied previously. The function $\tilde{F}$ is the analytic continuation of $\cos(2\delta_\pi)$ defined in the single channel unitarity region.

According to the analytic structure of $\tilde{F}$, as discussed above we can set up the following dispersion relation [24],

$$\begin{aligned} \cos(2\delta_\pi) = \tilde{F} = \tilde{\alpha} + \sum_i \frac{\beta_i}{2(s - s_i)} \\ + \sum_j \frac{1}{2S'(z_j^{II})(s - z_j^{II})} + \frac{1}{\pi}\int_L \frac{\mathrm{Im}_L \tilde{F}(s')}{s' - s}\mathrm{d}s' \\ + \frac{1}{\pi}\int_R \frac{\mathrm{Im}_R \tilde{F}(s')}{s' - s}\mathrm{d}s' \end{aligned} \tag{6}$$

where $\tilde{\alpha}$ is the subtraction constant and one subtraction to the cut integrals in the above expression is understood. The right hand cut R starts from $16m_\pi^2$ in principle but becomes important only when s approaches the $\bar{K}K$ threshold[2)]. Using Eqs. (6) and (3), we get an analytic expression of S on the complex s plane in terms of poles, dynamical cuts, and the kinematic factor:

$$S(z) \equiv \cos(2\delta_\pi(z)) + \mathrm{i}\sin(2\delta_\pi(z)) \tag{7}$$

One may use the definition $S(4m_\pi^2) = 1$ to re-express $\tilde{\alpha}$ in Eqs. (7) and (6) in terms of other parameters. The above expression respects all the known properties of S matrix theory. For example, the physical sheet $S(z)$ does not contain resonance poles though the phase motion of S is affected by resonance poles on the second sheet. Eq. (7), though simple to derive, is an exact relation. Eqs. (6) and (3) must satisfy a relation on the whole complex s plane:

$$\sin^2(2\delta_\pi) + \cos^2(2\delta_\pi) \equiv 1 \tag{8}$$

which is the analytic continuation of the single channel unitarity relation, $S^+S = 1$, on the complex s plane. Eq.

[2)] We neglect the 4π cut from now on in the text. According to the conventional wisdom the 4π cut becomes important only above, say, 1.2 GeV.

(8) is equivalent to the generalized unitarity condition $S(k)S(k^*)^* = 1$ in quantum mechanical scattering theory, and it contains all information about single channel unitarity and analyticity.

In Refs. [24] phenomenological studies based on Eqs. (3), (6) and (8) have been performed, on the $\pi\pi$ scattering phase shift in the $IJ = 00$ channel. In order to achieve this, one needs to estimate the two left hand cuts. A problem occurs here when using the χPT results on left-hand cut, concerning the value of $\mathrm{Im}_L \tilde{F}$ at $s = 0$. It is found that χPT results lead to a divergent dispersion integral on the left cut. This problem is studied in detail and solved in Ref. [25].

2.2 The generalized unitarity relation

To understand that Eq. (8) corresponds to the generalized unitarity relation in quantum mechanical scattering theory, first we have

$$S(s) = \tilde{F}(s) + \mathrm{i}\rho(s)F(s) \tag{9}$$

where various functions have the following reflection properties, i.e., $S^*(s) = S(s^*)$, $F^*(s) = F(s^*)$, $\tilde{F}^*(s) = \tilde{F}(s^*)$ and $\rho^*(s) = -\rho(s^*)$. Eq. (8) can be rewritten as

$$\tilde{F}^2(s) + (\rho F(s))^2 \equiv S(s) \times [\tilde{F}(s) - \mathrm{i}\rho(s)F(s)] = 1$$

Therefore $\tilde{F}(s) - \mathrm{i}\rho(s)F(s)$ is nothing but the proper definition of the S matrix on the second Riemann sheet, i.e.,

$$S^{II}(s) \equiv \tilde{F}(s) - \mathrm{i}\rho(s)F(s) \tag{10}$$

and Eq. (8) can be recasted as

$$S^I(s)S^{II}(s) \equiv 1 \tag{11}$$

This equation is not new but it is usually regarded as the definition of $S^{II}(s)$. What is new regarding to Eq. (8) is that we afford the analytic expression of S^{II}. Now we will prove that Eq. (11) is just the generalized unitarity equation in the language of scattering theory, for which we need to go to the momentum p plane. The upper half of p plane corresponds to first sheet of s plane and the lower half corresponds to the second sheet of s plane. On p plane one gets $\tilde{F}$ and F as, $\tilde{F}(p^*) = \tilde{F}^*(p)$, $F(p^*) = F^*(p)$. Further, it is important to notice that

$$\begin{aligned} S(p) &= S^I(s), \quad \mathrm{Im}[p] > 0 \\ S(p) &= S^{II}(s), \quad \mathrm{Im}[p] < 0 \end{aligned} \tag{12}$$

or

$$\begin{aligned} \rho(p) &= \rho^I(s) = \rho(s), \quad \mathrm{Im}[p] > 0 \\ \rho(p) &= \rho^{II}(s) = -\rho^I(s), \quad \mathrm{Im}[p] < 0 \end{aligned}$$

where $s = 4(m^2 + p^2)$ and $\rho(p) = \frac{p}{\sqrt{p^2+m^2}}$. Rewrite Eq. (9) in above in terms of p we have

$$S(p) = \tilde{F}(p) + \mathrm{i}\rho(p)F(p)$$

which has the following standard properties of the S matrix in scattering theory:

$$S(-p) = S^*(p) = 1/S(p)$$

when p is real. More importantly, assuming $\mathrm{Im}[p] > 0$, we have, following Eq. (12), $S(p^*) = S^{II}(s^*)$ since $\mathrm{Im}[p^*] < 0$. Therefore,

$$S^*(p^*) = S^{II*}(s^*) = S^{II}(s)$$

where the latter equality follows from real analyticity. Therefore we have

$$S(p)S^*(p^*) \equiv S(s)S^{II}(s) \equiv 1$$

2.3 Factorized S matrix elements and separable singularities

One tries to solve the generalized unitarity relation, Eq. (8). The simplest solutions, i.e., those with neither left nor right cut integrals and containing only a single pole or a pair of complex conjugate poles, can be obtained for unequal mass scatterings. The results are listed as below [27]:

i) A virtual state pole at s_0 with s_0 real. The S matrix can be expressed as

$$S(s) = \frac{1 + \mathrm{i}\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_0-s_L}{s_R-s_0}}}{1 - \mathrm{i}\rho(s)\frac{s}{s-s_L}\sqrt{\frac{s_0-s_L}{s_R-s_0}}} \tag{13}$$

If there is a bound state at s_0, then the above S reverses. Besides, a bound or virtual state exists only when $s_L < s_0 < s_R$, and s_L and s_R denote the branch point on the left and right, respectively.

ii) A pair of resonances at z_0 (having positive imaginary part) and z_0^*. The S matrix can be expressed as

$$S(s) = \frac{M^2(z_0) - s + \mathrm{i}\rho(s)sG}{M^2(z_0) - s - \mathrm{i}\rho(s)sG} \tag{14}$$

where

$$M^2(z_0) = \mathrm{Re}[z_0] + \frac{\mathrm{Im}[z_0]\,\mathrm{Im}[z_0\,\rho(z_0)]}{\mathrm{Re}[z_0\,\rho(z_0)]}$$

$$G = \frac{\mathrm{Im}[z_0]}{\mathrm{Re}[z_0\,\rho(z_0)]} \tag{15}$$

The properties of these "simplest" S matrices are quite interesting for discussion and we leave it in Ref. [27] for details.

Having obtained those "simplest" S matrices we notice that $S^{phys}/\prod S^{poles}$ is also unitary and can be denoted as S^{cut}, which, by construction, contains no poles but all the cuts that the original S^{phys} owns. Hence one constructs, using analyticity, unitarity, and partial wave dispersion relations, a factorized form for elastic scattering S matrix element [24–28]:

$$S^{phys} = \prod_i S^{R_i} \cdot S^{cut} \tag{16}$$

where S^{R_i} denotes the i-th *second* sheet pole contribution (or a physical sheet pole contribution, if there exists a bound state) and S^{cut} denotes the contribution from cuts or background. The information from higher sheet poles is hidden in the right hand integral which consists of one part of the total background contribution,

$$S^{cut} = \mathrm{e}^{2\mathrm{i}\rho f(s)}$$

$$f(s) = \frac{s}{\pi}\int_L \frac{\mathrm{Im}_L f(s')}{s'(s'-s)} + \frac{s}{\pi}\int_R \frac{\mathrm{Im}_R f(s')}{s'(s'-s)} \tag{17}$$

The "left hand" cut $L = (-\infty, 0]$ for equal mass scatterings and may contain a rather complicated structure for unequal mass scatterings. The right hand cut R starts from first *in*elastic threshold to positive infinity. It can be demonstrated that the dispersive representation for f is free from the subtraction constant, i.e., $f(0) = 0$. The detailed proof using S matrix theory and dispersion theory can be found in the appendix of Ref. [28].

Phenomenologically the right hand cut can be estimated from experimental input. Nearby left hand cut ($s > -32m_\pi^2$ for $\pi\pi$ scatterings if assuming Mandelstam representation) can in principle be estimated from experimental data as well, using Froissart–Gribov projection formula. Nevertheless, one expects such an estimate gives more or less the same effect as using χPT results when estimating the nearby left cut. There is no reliable way to estimate further left hand cut effects[3)], nevertheless physics around physical threshold should not be strongly affected by what might happen far away in the complex s plane. Moreover, further left cut contribution as defined by Eq. (17) is very mild which is understood by the fact that the integrand appeared in Eq. (17) is of logarithmic form. Numerical evaluation justifies this argument [26–28].

Estimates in various channels of $\pi\pi$ and πK scatterings reveal a common feature: all the background contributions as defined in Eq. (17) are numerically found to be negative! This fact is actually crucial to establish the existence of the σ and κ pole in the present approach and also helps greatly in stabilizing the pole location in the data fit. It is interesting to notice that, there actually exists a correspondence of Eq. (16) in quantum mechanical scattering theory, obtained more than sixty years ago [29]:

$$S(k) = \mathrm{e}^{-2\mathrm{i}pR}\prod_1^\infty \frac{p_n + p}{p_n - p} \tag{18}$$

where k is the (single) channel momentum and p_n pole locations on the complex p plane. The above formula is written down for any finite range potential, in s wave. It is amazing to notice that Eq. (18) automatically predicts a negative background contribution!

In this section we present the proof of the production representation, Eq. (16), in the context of quantum field theory. In the literature we also name it as the PKU representation. Previous results as discussed above, are however only limited to the single channel case. In Ref. [30], the couple channel situation is investigated but ends up with only very limited success, and no production representation were obtained in the couple channel situation.

2.4 Further theoretical developments

Before jumping to the last section where we will concentrate again on the physics of $f_0(600)$ and $K(800)$ resonances, here we would like to further report the theoretical development that can be achieved starting from the PKU representation, i.e., Eq. (16) – which can actually be understood as a simple combination of single channel unitarity and the partial wave dispersion relation [32].

Going back to Eq. (16), on the right hand side, we have S matrix elements parameterized in terms of resonance parameters, i.e., mass M_i and width Γ_i, and somewhat unpleasantly, the cut integrals. The cut integrals are difficult to estimate. However, if *assuming* all resonance's widths vanish in the large N_c limit, the cut integral becomes calculable in the leading order of $1/N_c$ expansion. The leading order term is actually solely contributed by crossed channel resonance exchanges at tree level. On the left hand side of Eq. (16), the physical S matrix elements can be calculated using χPT, near threshold. In this way, one gets the low energy constants (LECs) in the effective lagrangian expressed by resonance parameters, in narrow width approximation, or in the large N_c limit[4)]. In such a situation, one gets a series of relations without relying on the precise form of resonance effective lagrangian.

3) One may use Regge behavior to estimate the high energy contributions to the left hand cut integral.

4) In Ref. [31], it cautiously paid the attention to the situation that the resonance width may not vanish when $N_c \to \infty$. Modest results are still obtainable there.

These relations are obtained at $O(p^4)$ level in Ref. [32] and at $O(p^6)$ level in Ref. [33]. These new relations are helpful in phenomenology as well. E.g., they can be used to give a rather reliable estimate on the LECs at $O(p^6)$ level [34].

3 Physics of $f_0(600)$ and $K(800)$

3.1 The σ and κ pole locations

In our numerical study it is found that the production representation is sensitive to S matrix poles not too far away from physical threshold, hence it provides a powerful tool to explore the broad resonance $f_0(600)$ (or called σ) and $K(800)$ (or called κ). The "sensitivity" relies crucially on the fact, as already mentioned before, that the background contribution to the phase shift is negative and concave.

It is also found that crossing symmetry plays an important role in fixing the σ pole location [26]. For this we put the constraint given by the so-called BNR relations in the data fit and determine the σ pole location as $M_\sigma = 470 \pm 50$ MeV, $\Gamma_\sigma = 570 \pm 50$ MeV. Especially, in Eq. (21) of Ref. [26], we get $M_\sigma = 457 \pm 15$ MeV, $\Gamma_\sigma = 551 \pm 28$ MeV, in nice agreement with the determination using more sophisticated Roy equation analysis [35]. The application of Eq. (16) to πK scattering data [36] also unambiguously establish the existence of the κ meson with the pole location [28]: $M_\kappa = 694 \pm 53$ MeV, $\Gamma_\kappa = 606 \pm 59$ MeV, which are also in nice agreement with the later determination on κ pole parameters using Roy–Steiner equations [37]. Concerning the extraordinarily large widths of $f_0(600)$ and $K(800)$, it is very impressive and remarkable that orthogonal calculations agree with each other so satisfactorily.

3.2 The physical properties of $f_0(600)$ and $K(800)$

Though the existence of the broad σ and κ resonances have been firmly established, its nature still remains mysterious. Especially it is completely an open question on how to understand it from the underlining theory, QCD. Though it is natural to speculate that the $f_0(600)$ meson is related to the quantum excitation of the order parameter $\langle\bar{\psi}\psi\rangle$, a proof at fundamental level is still missing.

At the phenomenological level, many efforts have been made to explore the nature of $f_0(600)$ in the literature [38]. We argue that the $f_0(600)$ (and $K(800)$) may best be understood as the chiral partner of the pseudo-goldstone bosons, in a linear realization of chiral symmetry.

When exploring the nature of σ, it can be helpful to investigate the κ, $f_0(980)$, $a_0(980)$ simultaneously. One of the most challenging problem is to understand not only the mass spectrum but also the widely spread widths between these lightest scalars in different channels. An investigation using the ENJL model has been given [39]. It is found that the masses and widths of these lightest scalars, except the $f_0(980)$, can be understood simultaneously, taking them as the chiral partners of the $SU(3)$ pseudo-goldstone bosons, within linearly realized chiral symmetry [39]. The estimate is very crude because it is based upon a K-matrix unitarization approach. But one may hope the study may get the qualitatively correct picture of these lightest scalars.

There exist further evidences in supporting the above picture. A careful re-examination to the unitarized chiral perturbative amplitudes [40–42] reveals that the [1,1] Padé approximation leads to a "σ" pole falling down to the real s axis in the large N_c limit, though the pole does not fall straightforwardly down to the real axis, unlike the ρ meson. It is found that this bent structure of the σ pole trajectory with respect to N_c found in [1,1] Padé approximant is similar as what one finds in $O(N)$ σ model. Furthermore, the $f_0(600)\pi\pi$ coupling can be estimated, and it is found that $\mathrm{Re}[g^2_{f_0\pi\pi}] < 0$ [43, 44]. This is again found to be very similar to the behavior of the σ meson in $O(N)$ model with a low cutoff scale [45]. The $O(N)$ σ model is certainly not QCD, but possibly it may simulate rather well QCD in the $IJ = 00$ channel. Furthermore, the property of $f_0(600)$ may be understood by studying the $\gamma\gamma\pi\pi$ process [46]. It is estimated that the value of $g^2_{f_0\gamma\gamma}$ lies between the value calculated using a $\bar{q}q$ assumption and that of a $q^2\bar{q}^2$ assumption [47]. As emphasized in Ref. [38], a collective excitation may contain equally important $\bar{q}q$, $\bar{q}^2q^2$, $\bar{q}^3q^3$, $\cdots$ components. This is also consistent with the results obtained in the study of $\gamma\gamma \to \pi\pi$ process. The suggestion made in Ref. [38] on the property of $f_0(600)$ still remains to be reasonable.

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References and notes

1. M. H. Johnson and E. Teller, Classical field theory of nuclear forces, *Phys. Rev.*, 1955, 98(3): 783
2. M. Gell-Mann and M. Lévy, The axial vector current in beta decay, *Nuovo Cim.*, 1960, 16(4): 705
3. S. R. Coleman, J. Wess, and B. Zumino, Structure of phenomenological Lagrangians (I), *Phys. Rev.*, 1969, 177(5): 2239

4. C. G. Callan, S. R. Coleman, J. Wess, and B. Zumino, Structure of phenomenological Lagrangians (II), *Phys. Rev.*, 1969, 177(5): 2247

5. Y. Nambu and G. Jona-Lasinio, Dynamical model of elementary particles based on an analogy with superconductivity (I), *Phys. Rev.*, 1961, 122(1): 345

6. K. C. Chou, On the pseudovector current and lepton decays of baryons and mesons, *Soviet Physics JETP*, 1961, 12: 492

7. S. Weinberg, Phenomenological Lagrangians, *Physica A*, 1979, 96(1–2): 327

8. J. Gasser and H. Leutwyler, Chiral perturbation theory to one loop, *Ann. Phys.*, 1984, 158(1): 142

9. For a review, see, N. A. Tornqvist, Invited talk at *YITP Workshop on Possible Existence of the Sigma Meson and Its Implications to Hadron Physics*, Kyoto, Japan, 12–14 Jun. 2000, arXiv: hep-ph/0008135, and reference therein.

10. L. Y. Xiao, H. Q. Zheng, and Z. Y. Zhou, Lightest scalars as chiral partners of the Nambu–Goldstone bosons, *Nucl. Phys. Proc.*, 2007, 174(Suppl. 174): 142

11. G. Grayer, B. Hyams, C. Jones, P. Schlein, P. Weilhammer, W. Blum, H. Dietl, W. Koch, E. Lorenz, G. Lütjens, W. Männer, J. Meissburger, W. Ochs, and U. Stierlin, High statistics study of the reaction $\pi^-p\rightarrow \pi^-\pi^+n$: Apparatus, method of analysis, and general features of results at 17 GeV/c, *Nucl. Phys. B*, 1974, 75(2): 189

12. W. Ochs, Ph.D. thesis, Munich University, 1974

13. W. Maenner, in: Experimental Meson Spectroscopy – 1974 (Boston), Proceedings of the fourth International Conference on Meson Spectroscopy, edited by D. A. Garelick, *AIP Conf. Proc.* 21, 1974

14. L. Rosselet, P. Extermann, J. Fischer, O. Guisan, R. Mermod, R. Sachot, A. Diamant-Berger, P. Bloch, G. Bunce, B. Devaux, N. Do-Duc, G. Marel, and R. Turlay, Experimental study of 30000 K_{e4} decays, *Phys. Rev. D*, 1977, 15(3): 574

15. S. Pislak, et al. [The E865 Collaboration], High statistics measurement of K_{e4} decay properties, *Phys. Rev. D*, 2003, 67(7): 072004

16. Z. G. Xiao and H. Q. Zheng, Left-hand singularities, hadron form factors and the properties of the sigma meson, *Nucl. Phys. A*, 2001, 695(1–4): 273

17. Z. G. Xiao and H. Q. Zheng, Further analysis of scattering dispersion relations, *Chin. Phys. Lett.*, 2003, 20(3): 342

18. E. M. Aitala, et al. [E791 Collaboration], Experimental evidence for a light and broad scalar resonance in $D^+\rightarrow \pi^-\pi^+\pi^+$ decay, *Phys. Rev. Lett.*, 2001, 86(5): 770

19. M. Ablikim, et al. [BESII Collaboration], The sigma pole in $J/\psi\rightarrow\omega\pi^+\pi^- J/\psi\rightarrow\omega\pi^+\pi^-$, *Phys. Lett. B*, 2004, 598(3–4): 149

20. M. Ablikim, et al. [BES Collaboration], Evidence for κ meson production in process, *Phys. Lett. B*, 2006, 633(6): 681

21. D. V. Bugg, The kappa in $J/\Psi\mapsto K^+\pi$, *Eur. Phys. J. A*, 2005, 25(1): 107; Erratum-ibid, 2005, 26(1): 151

22. E. M. Aitala, et al. [E791 Collaboration], Dalitz plot analysis of the decay $D^+\rightarrow K^-\pi^+\pi^+$ and indication of a low-mass scalar Kπ resonance, *Phys. Rev. Lett.*, 2002, 89(12): 121801

23. J. M. Link, et al. [The FOCUS Collaboration], Evidence for new interference phenomena in the decay $D^+\rightarrow K^-\pi^+\mu^+\nu$, *Phys. Lett. B*, 2002, 535(1–4): 43

24. J. Y. He, Z. G. Xiao, and H. Q. Zheng, The constraints of unitary on pi pi scattering dispersion relations, *Phys. Lett. B*, 2002, 536(1–2): 59; Erratum-ibid, 2002, 549(3–4): 362

25. Z. G. Xiao and H. Q. Zheng, Further analysis of scattering dispersion relations, *Chin. Phys. Lett.*, 2003, 20(3): 342

26. Z.-Y. Zhou, G.-Y. Qin, P. Zhang, Z.-G. Xiao, H.-Q. Zheng, and N. Wu, The pole structure of the unitary, crossing symmetric low energy $\pi\pi$ scattering amplitudes, *J. High Energy Phys.*, 2005, 0502: 043

27. H. Q. Zheng, Z. Y. Zhou, G. Y. Qin, Z. G. Xiao, J. J. Wang, and N. Wu, The k resonance in s wave pK scatterings, *Nucl. Phys. A*, 2004, 733(3–4): 235

28. Z. Y. Zhou and H. Q. Zheng, An improved study of the kappa resonance and the non-exotic s wave pK scatterings up to of LASS data, *Nucl. Phys. A*, 2006, 775(3–4): 212

29. N. Hu, On the application of Heisenberg's theory of S-matrix to the problems of resonance scattering and reactions in nuclear physics, *Phys. Rev.*, 1948, 74(2): 131

30. Z. G. Xiao and H. Q. Zheng, Use of dispersion relations in pp and K [unknown entity & Kmacr] coupled-channel system, *Commun. Theor. Phys.*, 2007, 48(4): 685

31. Z. G. Xiao and H. Q. Zheng, Analyticity and the N_c counting rule of S matrix poles, *Mod. Phys. Lett. A*, 2007, 22(01): 55

32. Z. H. Guo, J. J. Sanz-Cillero, and H. Q. Zheng, Partial waves and large N_c resonance sum rules, *J. High Energy Phys.*, 2007, 0706: 030, arXiv: hep-ph/0701232

33. Z. H. Guo, J. J. Sanz-Cillero, and H. Q. Zheng, extension of the large-partial wave dispersion relations, *Phys. Lett. B*, 2008, 661(5): 342

34. Z. H. Guo and J. J. Sanz-Cillero, ππ-scattering lengths at O(p6) revisited, *Phys. Rev. D*, 2009, 79(9): 096006

35. I. Caprini, G. Colangelo, and H. Leutwyler, Mass and width of the lowest resonance in QCD, *Phys. Rev. Lett.*, 2006, 96(13): 132001

36. D. Aston, et al. [LASS Collaboration], A study of $K^-\pi^+$ scattering in the reaction $K^-p\rightarrow K^-\pi^+ n$ at 11 GeV/c, *Nucl. Phys. B*, 1988, 296(3): 493

37. S. Descotes-Genon and B. Moussallam, The K* 0(800) scalar resonance from Roy–Steiner representations of πK scattering, *Eur. Phys. J. C*, 2006, 48(2): 553

38. For an incomplete list of early references, see for example, H. Q. Zheng, Plenary talk given at conference *Chiral 07*, *Mod. Phys. Lett. A*, 2008, 23: 2218

39. M. X. Su, L. Y. Xiao, and H. Q. Zheng, On the scalar nonet in the extended Nambu–Jona-Lasinio model, *Nucl. Phys. A*, 2007, 792(3–4): 288

40. Z. X. Sun, L. Y. Xiao, Z. G. Xiao, and H. Q. Zheng, Model dependent analyses on the N_c dependence of the sigma pole trajectory, *Mod. Phys. Lett. A*, 2007, 22: 711

41. Z. G. Xiao and H. Q. Zheng, Analyticity and the N_c counting rule of S matrix poles, *Mod. Phys. Lett. A*, 2007, 22(01): 55

42. Z. H. Guo, L. Y. Xiao, and H. Q. Zheng, Is the $f_0(600)$ meson a dynamically generated resonance? A lesson learned from the $O(N)$ model and beyond, *Int. J. Mod. Phys. A*, 2007, 22(26): 4603

43. Y. Mao, X. G. Wang, O. Zhang, H. Q. Zheng, and Z. Y. Zhou, Dispersive analysis on the $f_0(600)$ and $f_0(980)$ resonances in $\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$ processes, *Phys. Rev. D*, 2009, 79(11): 116008

44. G. Mennessier, S. Narison, and W. Ochs, Glueball nature of the from pp and $\gamma\gamma$ scatterings, *Phys. Lett. B*, 2008, 665(4): 205

45. X. G. Wang, O. Zhang, L. C. Jin, H. Q. Zheng, and Z. Y. Zhou, Investigations on the property of $f_0(600)$ and $f_0(980)$ resonances in $\gamma\gamma \to \pi\pi$ process, arXiv: 0910.0341; talk given at Conference *Chiral Dynamics 09, PoS CD*, 2009, 09: 084

46. M. R. Pennington, T. Mori, S. Uehara, and Y. Watanabe, Amplitude analysis of high statistics results on $\gamma\gamma \to \pi^+\pi^-$ and the two photon width of isoscalar states, *Eur. Phys. J. C*, 2008, 56(1): 1

47. See for example the compilation in: M. Hoferichter, D. R. Phillips, and C. Schat, Roy–Steiner equations for $\gamma\gamma \to \pi\pi$, *Eur. Phys. J. C*, 2011, 71: 1743