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RESEARCH ARTICLE

Quantum nonthermal effect of the Vaidya–Bonner–de Sitter black hole

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Using the Hamilton–Jacobi equation of a scalar particle in the curve space-time and a correct-dimension new tortoise coordinate transformation, the quantum nonthermal radiation of the Vaidya–Bonner–de Sitter black hole is investigated. The energy condition for the occurrence of the Starobinsky–Unruh process is obtained. The event horizon surface gravity and the Hawking temperature on the event horizon are also given.

Keywords quantum nonthermal radiation, Hamilton–Jacobi equation, new tortoise coordinate transformation, Vaidya–Bonner–de Sitter black hole

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1 Introduction

In 1974, Hawking made a striking discovery that black holes could produce thermal radiation [1, 2]. This discovery has quickly attracted the attention of the researchers [3–6]. Recently, a semi-classical tunneling method has been put forward, which views Hawking radiation as a tunneling process across the event horizon [7]. It has attracted tremendous interests and made many progresses [8–16]. Subsequently, a semi-classical Hamilton–Jacobi method to research the black hole's tunneling radiation has also been put forward, which greatly simplifies the calculation of the tunneling method [17–21].

In the relativistic quantum mechanics, vacuum is the state of which all positive energy states are empty and all negative ones are filled with fermions. When the external electric field appears in the vacuum, so that caused a crossing of the positive and negative energy levels, negative energy particles in the vacuum can become positive energy particles that can move out by the quantum tunneling effect, which is Klein mechanism. The external cause of vacuum instability is outer field, and the internal one may be fluctuation of the vacuum level. Starobinsky [22] and Unruh [23] have proved that there is not only the thermal radiation but also nonthermal radiation to a rotating or charged black hole. The nonthermal radiation of black hole is caused by a crossing of the positive and negative vacuum levels near the horizon, which is independent of its temperature. The horizon of absolute zero still have this kind of radiation, and the horizon of neither tangential motion nor electrostatic potential will not produce the nonthermal radiation regardless of the temperature. The researchers have studied the nonthermal radiation in different rotating and charged black hole [24–27], and their results is consistent with Starobinsky and Unruh.

In this paper, we will study the quantum nonthermal radiation characteristics of a black hole near the event horizon by using the Hamilton–Jacobi equation, and work out the event horizon surface gravity and the Hawking temperature of the black hole.

The paper is organized as follows. In Section 2, we will give the line element of the Vaidya–Bonner–de Sitter black hole, and derive the surface equation of horizon. In Section 3, the quantum nonthermal radiation of the black hole is derived by using Hamilton–Jacobi equation. In Section 4, the event horizon surface gravity and the Hawking temperature of the black hole are obtained. A summary of the conclusions is given in the last section.

2 The Vaidya–Bonner–de Sitter black hole

The line element of the Vaidya–Bonner–de Sitter black hole can be expressed in the advanced Eddington–Finkelstein time coordinate and adopting signature $(-,+,+,+)$ as [28]

$$\mathrm{d}s^2 = -\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}-\frac{1}{3}\lambda r^2\right)\mathrm{d}\upsilon^2+2\mathrm{d}\upsilon\mathrm{d}r +r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2) \tag{1}$$

where $m=m(\upsilon)$ and $Q=Q(\upsilon)$ are the mass and charge of the black hole, which are the functions of the advanced Eddington–Finkelstein time coordinate υ. λ is the cosmological constant.

According to Eq. (1), the determinant and the nonzero contravariant components of metric can be written respectively as

$$g=-r^4\sin^2\theta \tag{2}$$

$$g^{10}=g^{01}=1,\quad g^{11}=1-\frac{2m}{r}+\frac{Q^2}{r^2}-\frac{1}{3}\lambda r^2$$
$$g^{22}=\frac{1}{r^2},\quad g^{33}=\frac{1}{r^2\sin^2\theta} \tag{3}$$

Because the space-time represented by Eq. (1) is spherically symmetric, the surface equation of event horizon can be written as

$$F(\upsilon,r)=0\ \ or\ \ r_H=r_H(\upsilon) \tag{4}$$

that should satisfy the null hypersurface equation

$$g^{\mu\nu}\frac{\partial F(\upsilon,r)}{\partial x^\mu}\frac{\partial F(\upsilon,r)}{\partial x^\nu}=0 \tag{5}$$

From Eq. (3) and Eq. (5), we can obtain

$$2\frac{\partial F}{\partial \upsilon}\frac{\partial F}{\partial r}+\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}-\frac{1}{3}\lambda r^2\right)\left(\frac{\partial F}{\partial r}\right)^2=0 \tag{6}$$

From Eq. (4) we can obtain

$$\frac{\partial F}{\partial r}\frac{\partial r}{\partial \upsilon}+\frac{\partial F}{\partial \upsilon}=0 \tag{7}$$

Substituting Eq. (7) into Eq. (6), we have

$$1-\frac{2m}{r_H}+\frac{Q^2}{r_H^2}-\frac{1}{3}\lambda r_H^2-2\dot{r}_H=0 \tag{8}$$

where

$$\dot{r}_H=\frac{\partial r_H}{\partial \upsilon} \tag{9}$$

Surface r_H which satisfies Eq. (8) is the event horizon of Vaidya–Bonner–de Sitter black hole. In which $\dot{r}_H$ denotes the event horizon changes with time.

3 The quantum nonthermal radiation of the black hole

In the curved space-time, the motion of the scalar particles with mass of μ and electric charge of e obeys the Hamilton–Jacobi equation

$$g^{\mu\nu}\left(\frac{\partial S}{\partial x^\mu}-eA_\mu\right)\cdot\left(\frac{\partial S}{\partial x^\nu}-eA_\nu\right)+\mu^2=0 \tag{10}$$

where S is Hamilton master function, A_μ are the four-dimensional electromagnetic potential, and $A_\mu=\left(-\frac{Q}{r},0,0,0\right)$.

Substituting Eq. (3) into Eq. (10), we can obtain

$$2\left(\frac{\partial S}{\partial \upsilon}\frac{\partial S}{\partial r}+e\frac{Q}{r}\frac{\partial S}{\partial r}\right)+\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}-\frac{1}{3}\lambda r^2\right)\left(\frac{\partial S}{\partial r}\right)^2 +\frac{1}{r^2}\left(\frac{\partial S}{\partial \theta}\right)^2+\frac{1}{r^2\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2+\mu^2=0 \tag{11}$$

A new tortoise coordinate can be written as [29, 30]

$$r_*=r+\frac{1}{2\kappa(\upsilon_0)}\ln\left[\frac{r-r_H(\upsilon)}{r_H(\upsilon)}\right]$$
$$\upsilon_*=\upsilon-\upsilon_0 \tag{12}$$

where κ is an adjustable parameter, and κ and υ_0 are all constants under tortoise coordinate transformation.

The differential operators can be expressed under the new tortoise coordinate transformation as

$$\begin{cases}\dfrac{\partial}{\partial r}=\dfrac{2\kappa(r-r_H)+1}{2\kappa(r-r_H)}\dfrac{\partial}{\partial r_*}\\ \dfrac{\partial}{\partial \upsilon}=\dfrac{\partial}{\partial \upsilon_*}-\dfrac{r\dot{r}_H}{2\kappa r_H(r-r_H)}\dfrac{\partial}{\partial r_*}\end{cases} \tag{13}$$

Through the tortoise coordinate transformation of Eq. (12), Eq. (11) can be rewritten as

$$2\left[\frac{\partial S}{\partial \upsilon_*}-\frac{r\dot{r}_H}{2\kappa r_H(r-r_H)}\frac{\partial S}{\partial r_*}\right]\cdot\left[\frac{2\kappa(r-r_H)+1}{2\kappa(r-r_H)}\frac{\partial S}{\partial r_*}\right] +\frac{2eQ}{r}\cdot\left[\frac{2\kappa(r-r_H)+1}{2\kappa(r-r_H)}\frac{\partial S}{\partial r_*}\right] +\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}-\frac{1}{3}\lambda r^2\right)\cdot\left[\frac{2\kappa(r-r_H)+1}{2\kappa(r-r_H)}\frac{\partial S}{\partial r_*}\right]^2 +\frac{1}{r^2}\left(\frac{\partial S}{\partial \theta}\right)^2+\frac{1}{r^2\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2+\mu^2=0 \tag{14}$$

Because of the spherical symmetry of space-time, there are two Killing vectors,

$$\frac{\partial S}{\partial \theta}=l\ (const),\quad \frac{\partial S}{\partial \varphi}=j\ (const) \tag{15}$$

Define

$$\omega=-\frac{\partial S}{\partial \upsilon_*} \tag{16}$$

It is the energy of the particle.

Substituting Eq. (15) and Eq. (16) into Eq. (14), we can obtain

$$\frac{A'}{2\kappa r_H(r-r_H)}\left(\frac{\partial S}{\partial r_*}\right)^2+B'\frac{\partial S}{\partial r_*}+2\kappa(r-r_H)C'=0 \tag{17}$$

where

$$A' = \left(1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{1}{3}\lambda r^2\right) r_H[2\kappa(r - r_H) + 1]^2 \\ -2r\dot{r}_H[2\kappa(r - r_H) + 1]$$

$$B' = -2\left(\omega - \frac{eQ}{r}\right)[2\kappa(r - r_H) + 1]$$

$$C' = \frac{l^2}{r^2} + \frac{j^2}{r^2 \sin^2\theta} + \mu^2$$

The solution of Eq. (17) is

$$\frac{\partial S}{\partial r_*} = \frac{\kappa r_H(r - r_H)}{A'}\left(-B' \pm \sqrt{B'^2 - \frac{4A'C'}{r_H}}\right) \tag{18}$$

Because both S and $\frac{\partial S}{\partial r_*}$ must be real numbers, we have $B'^2 - \frac{4A'C'}{r_H} \geqslant 0$, so the energy ω of the particle in the curve space-time described by Eq. (1) satisfies

$$\left(\omega - \frac{eQ}{r}\right)^2 - \frac{A'C'}{r_H[2\kappa(r - r_H) + 1]^2} \geqslant 0 \tag{19}$$

Adopting the equality in Eq. (19), we can obtain

$$\omega^{\pm} = \frac{eQ}{r} \pm \frac{1}{2\kappa(r - r_H) + 1}\sqrt{\frac{A'C'}{r_H}} \tag{20}$$

The distribution of the energy levels is given by

$$\omega \geqslant \omega^+ \quad \text{and} \quad \omega \leqslant \omega^- \tag{21}$$

while $\omega^- \leqslant \omega \leqslant \omega^+$ are the forbidden region. The width of the forbidden region is

$$\Delta\omega = \omega^+ - \omega^- = \frac{2}{2\kappa(r - r_H) + 1}\sqrt{\frac{A'C'}{r_H}} \tag{22}$$

Both Eq. (20) and Eq. (21) describe the distribution characteristics of the particle energy levels of the curve space-time described by Eq. (1).

When the radius r tends to infinity, the space-time approaches to the flat one and the electromagnetic field can be ignored. From Eq. (20), we can obtain

$$\lim_{r\to\infty} \omega^{\pm} \to \pm\mu \tag{23}$$

that is just the Dirac energy spectrum in the Minkowski space-time.

When the radius r tends to r_H near the event horizon, from Eq. (20) and Eq. (8), we can obtain

$$\begin{cases} \lim\limits_{r\to r_H, \upsilon\to\upsilon_0} \omega^+ = \lim\limits_{r\to r_H, \upsilon\to\upsilon_0} \omega^- = \omega_0 = \dfrac{eQ}{r_H} \\ \lim\limits_{r\to r_H, \upsilon\to\upsilon_0} \Delta\omega = 0 \end{cases} \tag{24}$$

This shows that the forbidden region vanishes on the event horizon. When $\mu < \omega_0$, there exists a crossing of the positive and negative energy levels near the event horizon. When the energy satisfies $\mu < \omega \leqslant \omega_0$, a negative energy particle can travel across the forbidden region and becomes a positive energy particle that can move out. This is a quantum tunneling effect which is independent of the temperature of the black hole, that is the Starobinsky–Unruh process of spontaneous radiation [31]. From Eq. (24) and Eq. (8), the energy of particle on the event horizon is related not only to the electric charge e of particle, the parameters (m, Q) of the black hole, but also to the cosmological constant.

4 The event horizon surface gravity and the Hawking temperature of the black hole

In Eq. (17) when r tends to r_H, the coefficient of $(\partial S/\partial r_*)^2$ is 0/0 type, so we can calculate the limit by the L'Hospital law. When we select the adjustable parameter κ as

$$\kappa = \frac{r_H - m - \frac{2}{3}\lambda r_H^3 - 3r_H\dot{r}_H}{2mr_H + \frac{1}{3}\lambda r_H^4 - Q^2} \tag{25}$$

then the limit of the coefficient equals 1. The κ is just the event horizon surface gravity, and is identical to the κ obtained by the Klein–Gordon equation [32].

In addition, according to $T = \kappa/(2\pi\kappa_{\mathrm{B}})$, we can obtain the Hawking temperature of the black hole on the event horizon

$$T = \frac{\kappa}{2\pi\kappa_{\mathrm{B}}} = \frac{1}{2\pi\kappa_{\mathrm{B}}} \cdot \frac{r_H - m - \frac{2}{3}\lambda r_H^3 - 3r_H\dot{r}_H}{2mr_H + \frac{1}{3}\lambda r_H^4 - Q^2} \tag{26}$$

where κ_{B} is Boltzmann's constant.

5 Conclusions

With an improved correct dimension new tortoise coordinate transformation, Quantum nonthermal radiation of the Vaidya–Bonner–de Sitter black hole is investigated by using Hamilton–Jacobi equation of scalar particles in the curve space-time. The nonthermal radiation has nothing to do with the temperature of the black hole, and it is a quantum effect. $\omega_0 = eQ/r_H$ is derived, which is consistent with the previous results [33, 34]. In view of using a new tortoise coordinate transformation with correct dimension, our results provides further support to Starobinsky and Unruh's point of view. The energy of particles comes from the rotation energy and electromagnetic energy of black hole in the process of nonthermal radiation, so Schwarzschild black hole does not exist Starobinsky–Unruh process.

In the end, the event horizon surface gravity κ and the Hawking temperature have been given, which are identical to the results obtained by the Klein–Gordon equation. So κ and T can be obtained in different ways, and they are identical under the same tortoise coordinate transformation.

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