

Perfect field concentrator using zero index metamaterials and perfect electric conductors

Mohammad Mehdi Sadeghi^{1,2}, Hamid Nadgaran², Huanyang Chen^{1,†}

¹School of Physical Science and Technology, Soochow University, Suzhou 215006, China

²Department of Physics, Shiraz University, Shiraz 71454, Iran

Corresponding author. E-mail: †chy@suda.edu.cn

Received July 28, 2013; accepted August 6, 2013

We design a perfect field concentrator from a singular radial mapping. Such a device can be implemented using alternating radial slices of zero index metamaterials and perfect electric conductors. Numerical simulations are performed to verify its functionality.

Keywords field concentrator, transformation optics, zero index metamaterials, perfect electric conductors

PACS numbers 42.25.Fx, 41.20.Jb

Transformation optics [1, 2] has been used to design various devices with extraordinary electromagnetic properties [3], such as invisibility cloaks [2], field rotators [4], field concentrator [5], etc. Some of these early designs have been implemented [6, 7], while field concentrator has not yet been realized. In this paper, we will show that a singular transformation can be used to design a perfect field concentrator. To realize such a device, only two kinds of isotropic and homogeneous metamaterials are required, i.e., zero index metamaterials (ZIMs) and perfect electric conductors (PECs). The current design paves a way for the eventual implementation of the field concentrator.

We start the work from the following radial transformation in Fig. 1: region I ($r' \in [0, R_2]$) is compressed into $r \in [0, R_1]$ and region II ($r' \in [R_2, R_3]$) is expanded into

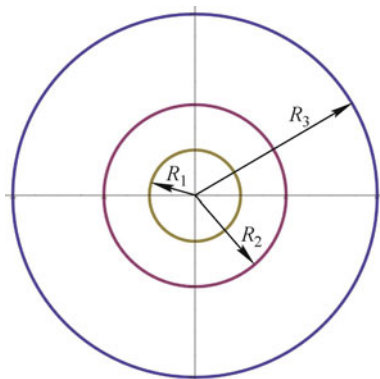


Fig. 1 The schematic plot of the radial mapping for the field concentrator.

$r \in [R_1, R_3]$. Specifically, we take a linear transformation as an example [5]:

$$r = f(r') = \begin{cases} \frac{R_1}{R_2} \cdot r', & 0 \leq r' \leq R_2 \\ \frac{R_3 - R_1}{R_3 - R_2} \cdot r' - \frac{R_2 - R_1}{R_3 - R_2} \cdot R_3, & R_2 \leq r' \leq R_3 \end{cases} \quad (1)$$

According to transformation optics, the material parameters of the field concentrator can be written as [5]:

$$\varepsilon = \mu = \begin{cases} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (R_2/R_1)^2 \end{pmatrix}, & 0 \leq r \leq R_1 \\ \begin{pmatrix} \eta_r & 0 & 0 \\ 0 & \eta_r^{-1} & 0 \\ 0 & 0 & (f/h)^2 \eta_r \end{pmatrix}, & R_1 \leq r \leq R_3 \end{cases} \quad (2)$$

in circular cylindrical coordinates, where $\eta_r = \frac{e}{f} \frac{R_3}{r} + 1$, $e = R_2 - R_3$, $f = R_3 - R_2$ and $h = R_3 - R_1$.

We will only focus on transverse magnetic (TM) modes (with magnetic field along z -direction) as we will see that one type of such field concentrators can be implemented using ZIMs and PECs.

Firstly, we will set $R_1 = 0.1$ m, $R_2 = 0.2$ m and $R_3 = 0.4$ m as an example [type I, see Fig. 2(a)]. All

the slopes of this transformation are positive, hence all the material parameters positive. As we will see, such a device has a concentrating efficiency $(\frac{R_2}{R_1})^2$ (defined by a factor of the input energy over the energy in the core $0 \leq r \leq R_1$). Therefore, as we increase the R_2 , we can enhance the concentrating efficiency. In principle, we can have a large R_2 in the virtual space. However, it will bring negative material parameters if $R_2 > R_3$ [8]. Such a field concentrator is not easy to implement and if we indeed success in designing one, the loss problem

of the negative index metamaterials will degrade the functionalities. There should be a suitable R_2 so that we will have a strong concentrating efficiency, meanwhile, with small loss. Taking all that into account, we have a compromised choice, i.e., to set $R_2 = R_3$ [type II, see Fig. 2(b)]. Such a field concentrator can be implemented using ZIMs and PECs, which can be easy to finish at least in microwave realm. Figure 3(a) shows the magnetic field distribution of type I field concentrator under the illumination of a TM incident plane wave from left

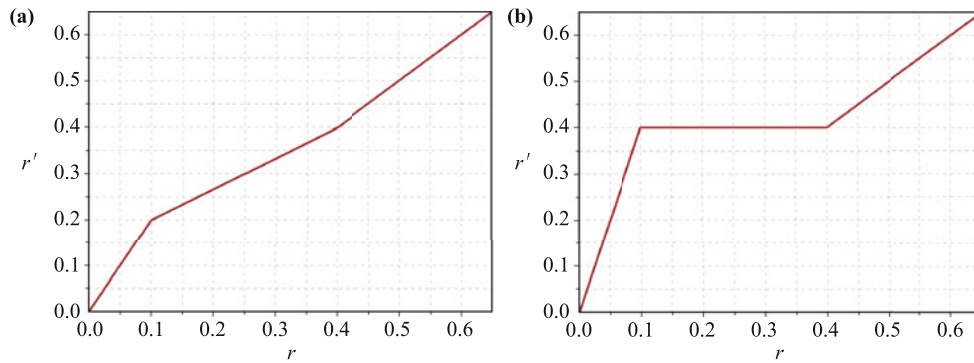


Fig. 2 The detailed mappings for the field concentrators with (a) $R_1 = 0.1$ m, $R_2 = 0.2$ m, and $R_3 = 0.4$ m; (b) $R_1 = 0.1$ m, $R_2 = R_3 = 0.4$ m.

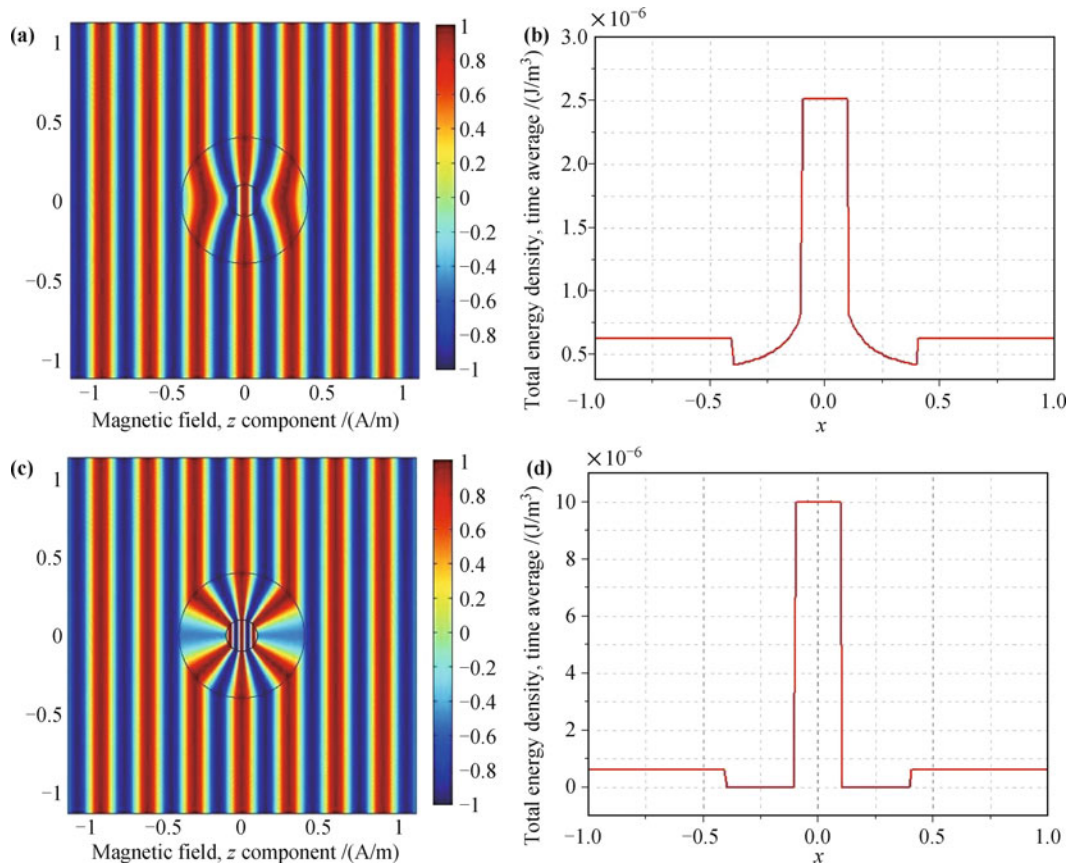


Fig. 3 (a) The magnetic field distribution and (b) the energy distribution along x -direction for type I field concentrator (with $R_1 = 0.1$ m, $R_2 = 0.2$ m, and $R_3 = 0.4$ m). (c) The magnetic field distribution and (d) the energy distribution along x -direction for type II field concentrator (with $R_1 = 0.1$ m, $R_2 = R_3 = 0.4$ m).

to right along x -direction. The wavelength is 0.3 meter. It is transparent and has a concentrating efficiency of 4, as shown in Fig. 3(b), the energy distribution along x -direction.

If we increase R_2 so that $R_2 = R_3$, the transformation will become singular [see Fig. 2(b)], and the material parameters will change into: $\epsilon_r = \epsilon_\theta = 1$ and $\mu_z = 16$ for $0 \leq r \leq R_1$; and $\epsilon_r = \infty$, $\epsilon_\theta = 0$ and $\mu_z = 0$ for $R_1 \leq r \leq R_3$. We plot the magnetic field distribution for this type II field concentrator in Fig. 3(c). The functionality is the same as that of type I but with a higher concentrating efficiency of 16, which can also be seen from the energy distribution along x -direction shown in Fig. 3(d). All the above results come from the calculations of COMSOL. Specifically, we set $R_2 = 0.399$ m during simulations.

Now we come to see how to implement such a singular field concentrator. Let us consider the configuration in Fig. 4. Two kinds of isotropic metamaterials are arranged in alternating radial slices with one's permittivity and permeability ϵ_A and μ_A , and the other ϵ_B and μ_B . The effective material parameters for such a configuration are

$$\begin{cases} \epsilon_r = f_A \epsilon_A + f_B \epsilon_B \\ \epsilon_\theta^{-1} = f_A \epsilon_A^{-1} + f_B \epsilon_B^{-1} \\ \mu_z = f_A \mu_A + f_B \mu_B \end{cases} \quad (3)$$

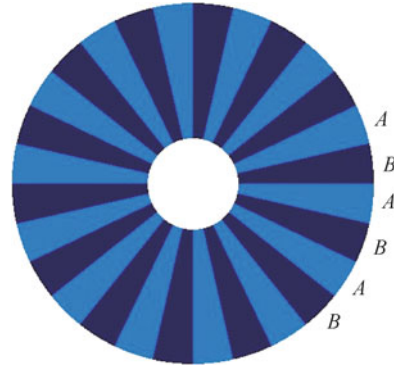


Fig. 4 The configuration to implement the part in $R_1 \leq r \leq R_3$ of the singular field concentrator with only two kinds isotropic and homogeneous metamaterials, in alternating radial slices.

when the working wavelength is much larger than the unit arc length at the outer boundary ($r = R_3$). f_A and f_B are the filling fractions for the two kinds of metamaterials. There has been a general design for field

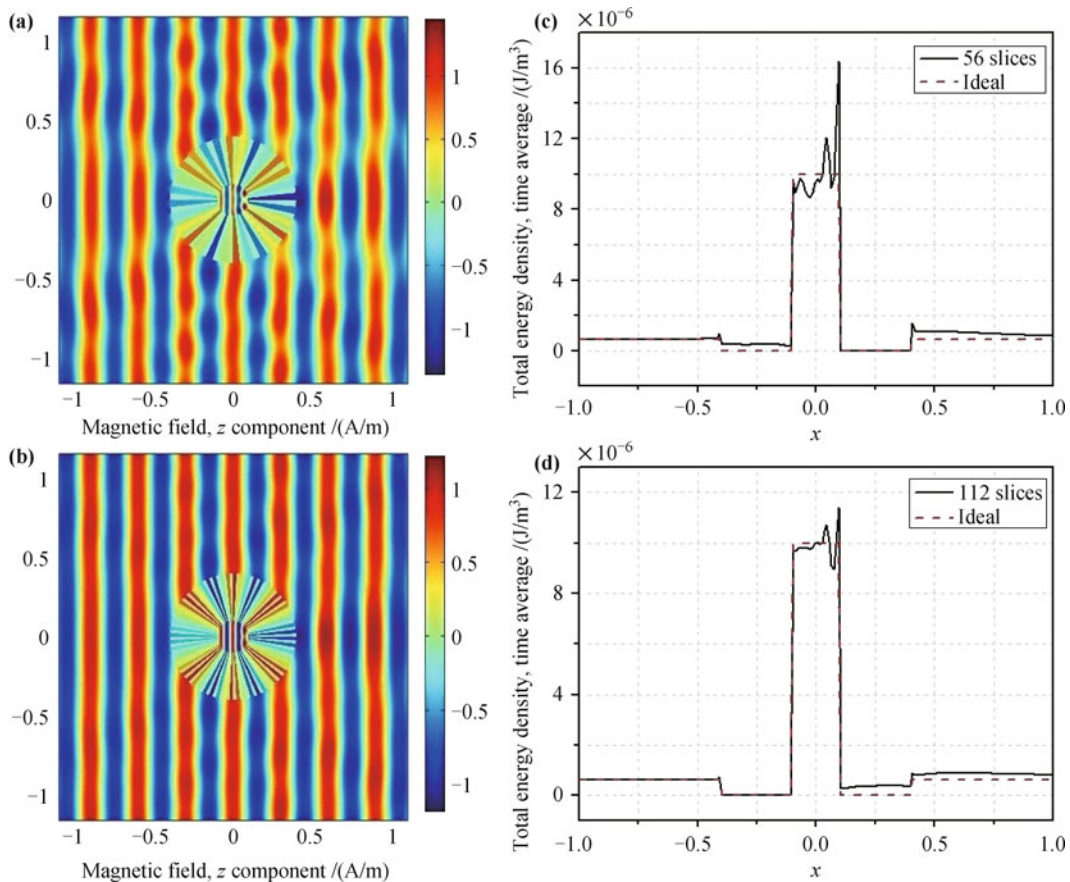


Fig. 5 The magnetic field distributions for (a) a 56-slice configuration and (b) a 112-slice configuration, and the energy distributions along x -direction for (c) the 56-slice configuration and (d) the 112-slice configuration. For comparison, we also plot the energy distribution along x -direction for an ideal case [the same as that in Fig. 3(b)].

concentrators by allowing ε_A , μ_A , ε_B and μ_B to be inhomogeneous [9]. However, in this paper, we will simplify the parameters so that only homogeneous metamaterials are required for an easier implementation in future.

In this design, we use two kinds of materials, one PECs (with $\varepsilon = \infty$ and $\mu = 1$) and the other, ZIMs (with $\varepsilon = \mu = 0$). Such an assembly has effective material parameters: $\varepsilon_r = \infty$, $\varepsilon_\theta = 0$, and $\mu_z = 0$. Although $\mu = 1$ for PECs, the effective $\mu_z = 0$ if the counterparts are ZIMs. The reason is that as $\varepsilon = \infty$, there are neither electric fields nor magnetic fields inside the PECs. To prove such an idea, we plot the magnetic field distributions for configurations with 56 slices in Fig. 5(a) and with 112 slices in Fig. 5(b). Figures 5(c) and (d) are the corresponding energy distributions along x -direction. We find that both configurations show excellent concentrating functionalities. For the configuration with more slices, the scattering is less, and the energy in the core is more stable (a better result in homogeneity of the concentrated waves). In simulations, we set $\varepsilon = -100\,000$ for PECs and $\varepsilon = \mu = 0.001$ for ZIMs. Notice that similar method has been utilized to design devices for static magnetic field [10] and thermodynamics [11–13]. For the concentrator of static magnetic field [10], only alternated layers of superconductors (with an infinite permeability) and soft ferromagnetic materials (with a zero permeability) are required. For thermal concentrator [13], the alternated layers are two kinds of materials, one with a very large thermal conductivity, and the other with a very small one. Our design here is different from those as we need to consider both the permittivities and permeabilities. Therefore it can function for finite frequencies.

In summary, we have designed a perfect field concentrator simply using two kinds of isotropic and homogeneous metamaterials, zero index metamaterials and perfect electric conductors, in alternative radial slices. Such a design is feasible to realize at microwave frequencies if we see the recent experimental progress of the epsilon-near-zero metamaterials [14, 15] and Dirac cone optics [16]. The potential application of such a field concentrator could be found in wireless power transfer in future [17].

Acknowledgements This work was supported by the Foundation for the Author of National Excellent Doctoral Dissertation of China (Grant No. 201217), the National Natural Science Foundation of China (Grant No. 11004147), the Natural Science Foundation of Jiangsu Province (Grant No. BK2010211), and the Priority Academic Program Development (PAPD) of Jiangsu Higher Education Institutions.

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