

Evolution of the single-mode squeezed vacuum state in amplitude dissipative channel

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Using the way of deriving infinitive sum representation of density operator as a solution to the master equation describing the amplitude dissipative channel by virtue of the entangled state representation, we show manifestly how the initial density operator of a single-mode squeezed vacuum state evolves into a definite mixed state which turns out to be a squeezed chaotic state with decreasing-squeezing and decoherence. We investigate average photon number, photon statistics distributions for this mixed state.

Keywords single-mode squeezed vacuum, decoherence, entangled state representation, sum representation

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1 Introduction

Squeezed states are such for which the noise in one of the chosen pair of observables is reduced below the vacuum or ground-state noise level, at the expense of increased noise in the other observable. The squeezing effect indeed improves interferometric and spectroscopic measurements, so in the context of interferometric detection and of gravitational waves the squeezed state is very useful [1]. In a very recently published paper, Agarwal [2] revealed that a vortex state of a two-mode system can be generated from a squeezed vacuum by subtracting a photon, and such a subtracting mechanism may happen in a quantum channel with amplitude damping. Usually, in nature every system is not isolated, and dissipation or dephasing usually happens when a system is immersed in a thermal environment, or a signal (a quantum state) passes through a quantum channel which is described by a master equation [3]. For example, when a pure state propagates in a medium, it inevitably interacts with the medium and evolves into a mixed state [4]. Dissipation or dephasing will deteriorate the degree of nonclassicality of photon fields, so physicists pay much attention to it [5–7].

In the present work we investigate how an initial single-mode squeezed vacuum state evolves in an ampli-

tude dissipative channel (ADC). When a system is described by its interaction with a channel with a large number of degrees of freedom, master equations are set up for a better understanding how quantum decoherence is processed to affect unitary character in the dissipation or gain of the system. In most cases people are interested in the evolution of the variables associated with the system only. This requires us to obtain the equations of motion for the system of interest only after tracing over the reservoir variables. A quantitative measure of nonclassicality of quantum fields is necessary for further investigating the system's dynamical behavior. For this channel, the associated loss mechanism in physical processes is governed by the following master equation [3]:

$$\frac{d\rho(t)}{dt} = \kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) \quad (1)$$

where ρ is the density operator of the system, and κ is the rate of decay. We have solved this problem with use of the thermo entangled state representation [8]. Our questions are: What kind of mixed state does the initial squeezed state turn into? How does the photon statistics distributions vary in the ADC?

Thus solving master equations is one of the fundamental tasks in quantum optics. Usually people use various quasi-probability representations, such as P-representation, Q-representation, complex P-

representation, and Wigner functions, etc. for converting the master equations of density operators into their corresponding c -number equations. Recently, a new approach [8, 9], using the thermal entangled state representation [10, 11] to convert operator master equations to their c -number equations is presented which can directly lead to the corresponding Kraus operators (the infinitive representation of evolved density operators) in many cases.

The work is arranged as follows. In Section 2 by virtue of the entangled state representation we briefly reviewed our way of deriving the infinitive sum representation of density operator as a solution of the master equation. In Section 3 we showed that a pure squeezed vacuum state (with squeezing parameter λ) will evolve into a mixed state (output state), whose exact form is derived, which turns out to be a squeezed chaotic state with decoherence. We investigated the average photon number, photon statistics distributions for this mixed state. The probability of finding n photons in this mixed state was obtained which turns out to be a Legendre polynomial function relating to the squeezing parameter λ and the decaying rate κ . In Section 4 we discussed the photon statistics distributions of the output state. In Sections 5 and 6 we discussed the Wigner function and tomogram of the output state, respectively.

2 Brief review of deducing the infinitive sum representation of $\rho(t)$

For solving the above master equation, in a recent review paper [12] we have introduced a convenient approach in which the two-mode entangled state [10, 11]

$$|\eta\rangle = \exp\left(-\frac{1}{2}|\eta|^2 + \eta a^\dagger - \eta^* \tilde{a}^\dagger + a^\dagger \tilde{a}^\dagger\right) |0\tilde{0}\rangle \quad (2)$$

is employed, where \tilde{a}^\dagger is a fictitious mode independent of the real mode a^\dagger , $[\tilde{a}, a^\dagger] = 0$. $|\eta = 0\rangle$ possesses the properties

$$\begin{aligned} a|\eta = 0\rangle &= \tilde{a}^\dagger|\eta = 0\rangle \\ a^\dagger|\eta = 0\rangle &= \tilde{a}|\eta = 0\rangle \\ (a^\dagger a)^n|\eta = 0\rangle &= (\tilde{a}^\dagger \tilde{a})^n|\eta = 0\rangle \end{aligned} \quad (3)$$

Acting the both sides of Eq. (1) on the state $|\eta = 0\rangle \equiv |I\rangle$, and denoting $|\rho\rangle = \rho|I\rangle$, we have

$$\begin{aligned} \frac{d}{dt}|\rho\rangle &= \kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)|I\rangle \\ &= \kappa(2a\tilde{a} - a^\dagger a - \tilde{a}^\dagger \tilde{a})|\rho\rangle \end{aligned} \quad (4)$$

so its formal solution is

$$|\rho\rangle = \exp[\kappa t(2a\tilde{a} - a^\dagger a - \tilde{a}^\dagger \tilde{a})]|\rho_0\rangle \quad (5)$$

where $|\rho_0\rangle \equiv \rho_0|I\rangle$, ρ_0 is the initial density operator.

Noticing that the operators in Eq. (5) obey the following commutative relation,

$$[a\tilde{a}, a^\dagger a] = [a\tilde{a}, \tilde{a}^\dagger \tilde{a}] = \tilde{a}a \quad (6)$$

and

$$\left[\frac{a^\dagger a + \tilde{a}^\dagger \tilde{a}}{2}, a\tilde{a}\right] = -\tilde{a}a \quad (7)$$

as well as using the operator identity [13]

$$e^{\lambda(A+\sigma B)} = e^{\lambda A} e^{\sigma(1-e^{-\lambda\tau})B/\tau} \quad (8)$$

(which is valid for $[A, B] = \tau B$), we have

$$e^{-2\kappa t\left(\frac{a^\dagger a + \tilde{a}^\dagger \tilde{a}}{2} - a\tilde{a}\right)} = e^{-\kappa t(a^\dagger a + \tilde{a}^\dagger \tilde{a})} e^{T' a\tilde{a}} \quad (9)$$

where $T' = 1 - e^{-2\kappa t}$.

Then substituting Eq. (9) into Eq. (5) yields [12]

$$\begin{aligned} |\rho\rangle &= e^{-\kappa t(a^\dagger a + \tilde{a}^\dagger \tilde{a})} \sum_{n=0}^{\infty} \frac{T'^n}{n!} a^n \tilde{a}^n |\rho_0\rangle \\ &= e^{-\kappa t a^\dagger a} \sum_{n=0}^{\infty} \frac{T'^n}{n!} a^n \rho_0 a^\dagger^n e^{-\kappa t \tilde{a}^\dagger \tilde{a}} |I\rangle \\ &= \sum_{n=0}^{\infty} \frac{T'^n}{n!} e^{-\kappa t a^\dagger a} a^n \rho_0 a^\dagger^n e^{-\kappa t a^\dagger a} |I\rangle \end{aligned} \quad (10)$$

which leads to the infinitive operator-sum representation of ρ ,

$$\rho = \sum_{n=0}^{\infty} M_n \rho_0 M_n^\dagger \quad (11)$$

where

$$M_n \equiv \sqrt{\frac{T'^n}{n!}} e^{-\kappa t a^\dagger a} a^n \quad (12)$$

We can prove

$$\begin{aligned} \sum_n M_n^\dagger M_n &= \sum_n \frac{T'^n}{n!} a^\dagger^n e^{-2\kappa t a^\dagger a} a^n \\ &= \sum_n \frac{T'^n}{n!} e^{2n\kappa t} : a^\dagger^n a^n : e^{-2\kappa t a^\dagger a} \\ &=: e^{T' e^{2\kappa t} a^\dagger a} : e^{-2\kappa t a^\dagger a} \\ &=: e^{(e^{2\kappa t} - 1) a^\dagger a} : e^{-2\kappa t a^\dagger a} = 1 \end{aligned} \quad (13)$$

where $::$ stands for the normal ordering. Thus M_n is a kind of Kraus operator, and ρ in Eq. (11) is qualified to be a density operator, i.e.,

$$\text{Tr}[\rho(t)] = \text{Tr}\left[\sum_{n=0}^{\infty} M_n \rho_0 M_n^\dagger\right] = \text{Tr}\rho_0 \quad (14)$$

Therefore, for any given initial state ρ_0 , the density operator $\rho(t)$ can be directly calculated from Eq. (11). The entangled state representation provides us with an elegant way of deriving the infinitive sum representation of density operator as a solution of the master equation.

3 Evolution of an initial single-mode squeezed vacuum state in ADC

It is seen from Eq. (11) that for any given initial state ρ_0 , the density operator $\rho(t)$ can be directly calculated. When ρ_0 is a single-mode squeezed vacuum state,

$$\rho_0 = \text{sech}\lambda \exp\left(\frac{\tanh\lambda}{2} a^{\dagger 2}\right) |0\rangle\langle 0| \exp\left(\frac{\tanh\lambda}{2} a^2\right) \quad (15)$$

we see

$$\begin{aligned} \rho(t) &= \text{sech}\lambda \sum_{n=0}^{\infty} \frac{T'^n}{n!} e^{-\kappa t a^\dagger a} a^n \\ &\times \exp\left(\frac{\tanh\lambda}{2} a^{\dagger 2}\right) |0\rangle\langle 0| \\ &\times \exp\left(\frac{\tanh\lambda}{2} a^2\right) a^{\dagger n} e^{-\kappa t a^\dagger a} \end{aligned} \quad (16)$$

Using the Baker-Hausdorff lemma [14],

$$e^{\lambda \hat{A}} \hat{B} e^{-\lambda \hat{A}} = \hat{B} + \lambda [\hat{A}, \hat{B}] + \frac{\lambda^2}{2!} [\hat{A}, [\hat{A}, \hat{B}]] + \dots \quad (17)$$

$$\begin{aligned} \rho(t) &= \text{sech}\lambda \sum_{n=0}^{\infty} \frac{T'^n}{n!} e^{-\kappa t a^\dagger a} a^n e^{\frac{\tanh\lambda}{2} a^{\dagger 2}} |0\rangle\langle 0| e^{\frac{\tanh\lambda}{2} a^2} a^{\dagger n} e^{-\kappa t a^\dagger a} \\ &= \text{sech}\lambda \sum_{n=0}^{\infty} \frac{(T' \tanh\lambda)^n}{2^n n!} e^{-\frac{2\kappa t}{2} a^{\dagger 2} \tanh\lambda} H_n\left(i\sqrt{\frac{\tanh\lambda}{2}} a^\dagger e^{-\kappa t}\right) |0\rangle\langle 0| H_n\left(-i\sqrt{\frac{\tanh\lambda}{2}} a e^{-\kappa t}\right) e^{-\frac{2\kappa t}{2} a^2 \tanh\lambda} \\ &= \text{sech}\lambda \sum_{n=0}^{\infty} \frac{(T' \tanh\lambda)^n}{2^n n!} : e^{-\frac{2\kappa t}{2} (a^2 + a^{\dagger 2}) \tanh\lambda} - a^\dagger a H_n\left(i\sqrt{\frac{\tanh\lambda}{2}} a^\dagger e^{-\kappa t}\right) H_n\left(-i\sqrt{\frac{\tanh\lambda}{2}} a e^{-\kappa t}\right) : \end{aligned} \quad (22)$$

Then using the following identity [16]

$$\begin{aligned} \sum_{n=0}^{\infty} \frac{t^n}{2^n n!} H_n(x) H_n(y) \\ = (1 - t^2)^{-1/2} \exp\left[\frac{t^2(x^2 + y^2) - 2txy}{t^2 - 1}\right] \end{aligned} \quad (23)$$

and $e^{\lambda a^\dagger a} = e^{(e^\lambda - 1)a^\dagger a}$, we finally obtain the expression of the output state

$$\rho(t) = W e^{\frac{\beta}{2} a^{\dagger 2}} e^{a^\dagger a \ln(\beta T' \tanh\lambda)} e^{\frac{\beta}{2} a^2} \quad (24)$$

we have

$$\begin{aligned} a^n \exp\left(\frac{\tanh\lambda}{2} a^{\dagger 2}\right) |0\rangle \\ = e^{\frac{\tanh\lambda}{2} a^{\dagger 2}} e^{-\frac{\tanh\lambda}{2} a^{\dagger 2}} a^n e^{\frac{\tanh\lambda}{2} a^{\dagger 2}} |0\rangle \\ = e^{\frac{\tanh\lambda}{2} a^{\dagger 2}} (a + a^\dagger \tanh\lambda)^n |0\rangle \end{aligned} \quad (18)$$

Further employing the operator identity [15]

$$\begin{aligned} (\mu a + \nu a^\dagger)^m \\ = \left(-i\sqrt{\frac{\mu\nu}{2}}\right)^m : H_m\left(i\sqrt{\frac{\mu}{2\nu}} a + i\sqrt{\frac{\nu}{2\mu}} a^\dagger\right) : \end{aligned} \quad (19)$$

where $H_m(x)$ is the Hermite polynomial, we know

$$\begin{aligned} (a + a^\dagger \tanh\lambda)^n \\ = \left(-i\sqrt{\frac{\tanh\lambda}{2}}\right)^n : H_n\left(i\sqrt{\frac{1}{2 \tanh\lambda}} a + i\sqrt{\frac{\tanh\lambda}{2}} a^\dagger\right) : \end{aligned} \quad (20)$$

From Eq. (18), it follows that

$$\begin{aligned} a^n e^{\frac{\tanh\lambda}{2} a^{\dagger 2}} |0\rangle \\ = \left(-i\sqrt{\frac{\tanh\lambda}{2}}\right)^n e^{\frac{\tanh\lambda}{2} a^{\dagger 2}} H_n\left(i\sqrt{\frac{\tanh\lambda}{2}} a^\dagger\right) |0\rangle \end{aligned} \quad (21)$$

On the other hand, noting $e^{-\kappa t a^\dagger a} a^\dagger e^{\kappa t a^\dagger a} = a^\dagger e^{-\kappa t}$, $e^{\kappa t a^\dagger a} a e^{-\kappa t a^\dagger a} = a e^{-\kappa t}$ and the normally ordered form of the vacuum projector $|0\rangle\langle 0| = e^{-a^\dagger a}$, we have

with

$$\begin{aligned} W &\equiv \frac{\text{sech}\lambda}{\sqrt{1 - T'^2 \tanh^2 \lambda}}, \quad \beta \equiv \frac{e^{-2\kappa t} \tanh\lambda}{1 - T'^2 \tanh^2 \lambda} \\ T' &= 1 - e^{-2\kappa t} \end{aligned} \quad (25)$$

By comparing Eq. (15) with (24) one can see that after going through the channel the initial squeezing parameter $\tanh\lambda$ in Eq. (15) becomes $\beta \equiv \frac{e^{-2\kappa t} \tanh\lambda}{1 - T'^2 \tanh^2 \lambda}$, and $|0\rangle\langle 0| \rightarrow \frac{1}{\sqrt{1 - T'^2 \tanh^2 \lambda}} e^{a^\dagger a \ln(\beta T' \tanh\lambda)}$, a chaotic

state (mixed state). Due to $T' > 0$, we can prove $\frac{e^{-2\kappa t}}{1-T'^2 \tanh^2 \lambda} < 1$, which means a squeezing-decreasing process. When $\kappa t = 0$, then $T' = 0$ and $\beta = \tanh \lambda$, and Eq. (22) becomes the initial squeezed vacuum state as expected.

It is important to check if $\text{Tr}\rho(t) = 1$. Using Eq. (22) and the completeness of coherent state $\int \frac{d^2z}{\pi} |z\rangle\langle z| = 1$ as well as the following formula [17]

$$\int \frac{d^2z}{\pi} e^{\zeta|z|^2 + \xi z + \eta z^* + f z^2 + g z^{*2}} = \frac{1}{\sqrt{\zeta^2 - 4fg}} e^{\frac{-\zeta\xi\eta + f\eta^2 + g\xi^2}{\zeta^2 - 4fg}} \quad (26)$$

whose convergent condition is $\text{Re}(\zeta \pm f \pm g) < 0$ and $\text{Re}\left(\frac{\zeta^2 - 4fg}{\zeta \pm f \pm g}\right) < 0$, we really see

$$\text{Tr}\rho(t) = W \int \frac{d^2z}{\pi} \langle z | e^{\frac{\beta}{2} a^{\dagger 2}} e^{a^\dagger a \ln(\beta T' \tanh \lambda)} e^{\frac{\beta}{2} a^2} | z \rangle$$

$$\begin{aligned} \text{Tr}(\rho(t) a^\dagger a) &= W \int \frac{d^2z}{\pi} \langle z | e^{\frac{\beta}{2} a^{\dagger 2}} e^{a^\dagger a \ln(\beta T' \tanh \lambda)} e^{\frac{\beta}{2} a^2} a^\dagger a | z \rangle = W \int \frac{d^2z}{\pi} \langle z | e^{\frac{\beta}{2} a^{\dagger 2}} e^{a^\dagger a \ln(\beta T' \tanh \lambda)} e^{\frac{\beta}{2} a^2} z a^\dagger | z \rangle \\ &= W \int \frac{d^2z}{\pi} z \langle z | e^{\frac{\beta}{2} a^{\dagger 2}} e^{a^\dagger a \ln(\beta T' \tanh \lambda)} (a^\dagger + \beta a) e^{\frac{\beta}{2} a^2} | z \rangle \\ &= W\beta \int \frac{d^2z}{\pi} z e^{\frac{\beta}{2}(z^{*2} + z^2)} \langle z | (a^\dagger T' \tanh \lambda + z) e^{a^\dagger a \ln(\beta T' \tanh \lambda)} | z \rangle \\ &= W\beta \int \frac{d^2z}{\pi} (|z|^2 T' \tanh \lambda + z^2) \exp\left[(\beta T' \tanh \lambda - 1)|z|^2 + \frac{\beta}{2}(z^{*2} + z^2)\right] \end{aligned} \quad (28)$$

In order to perform the integration, we reform Eq. (28) to

$$\begin{aligned} \text{Tr}(\rho(t) a^\dagger a) &= W\beta \left(T' \tanh \lambda \frac{\partial}{\partial f} + \frac{2}{\beta} \frac{\partial}{\partial s} \right) \times \int \frac{d^2z}{\pi} \exp\left[(\beta T' \tanh \lambda - 1 + f)|z|^2 + \frac{\beta}{2}(z^{*2} + (1+s)z^2)\right]_{f=s=0} \\ &= W\beta \left(T' \tanh \lambda \frac{\partial}{\partial f} + \frac{2}{\beta} \frac{\partial}{\partial s} \right) \frac{1}{\sqrt{(\beta T' \tanh \lambda - 1 + f)^2 - \beta^2(1+s)}} \Big|_{f=s=0} \\ &= \frac{W\beta^2 - W\beta^2 T'^2 \tanh^2 \lambda + W\beta T' \tanh \lambda}{[(\beta T' \tanh \lambda - 1)^2 - \beta^2]^{3/2}} = - \left[1 + \frac{\beta T' \tanh \lambda - 1}{(\beta T' \tanh \lambda - 1)^2 - \beta^2} \right] \end{aligned} \quad (29)$$

in the last step, we have used Eq. (27). Using Eq. (29), we present the time evolution of the average photon number in Fig. 1, from which we find that the average photon number of the single-mode squeezed vacuum state in the amplitude damping channel reduces gradually to zero as the decay time is extended.

5 Photon statistics distribution

Next, we shall derive the photon statistics distributions of $\rho(t)$. The photon number is given by $p(n, t) = \langle n | \rho(t) | n \rangle$. With $a^{\dagger m} | n \rangle = \sqrt{(m+n)!/n!} | m+n \rangle$ and by

$$\begin{aligned} &= W \int \frac{d^2z}{\pi} \exp\left[-(1 - \beta T' \tanh \lambda)|z|^2 + \frac{\beta}{2} z^{*2} + \frac{\beta}{2} z^2\right] \\ &= \frac{W}{\sqrt{(\beta T' \tanh \lambda - 1)^2 - \beta^2}} = 1 \end{aligned} \quad (27)$$

so $\rho(t)$ is qualified to be a mixed state, thus we see an initial pure squeezed vacuum state having evolved into a squeezed chaotic state with decreasing-squeezing after passing through an amplitude dissipative channel.

4 Average photon number

Using the completeness relation of coherent state and the normally ordering form of $\rho(t)$ in Eq. (22), and using $e^{\frac{\beta}{2} a^{\dagger 2}} a^\dagger e^{-\frac{\beta}{2} a^2} = a^\dagger + \beta a$, as well as $e^{a^\dagger a \ln(\beta T' \tanh \lambda)} a^\dagger e^{-a^\dagger a \ln(\beta T' \tanh \lambda)} = a^\dagger \beta T' \tanh \lambda$, we have

using the un-normalized coherent state $|\alpha\rangle = \exp[aa^\dagger]|0\rangle$ [18, 19], leading to $|n\rangle = \frac{1}{\sqrt{n!}} \frac{d^n}{d\alpha^n} |\alpha\rangle|_{\alpha=0}$, ($\langle\beta|\alpha\rangle = e^{\alpha\beta^*}$), as well as the normal ordering form of $\rho(t)$ in Eq. (22), the probability of finding n photons in the field is given by

$$\begin{aligned} p(n, t) &= \langle n | \rho(t) | n \rangle \\ &= \frac{W}{n!} \frac{d^n}{d\beta^{*n}} \frac{d^n}{d\alpha^n} \langle \beta | e^{\frac{\beta}{2} \beta^{*2}} e^{a^\dagger a \ln(\beta T' \tanh \lambda)} e^{\frac{\beta}{2} a^2} | \alpha \rangle_{\alpha, \beta^* = 0} \\ &= \frac{W}{n!} \frac{d^n}{d\beta^{*n}} \frac{d^n}{d\alpha^n} \exp(\beta^* \alpha \beta T' \tanh \lambda + \frac{\beta}{2} \beta^{*2} + \frac{\beta}{2} \alpha^2) |_{\alpha, \beta^* = 0} \end{aligned} \quad (30)$$

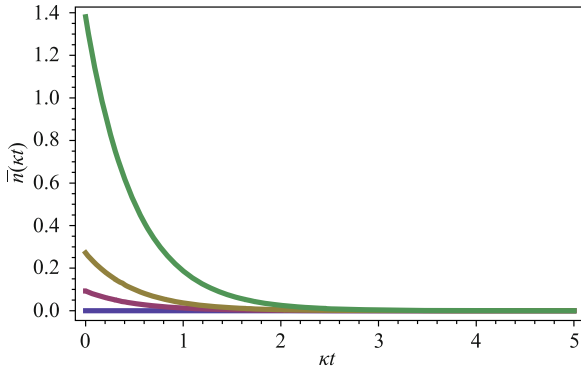


Fig. 1 The average $\bar{n}(\kappa t)$ as the function of κt for different values of squeezing parameter λ (from bottom to top $\lambda = 0, 0.1, 0.3, 0.5, 1$).

Note that

$$\begin{aligned} & [e^{\frac{\beta}{2}a^{\dagger 2}} e^{a^{\dagger}a \ln(\beta T' \tanh \lambda)} e^{\frac{\beta}{2}a^2}]^{\dagger} \\ &= e^{\frac{\beta}{2}a^{\dagger 2}} e^{a^{\dagger}a \ln(\beta T' \tanh \lambda)} e^{\frac{\beta}{2}a^2} \end{aligned}$$

so

$$\begin{aligned} \langle n | \rho(t) | n \rangle^* &= \langle n | \rho(t)^{\dagger} | n \rangle = \langle n | \rho(t) | n \rangle \\ & \frac{\partial^{n+n}}{\partial t^n \partial t'^n} \exp[2xtt' - t^2 - t'^2]_{t=t'=0} \\ &= 2^n n! \sum_{m=0}^{[n/2]} \frac{n!}{2^{2m} (m!)^2 (n-2m)!} x^{n-2m} \end{aligned} \quad (31)$$

and we derive the compact form for $\mathbf{p}(n, t)$, i.e.,

$$\begin{aligned} p(n, t) &= \frac{W}{n!} \left(-\frac{\beta}{2}\right)^n \frac{d^n}{d\beta^{*n}} \frac{d^n}{d\alpha^n} \exp(-2T' \tanh \lambda \beta^* \alpha - \beta^{*2} - \alpha^2) |_{\alpha, \beta^* = 0} \\ &= W(\beta T' \tanh \lambda)^n \sum_{m=0}^{[n/2]} \frac{n!}{2^{2m} (m!)^2 (n-2m)!} (T' \tanh \lambda)^{-2m} \end{aligned} \quad (32)$$

Using the new expression of Legendre polynomials found in Ref. [20]

$$x^n \sum_{m=0}^{[n/2]} \frac{n!}{2^{2m} (m!)^2 (n-2m)!} \left(1 - \frac{1}{x^2}\right)^m = P_n(x) \quad (33)$$

we can formally recast Eq. (32) into the following compact form, i.e.,

$$\begin{aligned} p(n, 0) &= \text{sech} \lambda (\tanh \lambda)^n \lim_{T' \rightarrow 0} \sum_{m=0}^{[n/2]} \frac{n!}{2^{2m} (m!)^2 (n-2m)!} (T' \tanh \lambda)^{n-2m} \\ &= \begin{cases} \frac{(2k)!}{2^{2k} k! k!} \text{sech} \lambda \tanh^{2k} \lambda, & n = 2k \\ 0 & n = 2k + 1 \end{cases} \end{aligned} \quad (34)$$

which just correspond to the number distributions of the squeezed vacuum state [21, 22]. From Eq. (34) it is not difficult to see that the photocount distribution decreases as the squeezing parameter λ increases. While for $\kappa t \rightarrow \infty$, we see that $p(n, \infty) = 0$. This indicates that there is no photon in a system interacting with a amplitude dissipative channel for enough long time, as expected. In Fig. 2, the photon number distribution is shown for different κt .

6 The Wigner functions

In this section, we shall use the normally ordering for density operators to calculate the analytical expression

$$p(n, t) = W(e^{-\kappa t} \sqrt{-\beta \tanh \lambda})^n P_n(e^{\kappa t} T' \sqrt{-\beta \tanh \lambda})$$

Note that since $\sqrt{-\beta \tanh \lambda}$ is pure imaginary, while $p(n, t)$ is real, so we must still use the power-series expansion on the right-hand side of Eq. (32) to depict figures of the variation of $p(n, t)$. In particular, when $t = 0$, Eq. (32) reduces to

of Wigner function. For a single-mode system, the WF is given by [23]

$$\begin{aligned} W(\alpha, \alpha^*, t) &= e^{2|\alpha|^2} \int \frac{d^2\beta}{\pi^2} \langle -\beta | \rho(t) | \beta \rangle \exp[-2(\beta\alpha^* - \beta^*\alpha)] \end{aligned} \quad (35)$$

where $|\beta\rangle$ is the coherent state [18, 19]. From Eq. (35) it is easy to see that once the normal ordered form of $\rho(t)$ is known, we can conveniently obtain the Wigner function of $\rho(t)$.

On substituting Eq. (24) into Eq. (35) we obtain the WF of the single-mode squeezed state in the ADC,

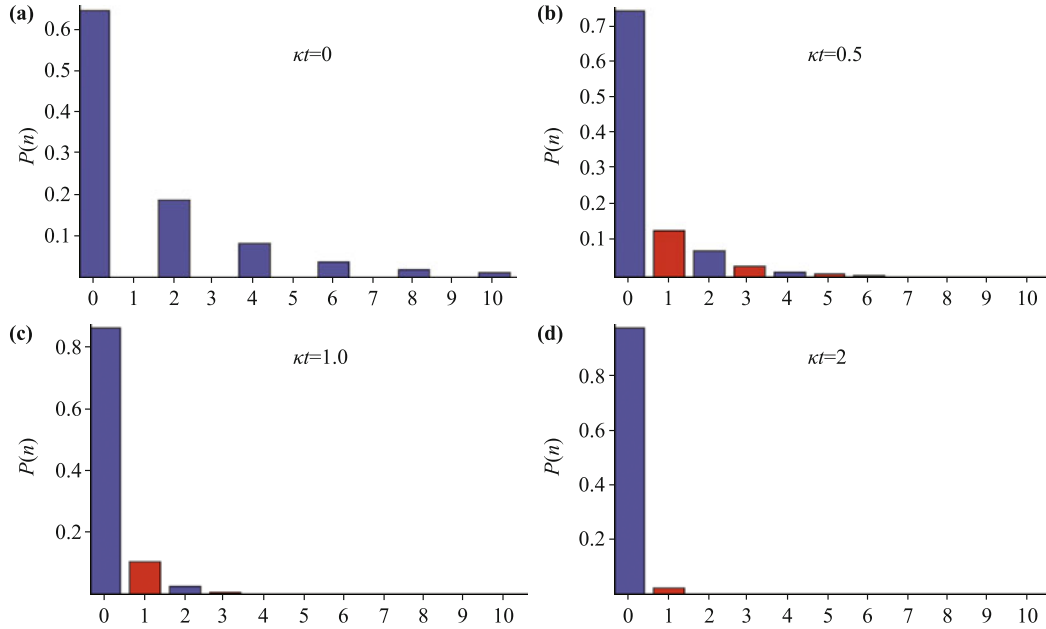


Fig. 2 Photon number distribution of the squeezed vacuum state in amplitude damping channel for $\lambda = 1$, and different κt : (a) $\kappa t = 0$, (b) $\kappa t = 0.5$, (c) $\kappa t = 1$ and (d) $\kappa t = 2$.

$$\begin{aligned}
 W(\alpha, \alpha^*, t) &= W e^{2|\alpha|^2} \int \frac{d^2\beta}{\pi^2} \exp \left[-(1 + \beta T' \tanh \lambda) |\beta|^2 - 2(\beta \alpha^* - \beta^* \alpha) + \frac{\beta}{2} \beta^{*2} + \frac{\beta}{2} \beta^2 \right] \\
 &= \frac{W}{\pi \sqrt{(1 + \beta T' \tanh \lambda)^2 - \beta^2}} \times \exp \left[2 \frac{-2(1 + \beta T' \tanh \lambda) |\alpha|^2 + \beta(\alpha^{*2} + \alpha^2)}{(1 + \beta T' \tanh \lambda)^2 - \beta^2} + 2|\alpha|^2 \right] \quad (36)
 \end{aligned}$$

In particular, when $t = 0$ and $t \rightarrow \infty$, Eq. (36) reduces to $W(\alpha, \alpha^*, 0) = \frac{1}{\pi} \exp[-2|\alpha|^2 \cosh(2\lambda) + (\alpha^{*2} + \alpha^2) \sinh(2\lambda)]$, and $W(\alpha, \alpha^*, \infty) = \frac{1}{\pi} \exp(-2|\alpha|^2)$, which are just the WF of the single-mode squeezed vacuum state and the vacuum state, respectively. In Fig. 3, the WF of the single-mode squeezed vacuum state in the amplitude damping channel is shown for different decay time κt .

7 Tomogram

As we know, once the probability distributions $P_\theta(\hat{x}_\theta)$ of the quadrature amplitude are obtained, one can use the inverse Radon transformation familiar in tomographic imaging to obtain the WF and density matrix [24]. Thus the Radon transform of the WF is corresponding to the probability distributions $P_\theta(\hat{x}_\theta)$. In this section we derive the tomogram of $\rho(t)$.

For a single-mode system, the Radon transform of WF,

denoted as \mathcal{R} , is defined by [25]

$$\begin{aligned}
 \mathcal{R}(q)_{f,g} &= \int \delta(q - fq' - gp') \text{Tr}[\Delta(\beta)\rho(t)] dq' dp' \\
 &= \text{Tr}[|q\rangle_{f,g} \langle q| \rho(t)] = {}_{f,g} \langle q | \rho(t) | q \rangle_{f,g} \quad (37)
 \end{aligned}$$

where the operator $|q\rangle_{f,g} \langle q|$ is just the Radon transform of single-mode Wigner operator $\Delta(\beta)$, and

$$|q\rangle_{f,g} = A \exp \left[\frac{\sqrt{2}qa^\dagger}{B} - \frac{B^*}{2B} a^{\dagger 2} \right] |0\rangle \quad (38)$$

as well as $B = f - ig$, $A = [\pi(f^2 + g^2)]^{-1/4} \exp[-q^2/2(f^2 + g^2)]$. Thus the tomogram of a quantum state $\rho(t)$ is just the quantum average of $\rho(t)$ in $|q\rangle_{f,g}$ representation (a kind of intermediate coordinate-momentum representation) [26].

Substituting Eqs. (24) and (38) into Eq. (37), and using the completeness relation of coherent state, we see that the Radon transform of WF of $\rho(t)$ is given by

$$\begin{aligned}
 \mathcal{R}(q)_{f,g} &= W_{f,g} \langle q | e^{\frac{\beta}{2} a^{\dagger 2}} e^{a^\dagger a \ln(\beta T' \tanh \lambda)} e^{\frac{\beta}{2} a^2} | q \rangle_{f,g} \\
 &= W A^2 \int \frac{d^2\alpha d^2\gamma}{\pi^2} \exp \left(-|\alpha|^2 + \frac{\sqrt{2}q}{B^*} \alpha - \frac{B}{2B^*} \alpha^2 + \frac{\beta}{2} \alpha^{*2} \right)
 \end{aligned}$$

$$\begin{aligned} & \times \exp\left(-|\gamma|^2 + \frac{\sqrt{2}q}{B}\gamma^* + \alpha^*\gamma\beta T' \tanh \lambda - \frac{B^*}{2B}\gamma^{*2} + \frac{\beta}{2}\gamma^2\right) \\ & = \frac{WA^2}{\sqrt{E}} \exp\left\{\frac{q^2\beta}{E|B|^4}[B^2 + (B^*)^2 + 2(T \tanh \lambda + \beta - \beta T^2 \tanh^2 \lambda)|B|^2]\right\} \end{aligned} \quad (39)$$

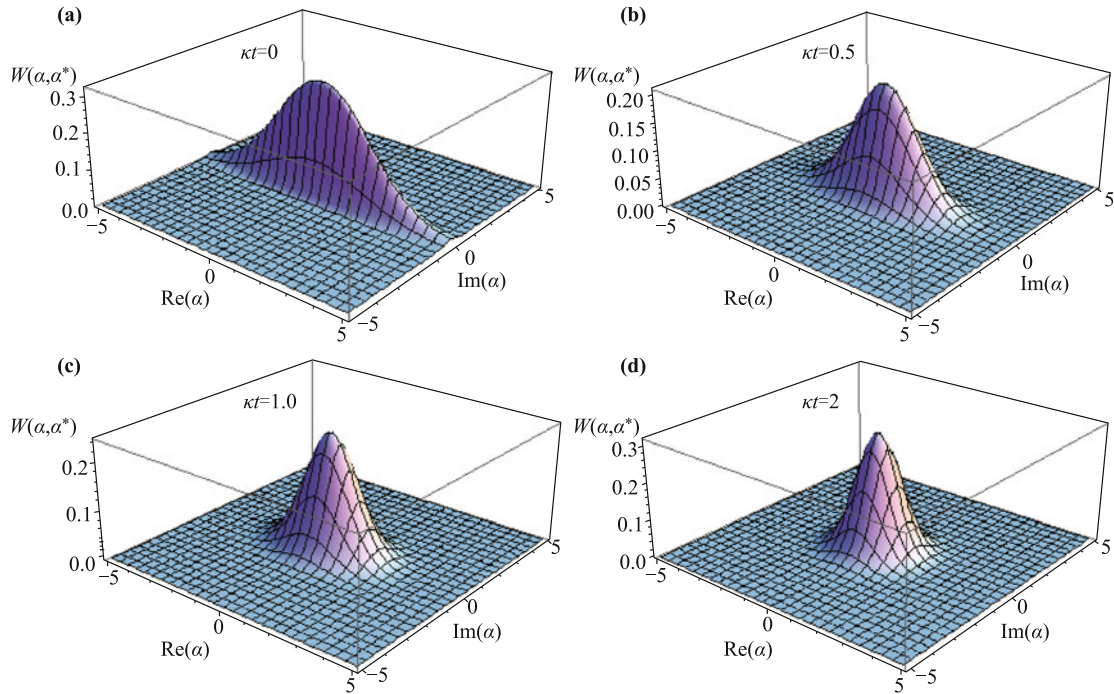


Fig. 3 Wigner function of the squeezed vacuum state in amplitude damping channel for $\lambda = 1.0$, different κt : (a) $\kappa t = 0.0$, (b) $\kappa t = 0.5$, (c) $\kappa t = 1$, and (d) $\kappa t = 2$.

where we have used the formula (26) and $\langle \alpha | \gamma \rangle = \exp[-|\alpha|^2/2 - |\gamma|^2/2 + \alpha^*\gamma]$, as well as

$$E = \left(1 + \beta \frac{B}{B^*}\right) \left[1 + \frac{B^*}{B}\beta - B^* \frac{(\beta T' \tanh \lambda)^2}{B^* + \beta B}\right]$$

$$= \left|1 + \frac{\beta B}{B^*}\right|^2 - (\beta T' \tanh \lambda)^2 \quad (40)$$

In particular, when $t = 0$ ($T = 0$), Eq. (39) reduces to $(\frac{B}{B^*} = e^{2i\phi})$

$$\mathcal{R}(q)_{f,g} = \frac{A^2 \text{sech} \lambda}{|1 + e^{2i\phi} \tanh \lambda|} \exp\left\{\frac{q^2 \tanh \lambda}{|1 + e^{2i\phi}|B|^4 \tanh \lambda|^2} (B^2 + (B^*)^2 + 2|B|^2 \tanh \lambda)\right\} \quad (41)$$

which is a tomogram of single-mode squeezed vacuum state.

In summary, using the way of deriving infinitive sum representation of density operator as a solution to the master equation describing the amplitude dissipative channel by virtue of the entangled state representation, we conclude that in this channel the initial density operator of a single-mode squeezed vacuum state evolves into a squeezed chaotic state with decreasing-squeezing. About more applications of entangled state representation, one can refer Ref. [27].

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