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RESEARCH ARTICLE

Mysteries of $R^{ik} = 0$: A novel paradigm in Einstein's theory of gravitation*

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Despite a century-long effort, a proper energy-stress tensor of the gravitational field, could not have been discovered. Furthermore, it has been discovered recently that the standard formulation of the energy-stress tensor of matter, suffers from various inconsistencies and paradoxes, concluding that the tensor is not consistent with the geometric formulation of gravitation [Astrophys. Space Sci., 2009, 321: 151; Astrophys. Space Sci., 2012, 340: 373]. This perhaps hints that a consistent theory of gravitation should not have any bearing on the energy-stress tensor. It is shown here that the so-called "vacuum" field equations $R^{ik} = 0$ do not represent an empty spacetime, and the energy, momenta and angular momenta of the gravitational and the matter fields are revealed through the geometry, without including any formulation thereof in the field equations. Though, this novel discovery appears baffling and orthogonal to the usual understanding, is consistent with the observations at all scales, without requiring the hypothetical dark matter, dark energy or inflation. Moreover, the resulting theory circumvents the long-standing problems of the standard cosmology, besides explaining some unexplained puzzles.

Keywords general relativity and gravitation – fundamental problems and general formalism, cosmological observations

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1 Introduction

The extensive development of the theory of general relativity (GR) and the possibility it offers to probe the various aspects of the Universe, constitute it a well-established mature scientific discipline, which describe accurately all gravitational phenomena ranging from the solar system to the Universe. However, there is a price for this success which is often ignored. More than 95 percent of the content of the energy-stress tensor has to be dark, in the form of inflaton, dark matter and dark energy, which do not have any non-gravitational or laboratory evidence. Moreover, the dark energy poses a serious confrontation between fundamental physics and cosmology.

It appears that the popular notions – inflation, non-baryonic dark matter and dark energy, have become more like liabilities than assets of the theory and may be the clearest clues that the properties of gravity are beyond the standard paradigm. We have perhaps misunderstood the true nature of a geometric theory of gravitation because of the way the ideas have evolved historically. This requires a critical examination of the basic formulation of the theory in order to have a better understanding, leading to a way out of the present crisis. A good place to start would be questioning the requirement of the energy-stress tensor in GR – one of the two very foundational building blocks of the theory, the "marble" (geometry) and the "wood" (the energy-stress tensor). This may raise eyebrows in the beginning, but later we shall realize that it was not such a bad idea. The motivation for this proposal comes from the following.

As is known, the dark energy and the accelerated expansion of the Universe are primarily required, by the standard ΛCDM cosmology, to explain the excessive faintness (for their redshift) of the supernovae of type Ia (SNeIa). It is however also known that the SNeIa observations can also be explained successfully in the framework of an unconstrained ΛCDM cosmology with $\Omega_m = 0 = \Omega_\Lambda$. This result is generally considered a co-

*A foundational review based on the discovery [1, 2] awarded "Honorable Mention" of the year 2012 by the Gravity Research Foundation.

incidence and is ignored by arguing that this "empty" model cannot be a viable theory of the real Universe. Nevertheless, as we shall see in the following (in Section 6), this model is consistent with other observations as well. Moreover, it also averts the long-standing problems of the standard cosmology and explains some hitherto unexplained puzzles of GR.

In this connection, it would be pertinent to mention that a foundational analysis has discovered some surprising inconsistencies and paradoxes in the formulation of the energy-stress tensor of matter, concluding that the formulation does not seem consistent with the geometric description of gravitation [3, 4].

Taken together, these two points make a strong case to examine whether it is not possible to interpret the real Universe (with matter) without having recourse to the energy-stress tensor. We shall see in the following that the so-called "vacuum" field equations do not represent an empty spacetime, since the energy, momenta and angular momenta of the gravitational and the material fields do exist in the metric field, whose effects are revealed through the geometry of these equations, *without including any formulation of the energy-stress tensor*. This explains why there does not exist a proper, flawless energy-stress tensor T^{ik}, implying that T^{ik} is a redundant part of Einstein's equations.

2 Gravity of the "vacuum" field equations $R^{ik} = 0$

In a geometric theory of gravitation, gravity is a manifestation of the curvature of spacetime whose source is matter. The relation between the geometry and the matter is specified by the field equations. For example, in the case of GR, this relation is governed by the Einstein field equations

$$R^{ik} - \frac{1}{2}R\, g^{ik} = -\frac{8\pi G}{c^4}T^{ik}, \quad i,k = 0,1,2,3 \tag{1}$$

where G is Newton's constant of gravitation, c is the speed of light in vacuum and the energy-stress tensor T^{ik}, representing the source matter, has the dimensions of the energy density. It should be noted that the tensor T^{ik} includes in it all the sources of energy, momenta and stresses including the cosmological constant or any other dark energy candidate, *except the energy, momenta and stresses of the gravitational field itself* (whose formulation does not exist in GR, as will be discussed later). Hence in the absence of T^{ik}, in which case Eq. (1) reduce to the so-called "vacuum" field equations

$$R^{ik} = 0 \tag{2}$$

one should expect essentially a flat spacetime solution. However, it is already known that this is not true in general, and a space with dimensions four or more can have curvature even for $T^{ik} = 0$. Let us consider the following well-known examples.

2.1 Curved solutions of $R^{ik} = 0$

Schwarzschild solution

Discovered by Karl Schwarzschild in 1915 immediately after GR was formulated, the solution forms the cornerstone of GR. It represents the spacetime structure outside an isotropic mass in an empty space. In the Schwarzschild coordinates [1], the solution reads

$$\mathrm{d}s^2 = \left(1 + \frac{K}{r}\right)c^2\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1 + K/r} - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\ \mathrm{d}\phi^2 \tag{3}$$

where K is a constant of integration. In fact, all the experiments which have so far been carried out to test GR, are based on the predictions by this solution (except the Gravity Probe B experiments, which are based on the predictions of the Kerr solution).

Kerr solution

Discovered by Roy Kerr in 1963, the solution describes the spacetime surrounding a spherical mass m spinning with angular momentum per unit mass $= \alpha$ (so that its total angular momentum $= m\alpha c$). In the Boyer-Lindquist coordinates [5], the solution takes the form

$$\mathrm{d}s^2 = \left(1 - \frac{r_{\mathrm{S}}r}{\rho^2}\right)c^2\mathrm{d}t^2 - \frac{\rho^2}{\Delta}\mathrm{d}r^2 - \rho^2\mathrm{d}\theta^2 - \left(r^2 + \alpha^2 + \frac{r_{\mathrm{S}}r\alpha^2}{\rho^2}\sin^2\theta\right)\sin^2\theta\ \mathrm{d}\phi^2 + \frac{2r_{\mathrm{S}}r\alpha}{\rho^2}\sin^2\theta\ \mathrm{d}\phi\ c\mathrm{d}t \tag{4}$$

where $\rho^2 = r^2 + \alpha^2\cos^2\theta$, $\Delta = r^2 - r_{\mathrm{S}}r + \alpha^2$ and $r_{\mathrm{S}} = 2Gm/c^2$ is the Schwarzschild radius. When $\alpha = 0$, the solution reduces to the Schwarzschild solution.

Kasner solution

This important cosmological solution, discovered by Edward Kasner in 1921 [6], is given by

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - t^{2p_1}\mathrm{d}x^2 - t^{2p_2}\mathrm{d}y^2 - t^{2p_3}\mathrm{d}z^2 \tag{5}$$

where the constants p_1, p_2 and p_3 satisfy

$$p_1 + p_2 + p_3 = 1, \quad p_1p_2 + p_2p_3 + p_3p_1 = 0$$

This curved Bianchi type I metric is believed to describe an "empty" homogeneous Universe which expands and contracts anisotropically at different rates in different directions (for example, for $p_1 = p_2 = 2/3$ and $p_3 = -1/3$, the space is expanding in two directions and contracting in the third). The metric was rediscovered, in a more useful form, by Narlikar and Karmarkar in 1946 [7]:

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - (1+nt)^{2p_1}\mathrm{d}x^2 - (1+nt)^{2p_2}\mathrm{d}y^2 - (1+nt)^{2p_3}\mathrm{d}z^2 \quad (6)$$

where n is an arbitrary constant. It may be checked that by the use of the transformations $1 + nt = n\bar{t},\ n^{p_1}x = \bar{x},\ n^{p_2}y = \bar{y},\ n^{p_3}z = \bar{z}$, the metric (6) takes the form (5) in the new coordinates.

2.2 Sources of curvature in $R^{ik} = 0$

Solutions (3)–(6) are intrinsically curved and they cannot be transformed to the flat Minkowski metric by any possible coordinate transformation, as the Riemann-Chistoffel curvature tensor $R_{hijk} \neq 0$ in these cases. Though T^{ik} is considered as the sole source of gravitation/curvature in Eq. (1), appearance of curved solutions in its absence, indicates that some (hidden) source must be present inherently in Eq. (2) itself. Let us unveil it.

Gravitational energy appears through geometry!

Let us first consider the Schwarzschild solution (3). The mystery of the presence of curvature in this solution is related to the central singularity of the spherically symmetric space represented by (3) (which is associated, through a correspondence between the Newtonian and Einsteinian theories of gravitation in the case of a weak field, to an isotropic mass sitting at the centre $r = 0$). It is thus believed that the source of the curvature in GR is either T^{ik} or a singularity in g_{ik}. However, it should be noted that the metric (3) represents space exterior to the central mass at $r = 0$ and *not* the point $r = 0$ itself, where the metric breaks down. So, how can a mass (represented by the singularity here) situated at the point $r = 0$ (which is not even represented by the metric) curve the space of (3) at the points for which $r > 0$? Obviously, one cannot expect the Newtonian theory of action-at-a-distance to work in the framework of GR which is a local theory.

A little reflection suggests that the agent responsible for the curvature in Eq. (3) at the points for $r > 0$, must be the gravitational energy[1)], which can definitely exist in an empty space.

A brief account of the history of the formulation of the gravitational energy in GR, would not be out of place. While formulating the field equations, Einstein tried to include the energy-stress tensor of the gravitational field in the right hand side of Eq. (1) emphasizing that "*the gravitational field must have an energy-stress tensor, like all other physical fields*". However, failing to find a proper energy-stress tensor of the gravitational field, he rejected this requirement in the final version of GR (in order to have a generally covariant theory) by declaring "*there may very well be gravitational fields without stress and energy density*". It can be said safely that despite the century-long dedicated efforts of many luminaries, like Einstein, Tolman, Papapetrou, Landau-Lifshitz, Moller and Weinberg, the attempts to discover a unanimous formulation of the energy-stress tensor of the gravitational field, have failed at least on three counts: (i) the non-tensorial character of the energy-stress "complexes" (pseudo-tensors) of the gravitational field; (ii) the lack of a unique agreed-upon formula for these pseudo-tensors, for example, the different formulae may lead to different distributions of energy-stress even in the same spacetime background; (iii) the inherent difficulty in the localization of the gravitational energy.

After failing to find out a proper formulation of the gravitational energy in a geometric theory of gravitation, it is sometimes argued that the gravitational energy may result from the geometry itself (which also seems consistent with our observations made above), through the non-linearity of the field equations. However, if this is true, one should be able to calculate the gravitational energy from metric (3). We can certainly do this by the following two simple observations:

(i) The metric (3) departs from the flat spacetime in the term K/r, implying that this term must be having the source of curvature.

(ii) We have shown that source of curvature in (3), at the points $r > 0$, must be the gravitational energy.

Taken together, these two points imply that K/r must be the gravitational energy (in the units with $c = 1$) in (3). This is in perfect agreement with the way the value of the constant K is determined. Let us recall that the constant K in Eq. (3) is specified in terms of the Newtonian gravitational potential energy (by requiring that in the case of a weak gravitational field, Newton's law should hold) giving $g_{00} = 1 + 2\psi/c^2$ where $\psi = -Gm/r$ is the gravitational energy (per unit mass) at a distance r from the central mass m producing the field. This fixes the constant K as

1) Of course, the ultimate source of this gravitational energy is the central singularity, which is the general relativistic analogue of the Newtonian central mass in this case.

$$K = -\frac{2Gm}{c^2} \tag{7}$$

It is thus established that the source of the curvature of spacetime in (3) is the energy of the gravitational field present at the points exterior to $r = 0$. This also establishes beyond doubts that the gravitational energy already exists in Eq. (2) implicitly in the geometry, through the non-linearity of the field equations, and no additional incorporation thereof is needed. This fits very well in the story of the failure to discover a true energy-stress tensor of the gravitational field. The energy-stress tensor of the gravitational field does not exist simply because the tensor is not needed in the geometric framework of gravitation, it already exists there inherently in the geometry! We should note that in the weak-field approximation, the GR equations do reduce to the usual Newtonian dynamical equations; gravitational energy, force, etc. do emerge respectively from the metric, the Christoffel symbol, etc., *without adding any formulation of the gravitational field energy to the Einstein equations.*

The above-gained insight about the futility of the energy-stress tensor of the gravitational field is also corroborated by the Kerr metric (4) wherein the angular momentum also contributes to its curvature. This is an entirely new concept for an energy-stress tensor (in the framework of GR), which has no place for the angular momentum[2)].

Matter too registers its presence through geometry!

One would not show much inhibition to agree that the gravitational energy is inherently present in Eq. (2) *without* incorporating any formulation thereof. However, one would maintain that Eq. (2) represent otherwise empty spacetime *outside* the matter source. This interpretation is supported by the fact that the Schwarzschild and Kerr solutions (3) and (4) represent space outside the source mass, and not the point where the mass exists. However, the interpretation of the curvature of the Kasner solution in terms of the gravitational energy in empty space, does not appear satisfactory, as we shall see in the following.

The conventional wisdom about the source of curvature in the Kasner solution (5) is obscure and questionable. As the solution is interpreted in terms of an empty homogeneous Universe, the only possible source (in the absence of matter) can be a singularity. Solution (5) does contain a singularity at $t = 0$ (at $t = -1/n$ in the case of (6), which is just a rescaling of time), but *not* at any other time. However, the solution is curved at all times! A past singularity, which does not exist now, fueling the gravitational energy now without any other source, does not seem compatible with our understanding of the gravitational energy.

Another source of curvature in the Kasner metric (5) can be a net non-zero momentum resulting from the anisotropic expansion/contraction of the homogeneous space, since the space expands and contracts at different rates in different directions in it. However, it does not make much sense to imagine of momentum resulting from the expanding/contracting *empty* space, and it does not make sense, in the first place, to think of expanding/contracting empty space *without* matter. Hence, the conventional explanations of this puzzle do not seem satisfactory, and the Kasner solution has remained an unsolved mystery.

An important point regarding the Kanser metric, which has not been paid attention to is that, unlike the Schwarzschild and Kerr metrics, the Kasner metric represents a cosmological solution, which is not expected to have any "outside". By taking into account this point and by recalling that the ultimate source of the gravitational field is matter, we are led to the conclusion that the (homogeneously distributed) matter source, present around the epoch of singularity in the Kanser metric, must be present at all other times as well, as it *must not* have been destroyed mysteriously! This simply means that the Kasner metric represents a homogeneous distribution of matter expanding/contracting anisotropically! However, if this is correct, we should be able to decipher the presence of this matter from the geometry, as we could do in the case of the Schwarzschild metric. This can similarly be done from the following two points.

(i) For $n = 0$, metric (6) reduces to the Minkowski metric[3)]. This implies that this term must be having the source of curvature in terms of a dynamic quantity.

(ii) The source of curvature in metric (6) is a net non-zero momentum, resulting from an anisotropic motion of the homogeneously distributed matter (space) expanding and contracting at different rates in different directions.

2) It may be mentioned that there is no place for the angular momentum in T^{ik} in the framework of Einstein's theory, which needs to be extended to non-Riemannian curved spacetime with torsion (as in the Einstein–Cartan theory) to support asymmetric Ricci and metric tensors, so that an asymmetric energy-stress tensor of spin can appear on the right hand side of the field equations. (However, when the right hand side is vanishing, the Ricci tensor need not be asymmetric and the Einstein-Cartan 'vacuum' equations reduce to equations $R^{ik} = 0$.)

3) Though vanishing p_1, p_2 and p_3 can also reduce (5) to the Minkowskian form, however they are pure numbers and cannot support a dynamic quantity as a source.

If this is correct, we should expect the constant n to be expressed in terms of the momentum density, say, P. It is indeed possible to express n in terms of P, G and c in order to meet its natural dimension (which is the dimension of the inverse of time). Written along the lines of (7), the expression for n finds a unique form

$$n^2 = \frac{GP}{c} \tag{8}$$

which is in agreement with the two points mentioned above. It is difficult to verify (8) with a classical analogue (as we did in the case of the Schwarzschild metric) since in the Newtonian gravitational theory, the field is independent of the motion of the source. This is similar to the lack of a classical analogue of the Kerr metric.

Does it then mean that like the energy, momentum and angular momentum of the gravitational field, those of the matter fields are also included in equations $R^{ik} = 0$ inherently whose effects are revealed through the geometry? We shall see in the next section that only an affirmative answer of this question, can provide a consistent explanation to the flatness of a homogeneous-isotropic solution of $R^{ik} = 0$.

3 A new paradigm in $R^{ik} = 0$

The above-discovered revolutionary result, that the gravitational as well as material fields are built-in ingredients of the geometry of equations $R^{ik} = 0$, has a powerful prediction. If the source of curvature in the Kasner solution is a net non-zero momentum arising from the anisotropic motion of the homogeneously distributed matter, one would expect that the momentum should be canceled out thoroughly if the homogeneously distributed matter expands or contracts *isotropically*, giving rise to a flat spacetime solution of Eqs. (2). This prediction is perfectly realized in the following cosmological solution, which provides a concrete evidence for the correctness of the above-gained result.

Obviously, the symmetries of a homogeneous matter distribution expanding or contracting isotropically, require the metric to be the Robertson–Walker one given by

$$ds^2 = c^2 dt^2 - S^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 \right) \tag{9}$$

where $S(t)$ is the scale factor of the homogeneous, isotropic Universe. For metric (9), Eq. (2) yield

$$R^0{}_0 = \frac{3}{c^2}\frac{\ddot{S}}{S} = 0 \tag{10}$$

$$R^1{}_1 = R^2{}_2 = R^3{}_3 = \frac{1}{c^2}\left(\frac{\ddot{S}}{S} + 2\frac{\dot{S}^2}{S^2} + 2kc^2\frac{1}{S^2} \right) = 0 \tag{11}$$

which uniquely determine

$$S = ct \quad \text{with} \quad k = -1 \tag{12}$$

so that the final solution reduces to

$$ds^2 = c^2 dt^2 - c^2 t^2 \left(\frac{dr^2}{1 + r^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 \right) \tag{13}$$

It may be noted that by the use of the transformations $\bar{t} = t\sqrt{1 + r^2}$, $\bar{r} = ctr$, the metric (13) can be brought to manifestly Minkowskian form (see page 140 in Ref. [8]), fulfilling the prediction.

It would be worthwhile to scrutinize the conventional explanations to the flatness of solution (13), vis-a-vis those given above. The conventional wisdom may argue that solution (13) is Minkowskian simply because T^{ik} is vanishing in Eqs. (2). However, if this is so, why do we get curved solutions (3)–(6) from the same equations[4]? If Eqs. (2) contain gravitational field sourcing the curvature present in solutions (3)–(6), they must do so in solution (13) also. Obviously, solutions (3)–(6) have singularities to fuel the gravitational field, while solution (13) does not. But, what is there to stop the singularity to occur in (13)? The only difference between solutions (13) and (3)–(6) is that they have different types of symmetries in their spacetime structures. While metric (13) is homogeneous and isotropic, metrics (3)–(6) are either inhomogeneous or/and anisotropic. However, how a relaxation in the homogeneity and/or isotropy can result in a singularity, cannot be answered by the conventional wisdom. A possible explanation to the present situation leads us to the following two possibilities.

(i) **Conventional wisdom:** Equations $R^{ik} = 0$ represent empty spacetime structure and can support curved as well as flat spacetime solutions. However, they are unable to explain how and when a solution acquires curvature or flatness. For example, solutions (6) and (13) represent similar spacetime structures with the only difference that while the homogeneous space in (6) is expanding and contracting in different directions at different rates, the same space is expanding or contracting isotrop-

[4] Equations $R^{ik} = 0$ cannot decipher, just from the symmetry of a metric, that it belongs to a spacetime structure outside a mass, since the same symmetry can also result inside a matter distribution.

ically in (13). How this difference accounts for their curved and flat states and controls the appearance of the singularity, cannot be explained by equations $R^{ik} = 0$.

(ii) **New paradigm:** The "empty" space of equations $R^{ik} = 0$ is not really empty, the geometry of the equations does contain built-in impressions of the gravitational as well as the matter fields, and no extra formulation thereof is required. Further, the structure of the geometry is determined by the net contribution from the material and the gravitational fields (of the chosen matter distribution) which may be manifested in the form of a singularity.

Though the second possibility appears new and orthogonal to the usual understanding, it provides not only a sensible meaning to the Kasner solution (in the absence of the energy-stress tensors of the gravitational or the matter fields), but also a reasonable explanation to the flatness of solution (13). Hence, the solution (13) is flat because it represents a homogeneously distributed matter throughout the space at all times, which is either expanding or contracting isotropically. As the positive energy of the matter field would be exactly balanced, point by point, by the negative energy of the resulting gravitational field (contrary to the case of the Schwarzschild solution where there is only the gravitational energy and no matter at the points represented by the metric), this would provide a net vanishing energy. Neither there would be any momentum contribution from the isotropic expansion or contraction of the material system (contrary to the case of the Kasner solution). Hence, in the absence of any net non-zero energy, momentum or angular momentum, the spacetime of (13) must not have any curvature.

This implies that it is the symmetry of the chosen spacetime structure, which determines whether a solution of $R^{ik} = 0$ will be curved (may have a singularity) or flat (without singularity). This is in perfect agreement with the appearance of different kinds of singularities in accordance with the gravitational situations: while the Schwarzschild solution (describing the spacetime structure exterior to a point mass) has a point singularity, the Kerr solution (describing the spacetime structure exterior to a rotating mass) has a ring singularity, the Kasner solution (in which the $t =$ constant hypersurfaces are expanding and contracting at different rates in different directions) presents a singularity of the oscillating kind at $t = 0$.

The discovery, of the net vanishing energy-momentum-angular momentum in a homogeneous distribution of matter expanding/contracting isotropically, appears consistent with several investigations and results which indicate that the total energy of the Universe is zero. Hawking and Milodinow have argued recently that the total energy of the Universe must always remain zero, as the positive energy of the matter can balance the negative gravitational energy [9]. Thus, a flat spacetime solution, which has so far been a notion of special relativity (SR), can be achieved in the real Universe in the presence of matter, which originates dynamically from the field equations, and is not just put by hand as in SR. Further, the appearance of a flat spacetime in the presence of matter is also not impossible in the conventional approach. For example, it has been shown in Ref. [10] that conformally coupled matter does not always curve spacetime.

As solution (13) arose naturally in Milne's kinematic theory, it is generally considered as the Milne model, which is not quite correct when viewed with the traditional meaning (of the empty Universe). It should be noted that the Milne's Universe is not empty, albeit its evolution dynamics being the same as that of the empty FRW model. Milne derived his model of the Universe from the kinematic relativity and the cosmological principle by considering the matter in the Minkowskian spacetime (the matter not interacting with the geometry, from the point of view of GR, due to some unknown reasons) [11]. Here we unravel the mystery why a homogeneous, isotropic real Universe (with matter) becomes Minkowskian.

4 Why do we need the new paradigm?

We have seen that the appearance of the gravitational energy through the geometry of equations $R^{ik} = 0$ (without including any formulation of the energy-stress tensor of the gravitational field), is vindicated by the history of the failure to discover the energy-stress tensor of the gravitational field. Do we have any similar experience from the energy-stress tensor of the matter fields? Yes! Recently, it has been shown that a critical analysis of the relativistic formulation of matter given by T^{ik}, reveals some surprising inconsistencies and paradoxes, implying that the relativistic formulation of matter is not compatible with the geometric description of gravitation [3, 4]. This view seems vindicated by the following additional inconsistencies in the basic formulation of T^{ik} which have not been realized so far.

(i) The general expression of the energy-stress tensor T^{ik} is obtained by deriving it first in the absence of gravity, i.e., in SR. It is then imported to the ac-

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tual case in the presence of gravity, by making use of an inertial observer, which exists admittedly at all points of spacetime (by courtesy of the principle of equivalence). Formulating a tensor representation of the fluid element in a small neighbourhood of the observer, the expression of the tensor in the presence of gravity is imported, from SR to GR, through a coordinate transformation. This is the standard way to derive T^{ik} [4]. However, the simple point which has not been noticed, which makes this derivation questionable, is that an inertial coordinate system is valid only at a point, and *not* in a neighbourhood, however small it is (Christoffel symbols can be made vanishing only at a point, and *not* in a neighbourhood). But a fluid element cannot be defined at a point, we do need a neighbourhood. Hence, at the best, the tensor T^{ik} may approximate a dust (with vanishing pressure) but cannot represent a fluid (with non-zero pressure) which requires more than one particle to generate pressure.

(ii) As the derivation of T^{ik} assumes its validity in the absence of gravitation (in the flat spacetime), this goes contradictory to the very notion of T^{ik} being the source of curvature. To exemplify this, let us note that in a flat spacetime, the left hand side of Eq. (1) vanishes automatically, but not the right hand side which has to be put equal to zero by hand. That is, the source of curvature can exist there without producing any curvature! Although Eq. (1), being a geometric formulation of gravitation, is expected to be valid in a curved spacetime, it must also reduce to the no-gravitation case consistently. This provides another reason why T^{ik} should not appear in the field equations of gravitation.

(iii) Angular momentum is a fundamental and unavoidable characteristic of matter, as is witnessed from the subatomic to the highest scales. As T^{ik} fails to include the angular momentum, it cannot be a true representative of the matter. We have already noticed that the angular momentum (which registers its inherent presence through the geometry) does appear as a source of curvature/gravitation in the Kerr solution.

It should be noted that being a geometric theory of gravitation, GR eliminates any possibility to represent gravitation in terms of a force. Rather the theory replaces the effects of the force as through geometry. Similarly, the effects of stresses, momenta, angular momenta and energy density too are revealed through geometry. In this view, the theoretical crisis in the formulation of T^{ik}, mentioned above, acquires a new meaning in the present context: the tensor is not compatible with the geometric formulation of gravitation simply because, like the gravitational field, the effects of the matter field are already present inherently in the geometry!

Einstein argued that *"On the basis of the general theory of relativity, space as opposed to 'what fills space', has no separate existence"* [12]. If this is true, a mere consideration of a spacetime structure (conditioned by the equations $R^{ik}=0$) must be equivalent to considering the accompanying fields (material and gravitational) as well, and there should be no need to add any extra formulation thereof to the field equations. The resultant non-zero energy-momentum-angular momentum of the two fields (which acts as the source of curvature), may then be reflected as a singularity in the metric.

This, in fact, leaves equation $R^{ik}=0$ as the only possibility for a consistent field equation of gravitation, wherein the matter fields as well as the gravitational field appear through the geometry, i.e., they are represented by the metric field. One may argue that a consistent field equation of gravitation is expected to reduce to Poisson's equation $\nabla^2\psi=4\pi G\rho$ in a weak stationary gravitational field. However, this requirement no longer seems mandatory, as it has already been compromised in the concordance ΛCDM cosmology. It should be noted that the Einstein field equations with a non-zero Λ do *not* fulfill this requirement (see, for example, page 155 in Ref. [13]). While, there is no scope in the standard paradigm to mend this shortcoming, it would not be correct to compare the new theory (being a fundamentally different theory wherein matter does not appear explicitly) with the Poisson equation (wherein matter appears explicitly).

We shall see, in the following, that the new theory governed by equation $R^{ik}=0$, has more attractive features, as it can obviate the long-standing problems of the standard paradigm.

5 Features of the new paradigm

The standard cosmology has well-known shortcomings, such as the problem of the initial singularity, the cosmological constant problem and the problems of the special initial conditions, such as the horizon problem and the flatness problem. Interestingly, the new theory circumvents these problems, without invoking the hypothetical inflaton field.

5.1 Machian

Although the Mach principle has been one of Einstein's guiding principles and has played a central role in the development of GR, the resulting theory is not completely Machian. Solutions of the field equations discovered by, for example, de Sitter and Godel, disobey Mach's principle.

In the new paradigm, however, the metric tensor is determined uniquely by the overall distribution of the gravitational and the material fields. This is what Einstein had envisioned (though could not accomplish)! So, there is no usual problem of the "occurrence of curved solutions (non-inertial worldlines) without the presence of matter" (when the matter is solely identified with T^{ik}), since the so-called "vacuum" solutions do not really describe a spacetime devoid of matter; what they describe is a spacetime in which the matter field is represented by the geometry (the metric field). So Mach's Principle does hold in general in the new theory, provided we characterize it by requiring the determination of the metric tensor from the overall distribution of the energy-momentum-angular momentum resulting from the material field, together with the accompanying gravitational field, and *not* from the matter alone.

5.2 No singularity

As the standard (Big Bang) cosmological model of the Universe contains a singularity at the start of time, there have been attempts to discover singularity-free cosmological solutions of Einstein equations. Generally this is achieved by violating the energy conditions, though there are also examples of the singularity-free cosmological solutions which do not violate the energy and causality conditions [14, 15].

It would be interesting to note that the cosmological model (stemming from the present theory), represented by solution (13), is singularity free, since the singularity present in it[5)] is merely a coordinate singularity which can be removed in its Minkowskian form by a redefinition of the coordinates. It may be mentioned that in the new time coordinate, the epoch corresponding to the big bang, is pushed back to the infinite past [11] and hence the age of the Universe becomes infinite.

5.3 No horizons

The horizon problem in the standard (ΛDCM) model arises from the observed uniformity of the cosmic microwave background (CMB) radiation, which has the same temperature everywhere (except for tiny, stochastic fluctuations), even in regions on opposite sides of the sky, which appear to lie outside of each other's causal horizon. Since no physical process propagating at or below light speed could have brought them into thermal equilibrium, it appears that the Universe required special initial conditions, which is supposed to be provided by inflation. Hence the problem, in essence, appears because a (particle) horizon exists in the standard cosmology. This can be checked from the following. The particle horizon is defined by

$$d_{\rm PH}(t_1) = cS(t_1)\int_0^{t_1} \frac{{\rm d}t'}{S(t')} \tag{14}$$

which measures the largest (comoving) proper distance from which light could have reached the observer in the age of the Universe at epoch t_1. As a finite value exists for $d_{\rm PH}$ in the standard cosmology, this means that the Universe has a horizon in the theory. However, this problem does not exist in the new paradigm as $d_{\rm PH} = \infty$ at any time, as can be easily checked from (12) and (14). Hence, the whole Universe is always causally connected, which explains the observed uniformity of CMB without invoking the hypothetical inflaton field.

Neither exists the event horizon in the new paradigm. The event horizon, which sets a limit on communications to the future, is the proper distance

$$d_{\rm EH}(t_1) = cS(t_1)\int_{t_1}^{t_{\max}} \frac{{\rm d}t'}{S(t')} \tag{15}$$

beyond which no observer can receive any signal, sent at $t = t_1$ from any other observer. Here $t_{\max}$ is the time-coordinate at the end of the Universe, which would be infinite in the case of an ever-expanding Universe, like the one in the present model. Clearly, $d_{\rm EH} = \infty$ in the present model and the event horizon neither exists in this model.

5.4 No cosmological constant or flatness problems

The origin of the cosmological constant problem lies in a conflict between the energy-stress tensor T^{ik} in Eq. (1) and the energy density of vacuum in the quantum field theory (QFT). The vacuum energy, according to the QFT, results from the quantum vacuum fluctuations which provide an energy contribution of the order of the Planck mass. In GR, the vacuum energy can be represented by the cosmological constant (through a particular equation of state $p_\Lambda/c^2 = \rho_\Lambda$ of T^{ik}). Friedman solution of Eq. (1) then provides an estimate of the vacuum energy in terms of H_0^2 , where H_0 is the present value of the Hubble parameter $H = \dot{S}/S$. This is, however,

[5)] Although the metric potentials in (13) do not blow up at $t = 0$, the volume of the spatial slices vanishes at $t = 0$. This results in a density-blow up, as the metric does not represent an empty Universe.

smaller than the QFT-value by a factor of $\approx 10^{120}$!

Interestingly, this problem does not appear in the new paradigm owing to the fact that the energy-stress tensor is absent and the matter, which is represented through the geometry, does not appear explicitly in the dynamical equations. Hence, the problem is evaded as the cosmological constant (or any other candidate of dark energy) is absent in the new theory. Neither exists the coincidence problem (associated with the cosmological constant problem).

The flatness problem of the standard cosmology requires the initial density of matter (in the energy-stress tensor) to have a value extremely fine-tuned to its critical value (corresponding to a flat universe). Even a tiny deviation from this value would have had drastic effects on the nature of the present Universe. However, the problem is circumvented in the new paradigm due to the absence of the energy-stress tensor, as has been explained above.

5.5 Limitations of the new theory

The field equation of gravitation, in the new theory, is $R^{ik} = 0$, which is simpler and free from any conceptual problems. Nevertheless, as matter does not appear explicitly in it, it provides limited scope for various studies which can be done in the framework of the standard paradigm. For example, it would be difficult to study the evolution of the structures resulting from the perturbation of matter; computation of anisotropies in the CMB resulting from the perturbation of matter; gravitational collapse of matter; etc. However, this should be considered as the limitation of a geometric theory of gravitation.

6 Observational support

The last words on a putative theory has to be spoken by observations and experiments. The consistency of the field Eqs. (2) has already been established for the local Universe, in the well-established standard tests of GR. Let us see how the new theory fairs against the cosmological observations. For this purpose, we shall use the cosmological solution (13) for a homogeneous and isotropic distribution of matter, as is expected from the observations in a cosmological scenario. As has been mentioned above, the resulting cosmological theory contains two time scales in the new paradigm. Since the standard interpretation of the cosmological observations is provided in the framework of an evolving Universe, for this reason, and also to compare the results of the cosmological tests performed on the present model vis-a-vis those on the standard cosmology, we consider the cosmic time scale given by (13).

6.1 Observations of supernovae of type Ia (including the most distant one)

In order to study the compatibility of Eq. (13) with the cosmological observations, let us first consider the observations of SNeIa.

The SNeIa have remarkably similar light curves, both in absolute peak magnitude and in their time scales, because of the near uniformity of the mass of the white dwarf stars, which is controlled by the Chandrasekhar limit. This allows them to be used as standard candles to measure the distance to their host galaxies because the apparent magnitude of the SNeIa depends primarily on the (luminosity) distance. As the distances are model-based concepts in cosmology and since different cosmological models generally deviate from one another at high redshifts, one can use the high-redshift SNeIa observations to test and compare the models.

In the geometry of spacetime given by (12), the apparent magnitude m of a light source of redshift z is given by

$$m = 5 \log d_{\rm L} + M + 25 \tag{16}$$

where M is the absolute magnitude of the source and the luminosity distance $d_{\rm L}$ to the source is given by

$$d_{\rm L} = cH_0^{-1}(1+z)\sinh(\ln(1+z)) \ \text{Mpc} \tag{17}$$

wherein H_0 is measured in km·s^{-1}·Mpc^{-1}. The same can also be derived from the standard cosmology by putting $\Omega_m = 0 = \Omega_\Lambda$. It is already known that the coasting model (12) is consistent with the observations of SNeIa *without requiring any dark energy* (see, for example, [16]). Moreover, Perlmutter *et al.* [17] found, from the fitting of their data, that the performance of the "empty" model ($\Omega_m = 0 = \Omega_\Lambda$) was practically similar to that of the best-fit unconstrained cosmology with a positive Λ. Let us consider a newer dataset[6)], for example, the "new gold sample" of 182 SNeIa [19], which is a reliable set of SNeIa with reduced calibration errors arising from the systematics. It can be checked that the model (17) provides an excellent fit to the data with a value of χ^2 per degrees of freedom (DoF) $= 174.29/181 = 0.96$ and a probability of the goodness of fit $Q = 63\%$. Obviously the concordance ΛCDM model has even a better fit as it has more free parameters:

6) Although various newer SNeIa datasets are available, however, the way they are analyzed has left little scope for testing a theoretical model with them. This issue has been addressed in Ref. [18].

χ^2/DoF = 158.75/180 = 0.88 and $Q = 87\%$ obtained for the values $\Omega_m = 1 - \Omega_\Lambda = 0.34 \pm 0.04$.

It would be worthwhile to include, in the fit, the recently discovered SN UDS10Wil at a redshift of 1.914, which is the most distant SNIa yet known [20]. This new SNIa has been discovered under the CANDELS multi-cycle treasury program on the Hubble Space Telescope and has been observed with a corrected magnitude as $m = 26.15 \pm 0.39$ and the absolute magnitude $M = -19.39$. After including this SN in the fit, the model (17) still provides a good fit with a value of χ^2/DoF = 178.28/182 = 0.98 and $Q = 56\%$. The concordance ΛCDM model registers a better fit with χ^2/DoF = 160.15/181 = 0.88 and $Q = 87\%$ obtained for the values $\Omega_m = 1-\Omega_\Lambda = 0.35\pm0.04$. These best-fitting models have been compared with the updated sample of data in Fig. 1.

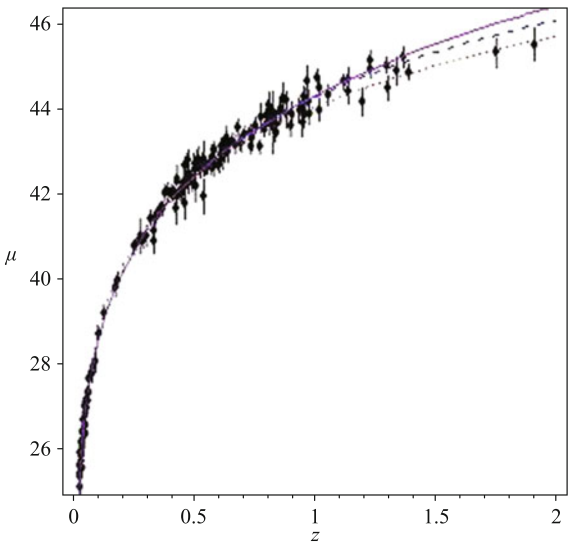

Fig. 1 The distance moduli μ (= $m - M$) for the "new gold sample" of 182 SNeIa from Riess *et al.* [19], added with the highest redshift SN (z = 1.914) yet discovered [20], are plotted against some best-fitting models. The solid curve corresponds to the present model and the dashed curve corresponds to the spatially-flat ΛCDM model $\Omega_m = 1 - \Omega_\Lambda = 0.35 \pm 0.04$. For comparison is also shown the Einstein-de Sitter model: $\Omega_m = 1$, $\Lambda = 0$ (*dotted curve*), which shows an unsatisfactory fit to the data with χ^2/DoF = 283.45/182 = 1.6 and $Q = 2 \times 10^{-6}$.

6.2 Observations of high-redshift radio sources

Next we consider the data on the angular size and redshift of 256 high-redshift quasars with their angular sizes of the order of a few milliarcseconds (ultracompact radio sources) and their redshift in the range 0.5–3.8, which were measured by the Very Long-Baseline Interferometry. These sources were selected by Jackson and Dodgson [21] from a bigger sample originally compiled by Gurvits [22]. The sources of the sample of Jackson and Dodgson are short-lived quasars deeply embedded inside the galactic nuclei, which are expected to be free from evolution on a cosmological time scale and thus comprise a set of standard rods, at least in a statistical sense. These sources are distributed into 16 redshift bins, each bin containing 16 sources. This compilation has earlier been used by many authors to test different cosmological models [16, 23–25]. In order to fit the data to the model, we derive the Θ–z relation in the following. The (apparent) angular size Θ of a source, of the proper diameter d, is given by

$$\Theta(z) = \frac{0.0688dh}{H_0 d_{\rm A}} \text{ milliarcseconds} \tag{18}$$

where d is measured in pc, h is the present value of the Hubble parameter in units of 100 km·s^{-1}·Mpc^{-1}, and the angular diameter distance $d_{\rm A} = d_{\rm L}/(1+z)^2$.

We find that the present model has a satisfactory fit to the data with χ^2/DoF = 20.78/15 = 1.39 and Q = 14%. In order to compare, we find that the best-fitting concordance model has a slightly better fit: χ^2/DoF = 16.03/14 = 1.15 and $Q = 31\%$ obtained for the values $\Omega_m = 1 - \Omega_\Lambda = 0.21 \pm 0.08$. These models are shown in Fig. 2.

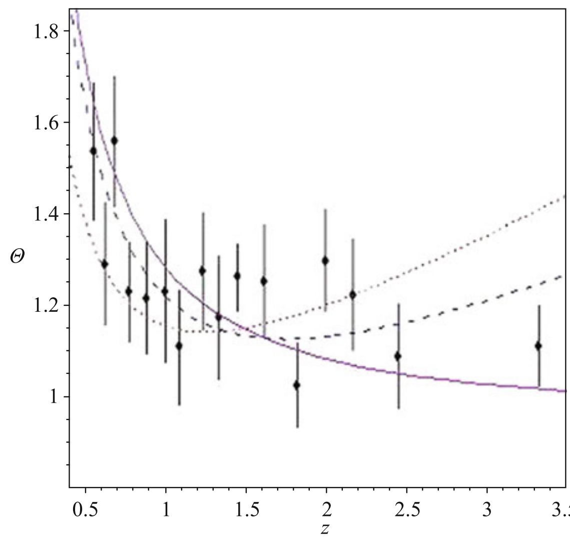

Fig. 2 The data on the ultra-compact radio sources compiled by Jackson and Dodgson [21] is compared with some best-fitting models. The solid curve corresponds to the present model and the dashed curve corresponds to the spatially-flat ΛCDM model $\Omega_m = 1 - \Omega_\Lambda = 0.21 \pm 0.08$. For comparison, we have also shown the Einstein-de Sitter model ($\Omega_m = 1$, $\Lambda = 0$), denoted by the dotted curve, which does not show a good fit to the data with χ^2/DoF = 28.82/15 = 1.92 and $Q = 1.7\%$.

The dataset compiled by Gurvits [22] was updated and extended by Gurvits *et al.* [26] as a sample of 330 sources distributed over a wide range of redshifts $0.011 \leqslant z \leqslant 4.72$. In order to minimize any possible dependence of linear size on luminosity and that of angular size on spectral index, they discarded lower values of luminosities and extreme values of spectral indices and selected only 145 sources with total radio luminosity $Lh^2 \geqslant 1026$ W·Hz^{-1} and $-0.38 \leqslant$ spectral index $\leqslant 0.18$, which are hoped to be free from the evolutionary effects and hence conceivably comprise a set of standard objects. This sub-sample, distributed into 12 bins, each bin containing 12 to 13 sources, was used to test different cosmological models [26, 27]. It would be interesting to note that both the models – the present one and the ΛCDM – have excellent fits to the new sample: while the present model provides the fitting result as χ^2/DoF $= 5.90/11 = 0.54$ and $Q = 88\%$, the concordance model $\Omega_m = 1 - \Omega_\Lambda = 0.20 \pm 0.34$ gives $\chi^2/\text{DoF} = 4.52/10 = 0.45$ and $Q = 92\%$.

6.3 Observations of H_0 and the oldest objects

Another powerful test of a cosmological theory can be devised by comparing the age of the Universe predicted by the theory and the age of the oldest objects in the Universe. The age of the Universe t_0 is the time elapsed since the Big Bang. It depends on the expansion dynamics of the model and is given by

$$t_0 = \int_0^\infty \frac{H^{-1}(z)}{1+z}\mathrm{d}z \tag{19}$$

Hence, the age of the Universe in a particular model depends on the expansion dynamics of the model, which in tern depends on the free parameters of the model. For example, by the use of the Friedmann equation, Eq. (19) reduces to the following in the standard cosmology:

$$t_0 = \frac{1}{H_0}\int_0^\infty \frac{\mathrm{d}z}{(1+z)\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda - \Omega_k(1+z)^2}} \tag{20}$$

Although t_0 is a model-based parameter, a lower limit is put on it by requiring that the Universe must be at least as old as the oldest object in it. This is done through $t_{\rm GC}$, the age of the globular clusters in the Milky Way which are among the oldest objects we so far know. However, the parameter H_0 can be estimated in a model-independent way, for example, from the observations of the low-redshift SNeIa, in which case the predicted magnitude does not depend on the model-parameters. One can use this value to calculate the age of the Universe in a particular theory which is to be compared with the age of the oldest objects.

Thus the measurements of H_0 and $t_{\rm GC}$ provide a powerful tool to test the underlying theory. For instance, by using the current measurements of $H_0 = 71 \pm 6$ km·s^{-1}·Mpc^{-1} from the Hubble Space Telescope Key Project [28] in Eq. (20), we get t_0 in the Einstein–de Sitter model as 9.18 Gyr. This cannot be reconciled with the age of the oldest globular cluster estimated to be $t_{\rm GC} = 12.5 \pm 1.2$ Gyr [29] and the age of the Milky Way as 12.5 ± 3 Gyr coming from the latest uranium decay estimates [30]. However, for the concordance ΛCDM model with $\Omega_m = 1 - \Omega_\Lambda = 0.27$ (as estimated by the WMAP project [31]), Eq. (20) gives a satisfactory age of the Universe $t_0 = 13.67$ Gyr which is well above the age of the globular clusters. The age of the Universe in the present model is given by $t_0 = H_0^{-1}$, as can be checked from (12). For the above-mentioned value of H_0, this gives $t_0 = 13.77$ Gyr which is even higher than the concordance model value.

6.4 Observations of starburst galaxies

Let us now consider the data on the apparent magnitude and redshift of starburst galaxies. Recent work has indicated that HII starbusrt galaxies might be considered as standard candles because of a correlation between their velocity dispersion, H luminosity, and metallicity (see, for example, [32] and the references therein). Siegel *et al.* [33] have compiled a sample of 15 HII-like starburst galaxies with redshifts in the range 2.17–3.39 (by using the data available in the literature) in order to constrain Ω_m. Mania and Ratra [32] modified this sample by excluding two HII galaxies (Q1700-MD103 and SSA22a-MD41) that show signs of a considerable rotational velocity component and used the resulting sample (shown in Table 1) to constrain the cosmological models of dark energy.

Table 1 Magnitudes, with uncertainties, of 13 HII starburst galaxies.

Galaxy	z	$\mu \pm \sigma$
Q0201-B13	2.17	$47.49^{+2.10}_{-3.43}$
Q1623-BX432	2.18	$45.45^{+1.97}_{-3.07}$
Q1623-MD107	2.54	$44.82^{+0.31}_{-1.58}$
Q1700-BX717	2.44	$46.64^{+0.31}_{-1.58}$
CDFa C1	3.11	$45.77^{+0.31}_{-1.58}$
Q0347-383 C5	3.23	$47.12^{+0.44}_{-0.32}$
B2 0902+343 C12	3.39	$46.96^{+0.71}_{-0.81}$
Q1422+231 D81	3.10	$48.81^{+0.38}_{-0.40}$
SSA22a-MD46	3.09	$46.76^{+0.56}_{-0.51}$
SSA22a-D3	3.07	$49.71^{+0.43}_{-0.41}$
DSF2237+116a C2	3.32	$47.73^{+0.25}_{-0.25}$
B2 0902+343 C6	3.09	$45.22^{+1.38}_{-1.76}$
MS1512-CB58	2.73	$47.49^{+1.22}_{-1.57}$

For this sample, the present model provides the minimum value of χ^2/DoF$= 53.54/12=4.46$ with $Q = 3.4 \times 10^{-7}$, whereas the standard ΛCDM model gives the minimum χ^2/DoF= 53.32/10=5.33 with $Q = 6.5 \times 10^{-8}$ for the values $\Omega_m = 0.19$, $\Omega_\Lambda = 0.98$. Interestingly, the Einstein-de Sitter model ($\Omega_m = 1$, $\Lambda = 0$) also shows a similar fit to the data with $\chi^2/\text{DoF} = 53.39/12 = 4.45$ and $Q = 3.5 \times 10^{-7}$. It would not be fair to claim that any of these models fits the data well, as is also clear from Fig. 3. Perhaps the inherent scatter of the data is large, and a large sample size is required to perform the test and to get any meaningful constraint on the cosmological parameters. However, this much is clear from the fitting results that the performance of the present model vis-a-vis that of the standard model, is better.

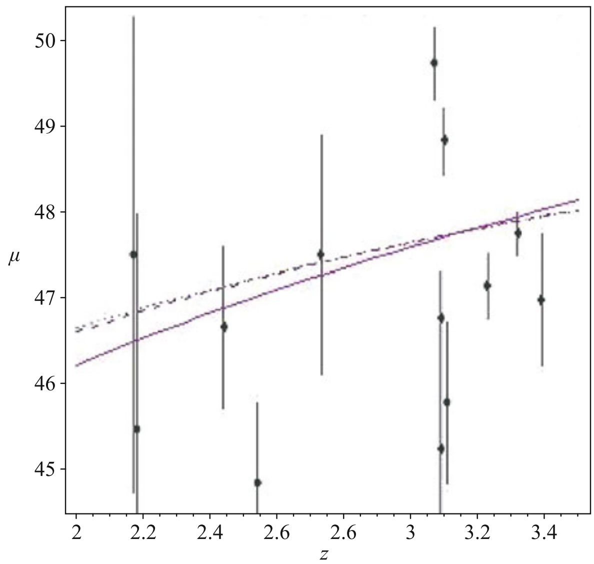

Fig. 3 The distance moduli μ of the HII-like starburst galaxies [32] are plotted against some best-fitting models. The solid curve corresponds to the present model and the dashed curve corresponds to the ΛCDM model $\Omega_m = 0.19$, $\Omega_\Lambda = 0.98$. The Einstein-de Sitter model is represented by the dotted curve.

6.5 Observations of CMB

Finally, let us study the compatibility of the present model with the observations of the CMB. As the CMB radiation carries the fingerprints of inhomogeneity the matter had during the epoch of decoupling, the observed anisotropy in the CMB temperature can be quantified in terms of the size of the structures on the surface of the "last scatter". For instance, a region which has a proper size L_{dec} on the surface of the last scatter, will subtend an angle θ at the observer today, given by

$$\theta = \frac{L_{\text{dec}}}{d_{\text{A}}(z_{\text{dec}})} \tag{21}$$

where $d_{\text{A}}(z_{\text{dec}})$ is the (angular diameter) distance to the surface of the last scatter at $z_{\text{dec}} = 1100$. If one considers L_{dec} to be equal to the Hubble radius $d_{\text{H}}(t_{\text{dec}}) = cH^{-1}(t_{\text{dec}})$, Eq. (21) gives $\theta \approx 1^{\text{o}}$ in the standard cosmology.

Although the standard cosmology gives the size of its horizon comparable to the Hubble radius d_{H}, it is not possible to explain the overall isotropy of the CMB in terms of some physical process operating under the principle of causality [13]. It is generally believed that inflation made the Universe smooth and left the seeds of structures, on the surface of the last scatter, of the order of the Hubble radius at that time[7)].

Moreover, if we interpret L_{dec}, appearing in (21), as the size of the largest coherent region (resulted from some homogenization process) on the last scattering surface, the standard cosmology would predict large anisotropies in CMB for angular scales greater than $\approx 1^{\text{o}}$ – a result that would be in contradiction with what is observed [8, 13].

Thus all one can say, permitted by the present situation, is that the CMB observations fix the length scale of the inhomogeneity on the surface of last scatter, which can be estimated in terms of the Hubble radius $d_{\text{H}}(t_{\text{dec}})$. For example, this length scale L_{dec} can be written as

$$L_{\text{dec}} = \eta d_{\text{H}}(t_{\text{dec}}) = \eta c H^{-1}(t_{\text{dec}}) \tag{22}$$

where the parameter η can be estimated from the observations of the CMB. Particularly, this can be estimated accurately by using the angular scale of the first peak in the observed angular power spectrum of the CMB, which is supposed to give, with a high precision, the physical scale of the density contrast during the epoch of decoupling. In terms of the Legendre multipole l, where $l = \pi/\theta$, the WMAP observations give the location of the first peak at $l = 220$ [31]. This is equivalent to $\theta = 0.82^{\text{o}}$. Hence, we have to solve Eq. (21), taken together with (22), for $\theta = 0.82^{\text{o}}$, which would be equivalent to fitting Eqs. (21) and (22) to the first peak observed by the WMAP.

This solution in the concordance ΛCDM model ($\Omega_m = 1 - \Omega_\Lambda = 0.27$), yields the value $\eta = 0.82$. By considering $H_0 = 71$ km·s^{-1}·Mpc^{-1}, this gives the size of the structures at t_{dec} as $L_{\text{dec}} = 182.4$ kpc. In the new paradigm, the solution yields $\eta = 7.86$, giving $L_{\text{dec}} = 30.14$ Mpc. The length scale L_{dec} is expected to grow, due to the cosmic expansion, to a proper length L_0 today, given by $L_0 = (1 + z_{\text{dec}})L_{\text{dec}}$. Thus the present size of our patch of homogeneity and isotropy, is 33.19 Gpc in the present model compared with 200.82 Mpc in the concordance

7) It may be a matter of debate that if inflation made everything else smooth, it was not expected to leave this significant signature [8]. It may also be noted that the growth of density perturbations, in the standard cosmology, pose difficulties for the inflationary predictions when confronted with the CMB observations [35].

model. A larger size of this patch is supported by the findings of Grishchuk [35] who concludes, by combining available observations with plausible statistical assumptions, that the present size of this patch is significantly bigger than the present Hubble radius.

We have noted that the overall isotropy of the CMB cannot be explained in the standard paradigm in terms of some homogenization process taken place in the baryon-photon plasma operating under the principle of causality. However, the new paradigm can successfully explain this, as the whole Universe is always causally connected in this model, as we have seen earlier. Thus the presence of the seeds of structures at the last scattering surface, finds its origin in the random quantum fluctuations in the primordial density of the early Universe (in a more natural way than the standard paradigm).

Although the angular power spectrum of the CMB also carries informations contained in other peaks, however, further study will be needed to achieve the level of sophistication in the finer details calculated in the standard cosmology.

6.6 Future observations

It is believed that precise measurements of gravitational waves (GW) will provide information about black holes and very early Universe which cannot be observed with light. It has been shown recently that the detection of GW will provide the best test of GR, if the advanced projects to detect them improve their sensitivity [36]. As the gravitational waves are indeed solutions of the field equation $R^{ik} = 0$, one expects that their detection would be a powerful endorsement for the new paradigm. However, before arriving at a definitive conclusion, we need to examine some issues at a foundational level. For instance, the so-called GW solutions of $R^{ik} = 0$ are plane GW in *empty* space, according to the conventional wisdom. As the new paradigm proves that equations $R^{ik} = 0$ do *not* represent an empty space, one needs to examine if the traditional GW solutions of $R^{ik} = 0$ still retain their meaning in the new paradigm. Or, the new findings of the novel paradigm have some consequences/constraints on the GW.

7 Conclusion

Theory of general relativity has been agreed upon by experts for almost a century and is believed to describe accurately all gravitational phenomena ranging from the solar system to the Universe. However, there is a price for this success which is often ignored. More than 95 percent of the content of the energy-stress tensor has to be dark, in the form of inflaton, dark matter and dark energy, which do not have any non-gravitational or laboratory evidence. Additionally, the dark energy poses a serious confrontation between fundamental physics and cosmology.

The present situation reminds us of Einstein's "biggest blunder" when he forced his theory to predict a static Universe, perhaps guided by his religious conviction that the Universe is one, eternal, and unchanging. It seems that we are making a similar blunder by forcing the energy-stress tensor into the field equations while the observations indicate that it is not needed. We seem to have a deep conviction that the spacetime will remain empty unless we fill it by the energy-stress tensor. However, we have been ignoring numerous evidences telling earnestly that the "vacuum" field equations $R^{ik} = 0$ do not necessarily represent an empty spacetime. Rather they are the consistent field equations of gravitation in the framework of a geometric theory. This result rests on the following logical ingredients.

1(a) No formulation of the energy-stress tensor of the gravitational field, is included in equations $R^{ik} = 0$. In fact, a proper energy-stress tensor of the gravitational field does not exist.

1(b) It is shown that the gravitational field appears, *through geometry*, as a source of curvature in the solutions (Schwarzschild, Kerr) of equations $R^{ik} = 0$.

2(a) Recently it has been discovered that the relativistic formulation of matter, given by the energy-stress tensor T^{ik}, is plagued with paradoxes and inconsistencies, implying that, like the energy-stress tensor of the gravitational field, an indisputable energy-stress tensor of matter neither exists.

2(b) It is shown that the matter field (in the form of the momentum density) too appears, *through geometry*, as a source of curvature in a cosmological solution (Kasner) of equations $R^{ik} = 0$.

3(a) There is no scope, in the framework of GR, to include the angular momentum in T^{ik}.

3(b) It is already known that the angular momentum too contributes to the curvature of spacetime, and it also appears, *through geometry*, for example, in Kerr solution of equations $R^{ik} = 0$.

This leaves equation $R^{ik} = 0$ as the only possibility for a consistent field equation of gravitation, wherein the matter fields as well as the gravitational field appear through the geometry (the metric field), without adding any formulation thereof to the field equations. The resultant non-zero energy-momentum-angular momentum of the two fields acts as the source of curvature.

This may appear orthogonal to the usual understanding, nevertheless, it is in striking agreement with the observations at all scales, without requiring the usual epicycles of the standard paradigm, such as inflaton, non-baryonic dark matter and dark energy. Moreover, it provides consistent explanations to the curvature of Kasner solution and the flatness of the homogeneous-isotropic solution of $R^{ik} = 0$, which have remained unexplained puzzles so far. Additionally, the resulting cosmological model, in the new theory, has many attractive features. For example, it is Machian and free from the problems of the standard cosmology – the singularity, horizon, flatness and the cosmological constant problems.

It appears that the quest for a better understanding of gravity is far from over! It was Einstein's obsession that the vibrant geometrical part of GR is "marble" and matter is "wood", and that all attempts should be directed to turn wood into marble. It, finally turns out that the "wood" is a redundant part of the theory, shunning which, enhances the beauty of the "marble" in the extreme simplicity of the field equations $R^{ik} = 0$!

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