Front. Phys., 2014, 9(1): 128–135
DOI 10.1007/s11467-013-0337-5

RESEARCH ARTICLE

Equilibrium state and non-equilibrium steady state in an isolated human system

Wen-Zhi Zheng (郑文智), **Yuan Liang** (梁源), **Ji-Ping Huang** (黄吉平)†

Department of Physics and State Key Laboratory of Surface Physics, Fudan University, Shanghai 200433, China

Corresponding author. E-mail: †jphuang@fudan.edu.cn

Received February 28, 2013; accepted March 20, 2013

The principle of increasing entropy (PIE) is commonly considered as a universal physical law for natural systems. It also means that a non-equilibrium steady state (NESS) must not appear in any isolated natural systems. Here we experimentally investigate an isolated human social system with a clustering effect. We report that the PIE cannot always hold, and that NESSs can come to appear. Our study highlights the role of human adaptability in the PIE, and makes it possible to study human social systems by using some laws originating from traditional physics.

Keywords equilibrium state, non-equilibrium steady state, human system, principle of increasing entropy, clustering effect, random network

PACS numbers 89.75.-k, 89.65.Gh, 05.70.-a

1 Introduction

Natural/Social systems contain molecules/humans as constructive units without/with adaptability to environmental changes due to the lack/existence of learning ability. Accordingly, the fundamental interactions underlying the two kinds of systems are distinctly different. For example, the interaction in natural systems satisfies Newton's laws of motion, but that in social systems does not. But, because entropy is a state parameter that describes a macroscopic state (say, disorderness or uncertainty) of a whole system, in spite of the microscopic interactions inside, the concept of entropy can be adopted in both natural [1] and social [2] systems. For natural systems, the most important law related to entropy is the principle of increasing entropy (PIE) [1] in statistical physics. Because statistical physics has started to offer new insights [3–21] into social systems, here we attempt to raise a question: How does the PIE behave in social systems? This question is non-trivial because social systems have many features (say, the clustering effect that will be explained below) that have no counterparts in natural systems. But it has never been experimentally touched in the literature. As the initial work, here we attempt to investigate the impact of the clustering effect upon the PIE in human social systems. This is not only because doing so can enrich the knowledge on the PIE, but also because one will be able to manage an actual human social system in the light of the PIE by increasing or decreasing the system's entropy appropriately.

According to the PIE [1], the entropy of an isolated natural system must gradually increase until the system reaches the equilibrium state with maximum entropy. If the system has not reached this equilibrium state, it is just called a non-equilibrium system. Meanwhile, if the non-equilibrium system has entropy that is unchanging in time, its state can be called a non-equilibrium steady state (NESS) [20]. To maintain a natural system in such a NESS, the system should be an open system that exchanges matter and energy with environments [20], which now is a common belief. The validation of the PIE means that a NESS must not exist in any isolated natural systems because of the lack of importation of matter and energy from environments.

It is known that the PIE is a statistical result as emerged from the global interactions among molecules in isolated natural systems. However, for human social systems, the situation could be quite different. For example, let us imagine an isolated social system by putting many strangers in a place. At first, they do not know each other. As time elapses, due to communications, small communities or clusters (each having a small number of strangers) are naturally formed in which they get to know each other. Eventually, all of them in the whole place know each other as a result of various kinds of human activities like introductions, communications, etc. Such a whole process is called the snowballing behavior, which apparently arises from human adaptability. For an isolated human system, the snowballing behavior is an intrinsic property due to the adaptability of human beings, and it actually describes the dynamic evolution of interactions in the system from no interactions (no one knows each other), to local interactions (a few of them know each other), and to global interactions (all of them know each other). In this article, as the initial work on the effect of snowballing behavior, for convenience, we shall only investigate how three separate sizes of clusters (small, middle and large) would influence the macrostate of the system; this is called "clustering effect".

To proceed, we investigate an isolated human social system based on the resource-allocation system [3, 6, 8, 9, 15, 17–19, 22], which is a variant of the classic model for collective behaviors of agents – the minority game [3]. In the minority game, the agents with well-defined strategies compete for limited resources in two rooms through adaptation. It behaves well when modelling various kinds of resource allocations in the realistic society, say, the El Farol bar problem [3, 22] and financial markets [8]. In our model, we introduced biased distribution of resources so that it is possible to study the equilibrium states and related phase transitions [15, 17, 21]. Here we focus on the clustering effect in this system by utilizing experiments and simulations.

2 Human experiments

2.1 Experimental design

For our purpose, the clustering effect is described by defining a probability, k ($0 \leqslant k \leqslant 1$), for each human to get to know the others. The increase of k can characterize the degree of clustering effect. The whole experiment was conducted on the basis of computers. In the experiment, we build an isolated system in which the total number of subjects is not changed. They compete with each other for the limited resources. Firstly, a random network is generated to indicate the relationship between the subjects. Each subject is represented by one node and there is a probability of k ($0 \leqslant k \leqslant 1$) for each node to connect the others in the network. Therefore, each node and its connected nodes will form its group. Next two different amounts of virtual resources M_1 and M_2 are respectively placed in Room 1 and Room 2 (denoted by two buttons on the screen of the computers). Each subject can choose to enter either of them (by clicking the corresponding button on her/his own screen) and share the virtual resource with the others in it. However, they know neither the values of M_1 and M_2 nor the numbers and identities of the subjects in their group. Otherwise, the information related to the total numbers of subjects respectively in two rooms in any rounds is also unnecessary to give subjects. The only information feedback to the subject is if she/he earned more resource than her/his group average in the last round. If so, she/he would receive a signal "win" and gets 1 score; otherwise, she/he would receive a signal "lose" and gets 0 score. As the role of price in economic activities, the binary signal (win or lose) reflects the degree of imbalance between the room capacity and the subjects in need of the resource. Except for the first round when the subjects have to choose their preferred room randomly (i.e., with 50% probability to choose either of the two rooms) because they have not received any signals, in the subsequent rounds, they would form their strategies based on the past records. In the duration, because of the learning ability, subjects can know, at least to some extent, which room has more amount of resources and then choose to enter the room with a probability larger than 50%.

For a given parameter set of k and M_1/M_2, each experiment was performed for 15 rounds. We have recruited 25 students and teachers from the Department of Physics in Fudan University to be the subjects. The scores they win in the experiments would be converted to cash according to the exchange rate: 1 score = 1 Chinese Yuan. In addition to this reward, the top three of them would get bonuses (150, 100, and 50 Chinese Yuan, respectively). Before the experiment, we clearly explain the rules and the rewards of the experiments.

2.2 Experimental results

During the experiment, each subject is considered to win or lose according to her/his performance within her/his own group. This should be reasonable because in real life, individuals often make decisions on the basis of their own neighbors or partial information. As a result, the whole society runs healthily or unhealthily. Hence, here we focus on the macroscopic behavior of the whole system, which is determined by individual behaviors. In order to study the evolution of the macroscopic state of the system and its mechanism related to the microscopic behaviors of the subjects, two quantities are introduced — entropy (s) and preference (μ) as follows.

To measure the special uncertainty associated with macroscopic arbitrage opportunities in our human system (as addressed below), we follow the mathematical expression for the information entropy coined by Shannon [23], and define the entropy (s) as

$$s = -(p_1 \ln p_1 + p_2 \ln p_2) \quad (1)$$

where $p_1 = \frac{\langle N_1\rangle/M_1}{\langle N_1\rangle/M_1+\langle N_2\rangle/M_2}$ and $p_2 = \frac{\langle N_2\rangle/M_2}{\langle N_1\rangle/M_1+\langle N_2\rangle/M_2}$. Here $\langle N_1\rangle$ (or $\langle N_2\rangle$) signifies the round average of the numbers that the subjects choose to enter Room 1 (or Room 2). As a feature of the present system (repeated experiments), averaging over more rounds makes the resource allocation more efficient because of human adaptability [15, 17, 21]. Thus, adopting more rounds not only makes both $\langle N_1\rangle$ and $\langle N_2\rangle$ reflect the time passage of the repeated experiment (with total 15 rounds) for a fixed parameter set of k and M_1/M_2, but also represents that the system has a higher efficiency of resource allocation with a higher degree of uncertainty associated with macroscopic arbitrage opportunities. Accordingly, p_1 and p_2 denote the probabilities of two micostates due to the existence of two rooms, which can be readily understood because $\langle N_1\rangle/M_1$ (or $\langle N_2\rangle/M_2$) represents the averaged number of subjects per amount of resources in Room 1 (or Room 2). It is worth noting that, when the number of rounds is large enough, $\langle N_1\rangle/M_1 = \langle N_2\rangle/M_2$ can appear as a special emergent result of the current system [15] that involves free competitions among the subjects (because any kind of communication is not allowed during the experiment). Clearly, as $\langle N_1\rangle/M_1$ tends to $\langle N_2\rangle/M_2$, the whole system gets closer to the equilibrium or balance state and s has the maximum value of 0.693 when $\langle N_1\rangle/M_1 = \langle N_2\rangle/M_2$ or $p_1 = p_2 = 1/2$. Therefore, overall speaking, the state with maximum entropy $s = 0.693$ means the macroscopic arbitrage opportunities are exhausted in the whole system, or equivalently, the degree of uncertainty associated with the macroscopic arbitrage opportunities reaches maximum. In addition, we know that the first round always tends to yield $N_1 = N_2$ due to lack of information, thus causing $\langle N_1\rangle/M_1 \neq \langle N_2\rangle/M_2$ when $M_1/M_2 > 1$. In this case, the macroscopic arbitrage opportunities are the most, and the degree of uncertainty associated with them is the lowest. Hence, we may conclude that the s defined above describes the degree of uncertainty associated with macroscopic arbitrage opportunities as time elapses for the repeated experiment. Incidentally, more remarks on justifying the above expressions of p_1 and p_2 will be given in the last paragraph of this section. It should be noted here that we calculate the entropy of the system from the result after round averaging rather than the result of a single round. This definition of a system state differs from that for thermodynamic entropy in statistical physics. Based on the characteristics of this resource allocation system, the efficiency of the resource allocation would be higher and higher as the system evolves from "State 1" to "State 15" according to our definition. So here "State N" includes the results from the first N rounds. In a word, the fact that the first N rounds of the repeated experiment are defined as one system state as a whole is just due to the features of this resource allocation system that includes finite rounds of repeated experiments.

On the other hand, we define another quantity to describe the overall preference (μ) of the subjects as

$$\mu = |1 - 2\rho| \times 100\% \quad (2)$$

where $\rho = \left(\sum_{m=1}^{N} \Psi_m^{(1)}\right)/(R_E N)$ denotes the ratio of the averaged times when they choose Room 1 and the total times when they need to make choices. Here N denotes the number of all the subjects ($N = N_1 + N_2$), and $\Psi_m^{(1)}$ is the number of times that the m-th subject chooses to enter Room 1 within the total rounds, R_E ($R_E = 15$ in this work), of the repeated experiment under a given set of k and M_1/M_2. Therefore, a larger μ indicates that the subjects are willing to enter one of the two rooms than the other. In particular, $\mu = 0\%$ means that the subjects show no preference to either of the two rooms.

Figure 1 shows the entropy (s) and the preference (μ) evolve as a function of the experimental period extension. This figure displays three values of k, namely, $k = 0.2$, 0.52, and 0.76, in order to represent the cases of local interactions from a lower to a higher strength. Figure 1(a)–(c) shows that the different settings influence both the macroscopic states (denoted by entropy, s) and the microscopic behaviors (reflected by preference, μ). While the entropies (s) of all the systems show an increasing trend as the experiment progresses, they (almost) stop increasing when hitting a certain threshold. However, only in the cases of $k = 0.76$ [Fig. 1(c)] which means the size of reference groups has spread over most parts of the whole system (thus having a high strength of local interactions), the entropies increase up to the maximum value of $s = 0.693$, no matter what degrees of the system complexity (indicated by M_1/M_2) the subjects face. When k is smaller ($k = 0.2$ and 0.52), the systems tend to stay at different NESSs (namely, s is almost independent of time, but has a separation from the maximum value of 0.693) except for the relatively simple case of $M_1/M_2 = 3$ in Fig. 1(b). In these states, macroscopic arbitrage opportunities exist but our subjects are unable to eliminate them because the size of reference groups is small (or, the local interactions among the subjects are relatively weak). On the other hand, the ratio of the resources, M_1/M_2, also contributes to the systems' evolutions to the equilibrium (balance) state and the NESS. In Fig. 1(c), the systems of different M_1/M_2's have different trajectories towards the equilibrium state. In general, the system of a small M_1/M_2 (i.e., a low degree of system complexity) can reach the equilibrium state faster than that of a large M_1/M_2. This means that subjects can adapt to the system faster for a small M_1/M_2 than for a large M_1/M_2, by eliminating macroscopic arbitrage opportunities. In Fig. 1(a), the three cases of $M_1/M_2 = 3$, 4, and 9 approach to different NESSs with different s's. Similar behavior happens in Fig. 1(b) ($M_1/M_2 = 4$ and 9). Particularly, in either Fig. 1(a) or (b), while the system of $M_1/M_2 = 4$ can evolve to the NESS that has a small gap with respect to the equilibrium state, there is always a big gap between the NESS of $M_1/M_2 = 9$ and the equilibrium state. This means the system with a lower degree of the system complexity (smaller M_1/M_2) reduces the requirement of the size of the reference group (or the strength of local interactions) to reach equilibrium.

Now let us turn to the preferences (μ) as displayed in Fig. 1. The preferences evolving over time display how the subjects are trained to adapt the system. We find that the microscopic behaviors of the subjects may provide an explanation for the above-mentioned macro phenomena. Seen from comparing the systems that finally evolve to the equilibrium state [$M_1/M_2 = 3$ in Fig. 1(b) and all M_1/M_2's in Fig. 1(c)] with those that approach to the NESS [all M_1/M_2's in Fig. 1(a) and $M_1/M_2 = 4$ and 9 in Fig. 1(b)], the subjects in the systems of a larger k generally show a higher degree of preference than those

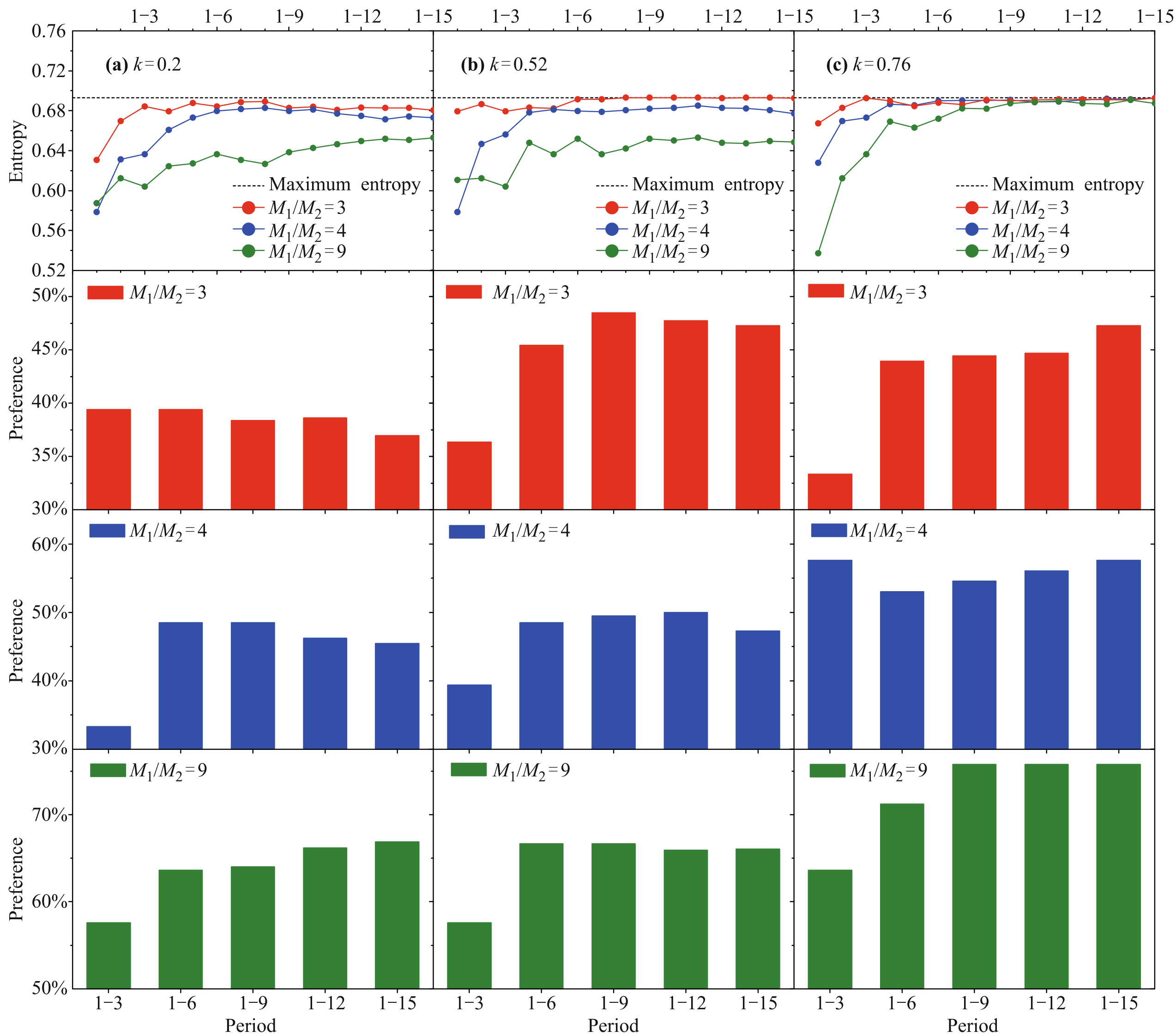

Fig. 1 Experimental results of entropy, s, and preference, μ, for 25 subjects and three M_1/M_2's, as a function of the period extension ("1–3", "1–6", "1–9", "1–12", and "1–15"): **(a)** $k = 0.2$, **(b)** $k = 0.52$, and **(c)** $k = 0.76$. For each parameter set of fixed M_1/M_2 and k, the repeated experiment was conducted for 15 rounds; here "1–3", "1–6", "1–9", "1–12", and "1–15" represent the first three, six, nine, twelve, and fifteen rounds, respectively. The horizontal dashed lines denote the equilibrium state with the maximum entropy of $s = 0.693$.

of a lower k when M_1/M_2 is given. That is, the emergence of the adaptability of preferences that matches with the system complexity causes the whole system to reach the equilibrium state and thus to obey the PIE. But when k is relatively small, the weak strength of local interactions inhibits the subjects from developing enough adaptability and then the inadaptability of subjects leads the whole system to the NESS. Accordingly, the PIE becomes invalid. In other words, for a given M_1/M_2, the probability, k, for each human to get to know the others has a threshold (k_c) above or below which the PIE holds or fails. According to Fig. 1, it is clear that smaller M_1/M_2 has a smaller value of k_c. Further, it is easy to conclude that the different degrees of inadaptability make the systems of different M_1/M_2's reach different NESSs once k is given.

Now we are in a position to add more remarks to show the validity of the above experimental results. Strictly speaking, to make the above results more solid, the experiments should be conducted under the same parameter set (M_1/M_2 and k), either for a specific group with many times or for many separated groups in the same time. After that, taking average over either the many times or the many groups will eliminate the influence of fluctuations. However, this can hardly be realized due to the following two reasons: i) human subjects in a specific group cannot afford being tested in the simple experiment again and again, and they will easily feel boring, thus yielding data distortion; and ii) using many separated groups to perform the experiment is practically impossible due to the limited resources like human resources and money. Nevertheless, there exist two exact values that can help to show the validity of the above experimental results obtained from Fig. 1. Let us take $M_1/M_2 = 3$ and $k = 0.76$ (or $k > k_c$) as an example. One exact value for it is the entropy $s_{1-1} = 0.562$ for the period 1–1 because taking the above-mentioned average must yield $\langle N_1\rangle/\langle N_2\rangle = 1$ for lack of information (or, human adaptability has not started to play a role). The other exact value is the maximum entropy $s_{1-\infty} = 0.693$ for the period 1–∞ because taking the above-mentioned average must yield $\langle N_1\rangle/\langle N_2\rangle = M_1/M_2 = 3$ due to the equilibrium or balance state emerged as a result of the system's efficiency (or, human adaptability has played a full role). Clearly, $s_{1-1} < s_{1-\infty}$. Thus, a single curve in Fig. 1(c) represents a specific realistic trajectory from s_{1-1} to $s_{1-\infty}$, indicating the trend, which suffices for our analysis of the PIE. This also implies that it is not only convenient, but also reasonable for us to have assumed that an occupation ratio equals to a probability as already used for determining p_1 and p_2 in the definition of s. Similar arguments hold for $M_1/M_2 = 4$ or 9, as also displayed in Fig. 1(c). For $M_1/M_2 = 4$, $s_{1-1} = 0.500$ and $s_{1-\infty} = 0.693$. For $M_1/M_2 = 9$, $s_{1-1} = 0.325$ and $s_{1-\infty} = 0.693$. Apparently, as $k < k_c$, human adaptability can only play a partial role. As a result, the system reaches the NESS with a time-independent value of entropy between s_{1-1} (for which human adaptability has not started to play a role) and $s_{1-\infty} = 0.693$ (for which human adaptability has played a full role); see Fig. 1(a) and (b).

In Fig. 1, we did not show the result of $k = 0$ (no interactions). Actually, according to Eq. (1), this case exactly yields three constant values of entropy, namely, $s = 0.562$ for $M_1/M_2 = 3$, $s = 0.500$ for $M_1/M_2 = 4$, and $s = 0.325$ for $M_1/M_2 = 9$, for the full period extension, because $k = 0$ corresponds to a situation where every subject chooses to enter a certain room randomly (due to the lack of a reference group) no matter what values of M_1/M_2 are taken. Clearly, when $k = 0$, the system with an arbitrary value of M_1/M_2 is always located in the NESS. On the other hand, $k = 1$ corresponds to the existence of global interactions, which has been investigated in Ref. [15] and naturally has the same framework as that of $k = 0.76$ for different values of M_1/M_2. That is, when $k = 1$, the system with an arbitrary value of M_1/M_2 always tends to reach the equilibrium state with the maximum entropy, thus satisfying the PIE.

In addition, Fig. 1 shows the results of M_1/M_2 =3, 4 and 9, three biased distributions of resources. We did not display the result of $M_1/M_2 = 1$, an unbiased distribution of resources, which has been extensively studied in the original minority game system [3,6] and its insightful extensions by other scholars [8, 9, 24–26]. In fact, according to Eq. (1), $M_1/M_2 = 1$ exactly yields the maximum value of entropy for various k's because each subject can simply choose to enter either of the two rooms randomly despite various values of k's. That is, in this case, the system is always located at the equilibrium (or balance) state for the full period extension as adopted in Fig. 1. In this sense, although the present system originates from the original minority game system [3, 6], neither the PIE nor the NESS holds in the original minority game system and its extensions.

3 Agent-based modeling and simulation results

Because the subjects are recruited from our university, it becomes necessary to design an agent-based model [15], in order to exclude the influence of the specific population. In the model, the configuration and the rules are the same as those in the human experiment. A decision-making system is established for the agents to mimic the human's behaviors in the experiment. There are also 25 agents in the model. λ strategies are allocated to each agent. In each round, one agent may encounter Σ possible situations [15]. For each of all the Σ possible situations the agents may face, one strategy for the resource-allocation problem provides a choice (Room 1 or Room 2). So a typical strategy is a two-column table — the left column is for the Σ possible situations and the corresponding choices are on the right column. The right column is filled in the following way in order to reflect the heterogeneous preference of the real human [15]:

- An integer M $(0 \leqslant M \leqslant \Sigma)$ is randomly picked up;
- Each element in the right column is filled in by 1 with probability M/Σ; and
- The rest of the elements are filled in by 2.

Bit 1 (or 2) denotes the choice of entering Room 1 (or Room 2). The decision-making system of one agent is composed of λ such strategies. After the simulation is started, each agent will score her/his strategies according to their performance. The agent will choose the best-performing strategy in each step. In the simulation, for the agents to identify the appropriate strategies in the scoring process, 20 simulation steps are adopted to represent one experiment round. For each parameter set used in the experiment (k and M_1/M_2), the agents also compete with each other for the total 15 rounds (namely, 300 simulation steps). As a result, Fig. 2 has a framework like Fig. 1. Accordingly, all the qualitative analysis and conclusions (appearing in the section of Experimental Results) remain valid for simulation results. That is, the simulations help us to confirm the above conclusions in

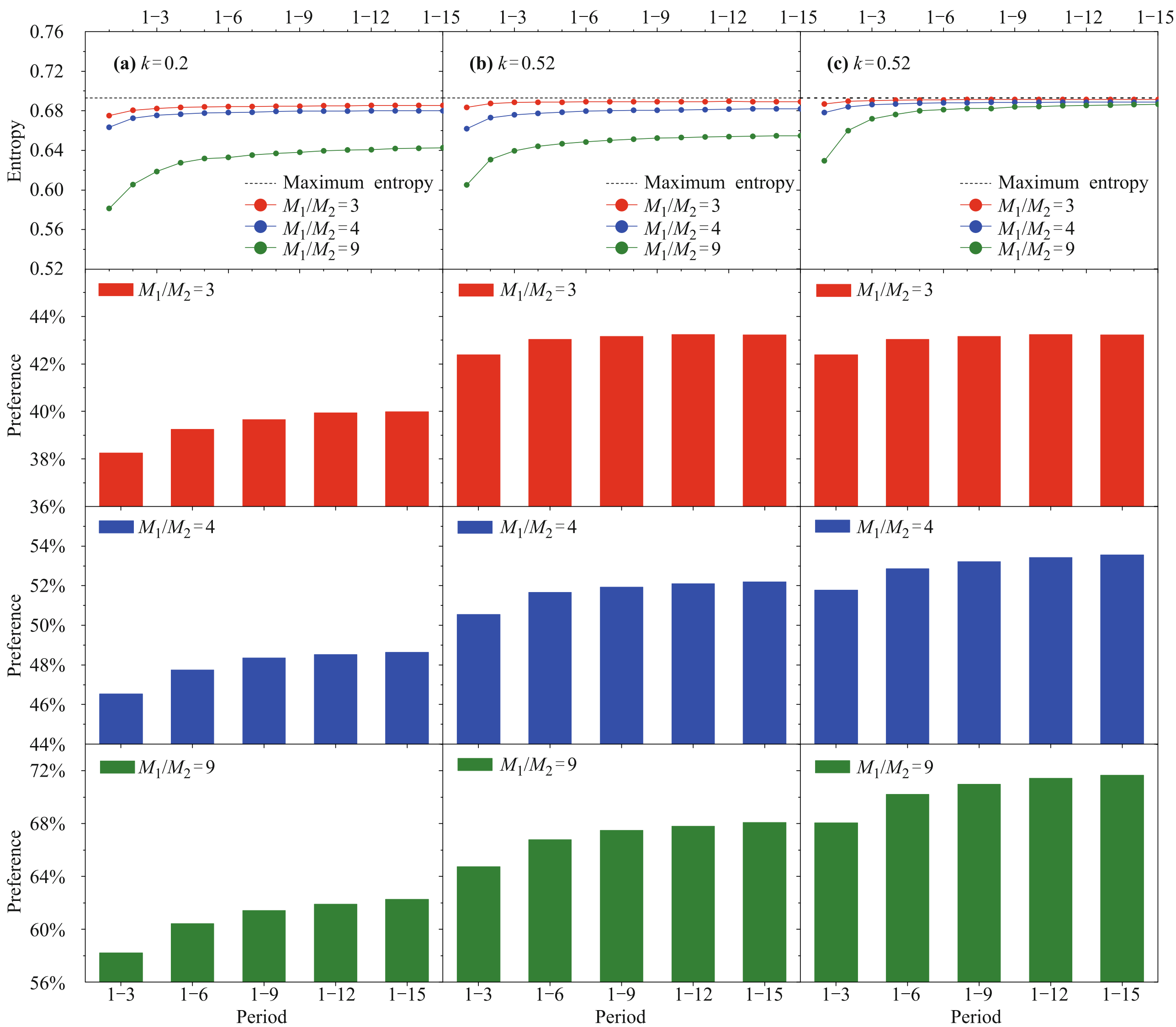

Fig. 2 Simulation results of entropy, s, and preference, μ, for 25 agents and three M_1/M_2's. Others are the same as Fig. 1. Parameters: $\Sigma = 72$ and $\lambda = 18$.

a more general sense indeed.

4 Discussion and conclusions

We have investigated an isolated human system with an intrinsic property — the clustering effect. The closeness of the system makes this system possess a counterpart of the conservation of energy that holds in all isolated natural systems where the PIE (principle of increasing entropy) works.

We have found that for the isolated human system, the probability, k ($0 \leqslant k \leqslant 1$), for each human to get to know the others has a threshold (k_c) above or below which the PIE holds or fails. This advances our previous knowledge that the PIE is valid for all isolated systems. This work also shows that NESSs (non-equilibrium steady states) exist in an isolated human system where low local interactions come to appear ($0 \leqslant k < k_c$). This is in sharp contrast to the common belief that NESSs can only exist in open systems because they maintain themselves in a NESS by the importation of materials and energy from environments [18]. Essentially, all of these results originate from a feature of our isolated human system, namely, interactions among constructive units can vary due to the adaptability of humans. The feature does not have a counterpart in any isolated natural systems (that involve non-adaptive molecules) where the PIE holds always. Thus, this feature (arising from human adaptability) serves as a necessary condition for the present results to be true.

Our results are expected to be valid in a more general sense due to the following arguments. Firstly, the experiment has been conducted on the basis of the resource-allocation system [3, 6, 8, 9, 15, 17–19], which behaves well when modelling various kinds of resource allocations in the realistic society, say, the El Farol bar problem [3, 22] and financial markets [8]. (It is worth noting that, although our system originates from the original minority game system [3, 6], neither the PIE nor the NESS holds in the original minority game system and its excellent extensions by other scholars [8, 9, 24–26]). Secondly, the computer simulations have confirmed the experimental results by excluding the impact of experimental limitations (say, specific population). Finally, being beyond the human social system discussed in this work, our results are also expected to work for other kinds of special social systems, for example, iterated prisoners' dilemma [27] and sequential bargaining [28], where, however, the entropy should be defined appropriately.

To sum up, this work indicates the important impact of human adaptability upon some laws of traditional physics, and makes it possible to reveal some universality or difference underlying natural systems and social systems by utilizing such laws even though the two kinds of systems have distinctly different fundamental interactions.

Acknowledgements We thank Dr. Y. Chen of Tokyo University for fruitful discussion. We acknowledge the financial support by the National Natural Science Foundation of China under Grant Nos. 11075035 and 11222544, the Fok Ying Tung Education Foundation under Grant No. 131008, the Program for New Century Excellent Talents in University (NCET-12-0121), the Shanghai Rising-Star Program (No. 12QA1400200), and the CNKBRSF under Grant No. 2011CB922004.

References

1. A. H. Carter, Classical and Statistical Thermodynamics, New Jersey: Prentice Hall, 2000: 96
2. K. D. Bailey, Social Entropy Theory, State University of New York Press, New York, 1990
3. D. Challet and Y. C. Zhang, Emergence of cooperation and organization in an evolutionary game, *Physica A*, 1997, 246(3–4): 407
4. P. Jefferies, M. L. Hart, P. M. Hui, and N. F. Johnson, From minority games to real-world markets, *Eur. Phys. J. B*, 2001, 20(4): 493
5. M. Hart, P. Jefferies, P. M. Hui, and N. F. Johnson, Crowdanticrowd theory of multi-agent market games, *Eur. Phys. J. B*, 2001, 20(4): 547
6. D. Challet, M. Marsili, and Y. C. Zhang, Minority Games: Interacting Agents in Financial Markets, Oxford: Oxford University Press, 2005
7. R. Mantegna and H. E. Stanley, Introduction to Econophysics, Cambridge: Cambridge University Press, 2000
8. N. F. Johnson, P. Jefferies, and P. M. Hui, Financial Market Complexity, Oxford: Oxford University Press, 2003
9. M. Anghel, Z. Toroczkai, K. E. Bassler, and G. Korniss, Competition-driven network dynamics: Emergence of a scale-free leadership structure and collective efficiency, *Phys. Rev. Lett.*, 2004, 92(5): 058701
10. K. Q. Yang, L. Yang, B. H. Gong, Z. C. Lin, H. S. He, and L. Huang, Geographical networks: Geographical effects on network properties, *Front. Phys. China*, 2008, 3(1): 105
11. L. Guo, Y. F. Chang, and X. Cai, The evolution of opinions on scale-free networks, *Front. Phys. China*, 2006, 1(4): 506
12. J. Zhou and Z. H. Liu, Epidemic spreading in complex networks, *Front. Phys. China*, 2008, 3(3): 331
13. C. M. Liu and J. W. Li, Small-world and the growing properties of the Chinese railway network, *Front. Phys. China*, 2007, 2(3): 364

14. S. Gheorghiu and M. Coppens, Heterogeneity explains features of "anomalous" thermodynamics and statistics, *Proc. Natl. Acad. Sci. USA*, 2004, 101(45): 15852

15. W. Wang, Y. Chen, and J. P. Huang, Heterogeneous preferences, decision-making capacity and phase transitions in a complex adaptive system, *Proc. Natl. Acad. Sci. USA*, 2009, 106(21): 8423

16. L. Pietronero, Statistical physics: Physicists get social, *Nat. Phys.*, 2010, 6(9): 641

17. L. Zhao, G. Yang, W. Wang, Y. Chen, J. P. Huang, H. Ohashi, and H. E. Stanley, Herd behavior in a complex adaptive system, *Proc. Natl. Acad. Sci. USA*, 2011, 108(37): 15058

18. S. Gourley, S. C. Choe, P. M. Hui, and N. F. Johnson, Effects of local connectivity in a competitive population with limited resources, *Europhys. Lett.*, 2004, 67(6): 867

19. R. T. Liu, F. F. Chung, and S. S. Liaw, Fish play minority game as humans do, *Europhys. Lett.*, 2012, 97(2): 20004

20. P. Glansdorff and I. Prigogine, Thermodynamic Theory of Structure, Stability, and Fluctuations, UK: Wiley-Interscience, 1971

21. Y. Liang, K. N. An, G. Yang, and J. P. Huang, Contrarian behavior in a complex adaptive system, *Phys. Rev. E*, 2013, 87(1): 012809

22. W. B. Arthur, Inductive reasoning and bounded rationality, *Am. Econ. Assoc. Papers Proc.*, 1994, 84: 406

23. C. E. Shannon, A mathematical theory of communication, *Bell Syst. Tech. J.*, 1948, 27: 379

24. Y. Y. Yu, C. Xu, G. Q. Gu, and P. M. Hui, Spies in the minority game, *Phys. Rev. E*, 2008, 77(1): 011106

25. Y. Baek, S. H. Lee, and H. Jeong, The market behavior and performance of different strategy evaluation schemes, *Phys. Rev. E*, 2010, 82(2): 026109

26. S. Biswas, A. Ghosh, A. Chatterjee, T. Naskar, and B. K. Chakrabarti, Continuous transition of social efficiencies in the stochastic-strategy minority game, *Phys. Rev. E*, 2012, 85(3): 031104

27. S. Van Segbroeck, J. M. Pacheco, T. Lenaerts, and F. C. Santos, Emergence of fairness in repeated group interactions, *Phys. Rev. Lett.*, 2012, 108(15): 158104

28. J. H. Kagel and A. E. Roth, The Handbook of Experimental Economics, New Jersey: Princeton University Press, 1995: 256–292